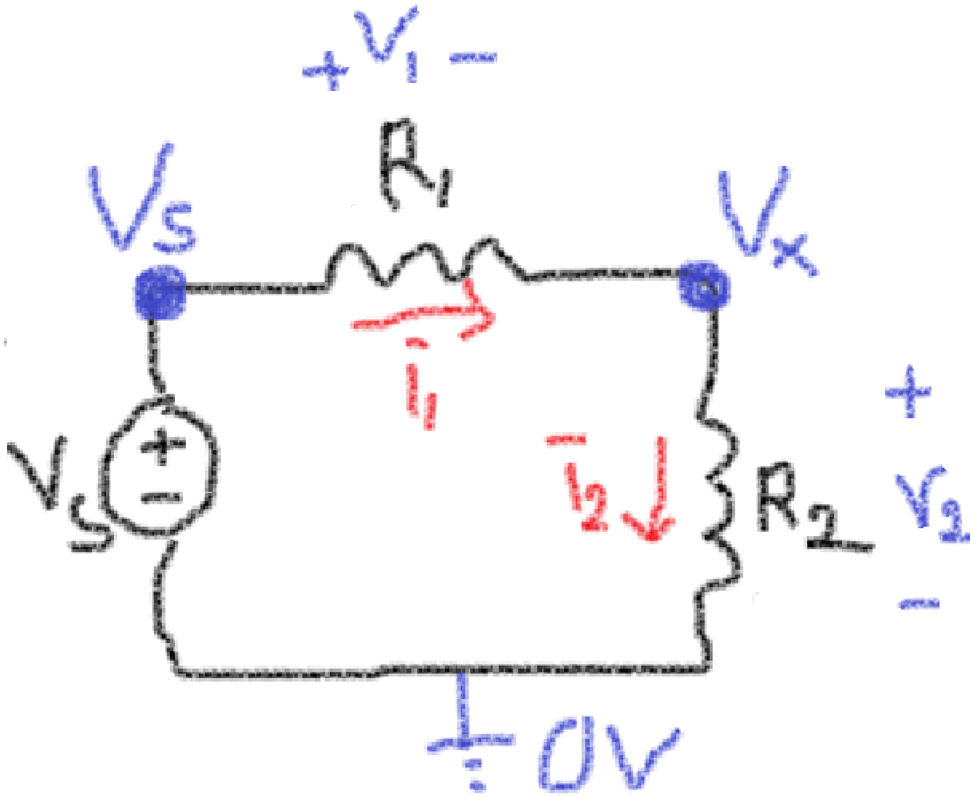
CET 141: Day 5

Dr. Noori KIM

AC *vs.* DC

- https://www.youtube.com/watch?v=BcIDRet787k (1min~)

What we did (and should have done) so far…



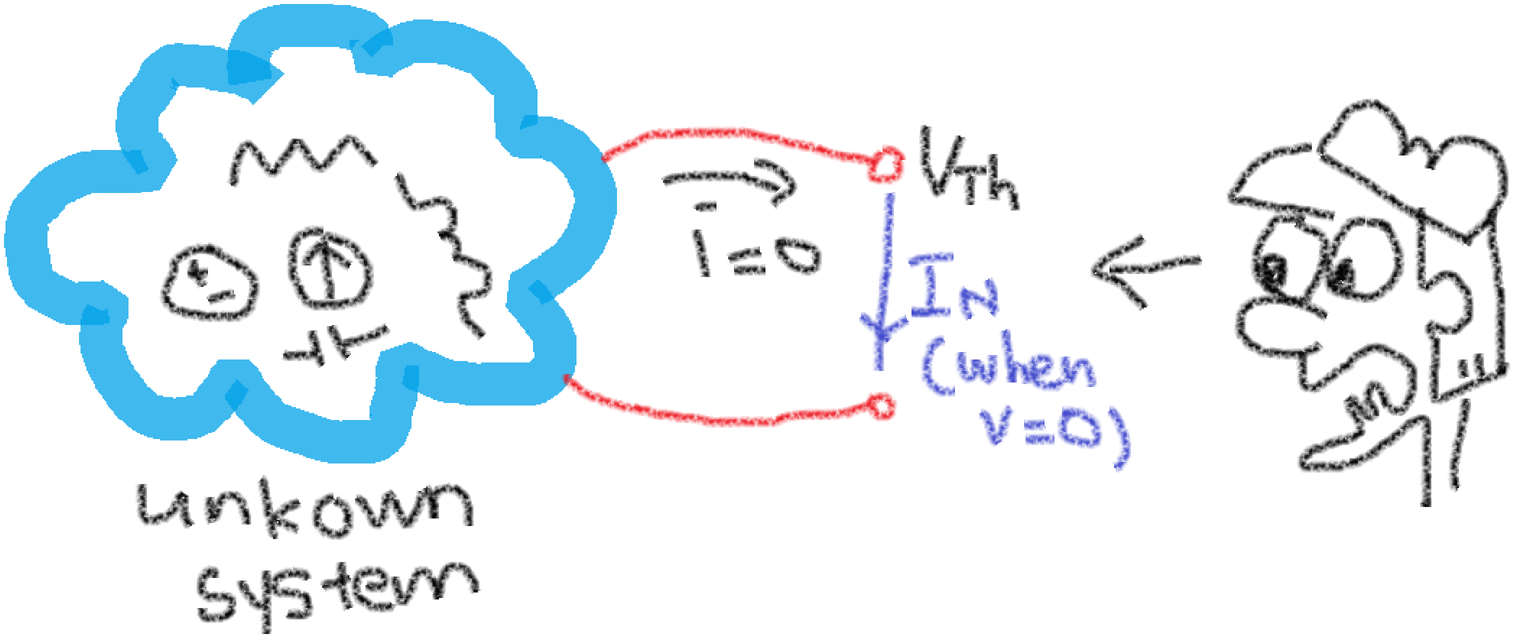
- Ohm's law
- KCL/KVL
- Nodal analysis

- Ohm's law: V1=R1*i1, V2=R2*i2
- KVL: Vs=V1+V2
- KCL: i1=i2 (=i)
- Nodal analysis: (Vs-Vx)/R1=(Vx-0)/R2

Note that v1=Vs-Vx
i=Vs/(R1+R2)=i1=V1/R1=i2=V2/R2

- Thevenin/Norton equivalents

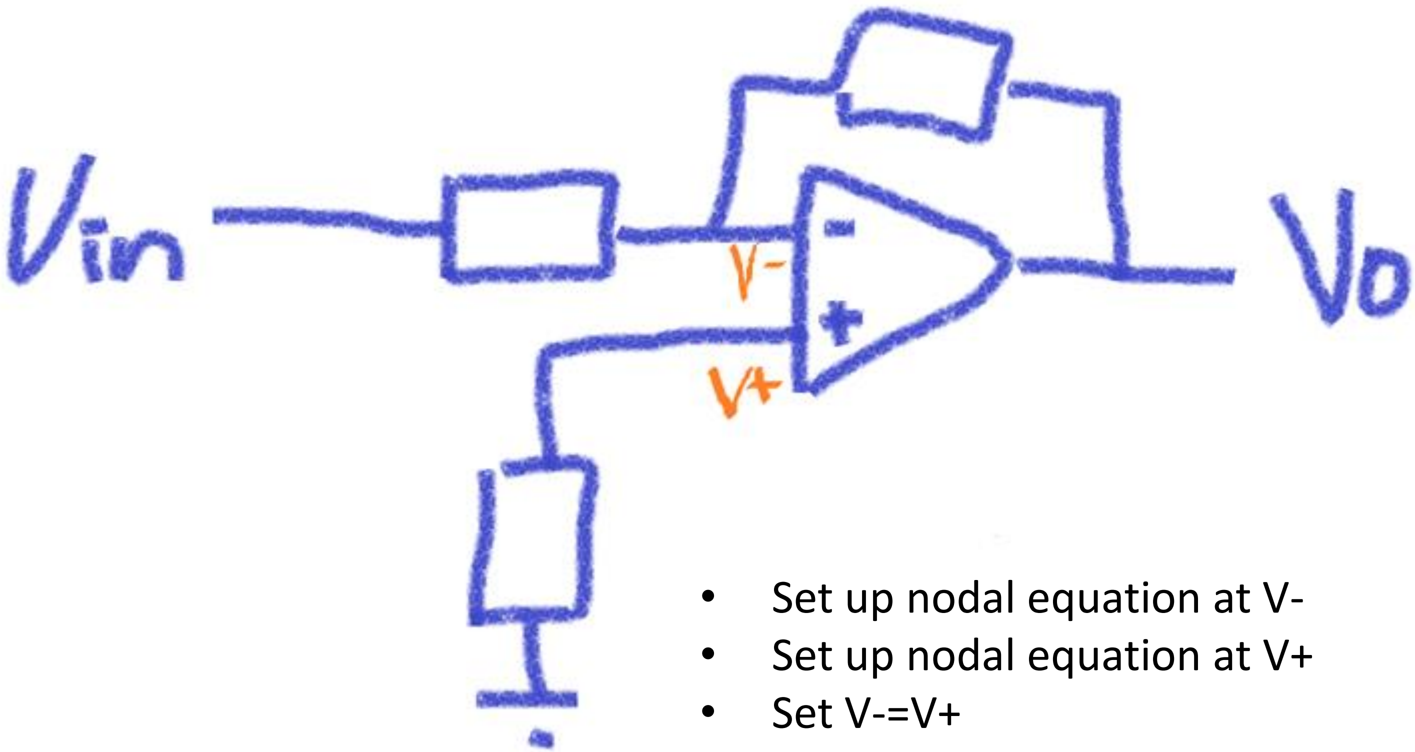
Source transformation



R_{th}

- Only independent sources: V short, I open
- With dependent sources: take a ratio (V_{th}/I_N)

- Op amp analysis



- Set up nodal equation at V-
- Set up nodal equation at V+
- Set V-=V+

- We will learn Caps and Inds components

a flow of electric charge

$$\Phi_B = N \quad B \quad A$$

$$\Phi_B = N \frac{\mu_0 N I}{\ell} A$$

$$\Phi_B = \underbrace{\frac{\mu_0 A N^2}{\ell}}_{L} I$$

Inductance and Capacitance

Dr. Noori Kim

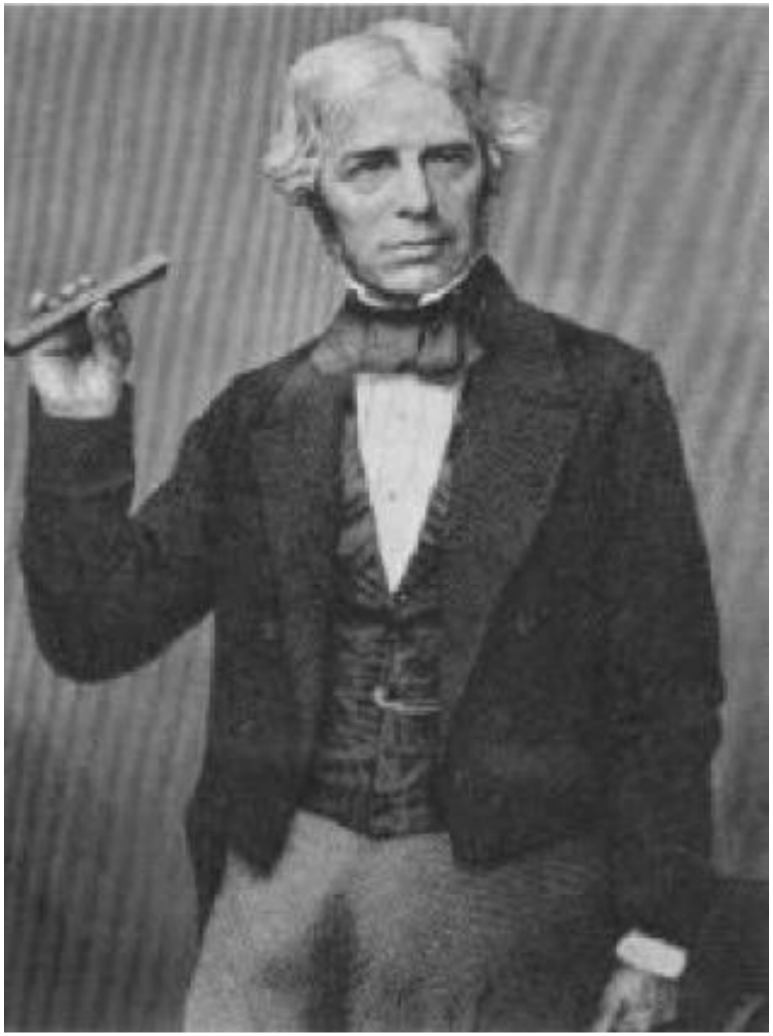
Chap. 6, Capacitors and Inductors

- Introduction
- Capacitors
- Series and Parallel Capacitors
- Inductors
- Series and Parallel Inductors

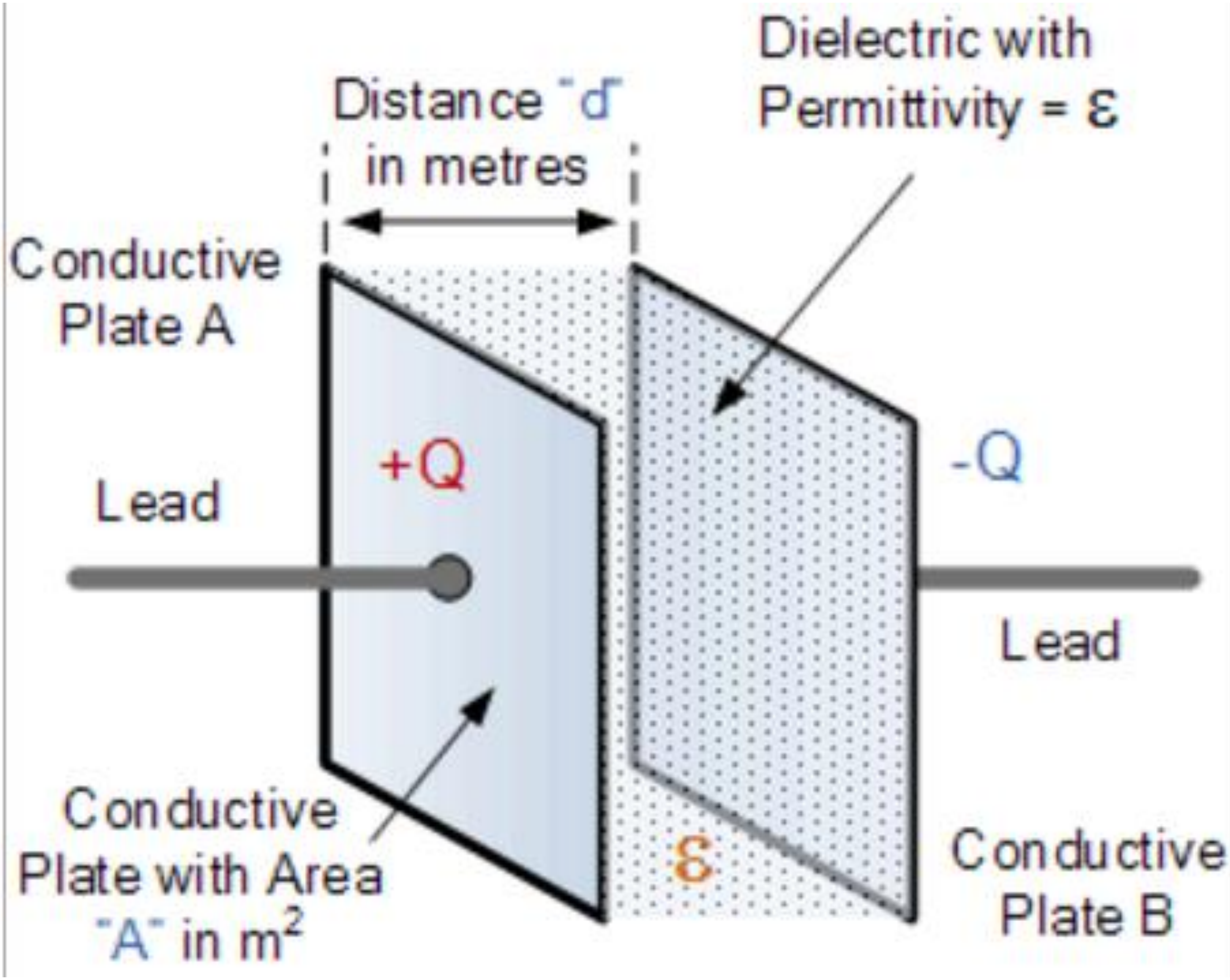
Introduction

- Resistor: a passive element which dissipates energy only
- Two other important **passive** linear circuit elements:
  1) Capacitor
  2) Inductor
- Ideal capacitors and inductors can **store energy** only and they can **neither generate nor dissipate energy**. They are not active.

Michael Faraday (1971-1867)



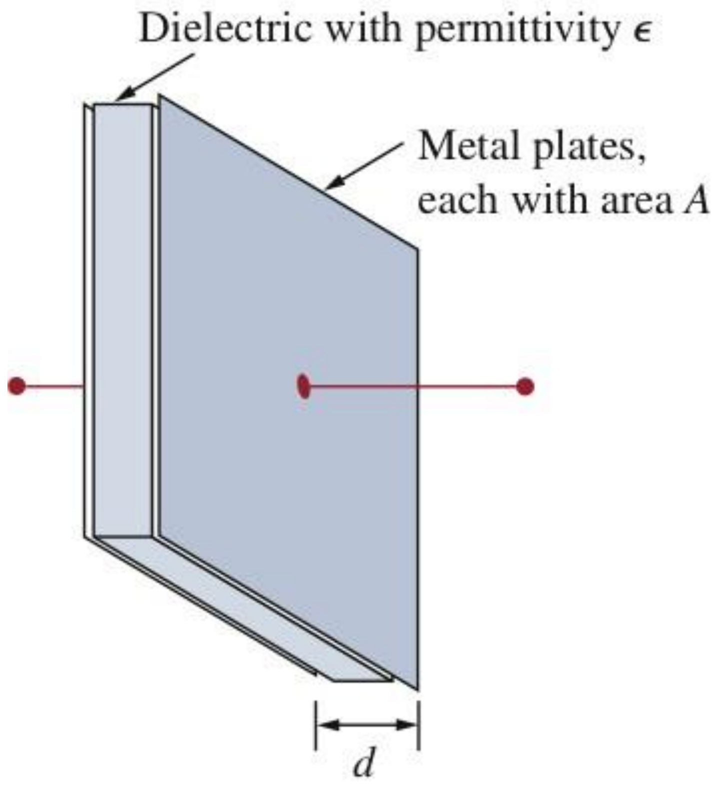
Capacitors



Dielectric with
Permittivity = ε
Distance "d"
in metres
Conductive
Plate A
+Q
-Q
Lead
Lead
Conductive
Plate with Area
"A" in m²
ε
Conductive
Plate B

Capacitors

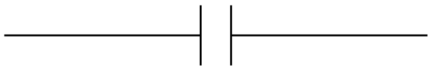
- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

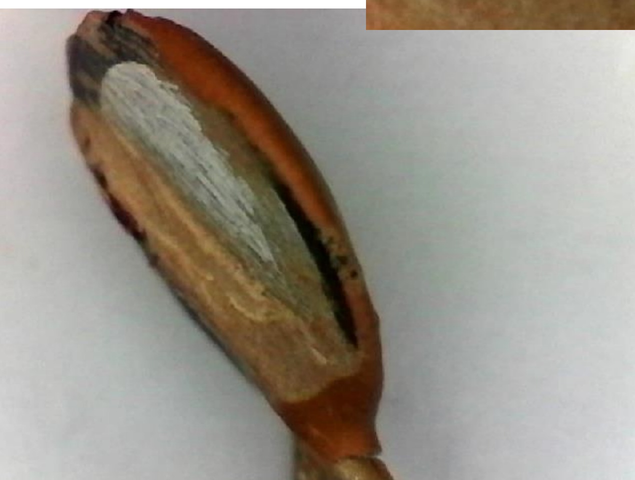


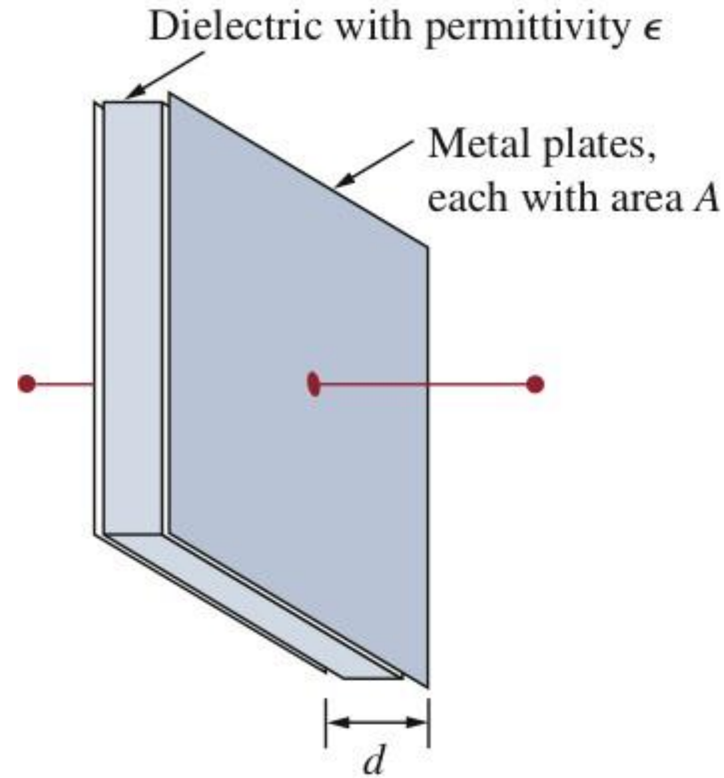
$$C = \frac{\varepsilon A}{d}$$

$$\varepsilon = \varepsilon_r \varepsilon_0$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \, (\mathrm{F/m})$$

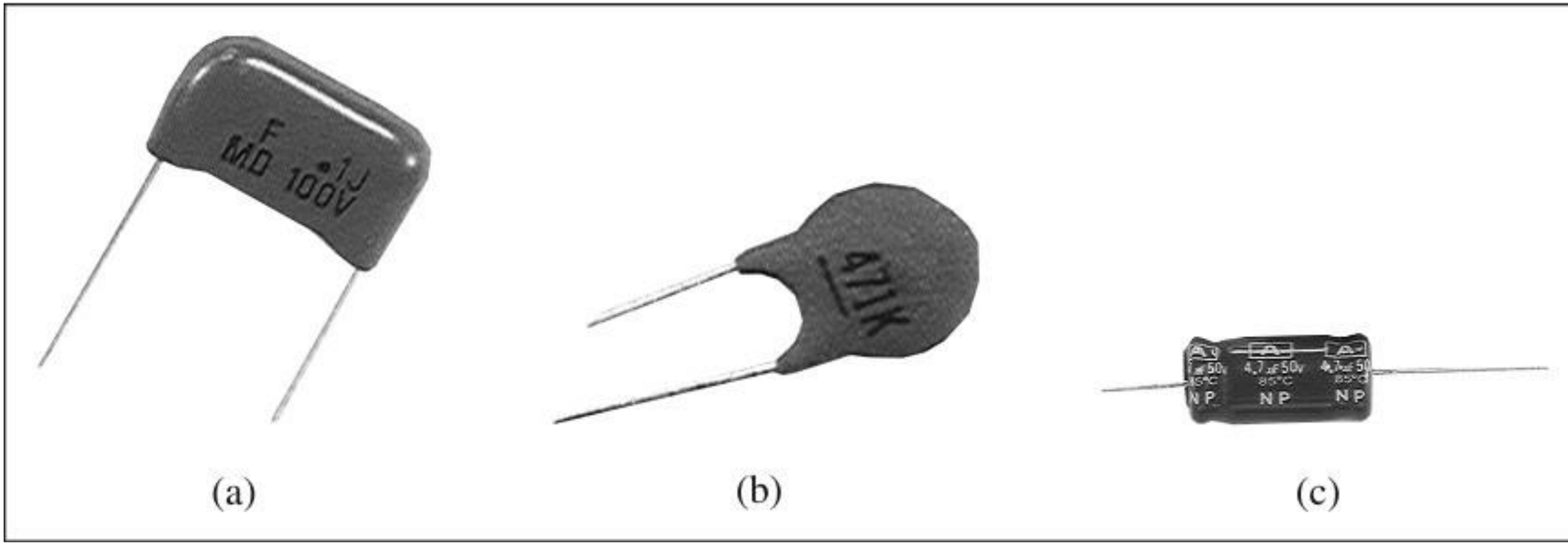





Dielectric with permittivity ϵ
Metal plates, each with area A
d

$$C = \frac{\varepsilon A}{d}$$

- Three factors affecting the value of capacitance:
  1. Area: the larger the area, the greater the capacitance.
  2. Spacing between the plates: the smaller the spacing, the greater the capacitance.
  3. Material permittivity: the higher the permittivity, the greater the capacitance.



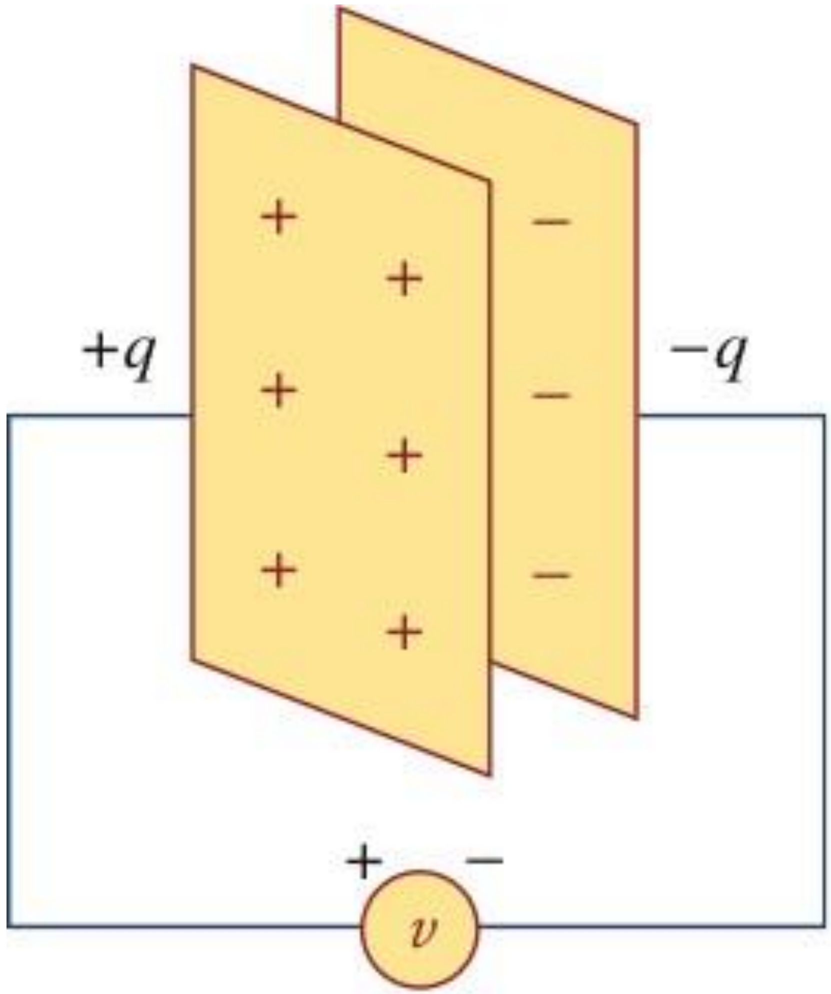
(a) Polyester capacitor, (b) Ceramic capacitor, (c) Electrolytic capacitor

(a)

(b)

Variable capacitors

Basic of Caps

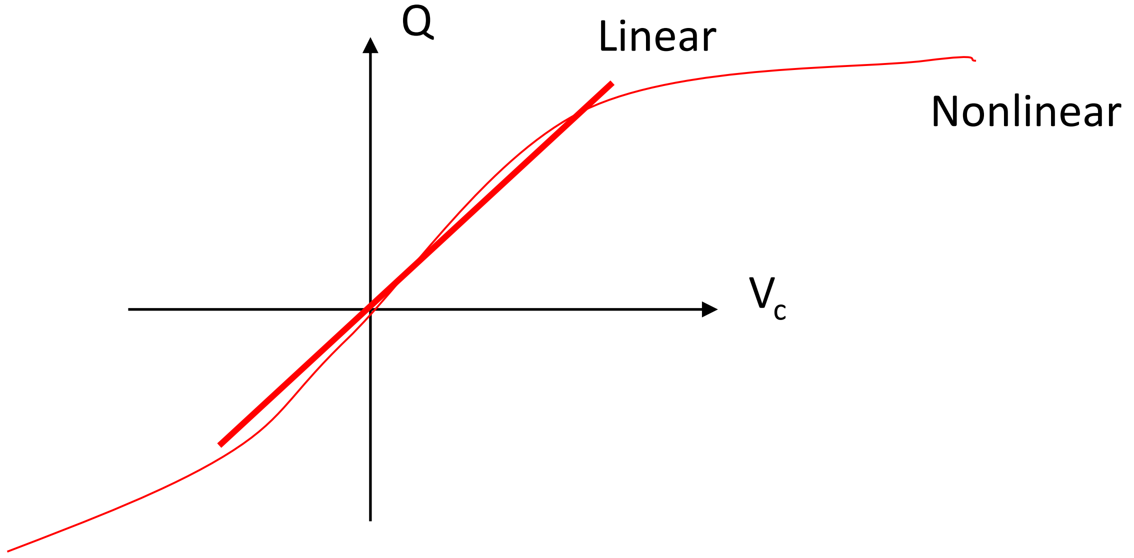


Charge in Capacitors

- The relation between the charge in plates and the voltage across a capacitor:

$$Q = CV_C$$

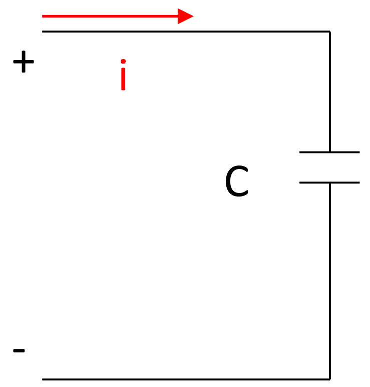
$$1\text{Farad} = 1\ \text{C/V}$$



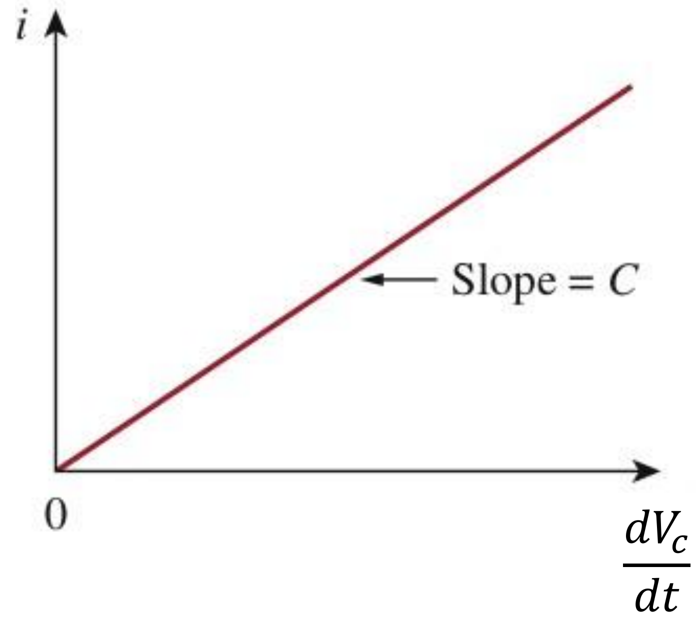
Voltage Limit on a Capacitor

- Since $Q=Cv$, the plate charge increases as the voltage increases. The electric field intensity between two plates increases.
- If the voltage across the capacitor is so large that the field intensity is large enough to break down the insulation of the dielectric, the capacitor is out of work.
- Hence, every practical capacitor has a maximum limit on its operating voltage.

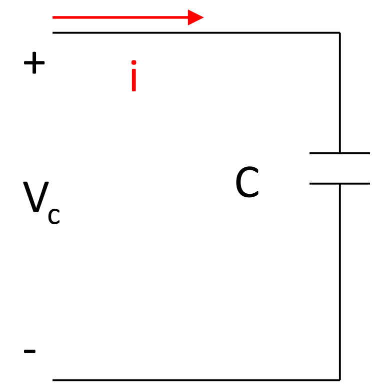
I-V Relation of Capacitor



$$Q = CV_c, i = \frac{dQ}{dt} = C\frac{dV_C}{dt}$$



Physical Meaning



$$i = \frac{dQ}{dt} = C\frac{dV_C}{dt}$$

- **V_c: a constant voltage across a capacitor**
- **When Vc** =0
  - The same potential between capacitor's two inner plates create no current through the capacitor,
  - the capacitor in this case: an **open circuit**.
- If voltage is abruptly changed
  - i: an infinite value
  - Impossible to have **an abrupt change in its voltage** except an infinite current is applied (practically impossible)

1. A capacitor is an open circuit (i=0) at DC (direct current, f=0, t=∞)

2. The voltage on a capacitor cannot change abruptly.

$$V_c(t_{0+}) = V_c(t_{0-})$$

$$i = \frac{dQ}{dt} = C\frac{dV_C}{dt}$$

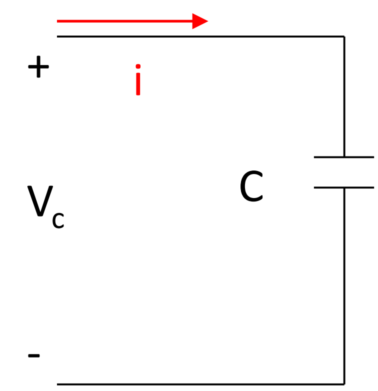
In other words, if this does not satisfy, current can't exist → as you can't differentiate V_c

A little more to think

$$i = C\frac{dV_C}{dt} \Longrightarrow V_C(t) = \frac{1}{C}\int_{t_o}^{t} i\,dt + V_C(t_o)$$

$$\Longrightarrow V_C(t) - V_C(t_o) = \frac{1}{C}\int_{t_o}^{t} i\,dt$$

$$\Longrightarrow \frac{Q(t)}{C} - \frac{Q(t_0)}{C} = \frac{1}{C}\int_{t_0}^{t} i\,dt$$



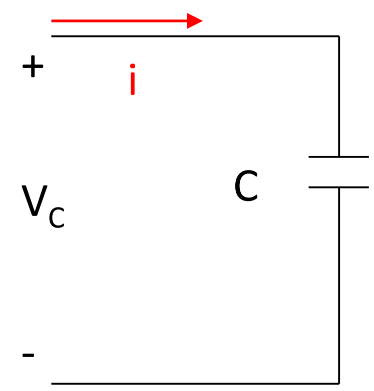
The charge on a capacitor is an integration of current through the capacitor (which makes sense)

Energy Storing in Capacitor

$$p = vi = CV_C \frac{dV_C}{dt} \qquad i = C\frac{dV_C}{dt}$$

$$w = \int_{-\infty}^{t} p\,dt = C\int_{-\infty}^{t} V_C \frac{dV_C}{dt}dt = C\int_{V_C(-\infty)}^{V_C(t)} V_C\,dV_C = \frac{1}{2}CV_C{}^2\Big|_{V_C(-\infty)}^{V_C(t)}$$

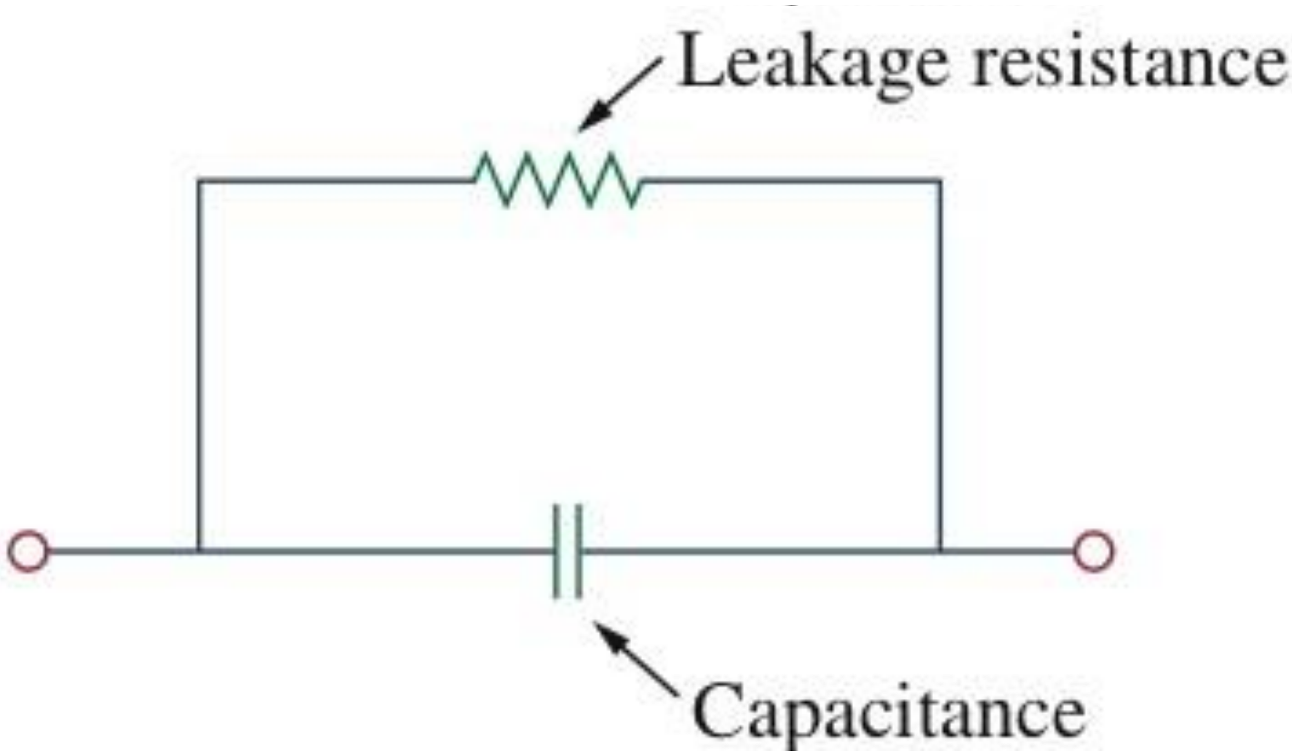
$$w(t) = \frac{1}{2}CV_C{}^2(t) \qquad (V_C(-\infty) = 0)$$



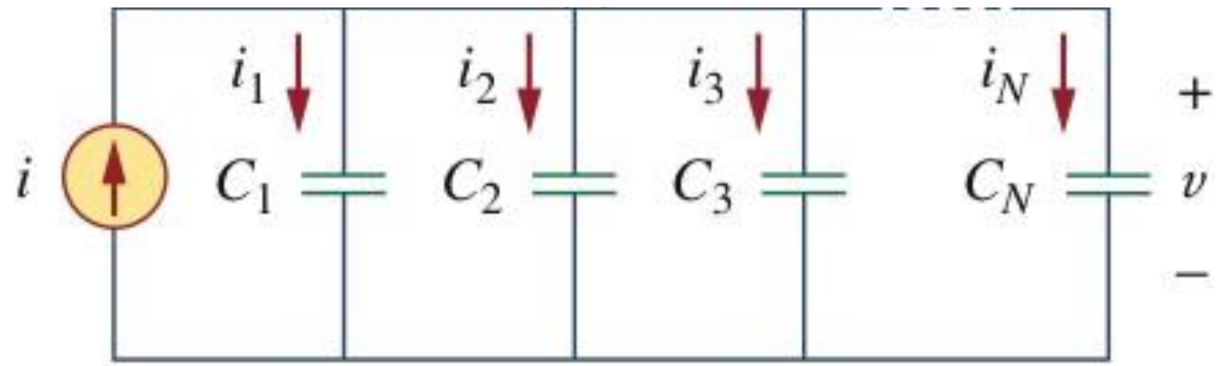
$$w(t) = \frac{Q^2(t)}{2C}$$

Eventually two plates potential will become same

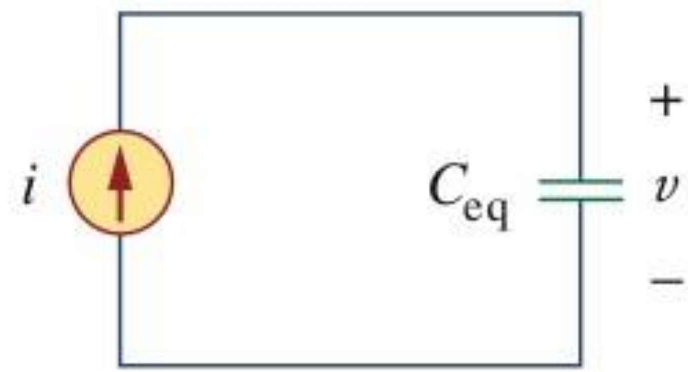
Model of Practical Capacitor



Series and Parallel Capacitors



(a)



$$C_{eq} = C_1 + C_2 + C_3 + ... + C_N$$

(b)

$$i = i_1 + i_2 + i_3 + \ldots + i_N$$

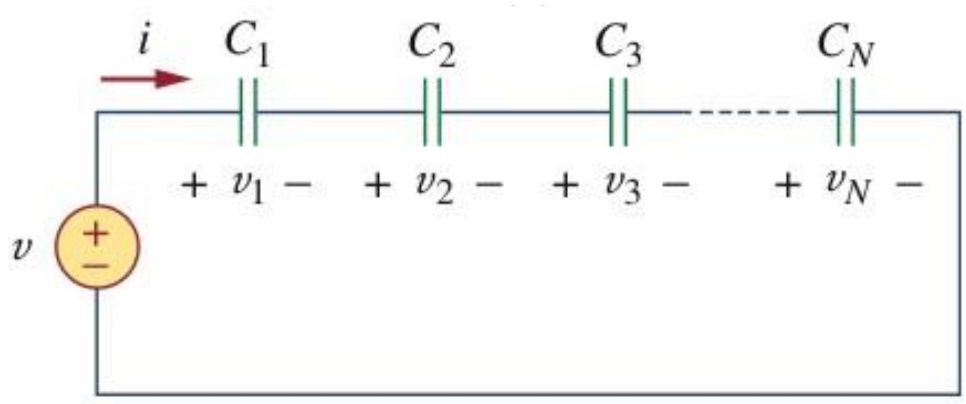
$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \ldots + C_N \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_K \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

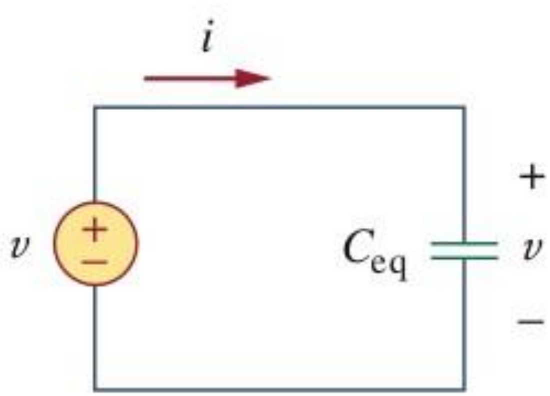
$$C_{eq} = C_1 + C_2 + C_3 + \ldots. + C_N$$

- The **equivalent capacitance** of *N* parallel-connected capacitors is the sum of the individual capacitance.

Series Capacitors



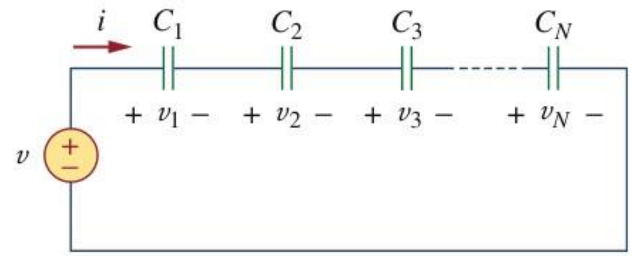
(a)



(b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_N}$$

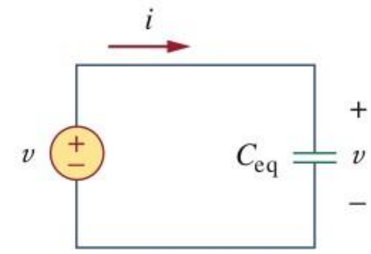
$$v(t) = v_1(t) + v_2(t) + \ldots + v_N(t)$$



(a)

$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i d\tau = (\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_N})\int_{-\infty}^{t} i d\tau$$

$$\frac{q(t)}{C_{eq}} = \frac{q(t)}{C_1} + \frac{q(t)}{C_2} + \cdots + \frac{q(t)}{C_N}$$



(b)

- The **equivalent capacitance** of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

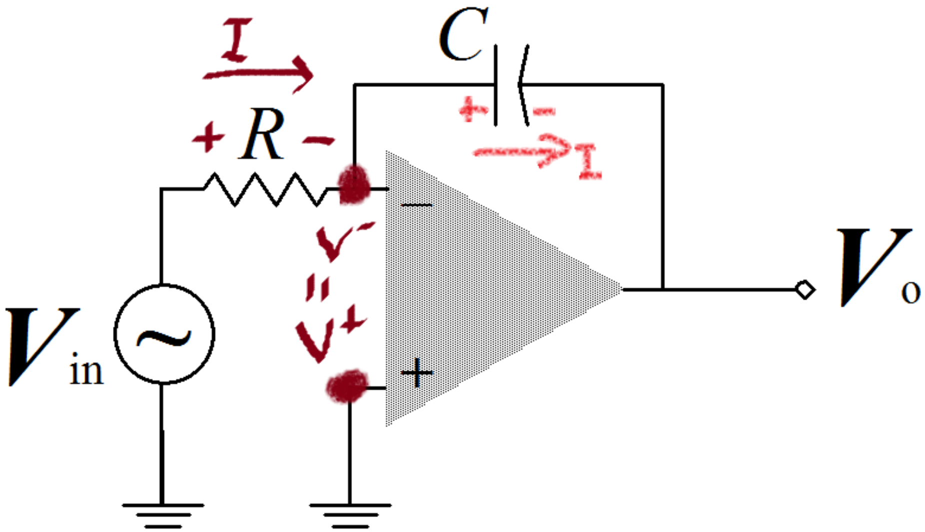
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

Summary-Capacitor

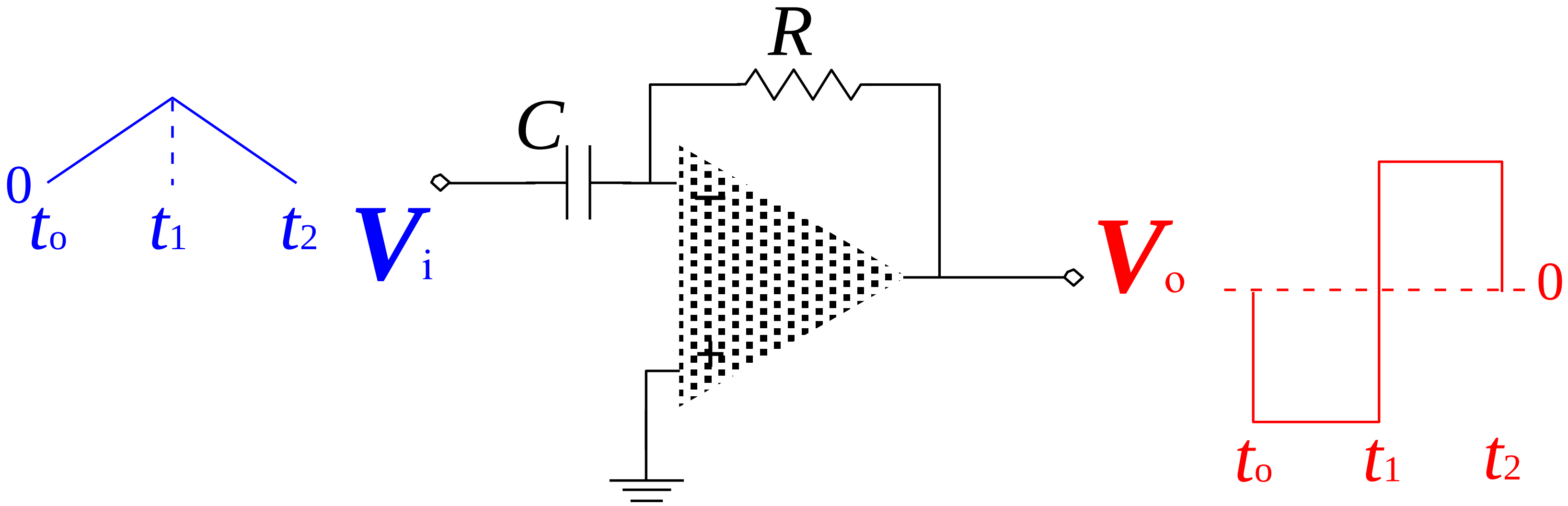
- These results enable us to look the capacitor in this way: **1/C has the equivalent effect as the eqv. resistance** calculation
- The equivalent capacitor of capacitors connected in parallel or series can be obtained via this point of view, so is the Y-△ connection and its transformation

Inverting Integrator



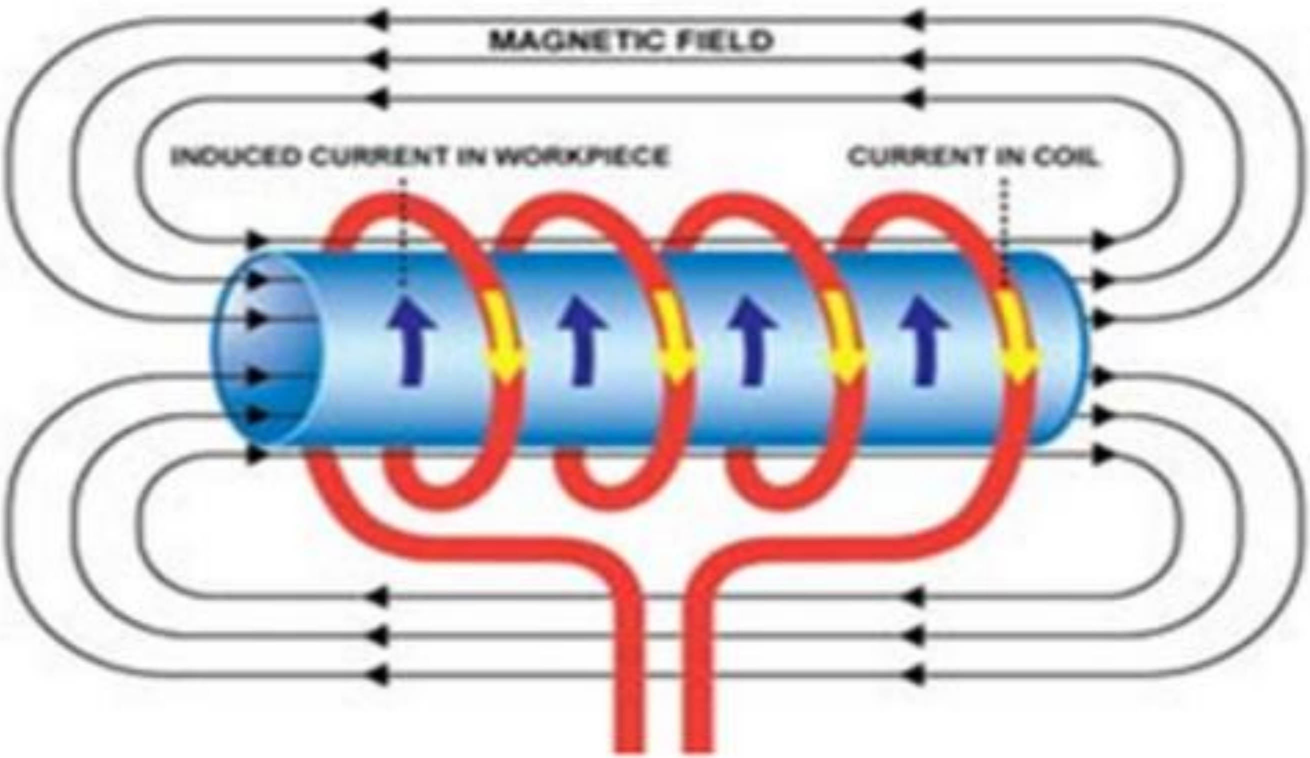
Vo in terms of Vin?

Op-Amp Differentiator



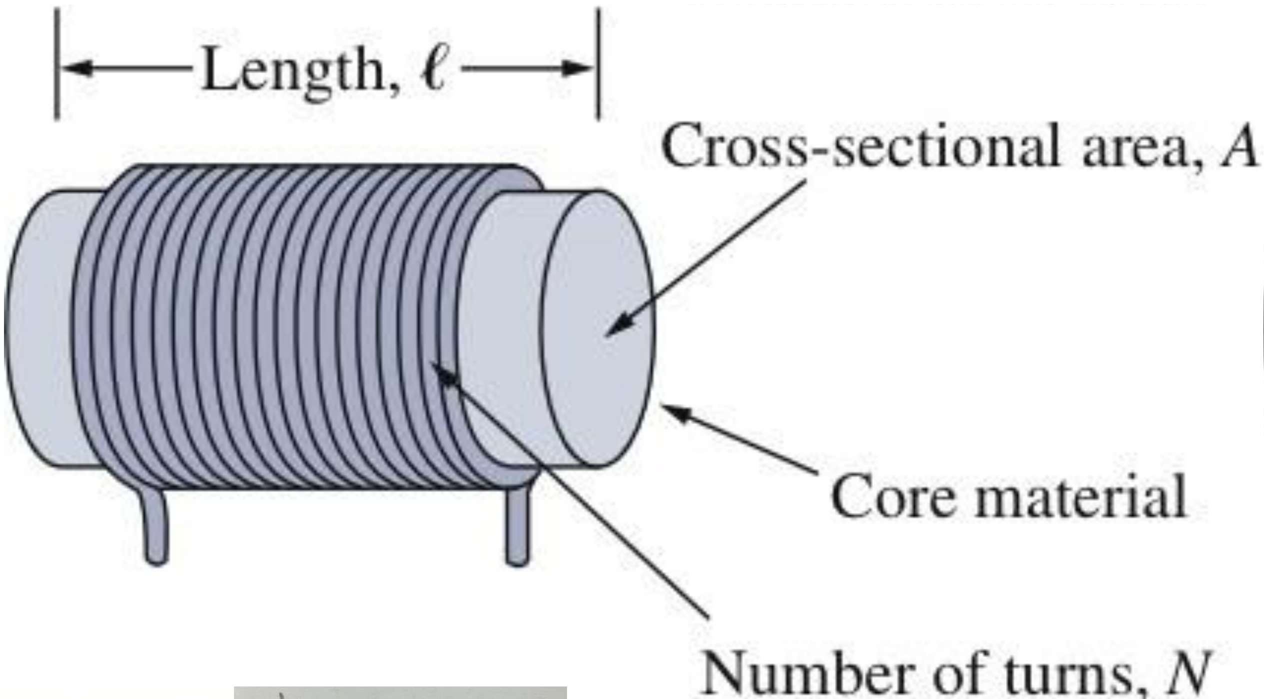
Vo in terms of Vin?

Joseph Henry (1779-1878)

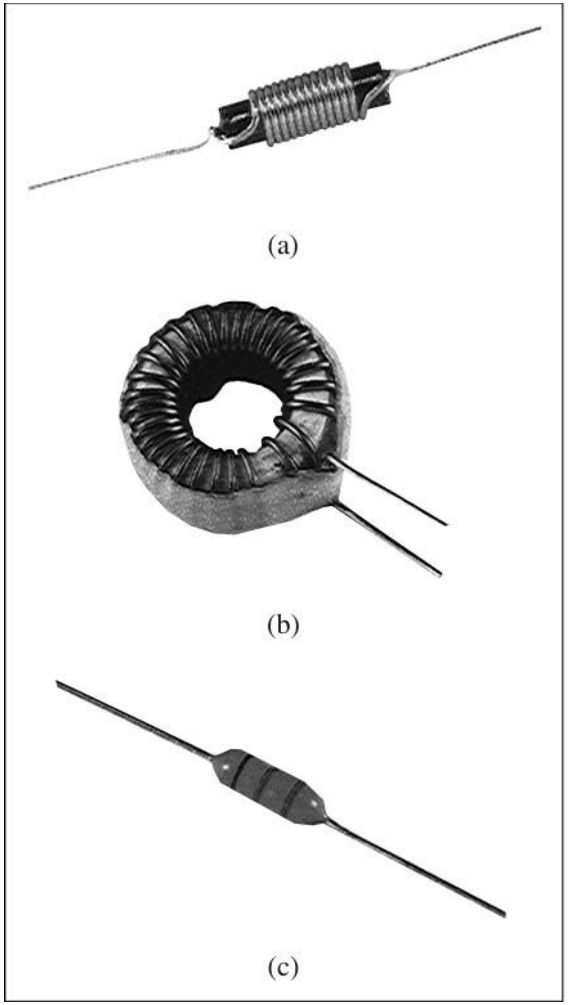

MAGNETIC FIELD
INDUCED CURRENT IN WORKPIECE
CURRENT IN COIL

Inductors

- An inductor is made of a coil of conducting wire



$$L = \frac{N^2 \mu A}{l}$$



$$L = \frac{N^2 \mu A}{l}$$

$$\mu = \mu_r \mu_0$$

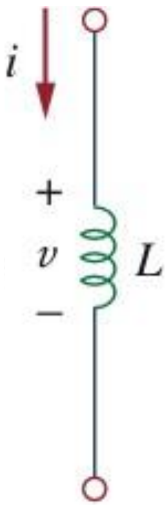
$$\mu_0 = 4\pi \times 10^{-7}\,(\mathrm{H/m})$$

N: number of turns.

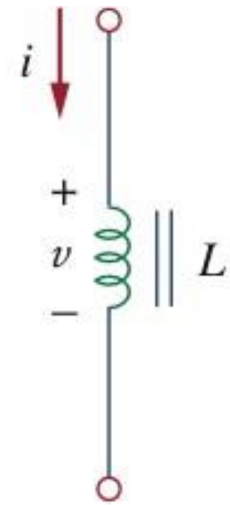
l: length.

A: cross – sectional area.

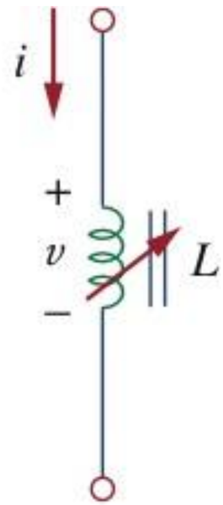
μ: permeability of the core



(a)



(b)



(c)

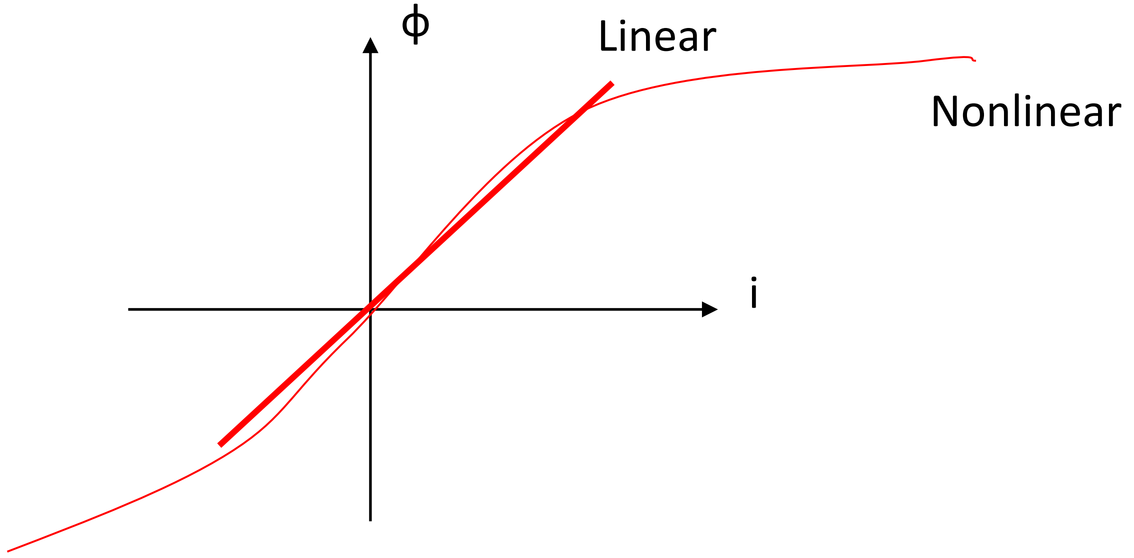
(a) air-core
(b) iron-core
(c) variable iron-core

Flux in Inductors

- The relation between the flux in inductor and the current through the inductor:

$$\phi = Li$$

$1\text{H} = 1\ \text{Weber/A}$



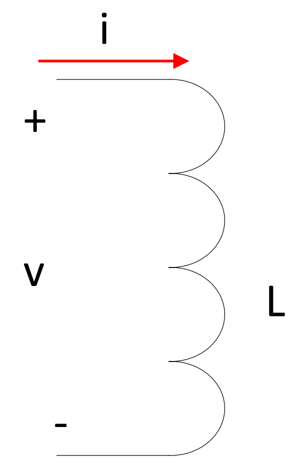
Energy Storage Forms: Cap vs. Ind

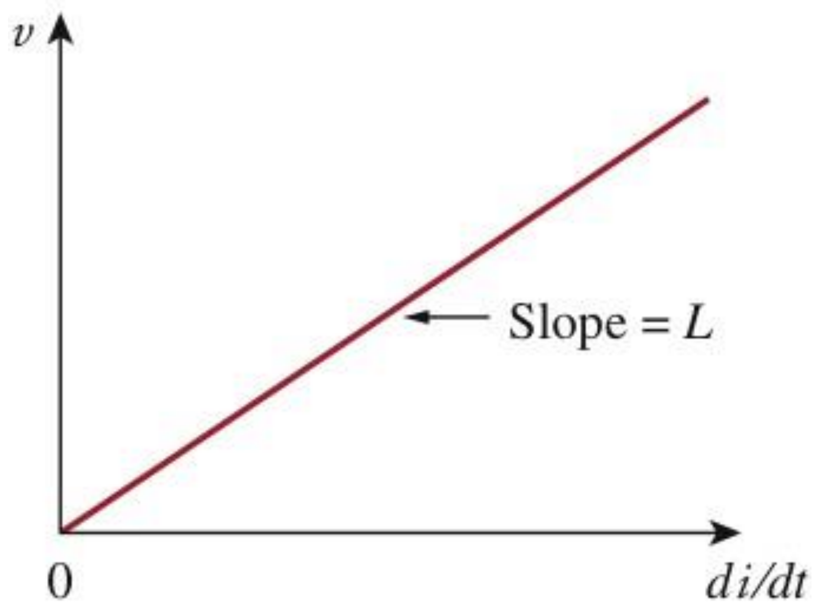
- An inductor is a passive element designed to store energy in the **magnetic** field while a capacitor stores energy in the **electric** field.

I-V Relation of Inductors

- An **inductor** consists of a coil of conducting wire.

$$v = \frac{d\phi}{dt} = L\frac{di}{dt}$$

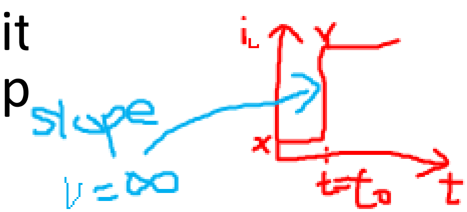




Physical Meaning

$$V = \frac{d\phi}{dt} = L\frac{di_L}{dt}$$

- **i_L : a constant current through an inductor**
- **When i_L=0**
  - There is no electron's flow through the inner inductor's spiral coil
  - The inductor in this case nothing but just a wire: **a short circuit → V=0**
- If current (i_L) is **abruptly changed**
  - V: and infinite value
  - Impossible to have an abrupt change in it n
    infinite voltage across the inductor is app
    impossible)



1. An inductor is a short circuit (V=0) at DC (direct current, f=0, t=∞)
2. The current through an inductor cannot change instantaneously.

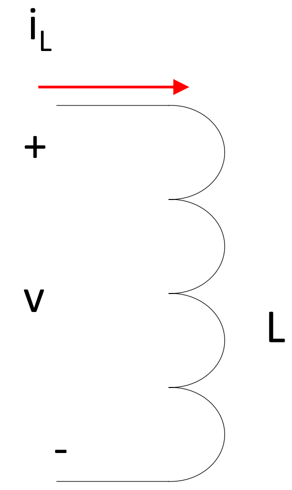
$$i_L(t_{0+}) = i_L(t_{0-})$$

$$V = \frac{d\phi}{dt} = L\frac{di_L}{dt}$$

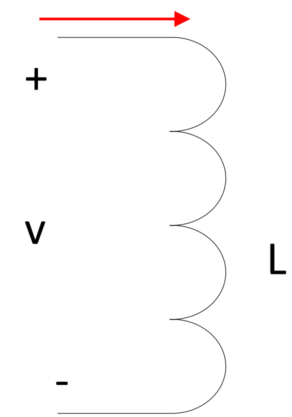
In other words, if this does not satisfy, voltage can’t exist ➔ as you can’t differentiate i_L

$$V(t) = L\frac{di_L}{dt}$$

$$\Rightarrow \quad i_L(t) = \frac{1}{L}\int_{t_o}^{t} v(t)\,dt + i_L(t_o)$$



Energy Stored in an Inductor



$$P = vi = \left(L\frac{di}{dt} \right) i$$

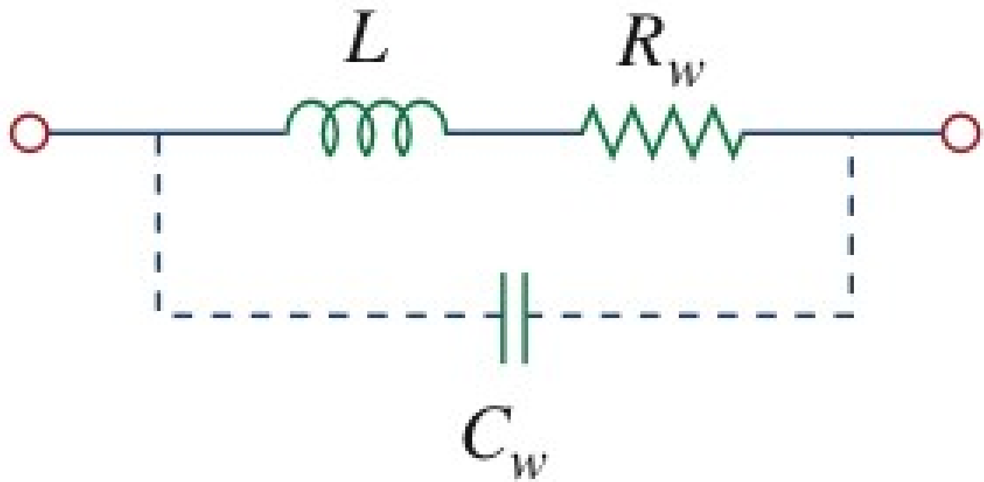
$$w = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t} \left(L\frac{di}{dt} \right) i\,dt$$

$$= L\int_{i(-\infty)}^{i(t)} i\,di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty) \quad i(-\infty) = 0,$$

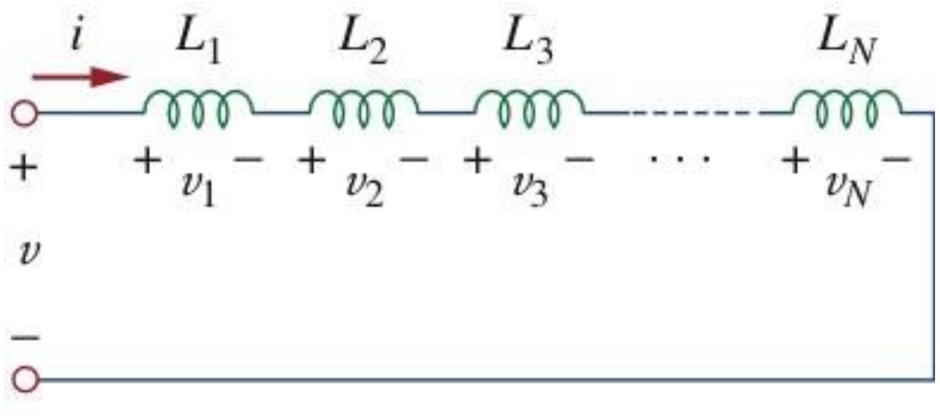
- The energy stored in an inductor

$$w(t) = \frac{1}{2}Li^2(t)$$

Model of a Practical Inductor

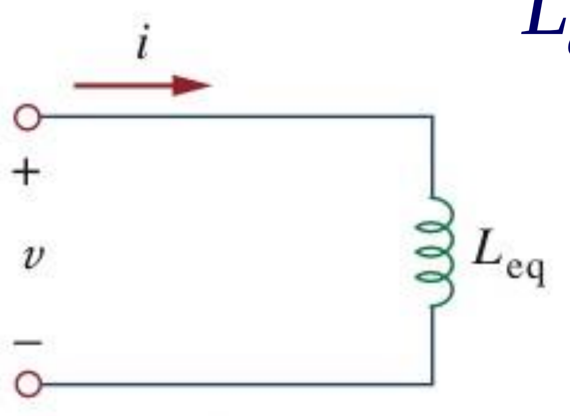


Inductors in Series



(a)

$$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N$$



(b)

- Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \ldots + v_N$$

- Substituting $v_k = L_k\, di/dt$ results in

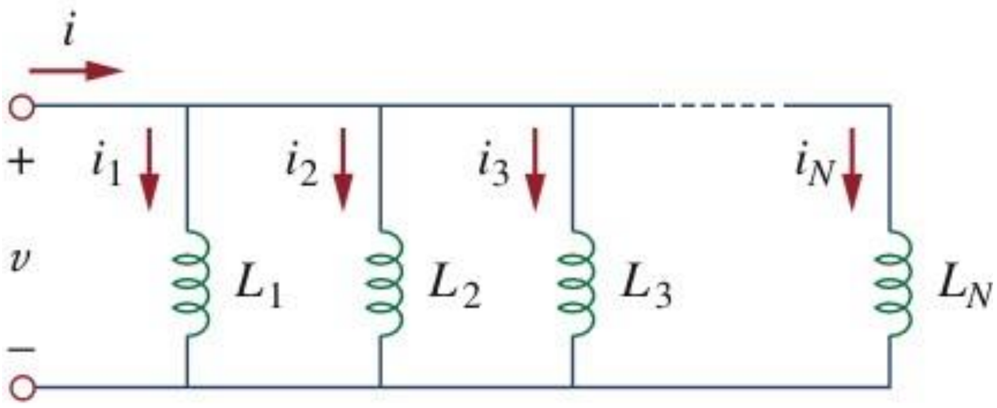
$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \ldots + L_N \frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \ldots + L_N) \frac{di}{dt}$$

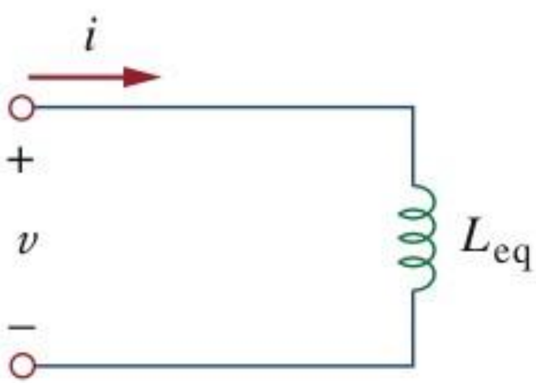
$$= \left(\sum_{K=1}^{N} L_K \right) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N$$

Inductors in Parallel



(a)



(b)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$$

- Using KCL, $i = i_1 + i_2 + i_3 + \ldots + i_N$
- But $i_k = \frac{1}{L_k}\int_{to}^{t} vdt + i_k(t_0)$

$$\therefore\ i = \frac{1}{L_k}\int_{t_0}^{t} vdt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} vdt + i_s(t_0)\ + \ldots + \frac{1}{L_N}\int_{t_0}^{t} vdt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}\right)\int_{t_0}^{t} vdt + i_1(t_0) + i_2(t_0) + \ldots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} vdt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} vdt + i(t_0)$$

- The inductor in various connection has the same effect as the resistor. Hence, the Y-Δ transformation of inductors can be similarly derived.

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} V\,dt + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i