CET 141: Day 7

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Agenda

- R, L, and C under AC conditions
- Complex plane and Phasor

All tools for today

1. Phasor
   - $v(t) = A\cos(\omega t + \theta)\ [V]$ ➔ $V_{phasor} = Ae^{j\theta}[V]$
2. Impedance (Z=R+jX [Ω])
   - $Z_R = R\ [\Omega]$
   - $Z_C = \frac{1}{j\omega C} = jX_C = -j\frac{1}{\omega C}\ [\Omega]$
   - $Z_L = jX_L = j\omega L\ [\Omega]$

Recap: R, L, and C under DC conditions

- DC sources (i.e, V=9V or I=3A),
  - Resistance R
    - Follows Ohm’s law, V=IR
  - Capacitance, C
    - Opened, I=0
  - Inductance, L
    - Shorted, V=0 (just a wire)

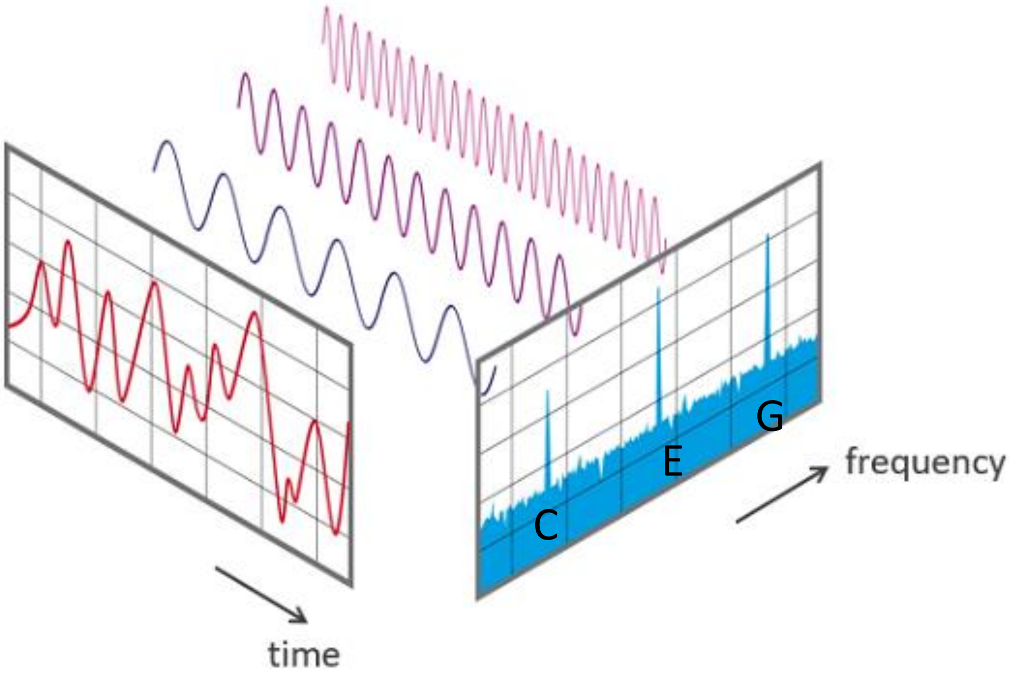
Complete R = Transient R + Steady-State R

Tell us everything that you remember about each response.

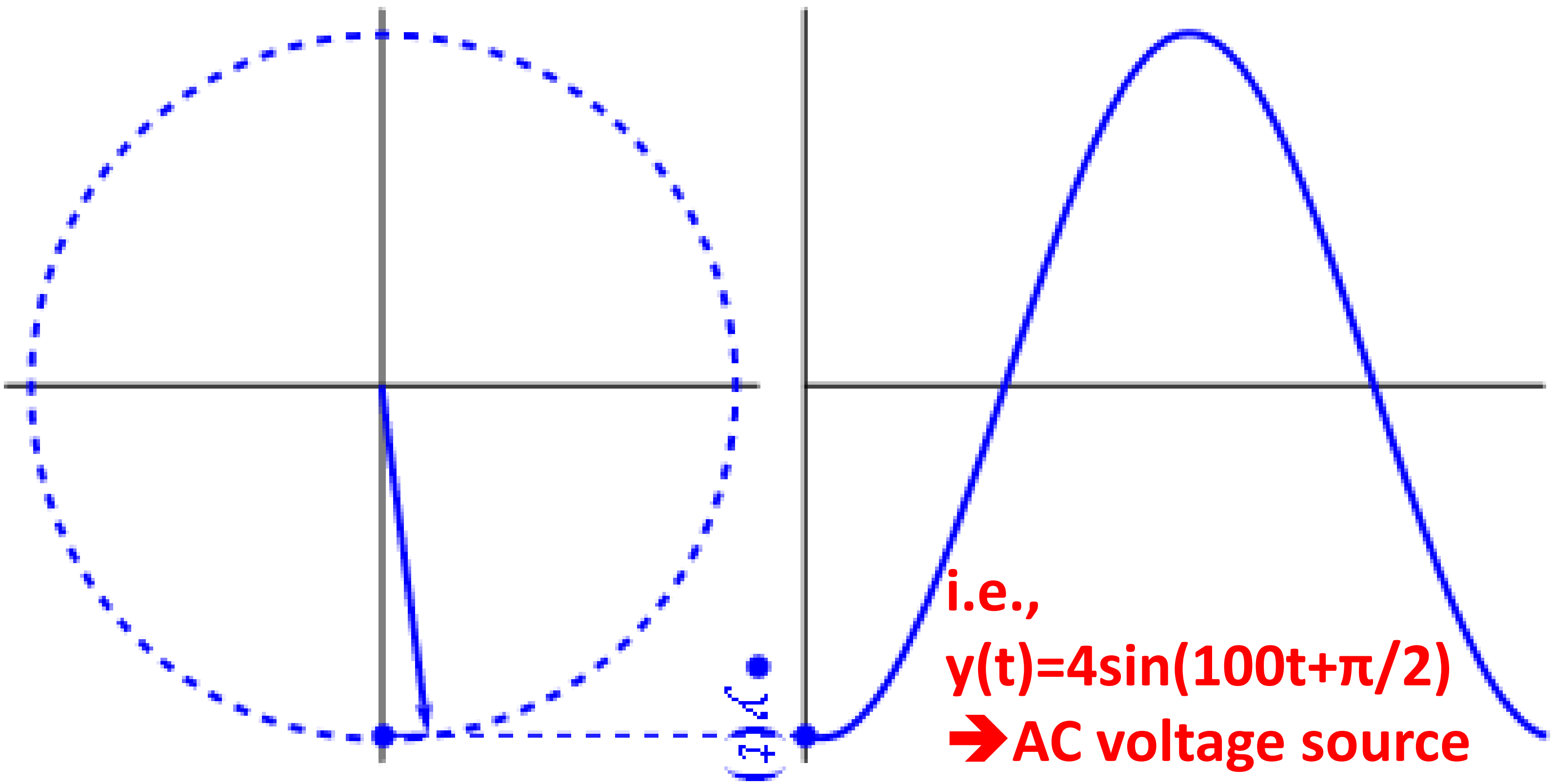
If a source is AC
→through RLC circuits
→Steady-State R is also AC
→phasor analysis can be used in ***frequency domain***

$\omega = 2\pi f$

- ω and f are both measures of frequency
- ω[rad/sec] and f [Hz]



