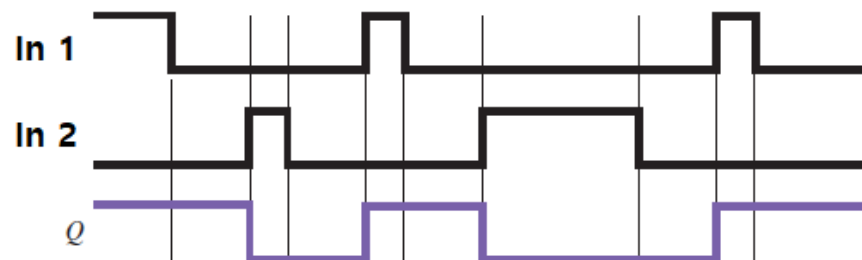


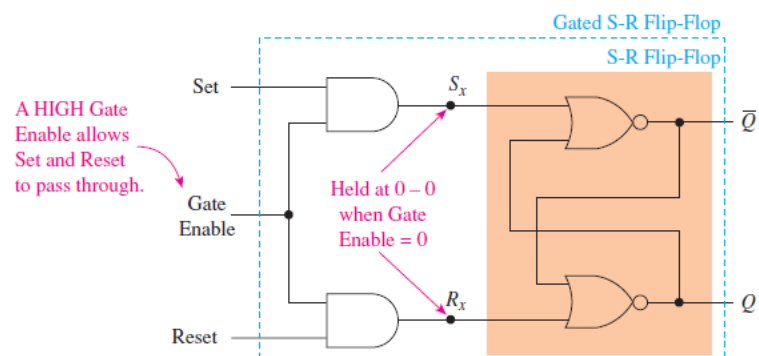
Questions

1. Given the Q output waveform shown in the figure below, determine the name of latch.



S-R latch

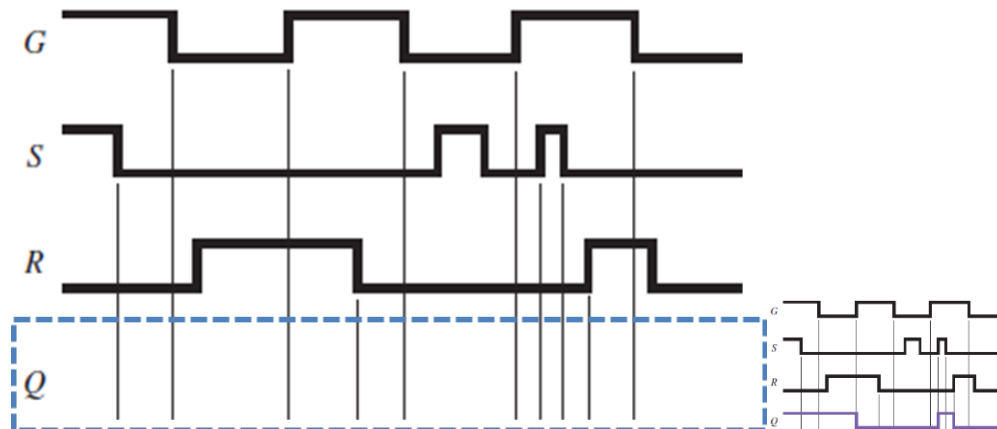
2. The following figure shows a Gated S-R flip-flop.



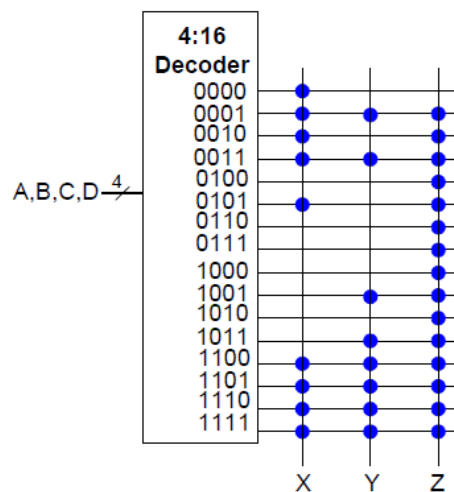
- a. Find its truth table (Inputs are S, G,R and output is Q)

G	S	R	Q
0	0	0	Q
0	0	1	Q
0	1	0	Q
0	1	1	Q
1	0	0	Q
1	0	1	0
1	1	0	1
1	1	1	0

- b. Feed the G, S, and R inputs in the gated S-R flip-flop, sketch the output wave at Q, and list the flip-flop functions



3. The following figure shows X, Y, Z functions' implementation using a single 16 X 3 ROM. Dot notations are used to indicate the ROM contents.



