Euclid's Axiomatic System (aka. Geometry)

❖ Undefined Terms

- Point (understood to be the set, S, of points in space. Points have no dimension). Point: ●
- Line (understood to be the set, L, (a subset of S) of points that stretch infinitely far in two directions. Lines have one dimension, length.)
  Line: ←——→
- Plane (understood to be the set, P, (a subset of S), of points that stretch infinitely far in all directions. Planes have two dimensions, length & width (but no height).)

❖ Definitions

- A **segment** is the portion of a line between two given points.

Looks like this: ●——●

- A **ray** is a portion of a line from one point (the endpoint) to infinity in one direction.

Looks like this: ●——→

- If a line M is a subset of a plane E, we shall say that L **lies in** E. Likewise, if a point T is a subset of a line M, then we shall say T **lies on** or **passes through** M. Similarly, we shall say that M **contains** T and E **contains** L.
- A **figure** is a set of points.
- Points lying on one line are called **collinear**.
- Points lying in one plane are called **coplanar**.
- Two lines are called **parallel** if they lie in the same plane but do not intersect.

❖ Postulates

- Every line contains at least two points; S contains at least 3 noncollinear points. Every plane contains at least three noncollinear points and S contains at least four noncoplanar points. (Existance postulate)
- All lines and planes are sets of points.
- Given any two different points, there is exactly one line containing them.
- Given any three different noncollinear points, there is exactly one plane containing them.
- If two points lie in a plane, then the line containing them lies in the plane.
- If two planes intersect, then their intersection is a line.
- Given a line, L, and a point P not on L, there is one and only one line that contains P and is parallel to L. (Euclid's 5^{th} Postulate, also known as Euclid's Parallel Postulate).

❖ Theorems

- Two different lines intersect in at most one point.
  - Proof?
- If a line intersects a plane not containing it, then the intersection is a single point.
  - Proof?
- Given a line and a point not on the line, there is exactly one plane containing both of them.
  - Proof?
- If two lines intersect, then their union lies in exactly one plane.
  - Proof?