Cycles (radian) and Frequency relation



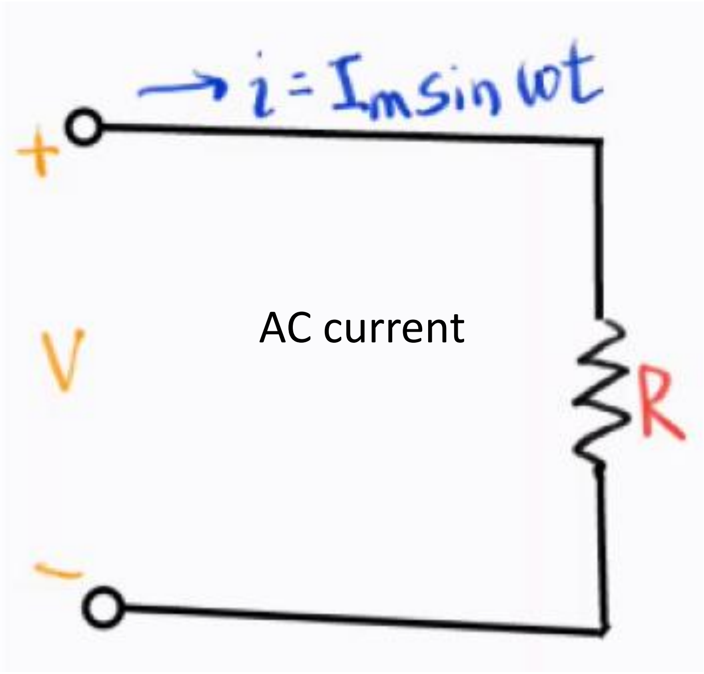
i.e.,
y(t)=4sin(100t+π/2)
➔AC voltage source

**Look at the time domain graph and think about why only steady-state R is considered in frequency domain analysis*

R under AC

- How do we calculate V?
  - Based on Ohm's law V=iR

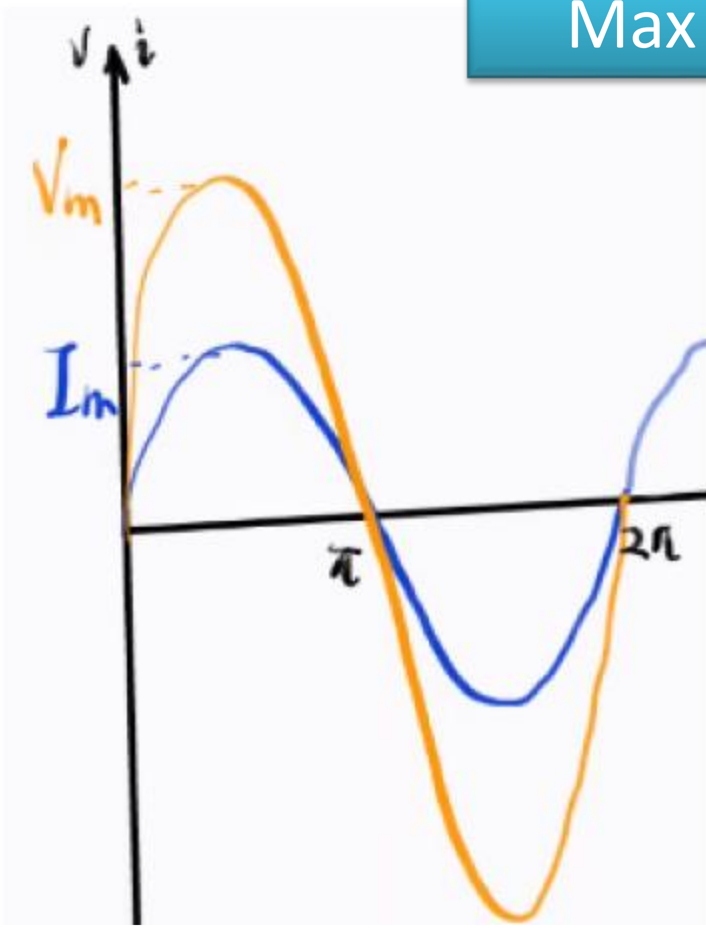
  $V = Ri = \boldsymbol{R} I_m \sin \omega t$ [V]
- R: **<u>a proportional constant</u>** between voltage v and current i
- If ***i*** is a sinusoidal form, then ***v*** is also a sinusoidal form ***<u>sharing the same ω (angular freq)</u>***
  - (+) No phase diff.
  - Same phase = same angle…



$$i = \mathbf{I_m} \sin \omega t \text{ [A]}$$
$$V = RI_m \sin \omega t$$
$$= \mathbf{V_m} \sin \omega t \text{ [V]}$$

$$i = I_m \sin \omega t\ [A],\ V = V_m \sin \omega t\ [V]$$

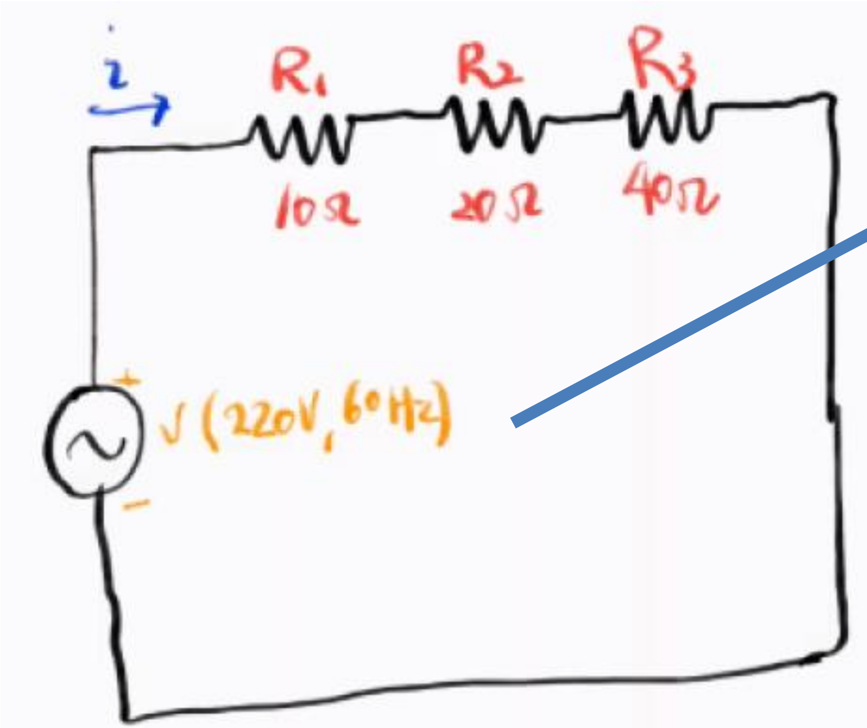
Max values!!!



- Whether you use a current source or a voltage source;
  - the current flows through R or
  - the voltage across the R

R Will not change.

Example: find i



The V is given in this format (think about all appliances that we have 110V-60Hz…110V-50Hz…)

- 220V is RMS amplitud, not V_{max}
  - $\sqrt{2} * 220 = V_{max}$
- $V = 220\sqrt{2}\sin(60 * 2\pi\, t)$ [V]

Note that $\omega = 2\pi f$

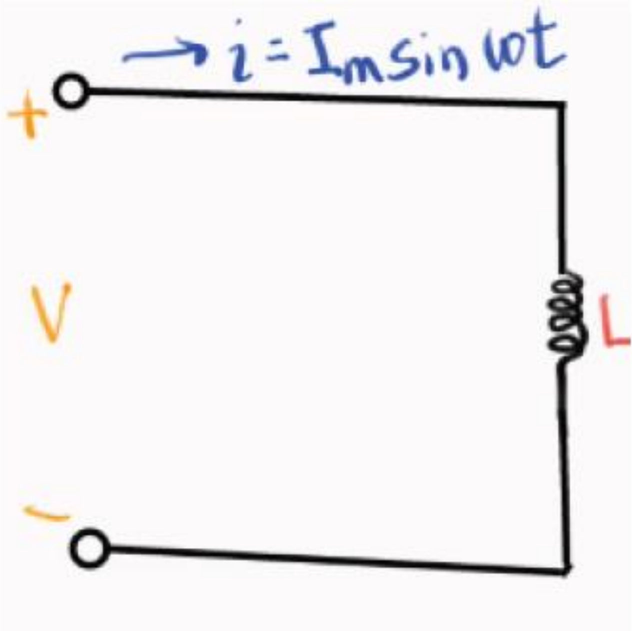
- R_{eq}=10+20+40=70Ω
- Therefore

$$i = 220\sqrt{2}\sin(60 * 2\pi t)/70 \text{ [A]}$$

L under AC

- How do we calculate V?

$$L=\frac{N^2\mu A}{l} \rightarrow \phi = L \cdot I \rightarrow V = \frac{\Delta\phi}{\Delta t} \rightarrow V = L \cdot \frac{d}{dt} I$$



$$\boldsymbol{v} = L\frac{d}{dt}(I_m \sin \omega t)$$
$$= \omega L I_m \cos \omega t$$
$$= \boldsymbol{\omega L I_m \sin(\omega t + 90^{\circ})\ [\mathrm{V}]}$$

$$\text{i=}\boldsymbol{I_m \sin \omega t\ [A]}$$

Do you see the clear difference compared to the R case?

A sidebar

$$\cos(\omega t + \phi) = \sin\left(\omega t + \phi + 90^\circ\right)$$

$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - 90^\circ\right)$$

$$-\sin\left(\omega t\right) = \sin\left(\omega t \pm 180^\circ\right)$$

$$-\cos\left(\omega t\right) = \cos\left(\omega t \pm 180^\circ\right)$$

$$-\cos\left(\omega t\right) = \sin\left(\omega t \pm 270^\circ\right)$$

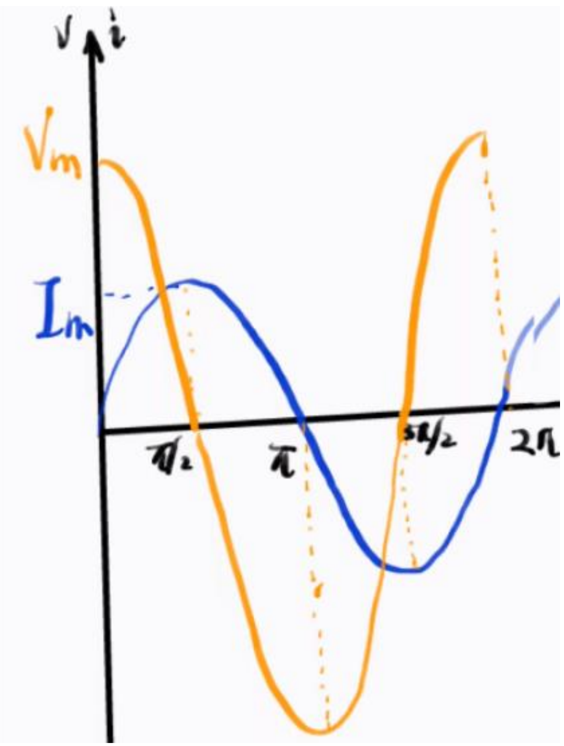
$$-\sin\left(\omega t\right) = \sin\left(-\omega t\right)$$

$$\cos\left(\omega t\right) = \cos\left(-\omega t\right)$$

https://www.electronics-tutorials.ws/accircuits/phase-difference.html

$$i = I_m \sin \omega t \; [A],$$
$$V = \omega L I_m \sin(\omega t + 90^{\circ}) \;\; [V]$$

OMG!!! There is a phase difference between the V and I!!!



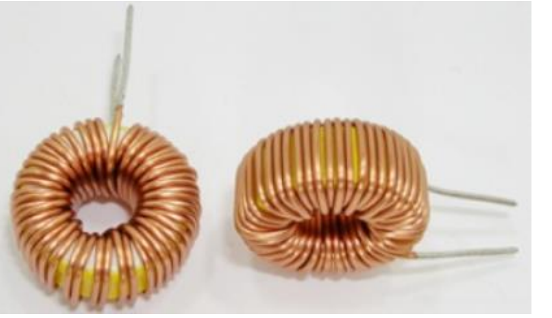
- ω is the same for V and I, <u>but affects magnitude of I</u>
- The V period is shifted $\frac{\pi}{2}$ from the I (to the left)
- The max V (V_m) is multiplied by ωL from the max I (I_m)

ωL: Inductive reactance $\mathbf{X_L}$

- Imag. Coeffic. of impedance Z with a unit of Ω-ohm
  - $Z=R+jX$ [Ω]
  - Z: impedance
  - R: resistance (real part), X: reactance (imaginary part) where units for both R and X are Ω
- Basically **it is a resistance (disturbing current flows)** but **depends on the frequency**

- Therefore if *frequency ($\omega=2\pi f$) increases*, *ωL increases*.
  - An Inductor: Blocking high frequency signals.
  - At DC status, $f=0 \rightarrow \omega = 0$:
    - Basically no inductive restiveness in the circuit, as $X_L=\omega L=0$
    - An Inductor behaves Just like a wire

Example find L_{min}



- A choke coil is an inductor used to block higher-frequency alternating current (AC) in an electrical circuit, while passing lower-frequency or direct current (DC).
- Determine the inductance of choke coil to exceed 8000Ω at 60Hz and above.

$$X_L = \omega L = 2\pi f L$$

$$L \geq \frac{X_L}{2\pi f} = \frac{8000}{2\pi * 60} \approx 21.22\ [H]$$

C under AC

- How do we calculate V?

$C = \frac{\varepsilon A}{d} \rightarrow Q = CV \rightarrow I = \frac{\Delta Q}{\Delta t} \rightarrow I_c = C \cdot \frac{d}{dt} V_c \rightarrow V_c = \frac{1}{C} \int I_c \, dt$

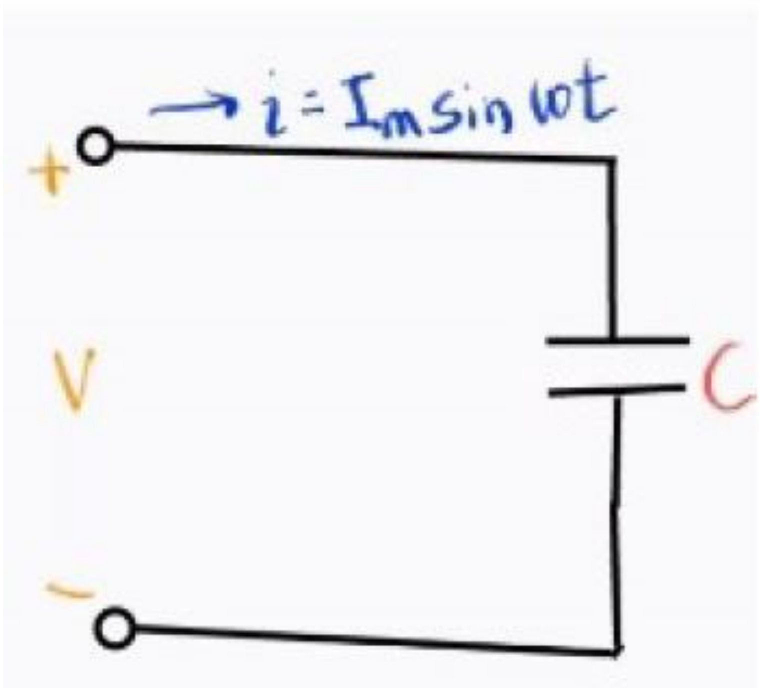
$$\boldsymbol{v} = \frac{1}{C} \int I_m \sin \omega t \, dt$$

$$= -\frac{1}{\omega C} I_m \cos \omega t$$

$$= \frac{\mathbf{1}}{\boldsymbol{\omega C}} \boldsymbol{I_m} \, \mathbf{sin}(\boldsymbol{\omega t} - \mathbf{90}^{\circ}) \; [\mathbf{V}]$$

$$\updownarrow$$

$$\text{i=}\boldsymbol{I_m} \, \mathbf{sin} \, \boldsymbol{\omega t} \; [A]$$



A sidebar

$$\cos(\omega t+\phi) = \sin\left(\omega t+\phi+90^{\circ}\right)$$

$$\sin(\omega t+\phi) = \cos\left(\omega t+\phi-90^{\circ}\right)$$

$$-\sin\left(\omega t\right) = \sin\left(\omega t \pm 180^{\circ}\right)$$

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$$-\cos\left(\omega t\right) = \sin\left(\omega t \pm 270^{\circ}\right)$$

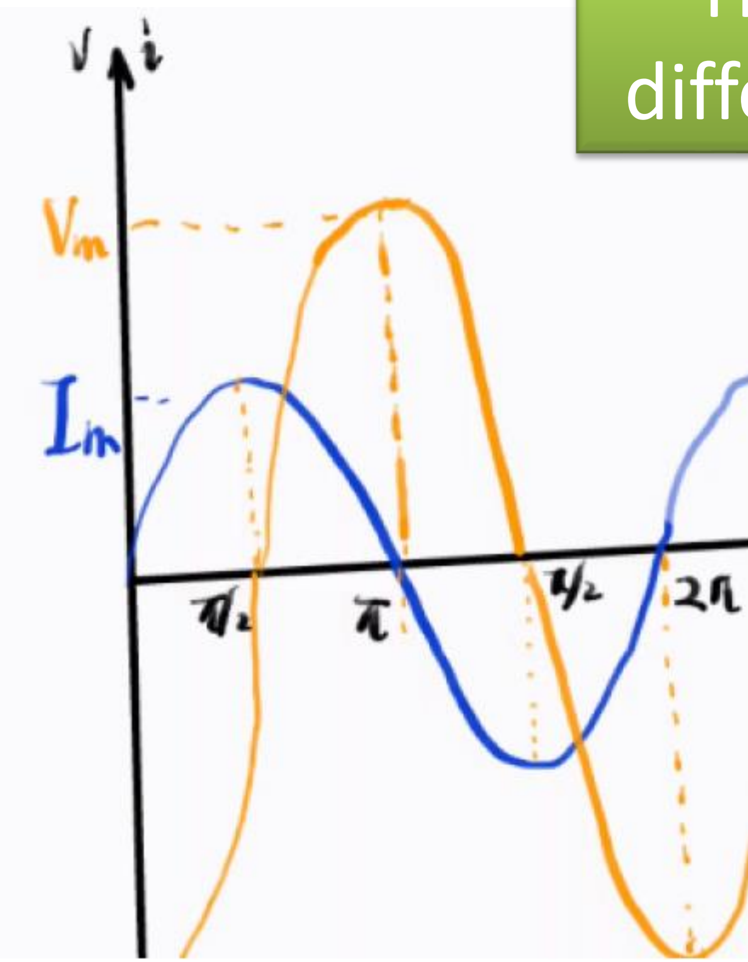
$$-\sin\left(\omega t\right) = \sin\left(-\omega t\right)$$

$$\cos\left(\omega t\right) = \cos\left(-\omega t\right)$$

https://www.electronics-tutorials.ws/accircuits/phase-difference.html i=$I_m\, sin\,\omega t\ [A]$,

$$V = \frac{1}{\omega C} I_m \sin(\omega t - 90^{\circ})\ \ [V]$$

Here we have another phase difference between the V and I!!!



- ω is same for both V and I, <u>but affects magnitude of V</u>
- The V period is shifted $\frac{\pi}{2}$ from the I (to the right)
- The max V (V_m) is multiplied by $\frac{1}{\omega C}$ from the max I (I_m)

$\frac{-1}{\omega C}$ or $\frac{1}{\omega C}$: Capacitive reactance **X_C**

- Imag. Coeffic. of impedance Z with a unit of Ω-ohm
  - Z=R+jX [Ω], where Z: impedance
  - R: resistance (real part), X: reactance (imaginary part) where units for both R and X are Ω
- Basically **it is a resistance (disturbing current flows)** but **depends on the frequency**

- Therefore if *frequency (ω=2πf) decreases*,

$$\frac{1}{\omega C}$$ *increases*.

  - A capacitor: Blocking low frequency signals.
  - At DC status, f=0 → ω =0:
    - Basically infinity capacitive restiveness in the circuit, as $X_C=\infty$, current can't flow this path.
    - A capacitor behaves Just like disconnected wires (O.C.)

LL pass
CH pass

Summary on the impedance

- An impedance (Z) is a concept to be defined for (in) the frequency domain
  - It is a sum of resistance (R) and reactance (X)
  - Z=R+jX → unit: "Ohm" → a measure of restiveness of the circuit → if Z is high, electrons are hard to flow (low current)
- A resistor (R)
  - has an impedance form of R → resistance
- An inductor (L)
  - has an impedance form of $j\omega L$ → reactance
- A Capacitor (C)
  - Has an impedance for of $\frac{1}{j\omega C}$ → reactance

Please, Do remember..

- Capacitive reactance X_c: $1/(\omega C)$
  - between V and I, ωt: no change only -90 phase diff.
- Inductive reactance X_L: ωL
  - between V and I, ωt: no change only +90 phase diff.
- Impedance $Z=R+jX$ [Ω], where
  - Series $Z_{eq}=Z_1+Z_2+Z_3$....
    - $Re\{Z_{eq}\}$: from resistors, $Im\{Z_{eq}\}$: inductors and caps
  - Parallel $1/Z_{eq}=1/Z_1+1/Z_2+1/Z_3$....
    - $Re\{Z_{eq}\}$ and $Im\{Z_{eq}\}$: combination of R, C, L

Sidebar

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\sin\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2}\right) \cdot \cos(x) + \cos\left(\frac{\pi}{2}\right) \cdot \sin(x)$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$
$$\cos\left(\frac{\pi}{2}\right) = 0$$

So we have :

$$\sin\left(\frac{\pi}{2} + x\right) = \cos(x)$$

$$\cos x = \sin(\frac{\pi}{2} - x) = \sin(-(x - \frac{\pi}{2}))$$
$$= -\sin(x - \frac{\pi}{2})$$

R under AC

$$i = \mathbf{I_m} \sin \omega t \text{ [A]}, V = \mathbf{V_m} \sin \omega t \text{ [V]}$$

Max values!!!

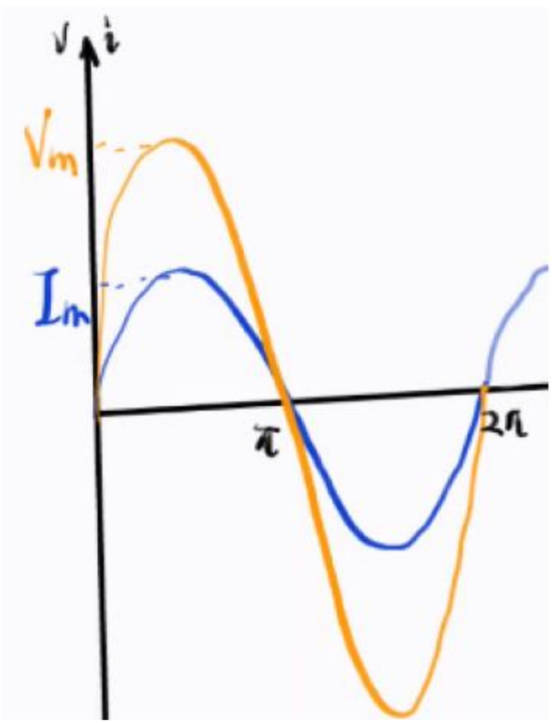
L under AC

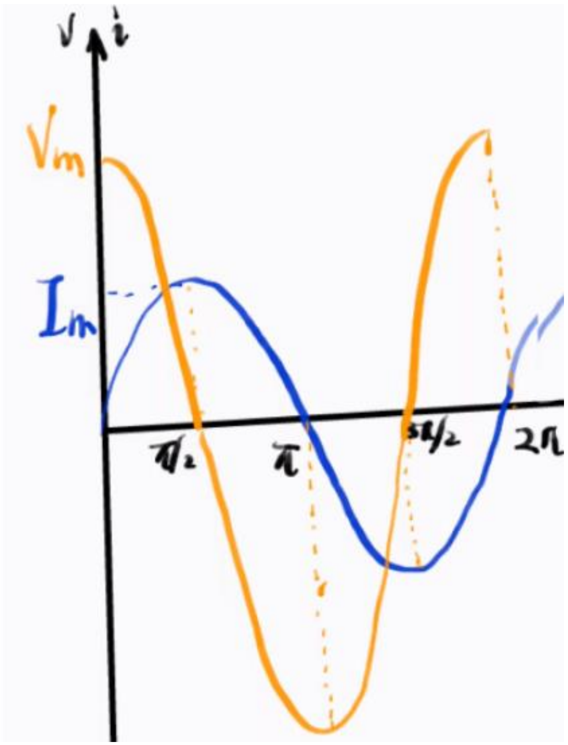
$$i = \boldsymbol{I_m}\, sin\, \omega t\ [A], V = \boldsymbol{\omega L I_m}\, sin(\omega t + \mathbf{90}^{\circ})\ [V]$$

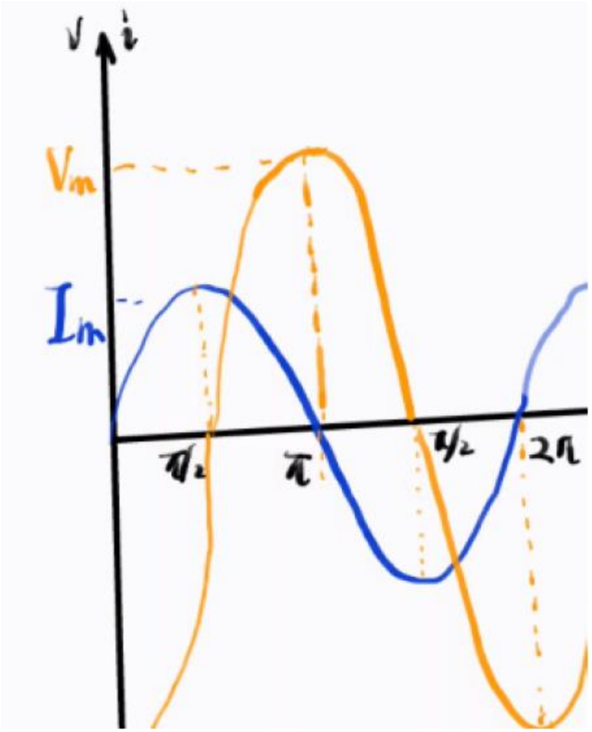
C under AC

$$i = \boldsymbol{I_m}\, sin\, \omega t\ [A], V = \boldsymbol{\frac{1}{\omega C} I_m} \sin(\omega t - 90^{\circ})\ [V]$$

R L C







To the frequency domain

COMPLEX PLANE AND PHASOR

- Before learning phasor, we need to review the complex plane
- There are two methods to represent <u>complex numbers</u>
  - Rectangular (Cartesian) coordinate
  - Polar (angular) coordinate
- **y=a+jb→ Re{Y}=a and Im{Y}=jb**

→|y|=$\sqrt{a^2+b^2}=r$, angle{y}=$\tan^{-1}\frac{b}{a}=\theta$

→y= $re^{j\theta}$

→y= $r(\cos\theta + j\sin\theta)$

Euler's formula: $e^{jx}=\cos x+\mathrm{j}\sin x$

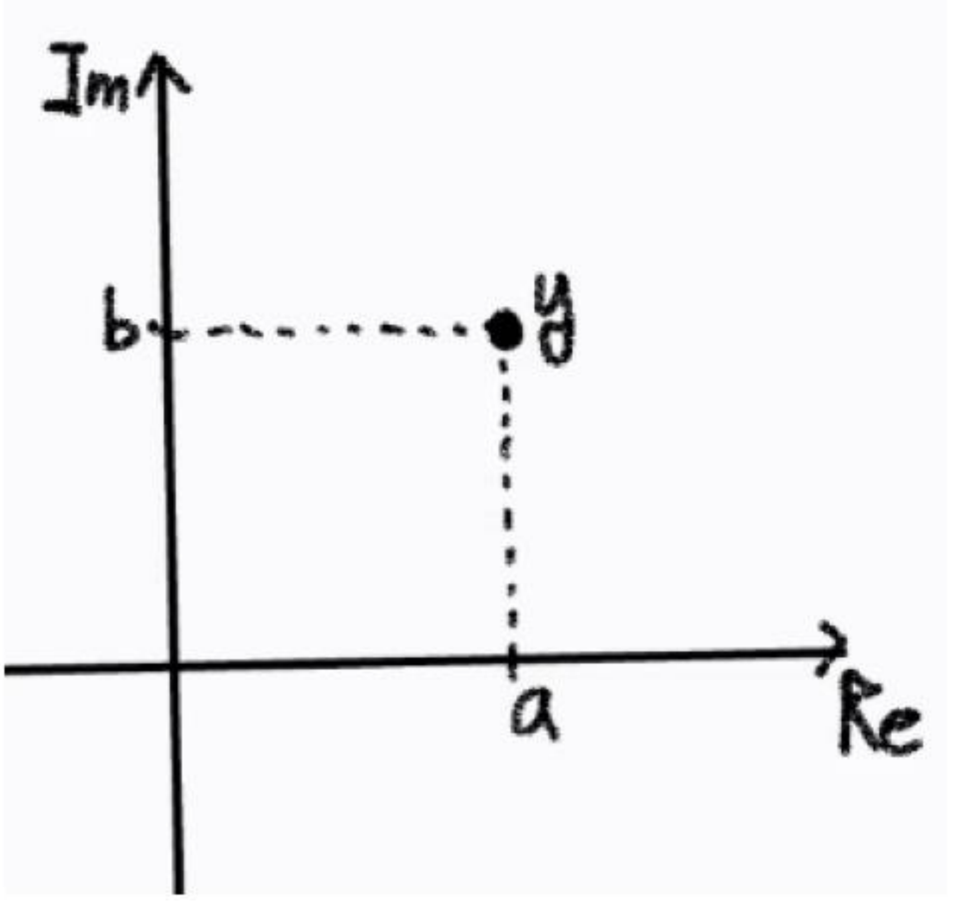
VIF: very important formula

Euler's formula: $e^{jx} = \cos x + \mathrm{j}\sin x$

Along with i_c and v_L

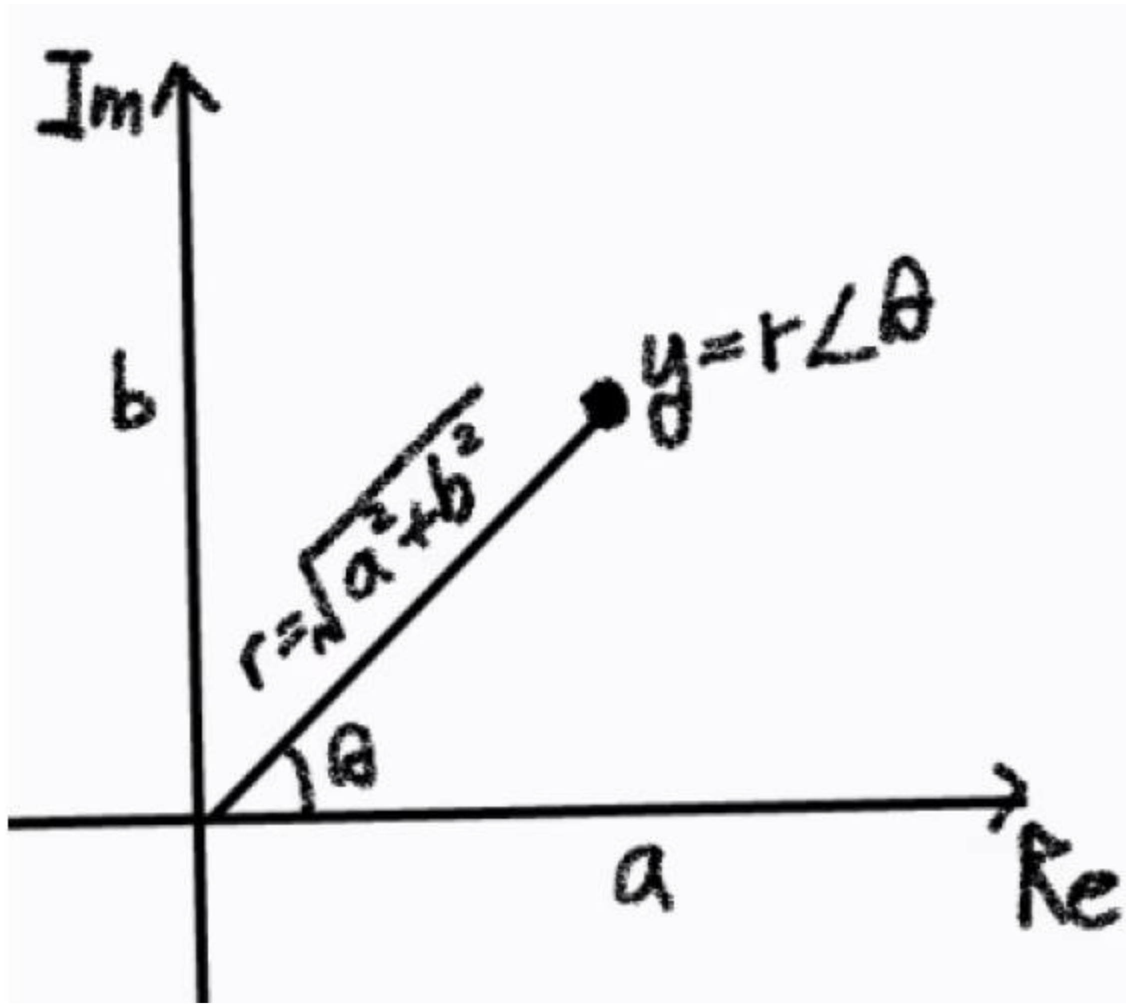
Rectangular (Cartesian) coordinate

- y=a+jb



Polar (angular) coordinate

- y=a+jb



A phasor

- A complex number, representing a sinusoidal function, whose
  - Amplitude M
  - Angular frequency ω
  - Initial phase θ
- Example
  - **A sinusoidal function x(t)**

    $$x(t) = M\cos(\omega t + \theta), \qquad -\infty < t < \infty$$

  - **A phasor form of x(t), namely $\bar{\mathbf{X}}$**

    $$\bar{X} = Me^{j\theta} = M\cos\theta + jM\sin\theta$$

Let's investigate the situation from $\mathrm{x(t)} \rightarrow \bar{X}$ or vice versa

$\mathbf{x(t)} = \boldsymbol{Re\{\bar{X}e^{j\omega t}\}}$ proof!!

$$x(t) = M\cos(\omega t + \theta) \Longleftrightarrow \bar{X} = Me^{j\theta} = M\cos\theta + jM\sin\theta$$

- $Re\{\bar{X}e^{j\omega t}\}$

$$= Re\{Me^{j\theta}e^{j\omega t}\}$$
$$= Re\{Me^{j(\theta+\omega t)}\}$$
$$= Re\{M\cos(\theta+\omega t) + jM\sin(\theta+\omega t)\}$$
$$= M\cos(\theta+\omega t)$$
$$= \mathbf{x(t)} = \boldsymbol{Re\{\bar{X}e^{j\omega t}\}}$$

- The sinusoidal signal **x(t) is the real part of phasor** taking back its original angular frequency
- When we represent a sinusoidal signal in phasor form, the complex exponential e^{iwt} factors out!!!

- Apply this to the sum of two sinusoidal **at the same frequency**

$$
\begin{aligned}
&A\cos(\omega t+\theta)+\mathrm{B}\cos(\omega t+\varphi) \\
&= Re\left\{Ae^{j(\omega t+\theta)}\right\}+Re\left\{Be^{j(\omega t+\varphi)}\right\} \\
&= Re\left\{Ae^{j(\omega t+\theta)}+Be^{j(\omega t+\varphi)}\right\} \\
&= Re\{e^{j\omega t}(Ae^{\theta}+Be^{\varphi})\}
\end{aligned}
$$

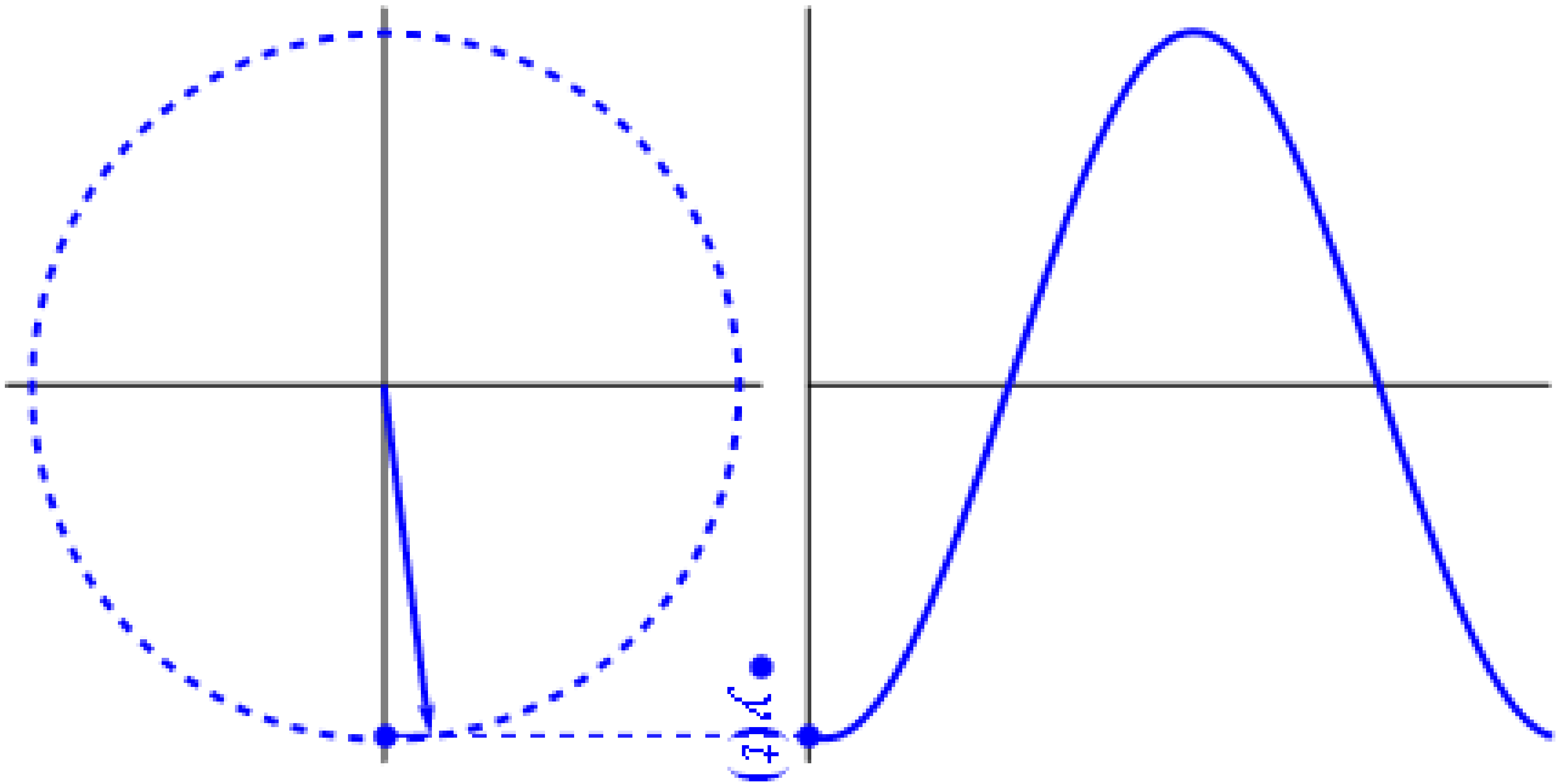
- Why do we learn this?
  – Do you remember V and I for L and C cases?