For example, X is a function which can be represented via following Boolean equation:

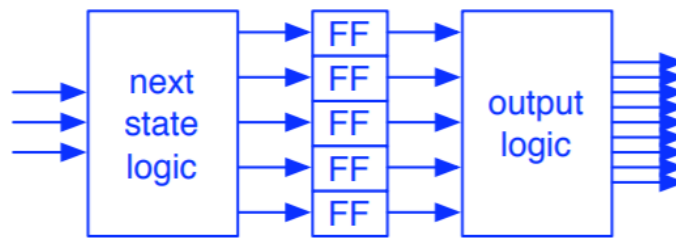
$$X = AB + B\bar{C}D + \bar{A}\bar{B}$$

Determine functions Y and Z accordingly. Provide simplified Boolean equations.

$$Y = AB + B'D$$

$$Z = A + B + C + D$$

4. The following figure shows a Moore machine which has five flip-flops, three inputs, and nine outputs. Assume a state encoding that makes efficient use of the flip-flops.



Answer the following questions:

- a. What are the minimum and maximum numbers of states in the state diagram?

The minimum number of states in the diagram is $2^{5-1} + 1 = 17$ and the maximum is $2^5 = 32$.

- b. A fully defined machine needs to know what action to take for all possible inputs. What are the minimum and maximum numbers of transition arrows starting at a particular state (originating from each state)?

At a minimum each node will have 2^3 (8) transitions originating from it. Similarly, it is not possible to have more than one transition on a particular input. Thus, the maximum number of outgoing transitions is also 2^3 (8)

Min=Max=8

- c. What are the minimum and maximum numbers of different binary patterns that can be displayed on the outputs? For the maximum case, a hint is this is a Moore machine. For the minimum case, assume that any output logic that is a function of the 5 state bits is valid.

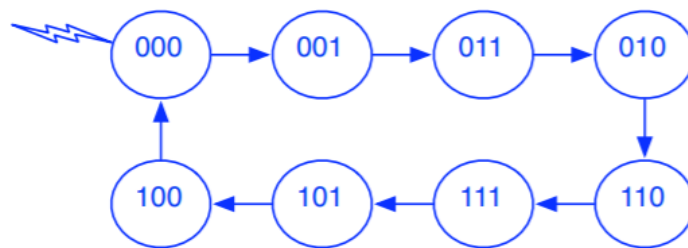
Because this is a Moore machine, the output is a function of only the 5 state bits. At most, the machine can display as many outputs as there are states, or 2^5 (32). The minimum number of output values depends on certain assumptions about the output logic. However, assuming, logic that assigns a constant value to the output no matter the state would produce the minimal number of output values (1).

Max=32, Min=1

5. Gray codes have a useful property in that consecutive numbers differ in only a single bit position. The following table lists a 3-bit Gray code representing the numbers 0 to 7.

Number	Gray code
0	0 0 0
1	0 0 1
2	0 1 1
3	0 1 0
4	1 1 0
5	1 1 1
6	1 0 1
7	1 0 0

Design a 3-bit modulo 8 Gray code UP counter FSM with no inputs and three outputs. The designed counter counts from 0, 1, 2, 3, 4, 5, 6 and 7, then repeats in Gray code pattern. When reset, the output should be 000. **On each clock edge, the output should advance to the next Gray code (meaning that T_n is simply S_n).** After reaching 100, it should repeat with 000. Draw a schematic for this counter using T flip-flops. A transition diagram with encoding is shown as a



starter of your design.

- a. State Transition Table with encoding (T_n are inputs of each T flip-flop, S_n are states, + indicates next state logic)

S_2	S_1	S_0	S_2'	S_1'	S_0'	T_2	T_1	T_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	0
0	1	1	0	1	0	0	0	1
0	1	0	1	1	0	1	0	0
1	1	0	1	1	1	0	0	1
1	1	1	1	0	1	0	1	0
1	0	1	1	0	0	0	0	1
1	0	0	0	0	0	1	0	0

- b. Simplify T_n equations using K-map or Boolean simplification.

