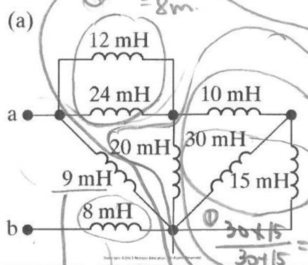
7. Assume that the initial energy stored in the inductors (a) and (b) is zero. Find the equivalent inductance with respect to the terminals a, b.

(a)



④ 12||24 = 8m

② 10+10 = 20m

① $\frac{30 \times 15}{30+15}$ = 10m

③ 20||20 = 10m

⑤ = ③+④ = 8m+10m = 18m

↑ working 0.2

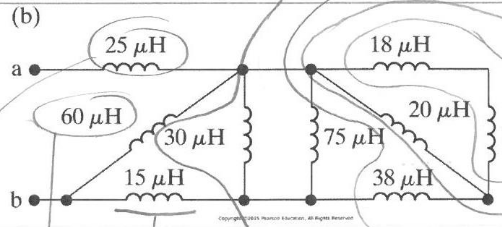
⑥ 9||18 = 6mH

⑦ 6+8 = 14mH ← ans 0.3

(0.5)

Answer:

(b)



① 18+12 = 30μ

② 30||20 = 12μ

③ 38+12 = 50μH

④ 75μ||50μ||30μ = 15μH

⑤ 15+15 = 30μH

⑥ 60||30 = 20μH

working 0.2

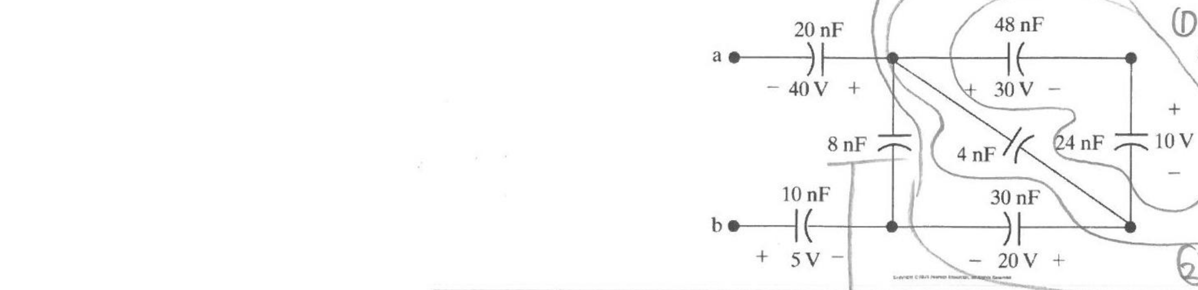
(0.5)

Answer:

⑦ 25+20 = 45μH

ans 0.3

8. Find the equivalent capacitance with respect to the terminals a, b.



① $\frac{1}{48n}+\frac{1}{24n}=\frac{1}{16n}=\frac{1}{C}$

$\Rightarrow 16nF$

② $4_n+16_n=20nF$

③ $\frac{1}{20n}+\frac{1}{30n}=\frac{1}{12}$

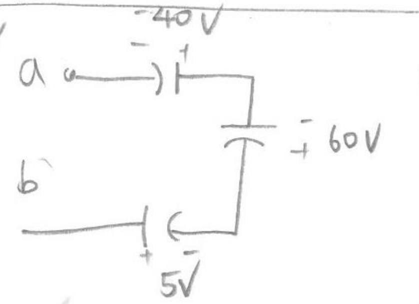
$\Rightarrow 12nF$

④ $12n+8_n=20nF$

⑤ $\frac{1}{C_{ab}}=\frac{1}{20_n}+\frac{1}{20_n}+\frac{1}{10_n}=\frac{1}{5_n}$

working 0.5

$\therefore C_{ab}=5_nF$ ⇐ answer ← 0.5



(1)

Equivalent capacitance at terminal a & b is 5nF with an initial voltage drop of 15V.

$= -40+60+5=15$

No need to show

Answer:

Questions

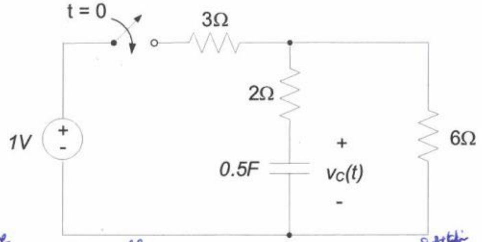
1. The For the circuit below, determine

"In general": RC circuit.

$(*)\ V_{Th} = \boxed{R_{Th} \cdot C} \frac{d}{dt} V_c + 1 * V_c$

⇓ Tau

a time constant.



a) $v_C(t)$, $t>0$, the cap is initially uncharged. (or the circuit switch was opened for a long time before t=0)

① Initial condition at $t=0$

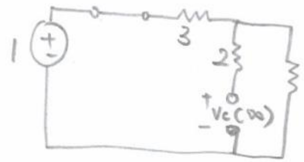
(steady state)

$V_c(0^-) = V_c(0^+) = 0[V] = V_c(0)$ ∵ the switch was opened for a long time before $t=0$

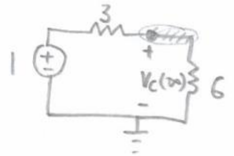
② Final condition at $t=\infty$ (steady state)

⇒ Cap's two plates charging status ⇒ Equilibrium!

⇒ no current $I_C = 0$ ⇒ open circuit



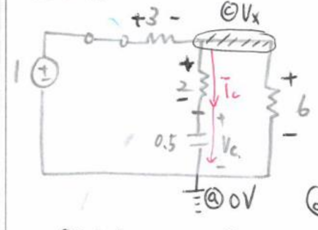
⇒



$V_c(\infty) = 1 \times \frac{6}{3+6}$

$= \frac{2}{3}$ [V]

③ $t>0$



ⓑ set polarity

ⓒ KCL at Vx

$\frac{1-V_x}{3} = \underbrace{0.5 \cdot \frac{d}{dt} V_c}_{I_c} + \frac{V_x - 0}{6}$

ⓓ $V_x = V_c + 2 * 0.5 \cdot \frac{d}{dt} V_c / 6$.

ⓔ Substitute ⓓ to ⓒ: V_x

$2 - 2V_c - 2\frac{d}{dt}V_c = 3\frac{d}{dt}V_c + V_c + \frac{d}{dt}V_c$

$2 = 6 \cdot \frac{d}{dt} V_c + 3V_c \Rightarrow \frac{2}{3} = 2 \frac{d}{dt} V_c + 1 V_c$

RC

④ Tau = RC = 2 (∵ (*))

particular sol. homo. sol.

⑤ Solution form: $V_c(t) = K_1 + K_2 \cdot e^{-t/\tau}$

Answer: ⑥ from ① & ②

$V_c(0) = 0 = K_1 + K_2$

$V_c(\infty) = \frac{2}{3} = K_1$] ⇒ $K_1 = \frac{2}{3}$, $K_2 = -\frac{2}{3}$

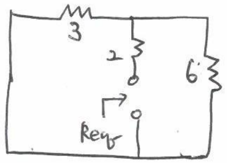
⑦ $V_c(t) = \frac{2}{3}(1 - e^{-t/2})$ [V] for $t>0$.

b) $v_c(t)$, $t>0$ if the capacitance is $1F$

⊛ double Check τ, steady-state response → for part(a)

Where $\tau = R_{eq} \cdot C_{eq} = R_{Th} \cdot C$. looking at V_c terminals ← capacitor's

(∵ our interest V_c)



$R_{eq} = 2 + (3 \| 6) = 4\Omega$

$\tau = 4 \times 0.5 = 2$. (V)

match with previou final answer.

Steady-state response.

$\cancel{\frac{d}{dt}V_c(\infty)}_0 + \frac{1}{2}V_c(\infty) = \frac{1}{3} \Rightarrow V_c(\infty) = 2/3\ (V)$

for this question ⇒ just changing capacitance to 1F.