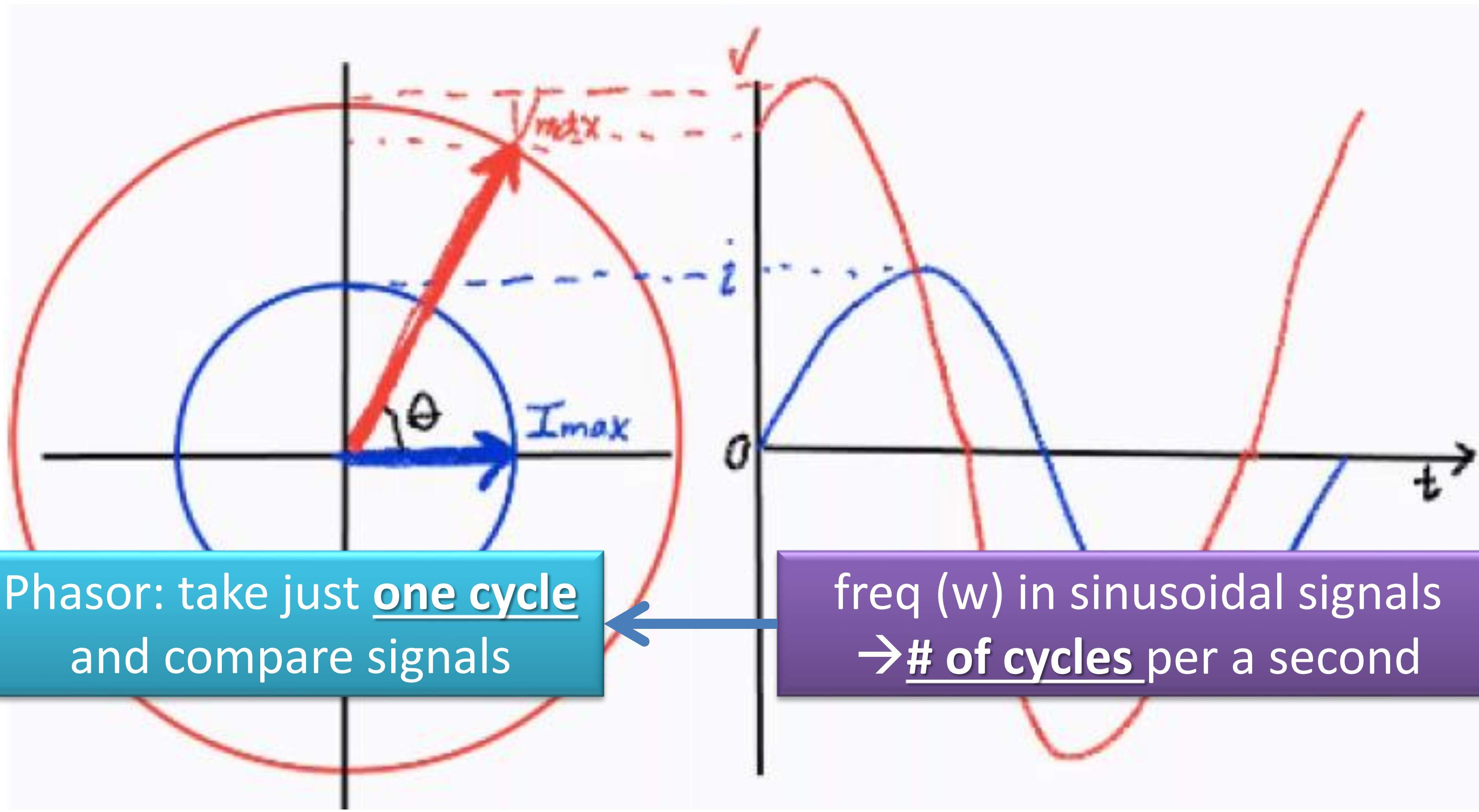
Recall

- Capacitive reactance X_c: $1/(\omega C)$
  - between V and I, **ωt: no change** only -90 phase diff.
- Inductive reactance X_L: ωL
  - between V and I, **ωt: no change** only +90 phase diff.

The phasor looks like a very useful tool for circuit analysis!!

Graphically, (reusing gif.)





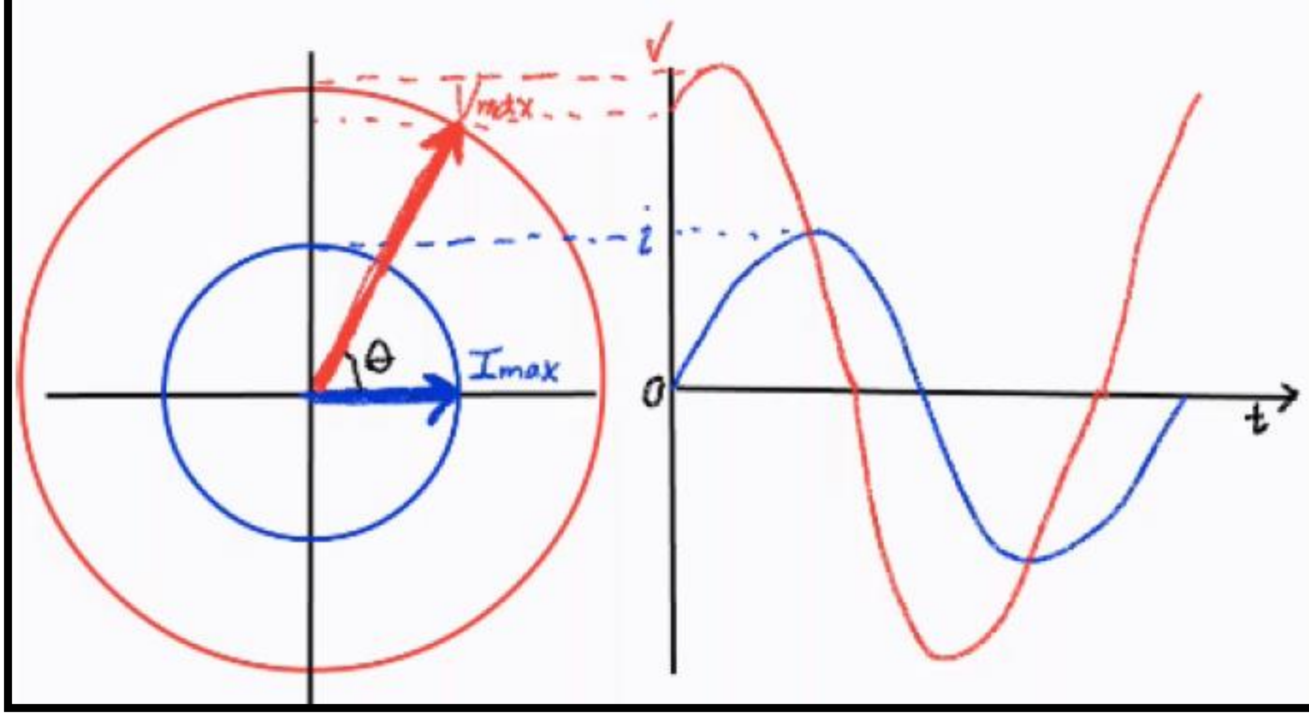
Phasor: take just **one cycle** and compare signals

freq (w) in sinusoidal signals → **# of cycles** per a second

Phasor graph, **at t=0**

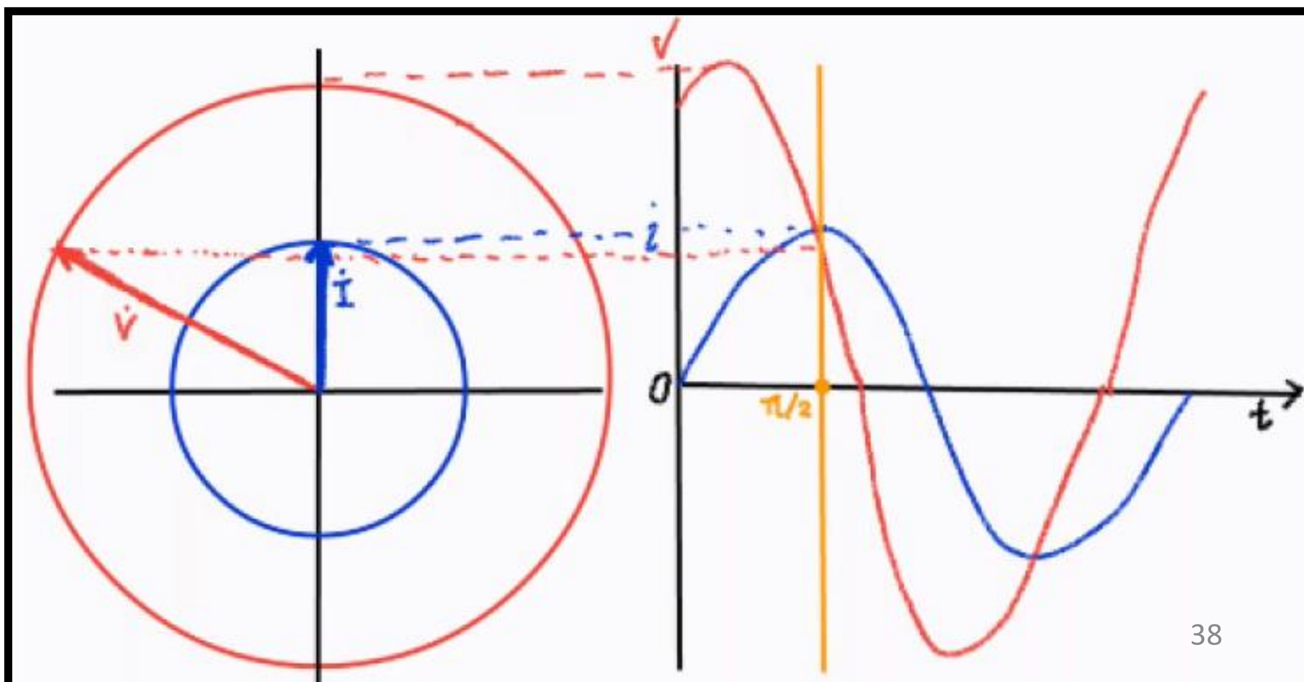
Sinusoidal graph

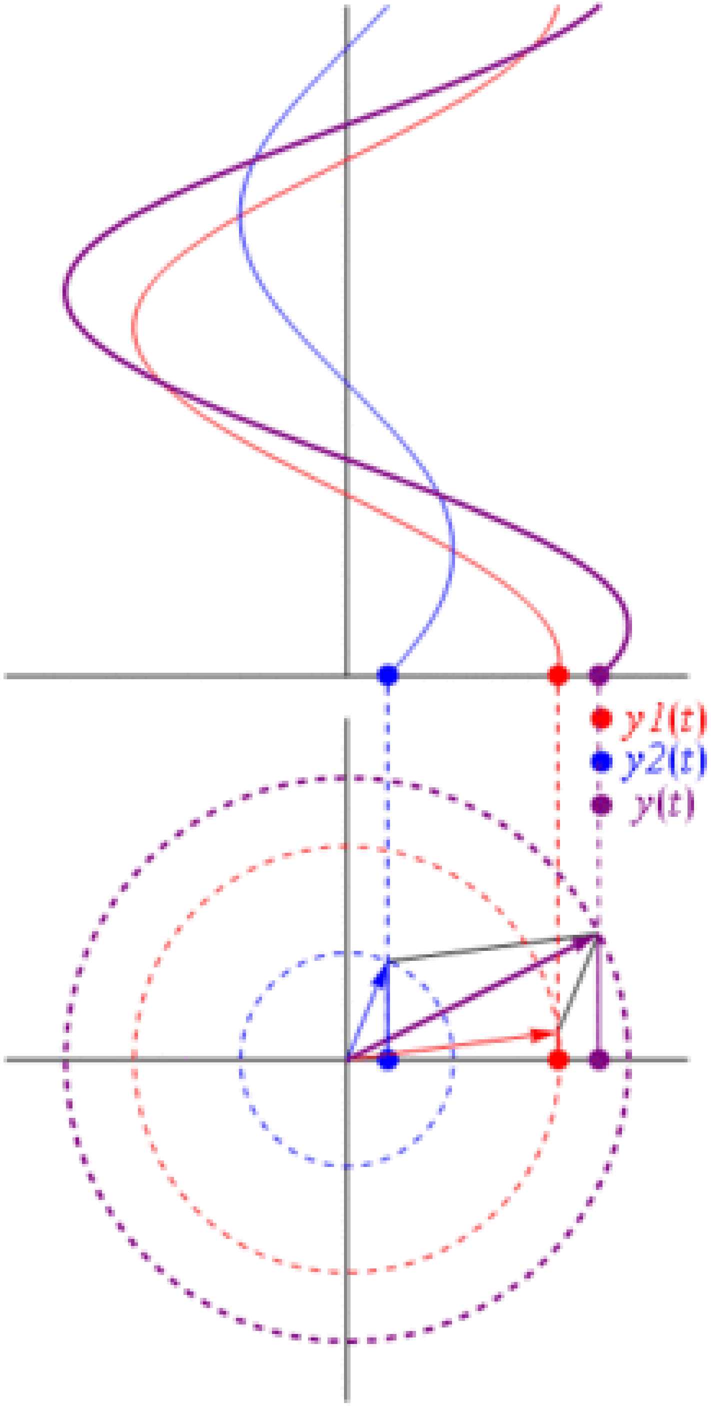
t=0



A phasor is meant to represent the magnitude and phase between V and I

t=π/2





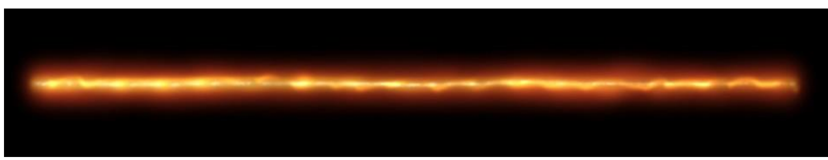
Phasor Ratio of y1 and y2 or y3 will be the same regardless of t ➔impedance

Summary of Phasor

Phasor Definition

$$\text{Time function}: \quad v_1(t) = V_1 \cos(\omega t + \theta_1)$$

$$\text{Phasor}: \quad \mathbf{V}_1 = V_1 \angle \theta_1 = V_1 e^{j\theta_1}$$



Phaser

Sidebar

Euler's Formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

$$\cos(\theta) = \mathrm{Re}(e^{j\theta})$$

$$\sin(\theta) = \mathrm{Im}(e^{j\theta})$$

$$\left|e^{j\theta}\right| = \sqrt{\cos^2(\theta) + \sin^2(\theta)} = 1$$

$$e^{j\theta} = 1\angle\theta$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1$$

$$e = 2.718281828459045235360287.... = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$$

$$\pi = 3.14159265358979323846264 3....$$

$$j = \sqrt{-1}$$

$$e^{j\pi} = -1$$

$$\sin\omega t = \cos\omega t - 90°$$
$$\cos\omega t = \sin\omega t + 90° = -\sin\omega t - 90°$$

$$\sin\omega t = \cos(\omega t - 90^o)$$