S_2	S_1S_0			
	00	01	11	10
0				1
1	1			

$$T_2 = S_2 \overline{S_1} \overline{S_0} + \overline{S_2} S_1 \overline{S_0}$$

S_2	S_1S_0			
	00	01	11	10
0				1
1	1			

$$T_1 = \overline{S_2} \overline{S_1} S_0 + S_2 S_1 S_0$$

S_2	S_1S_0			
	00	01	11	10
0	1		1	
1		1		1

$$T_0 = \overline{S_2} \overline{S_1} \overline{S_0} + \overline{S_2} S_1 S_0 + S_2 \overline{S_1} S_0 + S_2 S_1 \overline{S_0}$$

6. Check the following circuit whether it violates hold time constraint. Also determine the maximum clock frequency. Timing Characteristics of each gate and flip-flop are given as follows:

$$t_{ccq} = 30 \text{ ps}$$

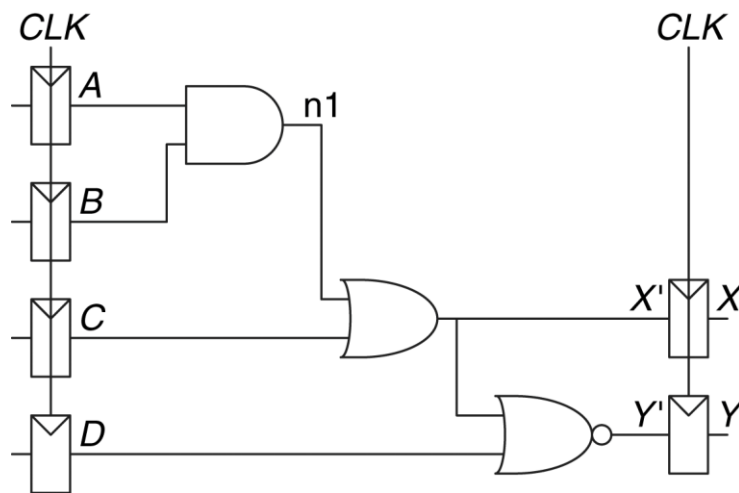
$$t_{pcq} = 80 \text{ ps}$$

$$t_{setup} = 50 \text{ ps}$$

$$t_{hold} = 60 \text{ ps}$$

$$t_{pd} = 40 \text{ ps}$$

$$t_{cd} = 25 \text{ ps}$$



$$t_{ccq} + t_{cd} > t_{hold}$$

$$30 + 25 = 55 > 60 (!)$$

Violates!

$$T_c \geq t_{pcq} + 3 \cdot t_{pd} + t_{setup} = 80 + 3 \cdot 40 + 50 = 250 \text{ ps}$$

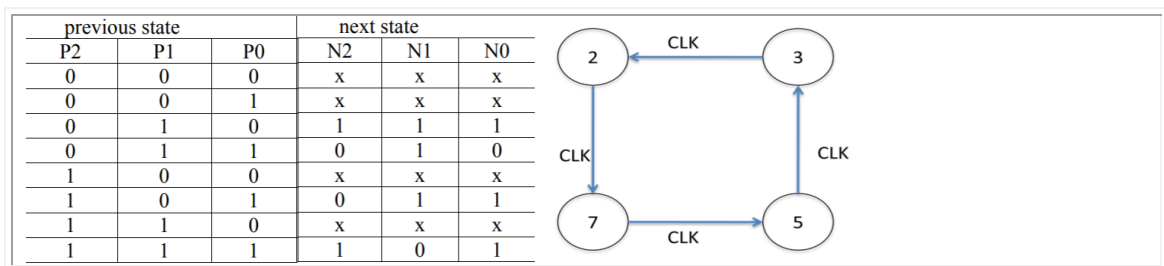
$$F = 1/T_c = 4 \text{ GHz}$$

7. Design a 3-bit down counter (synchronous) that counts prime numbers (i.e., 2, 3, 5, 7). An initial 3-bit prime number is 010_2 . Use three D-FFs for your answer.

a. Write the next state's state transition

Previous state			Next state		
P2	P1	P0 (LSB)	N2	N1	N0 (LSB)
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

b. Draw the transition diagram.



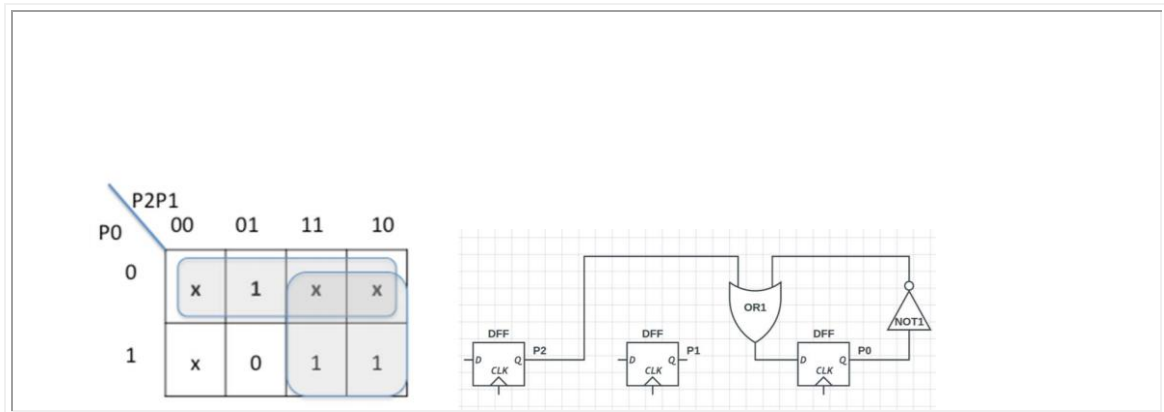
c. Boolean equations for the next state using K-maps (all three bits).

$$N2 = P2P1 + P0'$$

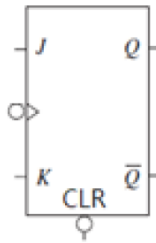
$$N1 = P2' + P1'$$

$$N0 = P2 + P0'$$

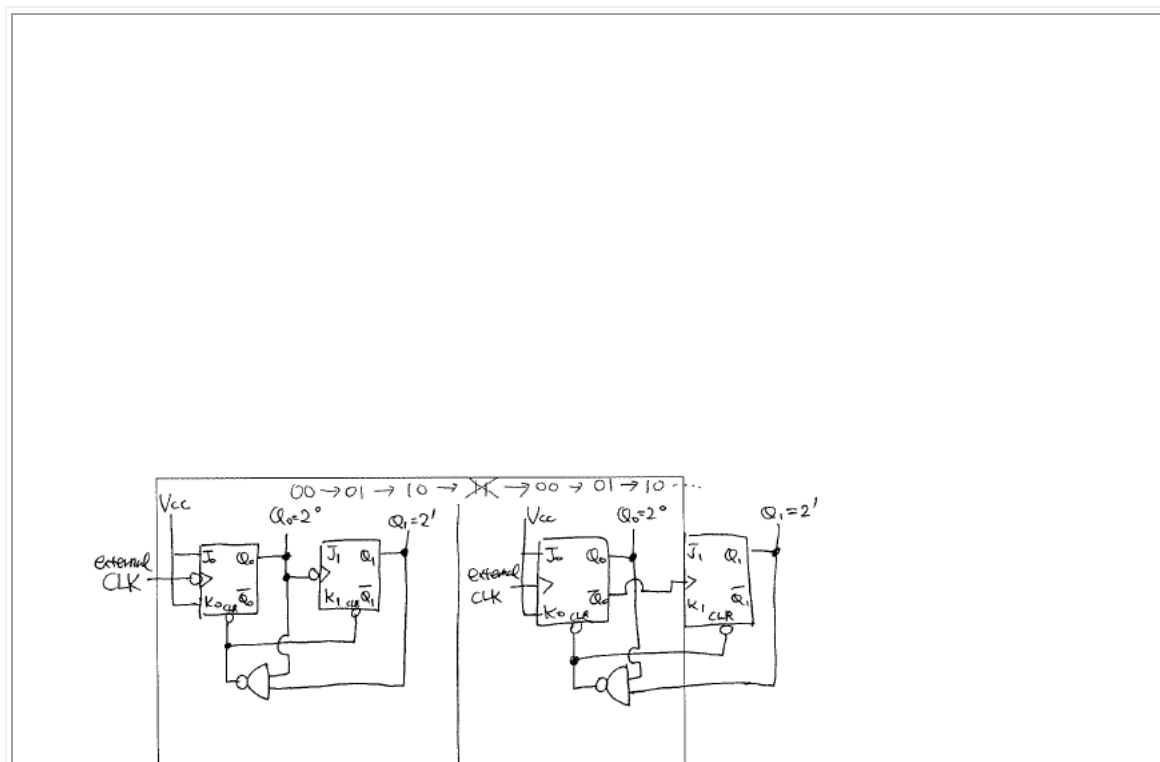
d. Design the circuit for the N0 - LSB of the next state



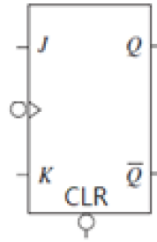
8. Design an asynchronous MOD3 up counter using JK FFs.



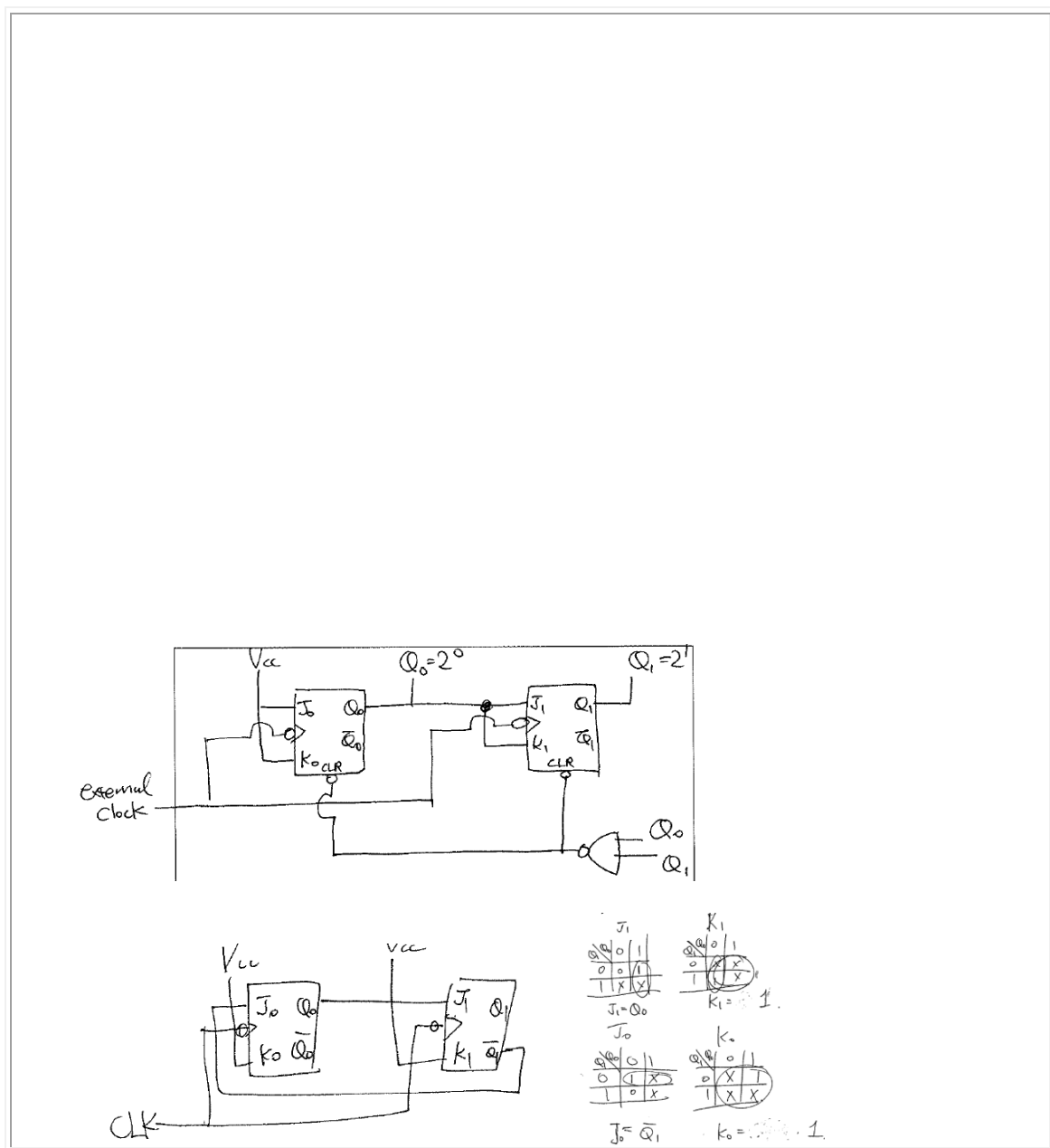
- * A symbol of JK FF is given above. Use an additional logic gates when you need.
- * Put a special consideration on CLR (clear). Due to "the bubble" placed on the CLR pin.
- * Clearly label each JK FFs bit position (make LSB inputs as J0, K0 and LSB outputs: Q0 and Q0')
- * Clarify wiring connection V_{cc} (or 1), GND (or 0), and an external clock if it needs.
- * To check your counter working, assume that you connect LEDs on Q (not on Q').



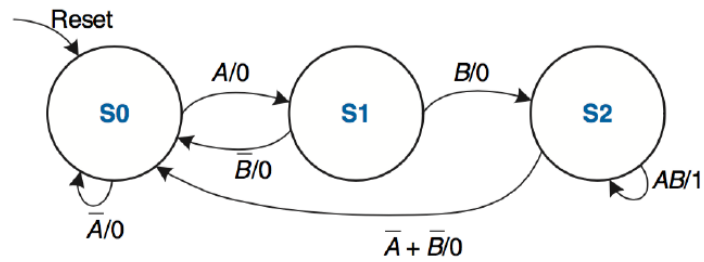
9. Design a synchronous MOD3 up counter using JK FFs.



- * A symbol of JK FF is given above. Use an additional logic gates when you need.
- * Put a special consideration on CLR (clear). Due to "the bubble" placed on the CLR pin.
- * Clearly label each JK FFs bit position (make LSB inputs as J_0 , K_0 and LSB outputs: Q_0 and \bar{Q}_0)
- * Clarify wiring connection V_{cc} (or 1), GND (or 0), and an external clock if it needs.
- * To check your counter working, assume that you connect LEDs on Q (not on Q').



10. Find a state transition table and output table for the FSM below. Write Boolean equations for the next state and output.



S_1	S_0	A	B	S_1^+	S_0^+	Q
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	0	0
0	1	0	1	1	0	0
0	1	1	0	0	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	1	0	1
1	1	0	0	X	X	X
1	1	0	1	X	X	X
1	1	1	0	X	X	X
1	1	1	1	X	X	X

$$S_1^+ = S_0 \cdot B + S_1 \cdot A \cdot B$$

$$S_0^+ = \bar{S}_1 \cdot \bar{S}_0 \cdot A$$

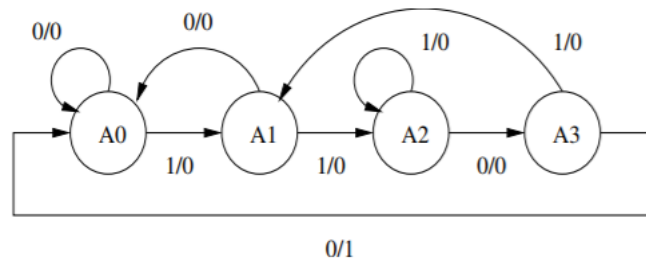
$$Q = S_1 \cdot A \cdot B$$

11. A sequence detector has a serial input X and an output Z. Whenever the circuit receives a sequence of 1100 on input X during four consecutive clock cycles, it produces an output Z = 1 at the same time as the fourth input. At all other times, the output Z = 0. To further clarify the problem, an example of input/output sequence is shown below:

X: 00110111000011110001100111...

Z: 00000000010000000100001000...

- a. Obtain a state transition diagram for this problem. Use Mealy FSM. Label the starting state as A0 and other states as A1, A2, etc.

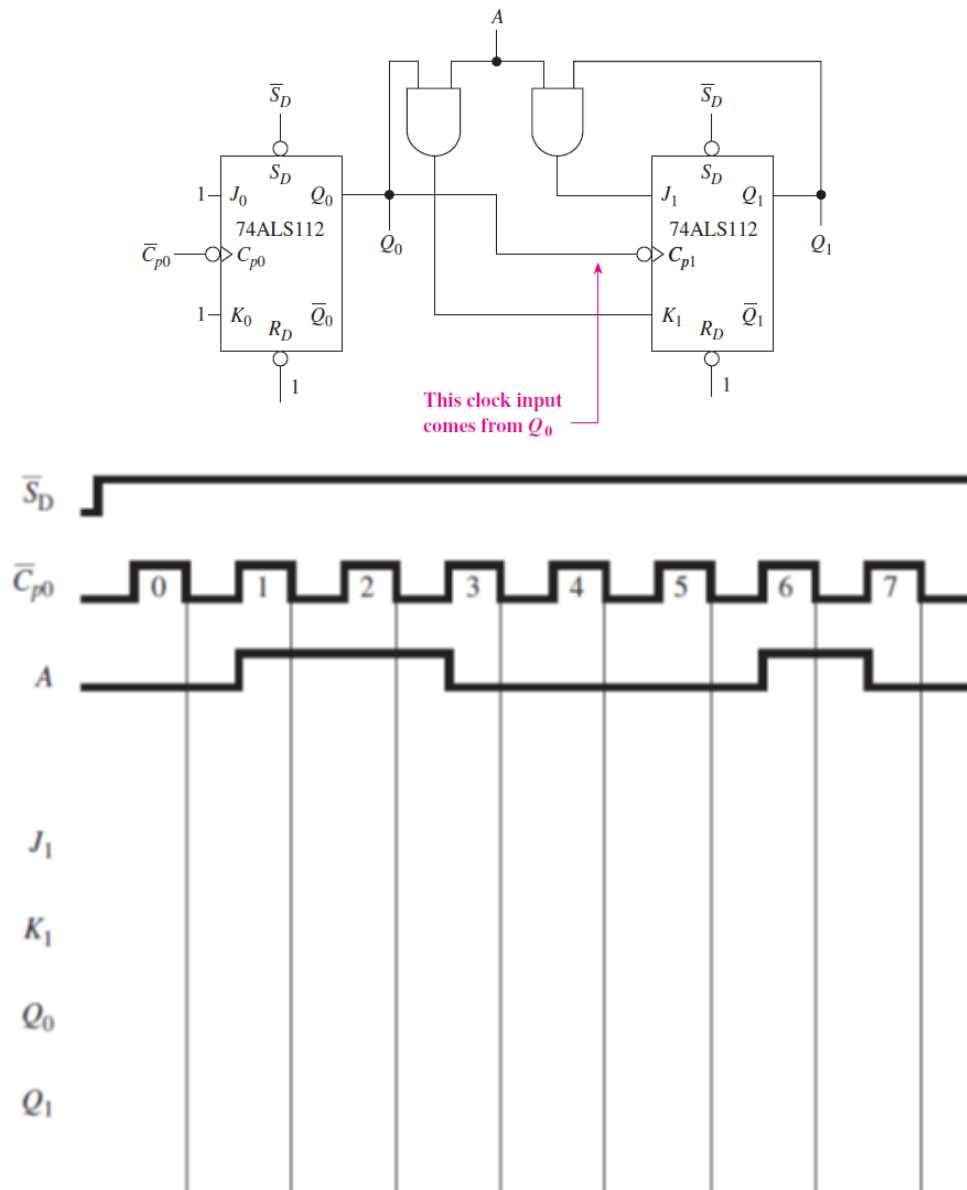


- b. Derive the state transition table.

S_1	S_0	X	S_1^+	S_0^+	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	1	1	0
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	0	1	0

Pres State	Next State, Z	
	X = 0	X = 1
A ₀	A ₀ , 0	A ₁ , 0
A ₁	A ₀ , 0	A ₂ , 0
A ₂	A ₃ , 0	A ₂ , 0
A ₃	A ₀ , 1	A ₁ , 0

12. This circuit uses negative edge-triggered JK flip-flops. Complete Input and output waveforms. Note that the clock input from the second FF comes from Q_0 .



Solution:

