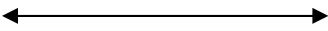
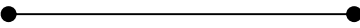
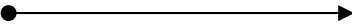


## Euclid's Axiomatic System (aka. Geometry)

### ❖ Undefined Terms

- Point (understood to be the set,  $S$ , of points in space. Points have no dimension). Point: ●
- Line (understood to be the set,  $L$ , (a subset of  $S$ ) of points that stretch infinitely far in two directions. Lines have one dimension, length.)  
Line: 
- Plane (understood to be the set,  $P$ , (a subset of  $S$ ), of points that stretch infinitely far in all directions. Planes have two dimensions, length & width (but no height).)

### ❖ Definitions

- A **segment** is the portion of a line between two given points.  
Looks like this: 
- A **ray** is a portion of a line from one point (the endpoint) to infinity in one direction.  
Looks like this: 
- If a line  $M$  is a subset of a plane  $E$ , we shall say that  $L$  **lies in**  $E$ . Likewise, if a point  $T$  is a subset of a line  $M$ , then we shall say  $T$  **lies on** or **passes through**  $M$ . Similarly, we shall say that  $M$  **contains**  $T$  and  $E$  **contains**  $L$ .
- A **figure** is a set of points.
- Points lying on one line are called **collinear**.
- Points lying in one plane are called **coplanar**.
- Two lines are called **parallel** if they lie in the same plane but do not intersect.

### ❖ Postulates

- Every line contains at least two points;  $S$  contains at least 3 noncollinear points. Every plane contains at least three noncollinear points and  $S$  contains at least four noncoplanar points. (Existence postulate)
- All lines and planes are sets of points.
- Given any two different points, there is exactly one line containing them.
- Given any three different noncollinear points, there is exactly one plane containing them.
- If two points lie in a plane, then the line containing them lies in the plane.
- If two planes intersect, then their intersection is a line.
- Given a line,  $L$ , and a point  $P$  not on  $L$ , there is one and only one line that contains  $P$  and is parallel to  $L$ . (Euclid's 5<sup>th</sup> Postulate, also known as Euclid's Parallel Postulate).

### ❖ Theorems

- Two different lines intersect in at most one point.
  - Proof?
- If a line intersects a plane not containing it, then the intersection is a single point.
  - Proof?
- Given a line and a point not on the line, there is exactly one plane containing both of them.
  - Proof?
- If two lines intersect, then their union lies in exactly one plane.
  - Proof?