Questions

1. A sinusoidal current is given by the expression

$$i = 100\cos(600t + 45°)\text{mA}$$

a) Find f in hertz

0.1 Answer: $\omega = 2\pi f = 600$ rad/s $\quad f = \frac{600}{2\pi} = 95.54$ Hz

b) T in milliseconds

0.1 Answer: $T = \frac{1}{f} = 10.47$ ms

c) I_m (magnitude)

0.1 Answer: 100 mA

d) i(0)

0.1 Answer: $i(0) = 100\cos(45°) = 100 \times \frac{\sqrt{2}}{2} = 70.72$ mA

e) ϕ in degrees and radians

0.1 Answer: $45° \Rightarrow \pi/4 = 0.7857$ rad

f) The smallest positive value of t at which i=0

0.1 Answer: $i = 0$ when $600t\,[\text{rad/s}] + 45° = 90°$ ($\because \cos 90° = 0$)

$600t\,[\text{rad/s}] = 45° \Rightarrow 600t = 0.7853\ [\text{rad/s}]$

$t = 1.308$ msec

g) The smallest positive value of t at which di/dt=0

0.2 Answer: $\frac{di}{dt} = -100 \cdot 600 \cdot \sin(600t + 45°)$ $\Rightarrow$ i max when $\frac{di}{dt} = 0$

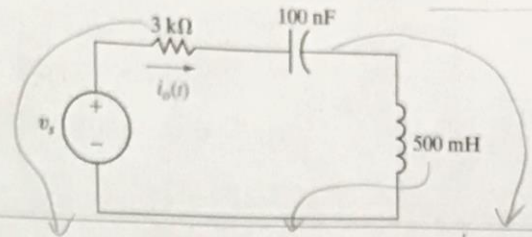
$\Rightarrow 600t + 45° = 180°$

$\therefore 600t + 45° = 180°$

$t = 3.926$ msec

2. Find the steady-state expression for $i_o(t)$ in the circuit if
$v_s = 80\cos 2000t$ V

① → $\bar{V}_s = 80\angle 0°$



0.8

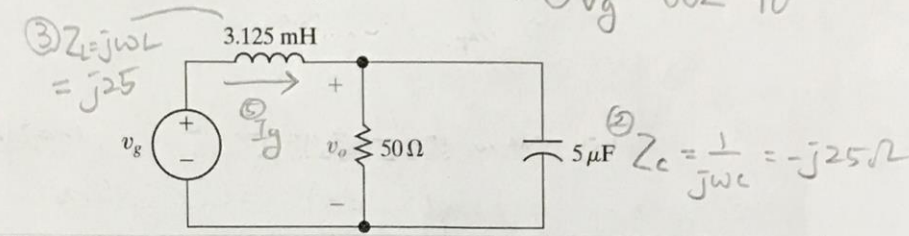
② $Z = 3000 + j(2000)(0.5) - j\frac{1}{(2000)(100\times10^{-9})} = 3000 - j4000\ \Omega$

③ $I_o = \frac{\bar{V}_s}{Z} = \frac{80\angle 0°}{3000-j4000} = 16\angle 53.13°$ mA

④ $i_o(t) = 16\cos(2000t + 53.13°)$ mA

Answer:

3. Find the steady-state expression for $v_o(t)$ in the circuit if
$v_g = 60\sin 8000t$ V ⇒ ① $\bar{V}_g = 60\angle -90°$

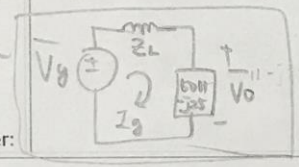


0.8

④ $Z_{total} = j25 + (50 \| -j25) = 10 + 5j\ \Omega$

⑤ $I_g = \frac{\bar{V}_g}{Z_{total}} = \frac{60\angle -90°}{10+j5} = -2.4 - j4.8$ A

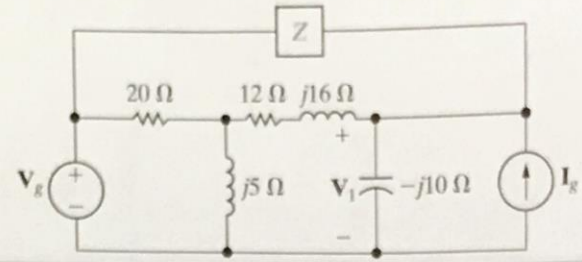
⑥ $\bar{V}_o = (50 \| -j25) I_g = -120$ V



⑦ $v_o(t) = -120\cos 8000t$ V

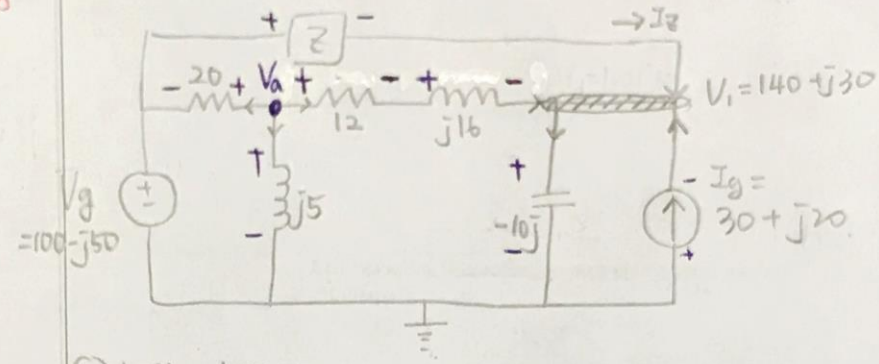
Answer:

4. Find the value of Z in the circuit if $V_g = 100 - j50$ V, $I_g = 30 + j20$ A, and $V_1 = 140 + j30$ V.



0.8

① Redraw the circuit in Phasor Domain



② KCL at V_a

$$\frac{V_a - (100 - j50)}{20} + \frac{V_a}{j5} + \frac{V_a - (140 + j30)}{12 + j16} = 0 \Rightarrow V_a = 40 + j30$$

③ KCL at V_1

$$I_Z + (30 + j20) + \frac{(40 + j30) - (140 + j30)}{12 + j16} = \frac{140 + j30 - 0}{-j10}$$

(I_g)

$$\Rightarrow I_Z = -30 - j10$$

(V_g, V_1)

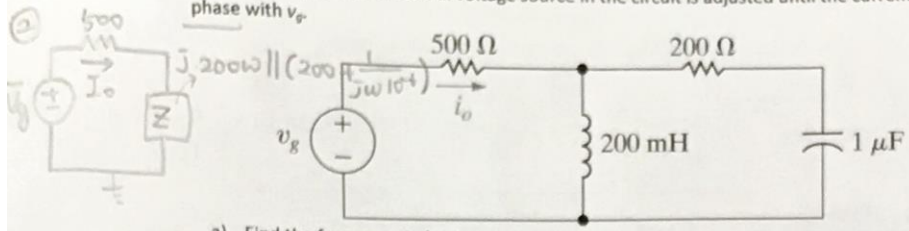
④ $$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\ \Omega$$

Answer:

ECE 200 Electric Circuits
Assignment #8

① Let $A = |A|\cos(\omega t + \theta_1) \Rightarrow \bar{A} = |A| \angle \theta_1^\circ$
$B = |B|\cos(\omega t + \theta_2) \Rightarrow \bar{B} = |B| \angle \theta_2^\circ$
If A & B are in phase $\theta_1 = \theta_2$
$\therefore \frac{\bar{A}}{\bar{B}} = \frac{|A| \angle \theta_1^\circ}{|B| \angle \theta_2^\circ} = \frac{|A|}{|B|}$ ← purely real

5. The frequency of the sinusoidal voltage source in the circuit is adjusted until the current i_o is in phase with v_g.



a) Find the frequency in hertz.

04 ③ If $\bar{V}_g$ and $\bar{I}_o$ are in phase, the ratio (impedance) must be purely real (∵ ①)

④ $\frac{\bar{V}_g}{\bar{I}_o} = Z_{eq} = 500 + Z = 500 + (j\omega 200_m \| (200_m + \frac{1}{j\omega 10^{-6}}))$ (Z_{eq}; $\frac{-j}{\omega}10^6$)
↑ real already, no need to consider.

⑤ to make Z_{eq} "real"

$Z_{eq} = \frac{(j\omega 200_m \times (200 - j/\omega \times 10^6)) \times \omega}{(j\omega 200_m - j/\omega 10^{-6} + 200) \times \omega} = \frac{j\omega 200_m (200_m \omega - j 10^6)}{j(\omega^2 200_m - 10^6) + 200_m \omega} = \frac{a+bj}{c+dj}$

$= \frac{(a+bj)(c-dj)}{(c+dj)(c-dj)} = \frac{1}{c^2+d^2}(ac+bd+j(bc-ad))$ If this is zero then Z_{eq} = purely real.

⑥ $\Rightarrow \omega 0.2(1 - \omega^2 0.2\times 10^{-6}) + \omega^3 200^2 \times 0.2 \times (10^{-6})^2 = 0$

$\omega^2 = \frac{1}{0.2\times 10^{-6} - 200^2 (10^{-6})^2} = 6{,}250{,}000 \Rightarrow \omega = 2500$ rad/s

⑦ $f = \frac{\omega}{2\pi} = 396.9$ Hz.

Answer:

b) Find the steady-state expression for i_o (at the frequency found in [a]) if $v_g = 90\cos\omega t$ V.

0.4

$\bar{Z} = 500 + Z_{eq} = 500 + j0.2\times 2500 \| (200 - \frac{j}{2500\times 10^{-6}})$

$= 500 + \frac{500j(200 - 400j)}{500j + 200 - 400j}$

$= 1500\,\Omega$

$I_g = \frac{\bar{V}_g}{\bar{Z}} = \frac{90\angle 0^\circ}{1500} = 60$ mA

$i_g(t) = 60\cos 2500t$ mA

Answer: