

Examples for Day 8

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The Karnaugh Map

Practice $\overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$

		C	
		0	1
AB	00		1
	01	1	
	11	1	1
	10		

Practice

$$\overline{A} + A\overline{B} + AB\overline{C}$$

		C	
		0	1
AB	00	1	1
	01	1	1
	11	1	
	10	1	1

Grouping the 1s

AB \ C	C	
	0	1
00	1	
01		1
11	1	1
10		

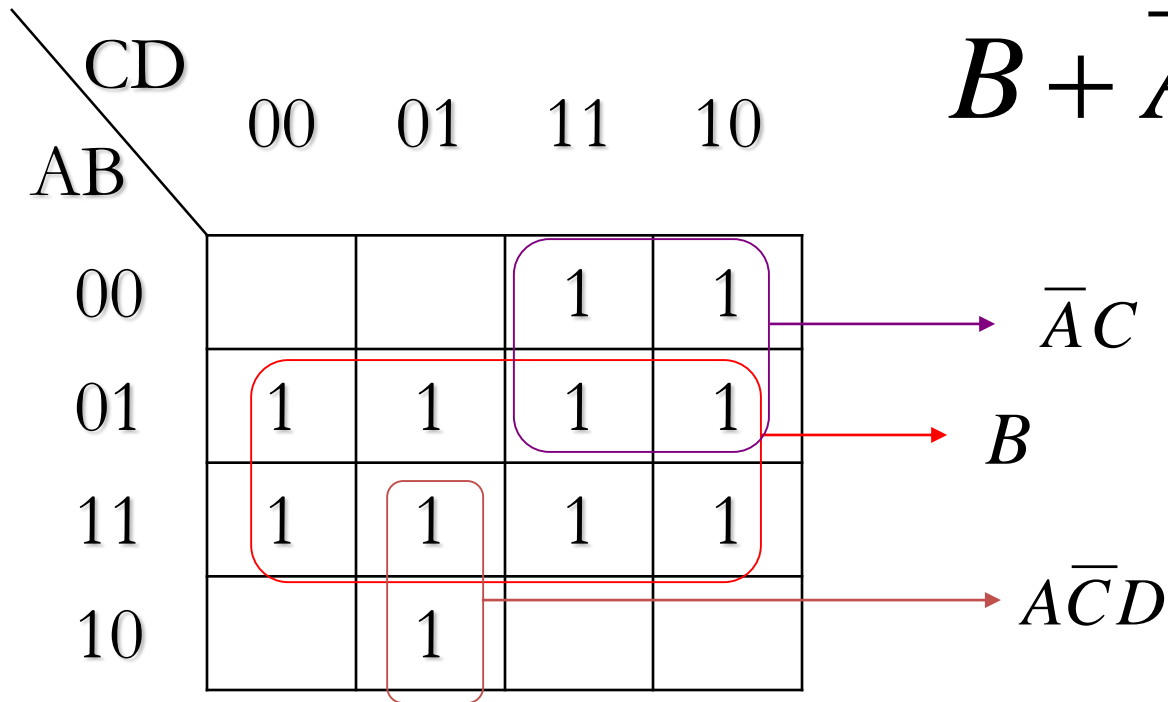
AB \ C	C	
	0	1
00	1	1
01	1	
11		1
10	1	1

Grouping the 1s

		CD			
		00	01	11	10
AB	00	1	1		
	01	1	1	1	1
	11				
	10		1	1	

		CD			
		00	01	11	10
AB	00	1			1
	01	1	1		1
	11	1	1		1
	10	1		1	1

Determining the Minimum SOP Expression from the Map



$$B + \bar{A}C + A\bar{C}D$$

Determining the Minimum SOP Expression

		C	
		0	1
AB	00	1	
	01		1
	11	1	1
	10		

$$AB + BC + \overline{A}\overline{B}\overline{C}$$

		C	
		0	1
AB	00	1	1
	01	1	
	11		1
	10	1	1

$$\overline{B} + \overline{A}\overline{C} + AC$$

Determining the Minimum SOP Expression

		CD			
		00	01	11	10
AB	00	1	1		
	01	1	1	1	1
	11				
	10		1	1	

$$\bar{A}B + \bar{A}\bar{C} + A\bar{B}D$$

		CD			
		00	01	11	10
AB	00	1			1
	01	1	1		1
	11	1	1		1
	10	1		1	1

$$\bar{D} + A\bar{B}C + B\bar{C}$$

Mapping Directly from a Truth Table

$$\overline{A}\overline{B}\overline{C} + A\overline{B}\overline{C} + AB\overline{C} + ABC$$

I / P			O / P
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		C	
		0	1
AB	00	1	
	01		
	11	1	1
	10	1	

$$AB + B'C'$$

“Don’t Care” Conditions

- Sometimes a situation arises in which some input variable combinations are not allowed:
 - i.e., six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111.
- Since these unallowed states will never occur in an application → they can be treated as “don’t care” terms with respect to their effect on the output.
- The “don’t care” terms can be used to advantage on the K-map (how? see the next).

“Don’t Care” Conditions

(i.e., six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111)

INPUTS				O/P
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	x
1	0	1	1	x
1	1	0	0	x
1	1	0	1	x
1	1	1	0	x
1	1	1	1	x

CD \ AB				
	00	01	11	10
00				
01			1	
11	x	x	x	x
10	1	1	x	x

Without “don’t care”

$$Y = A\bar{B}\bar{C} + \bar{A}BCD$$

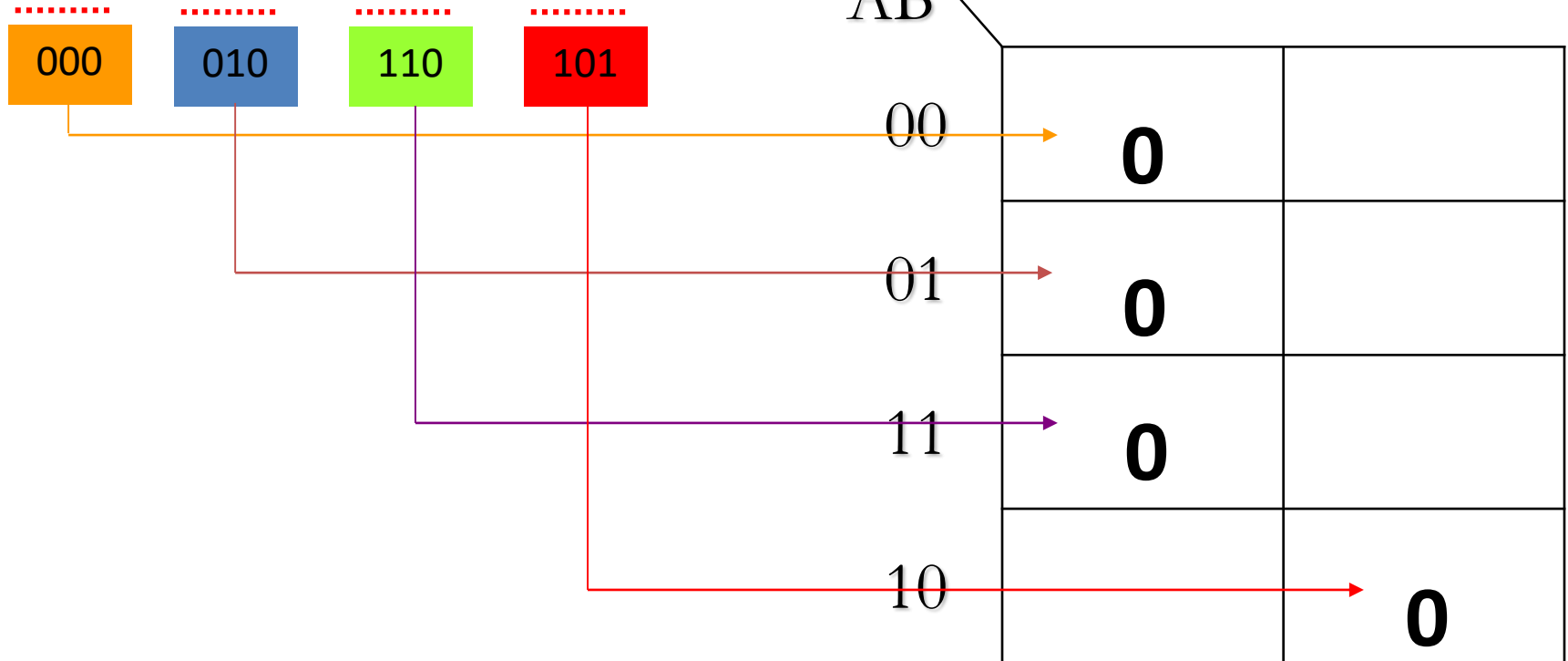
With “don’t care”

$$Y = A + BCD$$

Mapping a Standard POS

The expression:

$$(A+B+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+C)(\bar{A}+B+\bar{C})$$

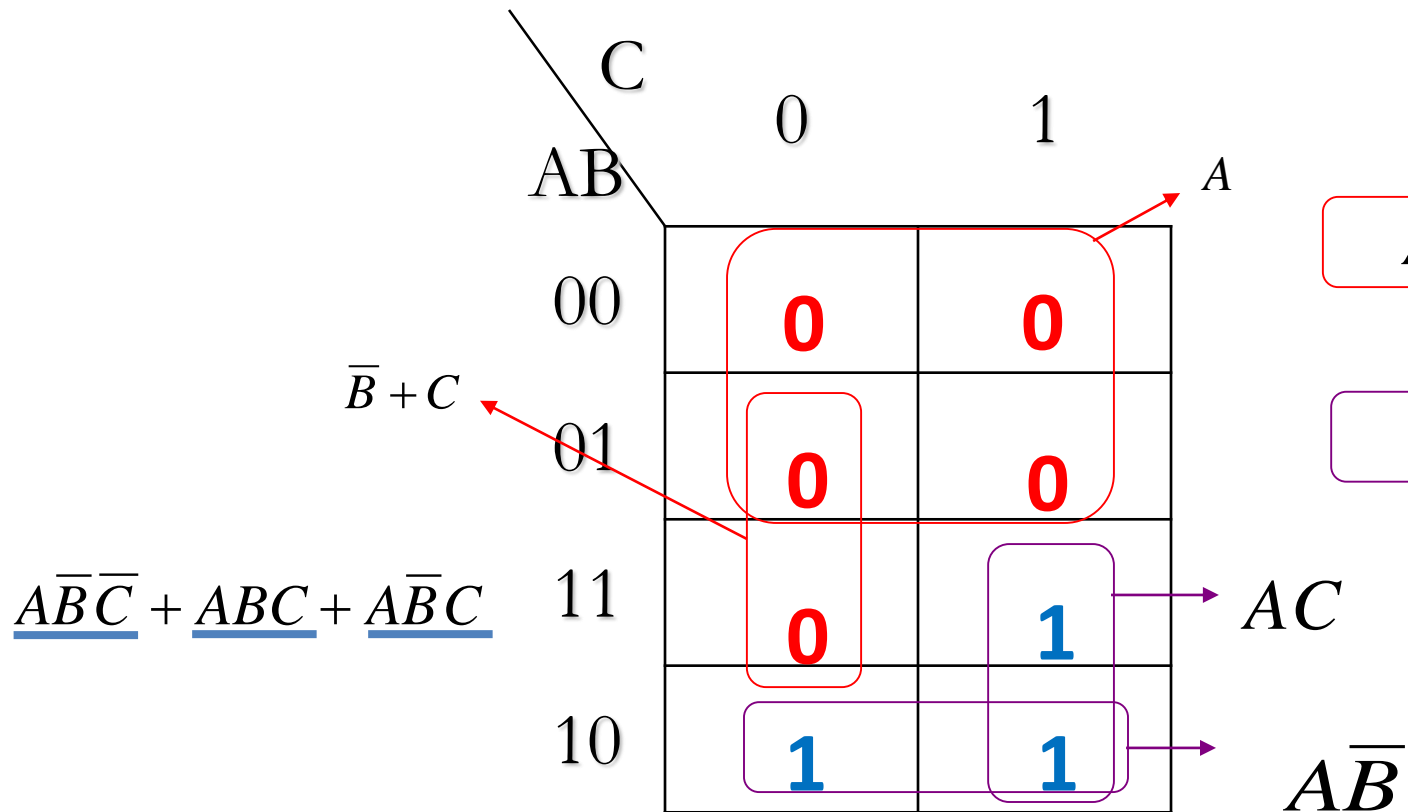


Mapping a Standard POS/SOP

$$\underline{(A + B + C)} \underline{(A + B + \bar{C})} \underline{(A + \bar{B} + C)} \underline{(A + \bar{B} + \bar{C})} \underline{(\bar{A} + \bar{B} + C)}$$

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

POS: 000,001,010,011,110 → missing 100,111,101 : SOP

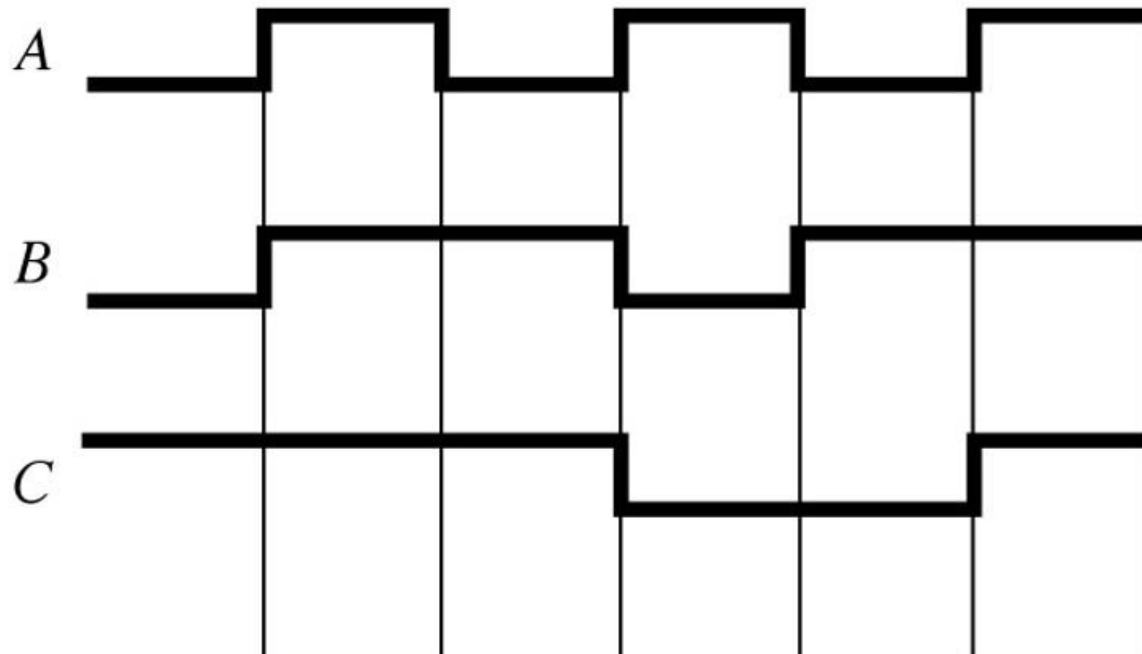
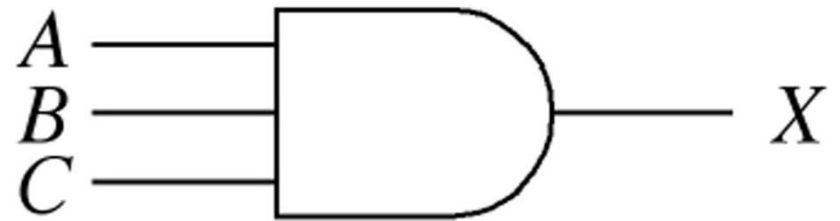


$$A(\bar{B} + C)$$

$$A\bar{B} + AC$$

Timing Analysis

- Determine the gate output for the input waveforms below.



Sketch the output waveforms for the circuit shown in Figure 3–58.

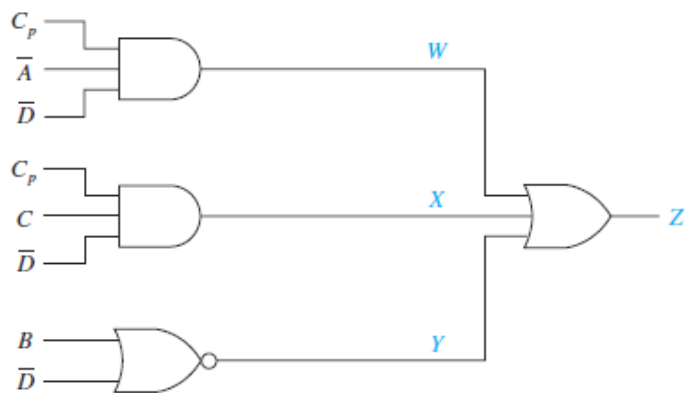
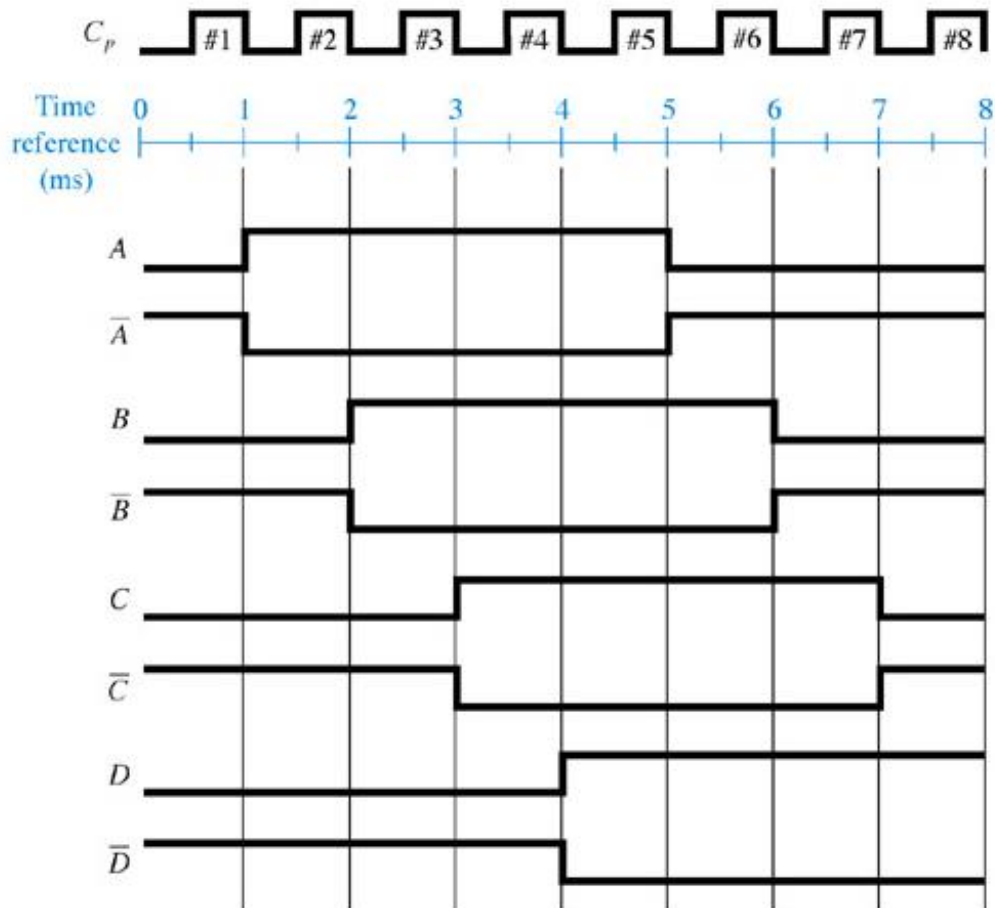


Figure 3–58



Simplification Using Boolean Algebra

1. $AB' + AB + BC = A(B' + B) + BC = A + BC$
2. $AB'D + AB'D' = AB' (D + D') = AB'$
3. $(A' + B)(A + B) = A'A + AB + A'B + BB = 0 + B(A + A') = B$
4. $ACD + A'BCD = CD(A + A'B) = CD(A + B) = ACD + BCD$
5. $AC' + ABC' = AC'(1 + B) = AC'$
6. $A'B'CD' + A'B'C'D' = A'B'D' (C + C') = A'B'D'$
7. $A'D + ABD = D(A' + AB) = D(A' + B)$
8. $((A' + C)(B + D'))' = AC' + B'D$

A note for K-map.

3. $(A' + B)(A + B)$: POS: $F = B$;

1. $AB' + AB + BC$: SOP: $F = A + BC$