Ex. For capacitors

Example 1

(a) Calculate the charge stored on a 3-pF capacitor with 20V across it.

(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60\text{pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600\text{pJ}$$

Example 2

- The voltage across a 5- μF capacitor is

$$v(t) = 10\cos 6000t \ \text{ V}$$

Calculate the current through it.

Solution:

- By definition, the current is

$$i = C\frac{dv}{dt} = 5\times10^{-6}\frac{d}{dt}(10\cos 6000t)$$

$$= -5\times10^{-6}\times6000\times10\sin 6000t = -0.3\sin 6000t \ \text{ A}$$

Example 3

- Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000\,t}\,\text{mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

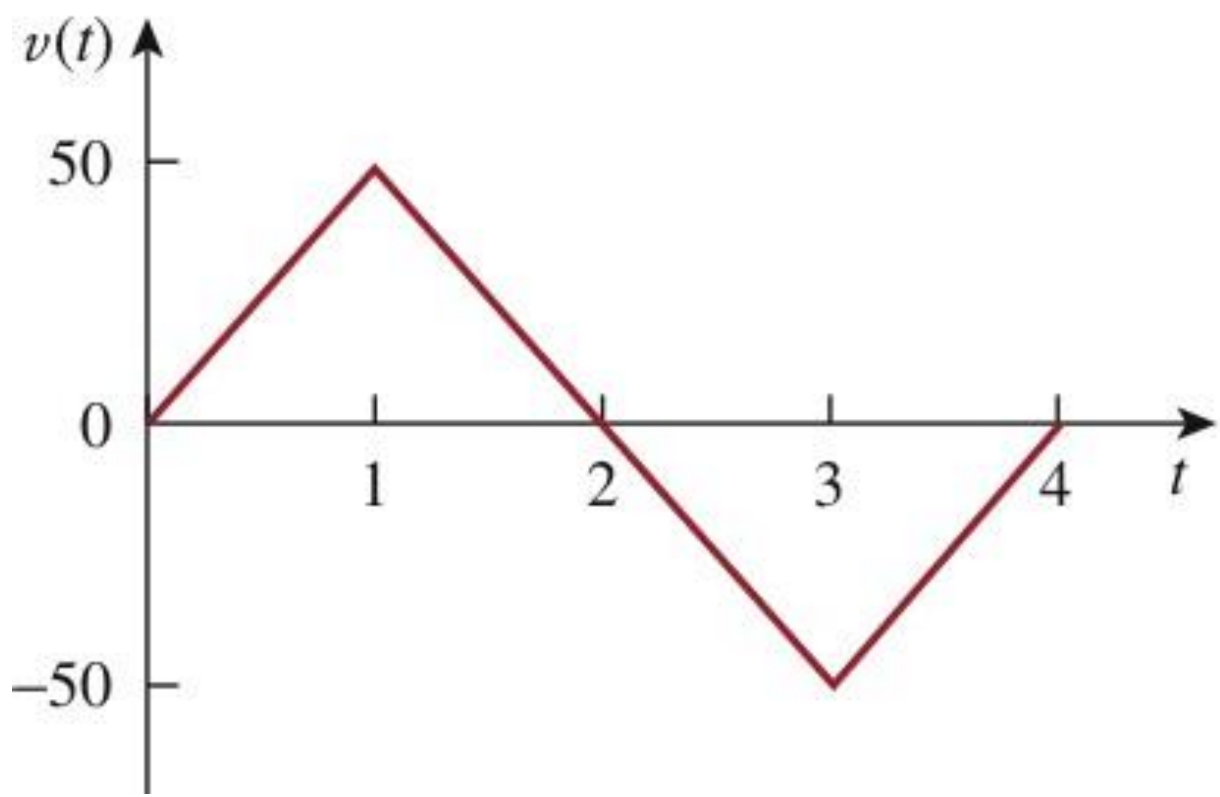
- Since $v = \dfrac{1}{C}\int_0^t i\,dt + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2\times 10^{-6}}\int_0^t 6e^{-3000t}\,dt \cdot 10^{-3} = \frac{3\times 10^{3}}{-3000}e^{-3000t}\Big|_0^t$$

$$= (1 - e^{-3000t})\,\text{V}$$

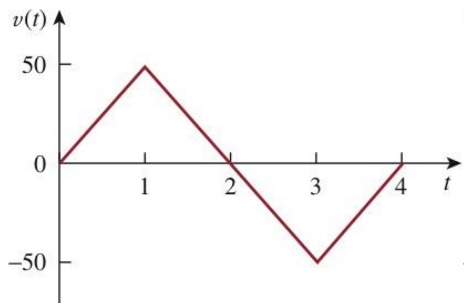
Example 4

- Determine the current through a 200- μF capacitor whose voltage is shown below.



Solution:

- The voltage waveform can be described mathematically as

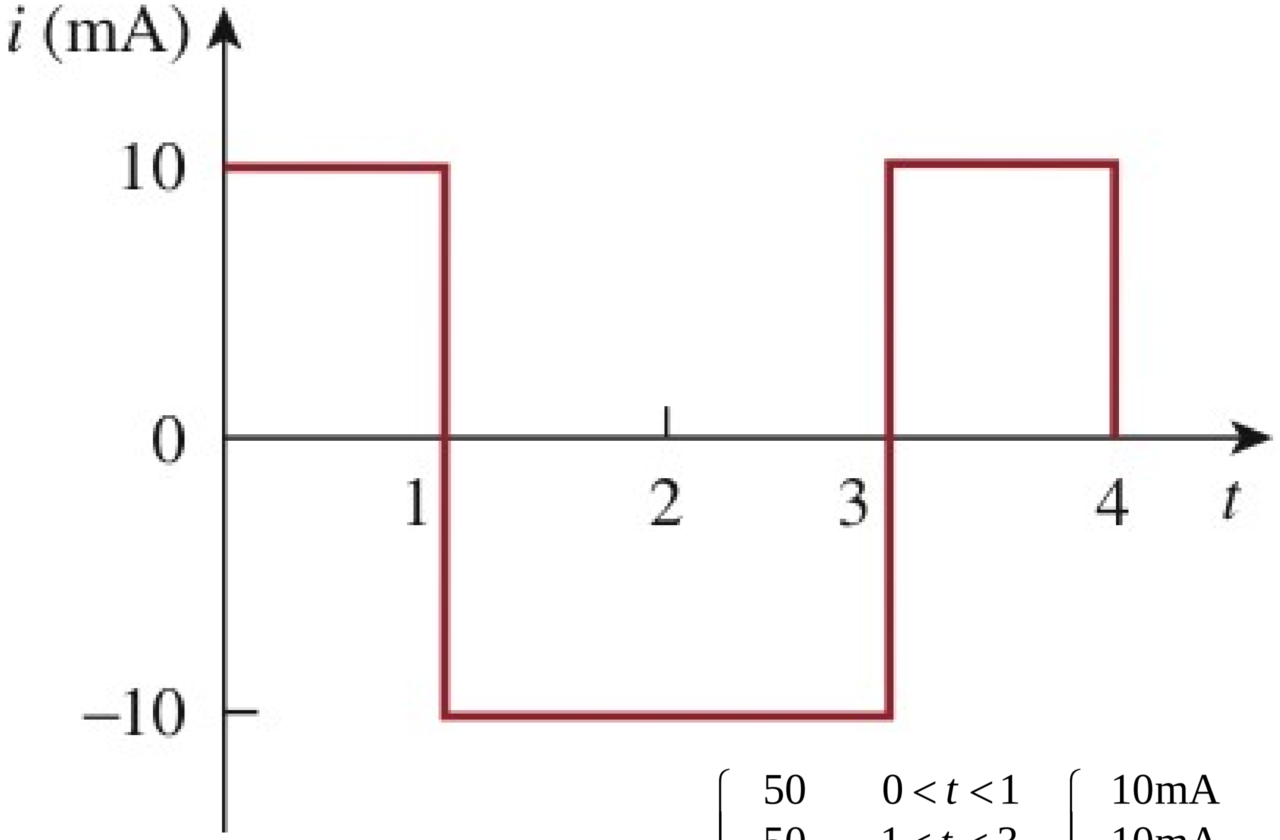


$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

- Since i = C dv/dt and C = 200 μF, we take the derivative of to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 10\text{mA} & 0 < t < 1 \\ -10\text{mA} & 1 < t < 3 \\ 10\text{mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

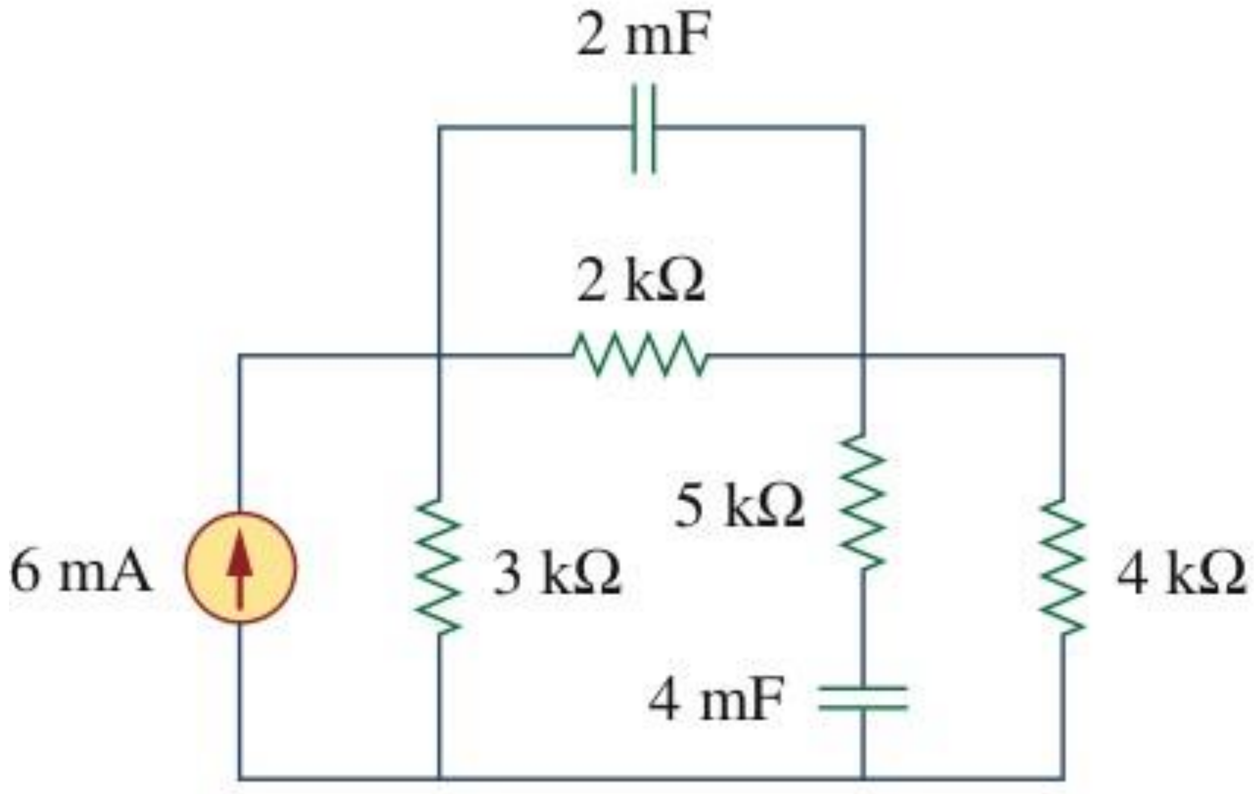
$$v(t)=\begin{cases} 50t\ \text{V} & 0<t<1 \\ 100-50t\ \text{V} & 1<t<3 \\ -200+50t\ \text{V} & 3<t<4 \\ 0 & \text{otherwise} \end{cases}$$



$$i(t)=200\times10^{-6}\times\begin{cases} 50 & 0<t<1 \\ -50 & 1<t<3 \\ 50 & 3<t<4 \\ 0 & \text{otherwise} \end{cases}=\begin{cases} 10\text{mA} & 0<t<1 \\ -10\text{mA} & 1<t<3 \\ 10\text{mA} & 3<t<4 \\ 0 & \text{otherwise} \end{cases}$$

Example 5

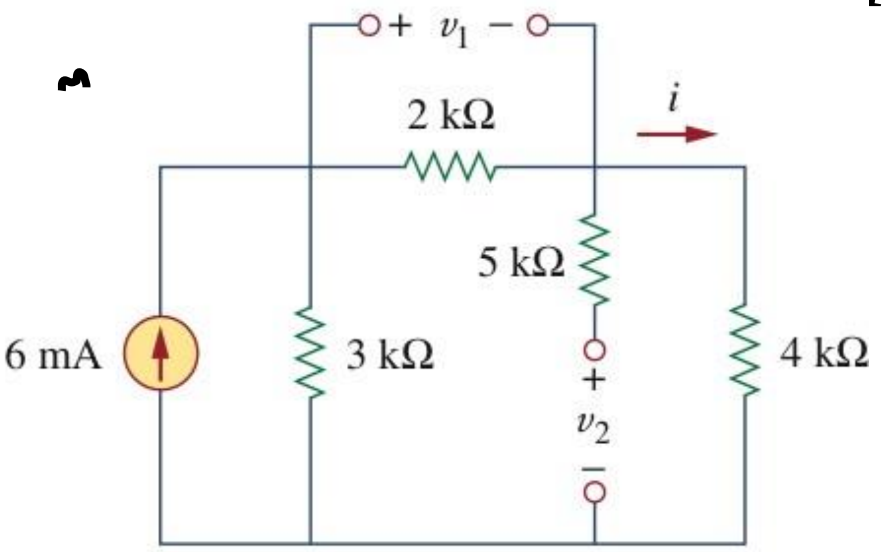
- Obtain the energy stored in each capacitor under dc condition (freq=0,t=∞).



Solution:

- DC condition: each capacitor→an open circuit.
  - By current division,

$$i = \frac{3}{3+2+4}(6\text{mA}) = 2\text{mA}$$



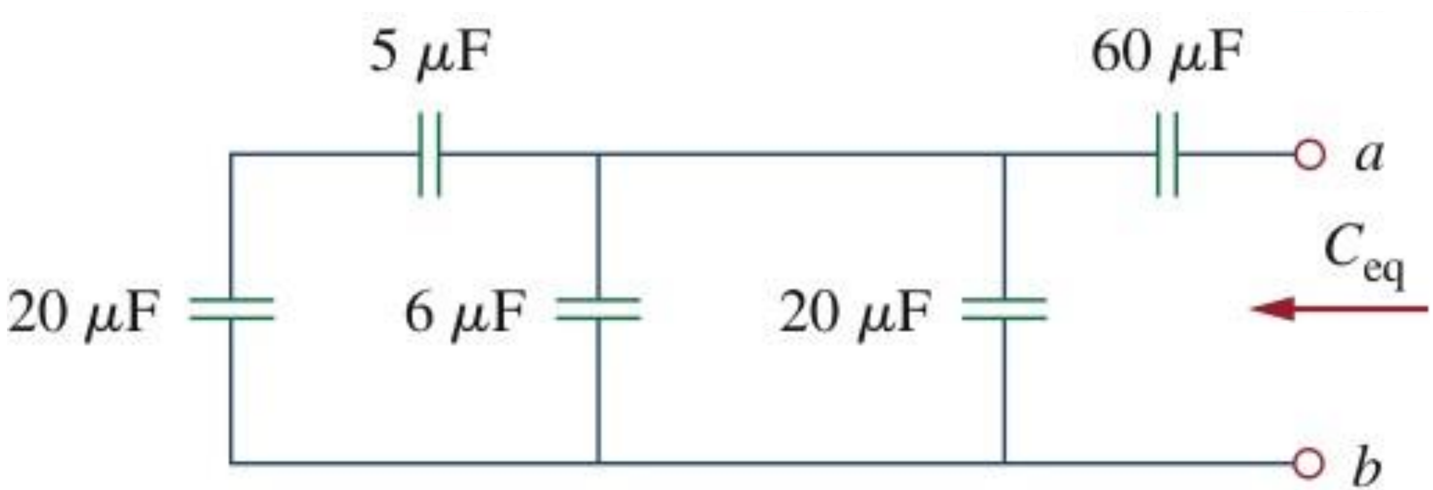
$$\therefore v_1 = 2000i = 4\text{V},$$
$$v_2 = 4000i = 8\text{V}$$

$$\therefore w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2\times10^{-3})(4)^2 = 16\text{mJ}$$

$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4\times10^{-3})(8)^2 = 128\text{mJ}$$

Example 6

- Find the equivalent capacitance seen between terminals a and b of the circuit



Solution:

- $20-\mu\text{F}$ and $5-\mu\text{F}$ capacitors are in series:

$$\therefore \frac{20\times 5}{20+5} = 4\mu\text{F}$$

- $4-\mu\text{F}$ capacitor is in parallel with the $6-\mu\text{F}$ and $20-\mu\text{F}$ capacitors:

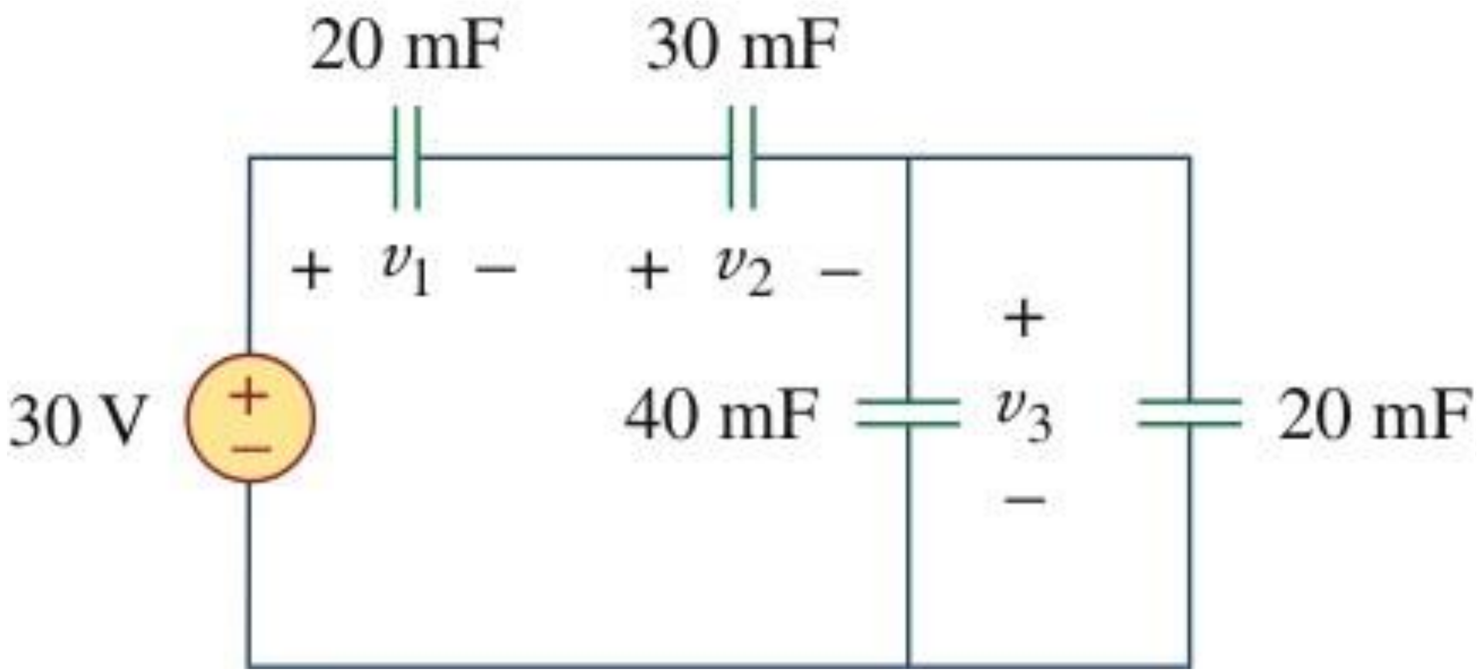
$$\therefore 4+6+20 = 30\mu\text{F}$$

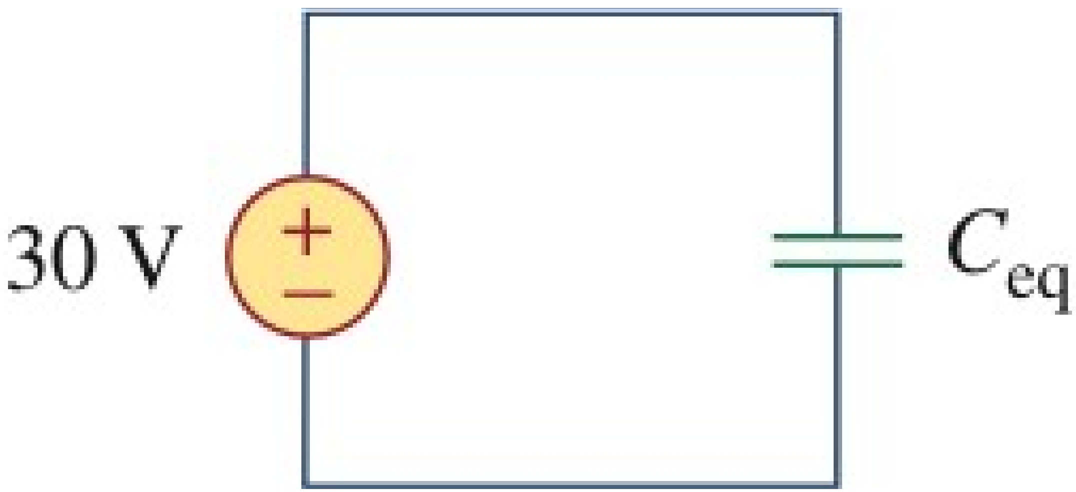
- $30-\mu\text{F}$ capacitor is in series with the $60-\mu\text{F}$ capacitor.

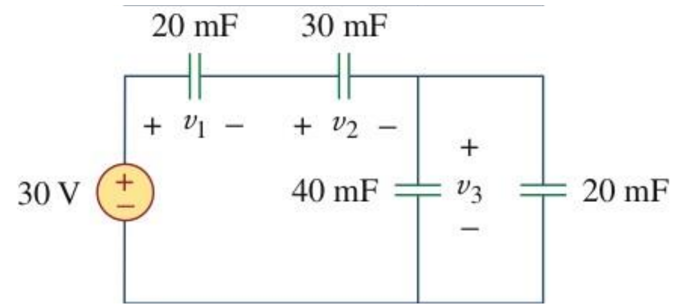
$$C_{eq} = \frac{30\times 60}{30+60}\mu\text{F} = 20\mu\text{F}$$

Example 7

- Find the voltage across each capacitor.




30 V
+
−
C_{eq}



Solution:

- Two parallel capacitors:

$$\therefore C_{eq} = \frac{1}{\frac{1}{60}+\frac{1}{30}+\frac{1}{20}} \text{mF} = 10\text{mF}$$

- Total charge

$$q = C_{eq} v = 10 \times 10^{-3} \times 30 = 0.3\text{C}$$

- This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-v source.
  - A crude way to see this is to imagine that charge acts like current, since *i* = *dq*/*dt*

- Therefore, $$v_1 = \frac{q}{C_1} = \frac{0.3}{20\times10^{-3}} = 15\text{ V},$$
$$v_2 = \frac{q}{C_2} = \frac{0.3}{30\times10^{-3}} = 10\text{ V}$$
- Having determined v_1 and v_2, we now use KVL to determine v_3 by
$$v_3 = 30 - v_1 - v_2 = 5\text{V}$$
- Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is 40+20=60mF.

$$\therefore\ v_3 = \frac{q}{60\text{mF}} = \frac{0.3}{60\times10^{-3}} = 5\text{V}$$

Ex. For inductors

Example 1

- The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

$$\text{Since } v = L\frac{di}{dt} \text{ and } L = 0.1\text{H},$$

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1-5t)\text{V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t}\text{J}$$

Example 2

- Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also find the energy stored within 0 < *t* < 5s. Assume *i*(*0*)=0.

Solution:

$$\text{Since } i = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0) \text{ and } L = 5\text{H}.$$

$$i = \frac{1}{5}\int_0^t 30t^2 dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\Bigg|_0^5 = 156.25\,\text{kJ}$$

Alternatively, we can obtain the energy stored using

$$w(5) - w(0) = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0)$$

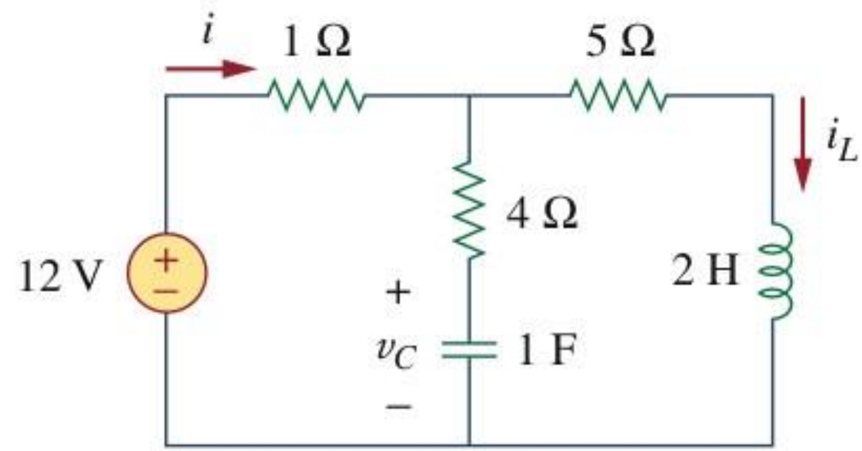
$$= \frac{1}{2}(5)(2\times 5^3)^2 - 0 = 156.25\,\text{kJ}$$

Example 3

- Consider the circuit, under dc conditions, find:

  (a) i, v_C, and i_L.

  (b) the energy stored in the capacitor and inductor.

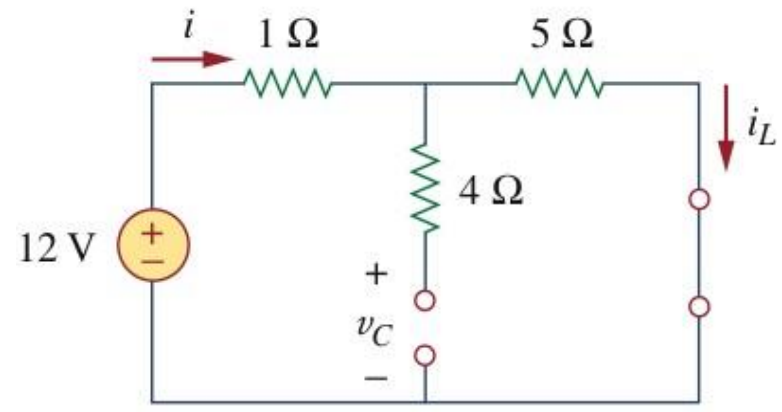


Solution:

(a) Under dc condition :

capacitor $\rightarrow$ open circuit

inductor $\rightarrow$ short circuit



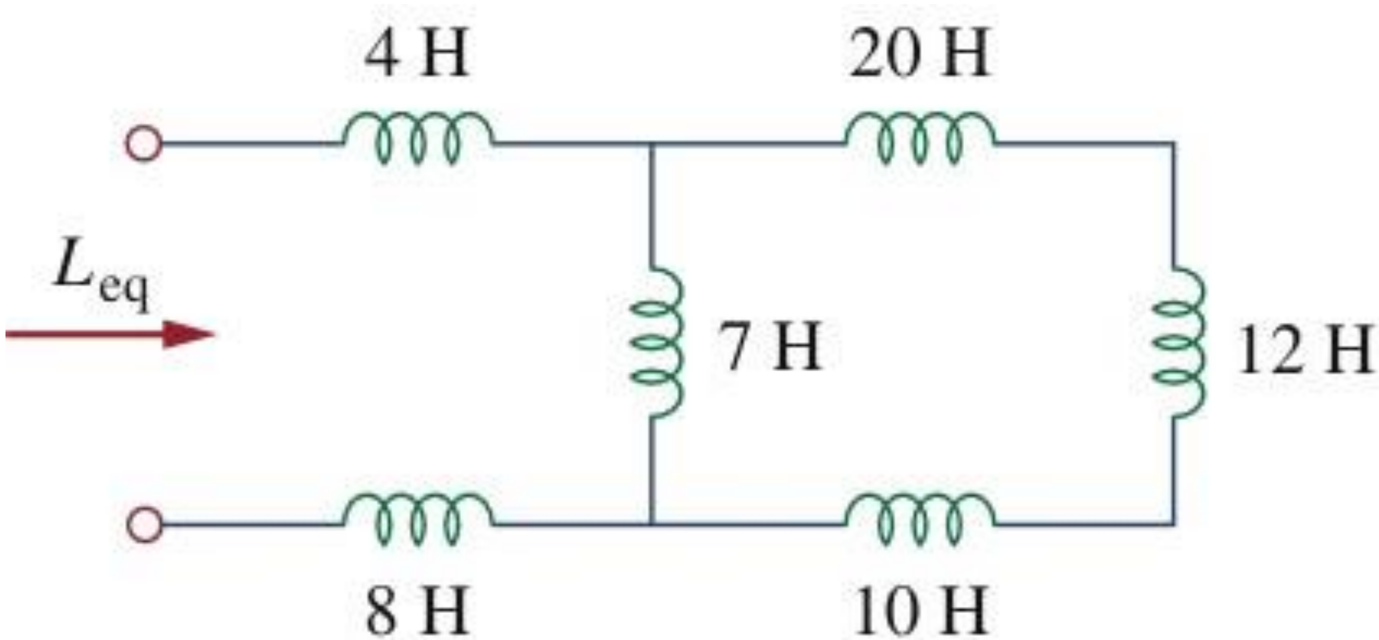
$$i = i_L = \frac{12}{1+5} = 2A,\ v_c = 5i = 10\,\text{V}$$

(b)
$$w_c = \frac{1}{2}Cv_c^{\ 2} = \frac{1}{2}(1)(10^2) = 50\text{J},$$

$$w_L = \frac{1}{2}L_i^{\ 2} = \frac{1}{2}(2)(2^2) = 4\text{J}$$

Example 4

- Find the equivalent inductance of the circuit.



- **Solution:**

$$\text{Series}: \ 20\text{H}, 12\text{H}, 10\text{H}$$

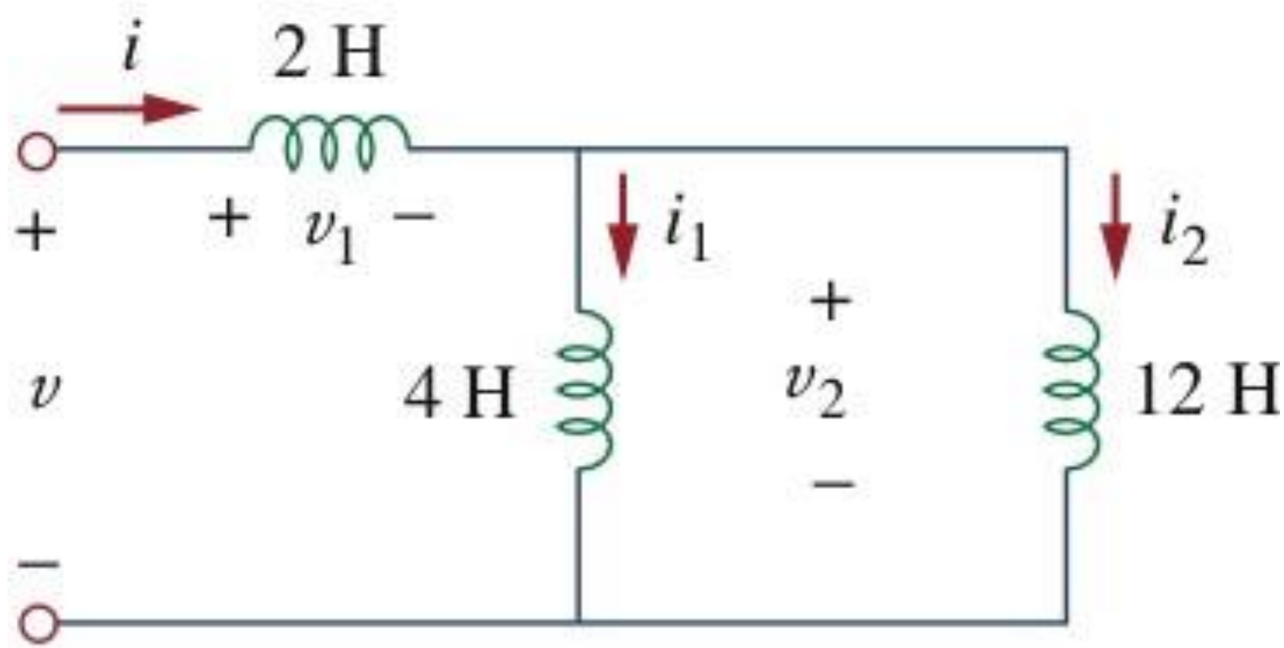
$$\rightarrow 42\text{H}$$

$$\text{Parallel}: \ \frac{7 \times 42}{7 + 42} = 6\text{H}$$

$$\therefore L_{eq} = 4 + 6 + 8 = 18\text{H}$$

Example 5

- $i(t) = 4(2 - e^{-10t})\text{mA}.$ If $i_2(0) = -1\,\text{mA},$
  - find : (a) $i_1(0)$
    (b) $v(t)$, $v_1(t)$, and $v_2(t)$;
    (c) $i_1(t)$ and $i_2(t)$



Solution:

(a) $i(t) = 4(2 - e^{-10t})\text{mA} \rightarrow i(0) = 4(2-1) = 4\text{mA}.$

$\therefore i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5\text{mA}$

(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \,\|\, 12 = 2 + 3 = 5\text{H}$$

$$\therefore v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t}\text{mV} = 200e^{-10t}\text{mV}$$

$$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t}\text{mV} = 80e^{-10t}\text{mV}$$

$$\therefore v_2(t) = v(t) - v_1(t) = 120e^{-10t}\text{mV}$$

$$\text{(c) } i = \frac{1}{L}\int_0^t v(t)\,dt + i(0) \Rightarrow$$

$$i_1(t) = \frac{1}{4}\int_0^t v_2\,dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t}\,dt + 5\,\text{mA}$$

$$= -3e^{-10t}\Big|_0^t + 5\,\text{mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t}\ \text{mA}$$

$$i_2(t) = \frac{1}{12}\int_0^t v_2\,dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t}\,dt - 1\,\text{mA}$$

$$= -e^{-10t}\Big|_0^t - 1\,\text{mA} = -e^{-10t} + 1 - 1 = -e^{-10t}\ \text{mA}$$

Note that $i_1(t) + i_2(t) = i(t)$

Example 6

- Find the equivalent inductance of the circuit.