⇒ only affects the time constant & the equivalent resistance remains the same.

$\therefore \tau = R_{eq} \cdot C = 4 \times 1 = 4$ [sec]

$\Rightarrow V_c(t) = 2/3\,(1 - e^{-t/4})$ [V] for $t>0$.

Answer:

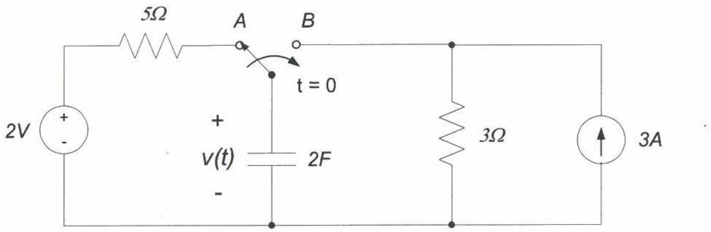
c) $v_c(t)$, $t>0$ if the capacitance is $0.25F$

$\tau = R_{eq} \cdot C = 4 \times 0.25 = 1$ [sec]

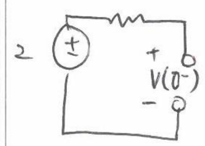
$\therefore V_c(t) = 2/3\,(1 - e^{-t})$ [V], $t>0$.

Answer:

3. For the circuit shown, the switch moves from position A to position B at time t = 0. Find *v(t)*, *t>0*.

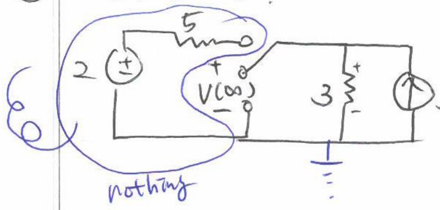


① for $t<0$, initial condition.



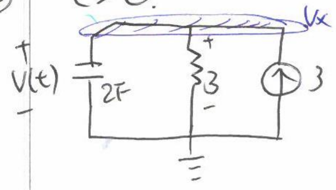
$V(0^-) = V(0^+) = V(0) = 2$ [V]

② for $t \to \infty$.



$V(\infty) = 3 \times 3 = 9$ V.

③ $t>0$.



KCL at Vx

$3 = 2 \cdot \frac{d}{dt}V(t) + \frac{V(t)}{3}$

$\Rightarrow 9 = 6 \frac{d}{dt}V(t) + 1V(t)$

$RC = \tau$

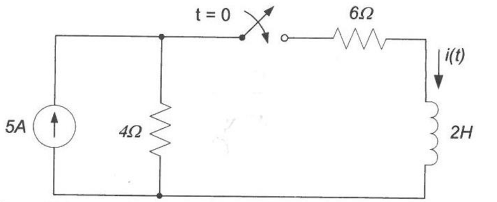
④ Sol form : $V(t) = K_1 + K_2 \cdot e^{-t/\tau} = K_1 + K_2 \cdot e^{-t/6}$ [V]

⑤ from ① & ②, $2 = K_1 + K_2 \Rightarrow K_1 = 9,\ K_2 = -7$.

⑥ $V(t) = 9 - 7 \cdot e^{-t/6}$ [V], $t>0$.

Answer:

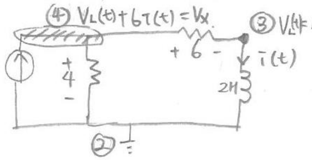
4. For the circuit shown, the switch closes at time $t = 0$.



a) Write the differential equation governing $i(t)$, $t>0$.

① $t>0 \Rightarrow$ The switch closes.

④ $V_L(t)+6i(t)=V_x$ ③ $V_L(t)=L\cdot\frac{d}{dt}i(t)$



⑤ KCL at V_x

$$5=\frac{V_x}{4}+i(t) \quad ⊛$$

Answer: Where $V_x=V_L(t)+6i(t)$

where $V_L(t)=L\cdot\frac{d}{dt}i(t)$

⑥ ⊛ rewrite

$$5=\frac{1}{4}\times\left(2\cdot\frac{d}{dt}i(t)+6i(t)\right)+i(t)$$

$$5=\frac{1}{2}\left(\frac{d}{dt}i(t)+3i(t)\right)+i(t)$$

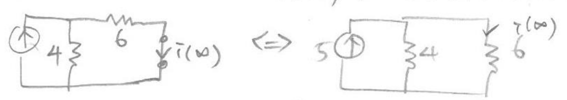
$$10=\frac{d}{dt}i(t)+3i(t)+2i(t)$$

$$\boxed{10=\frac{d}{dt}i(t)+5i(t)}$$ ✓

b) Determine initial ($t = 0$) and final ($t\to\infty$) conditions on the current $i(t)$. You may assume that no energy is stored in the inductor before $t=0$.

① If the switch was opened for a long time $\Rightarrow$ $i(0)=0\text{ A}$ for $t=0$

② find when $t\to\infty$ $i(\infty)\Rightarrow$ an inductor $\Rightarrow$ short



$$\Rightarrow \quad i(\infty)=5\times\frac{4}{4+6}=2\text{ A for } t\to\infty$$

Answer:

c) Find $i(t)$, $t>0$.

① sol form : $i(t)=K_1+K_2\cdot e^{-t/\tau}$

② from a). $10=\frac{d}{dt}i(t)+5i(t) \Rightarrow 2=\frac{1}{5}\frac{d}{dt}i(t)+i(t)$

$\boxed{\tau=L/R}$

③ plug in $i(\infty)$, $i(0)$ to ①

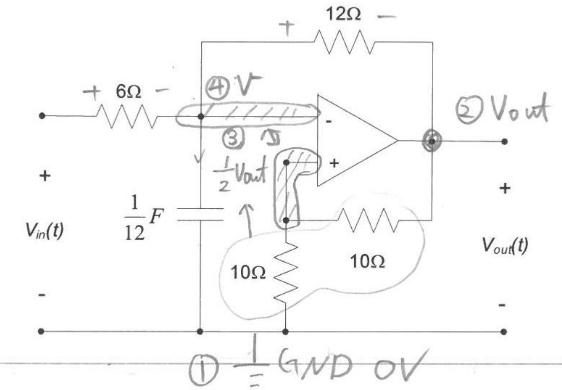
$i(\infty)\Rightarrow K_1+K_2\cdot \cancel{e^{-\infty/\tau}}=2 \Rightarrow K_1=2$

Answer: $i(0)\Rightarrow K_1+K_2\cdot e^{-0/\tau}=0 \Rightarrow K_2=-2$

$K_1=-K_2$

④ $\therefore i(t)=2\left(1-e^{-5t}\right)$ [A] for $t>0$

5. For the circuit below, determine the differential equation relating $V_{out}(t)$ and $V_{in}(t)$.



⑤ KCL at V^- $6\Omega \to$, $\to 12\Omega$, $i = C \cdot \frac{d}{dt} V$

$$\left(\frac{V_{in} - \frac{V_{out}}{2}}{6} = \frac{1}{12} \cdot \frac{d}{dt}\left(\frac{1}{2} V_{out}\right) + \frac{\left(\frac{1}{2}V_{out} - V_{out}\right)}{12} \right) \times 12$$

⑥ $$2\left(V_{in} - \frac{V_{out}}{2}\right) = \frac{1}{2}\frac{d}{dt}V_{out} + \frac{1}{2}V_{out}$$

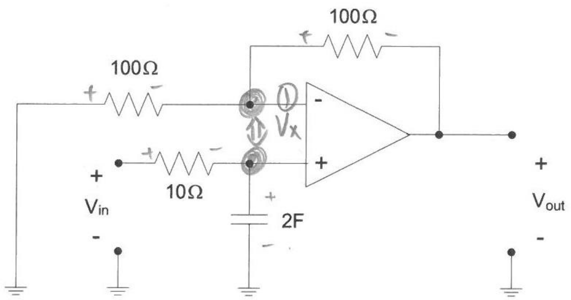
$$2V_{in} = V_{out} - \frac{1}{2}V_{out} + \frac{1}{2}\frac{d}{dt}V_{out}$$

$$2V_{in} = \frac{1}{2}V_{out} + \frac{1}{2}\frac{d}{dt}V_{out}$$

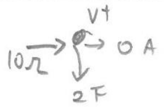
$$\underline{4V_{in}(t) = \frac{d}{dt}V_{out}(t) + V_{out}(t)} \;//$$

Answer:

6. Determine the differential equations relating V_{out} and V_{in} for the circuit below.

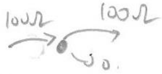


② KCL at V^+



$$\frac{V_{in} - V_x}{10} = 2 \cdot \frac{d}{dt} V_x$$

③ KCL at V^-



$$\frac{0 - V_x}{100} = \frac{V_x - V_{out}}{100}$$

$$2V_x = V_{out} \Rightarrow V_x = \frac{1}{2} V_{out}$$

④ ③ → ②

$$V_{in} - \frac{1}{2} V_{out} = 20 \frac{d}{dt}\left(\frac{1}{2} V_{out}\right)$$

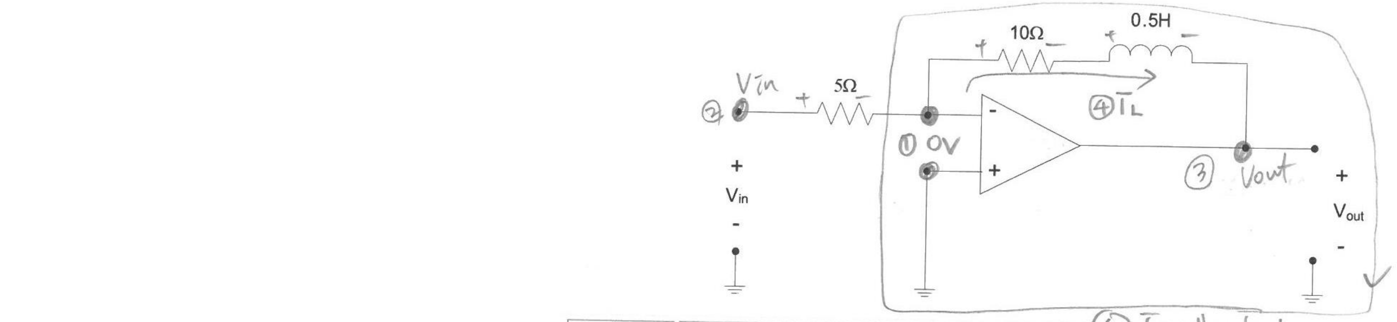
$$V_{in} = \frac{1}{2} V_{out} + 10 \frac{d}{dt} V_{out}$$

or

$$\frac{V_{in}}{10} = \frac{V_{out}}{20} + \frac{d}{dt} V_{out}$$

Answer:

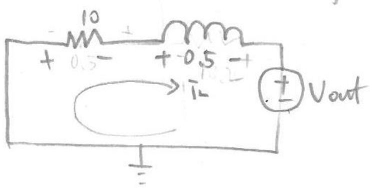
8. Determine the differential equations relating Vout and Vin for the circuit below.



⑤ KCL at V^-

$$\frac{V_{in} - 0}{5} = i_L \qquad \boxed{i_L = \frac{V_{in}}{5}}$$

⑦ Redraw the feed-back loop.



⑧ KVL for i_L Loop.

Gain drop

$$0 = 10 \cdot i_L + 0.5 \cdot \frac{d}{dt} i_L + V_{out}$$

⑨ plug in ⑤ to ⑧

$$-V_{out} = \cancel{10}^{2} \cdot \frac{V_{in}}{5} + \frac{0.5}{5}\frac{d}{dt} V_{in}$$

$$\boxed{-V_{out} = 2V_{in} + 0.1\frac{d}{dt} V_{in}}$$

Answer: