# Game-Theoretic Optimisation of EV Charging Network: Placement and Pricing Strategies via Atomic Congestion Game

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**Abstract:** The "chicken-and-egg problem" in Electric Vehicle (EV) charging reflects the interdependence between sufficient infrastructure and the demand needed to justify it, a challenge heightened by the UK's 2030 ban on new combustion engine vehicles. To address this, we propose a joint optimisation model that determines the optimal number of charging points and pricing at each station, while accounting for traffic patterns. From a policy perspective, our model seeks to maximise public benefit by reducing EV users' social costs, travel and queuing time, and charging fees, while ensuring station operator profitability. We model driver decisions as two interconnected congestion games, one on roads and one at charging stations (CS), and solve for stable outcomes using Nash Equilibrium (NE) strategies. To ensure tractability, we develop an efficient approximation algorithm for the Mixed-Integer Nonlinear Program (MINLP) and introduce a generalisation technique that targets charger placement at high-impact locations, enhancing scalability to larger Transportation Networks (TN). Applied to a benchmark case, the model reduces overall social cost by at least 14% compared to methods that optimise placement or pricing alone. This study tackles an AI challenge in modelling infrastructure with multi-agent behaviour, using game theory and optimisation to simulate interactions and enable learning-based approaches in transportation systems.

**Keywords:** Electric Vehicle Charging Network, Pricing, Placement, Game Theory, Joint Optimisation

## 1. Introduction

### 1.1. Motivation

Switching to EVs is essential for improving air quality, reducing emissions, enhancing energy security, and supporting sustainable transportation. EVs significantly reduce reliance on fossil fuels and carbon emissions, making them crucial in combating climate change [1]. To support this shift, the UK government aims to phase out new petrol and diesel cars by 2030 as part of its sustainability strategy [2].

As EV development advances, a reliable charging infrastructure is crucial for widespread adoption [3]. A recent survey underscores the urgent need for collaboration among all stakeholders, including industry and government, to enhance the UK's charging infrastructure and expand capacity in line with the rapid growth of EV adoption [4]. In response, the UK plans to install 6,000 rapid charging points by 2035, emphasising the importance of strategic planning [5]. Effective charger placement and pricing play a key role in ensuring a dependable public network while maintaining profitability for operators. However, these factors are often considered separately, missing potential synergies. This study introduces a joint optimisation model for EV charging station placement and pricing, assisting governments in network planning and operators in setting efficient, profitable pricing schemes. Key to this is understanding how both EV and non-EV traffic respond to infrastructure changes. The placement and pricing create a competitive environment where drivers engage in two intertwined congestion games: one for road congestion and another for CS congestion involving only EV drivers. In this paper, we first analyse intertwined congestion games and then present a joint optimisation model for the placement and pricing of CSs. To improve scalability, we propose a technique that incrementally expands candidate nodes for charger placement, optimising a smaller subset first to reduce complexity while remaining efficient in large networks.

While our framework is grounded in optimisation, its design reflects broader AI challenges related to agent-based modelling and strategic decision-making in multi-agent systems. By incorporating a coupled congestion game formulation, we capture the interplay between autonomous agents (drivers) and infrastructure in a way that mirrors real-world AI planning problems. This makes our approach well-suited to serve as a benchmark or hybrid layer within machine learning and reinforcement learning pipelines, particularly in the context of intelligent mobility, decentralised resource planning or adaptive pricing strategies.

### 1.2. Literature Review

EV CS placement and pricing have been widely studied, though often in isolation. Placement studies, such as Metais et al. [6], explore objectives like minimising queuing times [7, 8], maximising traffic flow [9], or reducing travel time, waiting time, and power loss [10, 11]. In contrast, our approach integrates travel and queuing time, and charging costs into a single objective, offering a realistic, comprehensive model of driver behaviour. Unlike Shafipour et al. [12], who focus on recommending existing stations to drivers, we support policymakers in deciding charger locations, numbers, and pricing.

Most pricing optimisation studies focus on maximising profit or minimising costs for station owners, often using reinforcement learning for dynamic pricing [13, 14]. Zhao et al. [15] aim to reduce waiting times and improve customer satisfaction, while Fescioglu and Aktas [16] target enhancing EV user utility. However, many studies neglect the interaction between pricing strategies and station placement, missing potential synergies. Few consider both aspects together. For example, Adil et al. [17] explore a Stackelberg model based on energy predictions, while our approach integrates the problem into a transportation network, better in capturing driver behaviour and addressing gaps in the literature [14, 18, 19]. Similarly, Wang et al. [19] use a sequential approach, first determining station placement and then applying reinforcement learning for pricing. However, it assumes drivers maximise utility without considering others' decisions, which we address by modelling the interaction between drivers as two intertwined congestion games and studying the NE to capture competitive dynamics. Congestion games analyse competition for limited resources, with costs depending on usage [20]. They are widely utilised in traffic problems like [21–23]. In EV CS context, Sonmez et al. [18] examine road congestion, while Xiong et al. [10] optimise both road and charging station usage for placement, and Xiong et al. [24] applies congestion games to charging pricing. Our work builds on these by integrating congestion games for both road and CS usage into a joint optimisation framework for placement and pricing, an approach not previously explored.

While most existing work on EV infrastructure stems from operations research or transport economics, there is a growing intersection with AI, especially in multi-agent systems, reinforcement learning, and computational game theory. Our joint optimisation approach contributes to this emerging area by providing a scalable, transparent framework for reasoning about strategic interactions in shared infrastructure, relevant to AI researchers tackling real-world, high-stakes planning problems.

## 2. Model Description

This section begins with the formulation of the TN, followed by a description of the participating agents and their respective features. It then introduces the behavioural factors influencing driver decisions, which are integrated into an optimisation problem aimed at minimising social cost from the Charging Station Owner (CSO) perspective. The optimisation model incorporates all relevant constraints to ensure feasibility and realism. The section concludes by presenting the game-theoretic structure underlying the model.

## 2.1. Charging Network Architecture

**Network:** The TN is represented as a directed graph $G = [N, A]$, where $N$ is the set of nodes (key urban locations) and $A$ is the set of directed links. Each link $l \in A$ has a distance $d_l$ and capacity $c_l$, which affects congestion. Vehicles travel from an origin $O$ to a destination $D$, selecting a CS if needed. For each O-D pair $\omega$, the set of feasible routes is $R_\omega$ with routes $r \in R_\omega$; We later define *extended path* represented by $P_\omega$ for each O-D pair $\omega$ as a set of routes that includes a selected charging node, with extended paths $p \in P_\omega$. Our goal is to optimise CS placement and pricing to enhance network efficiency.

**Agents:** Three types of agents interact within this network:

1. **Charging Station Owner (CSO):** The CSO decides CS placement and pricing, where $x_i$ denotes the number of CS at node $i$ and $y_i \geq 0$ represents the charging price. These decisions depend on location-based costs, including electricity prices ($e_i$) and maintenance/rental fees ($T_i$). CSO operates within a budget $B$, funded by the government. This study assumes a single CSO, with future work considering multiple CSOs.
2. **Non-Charging Drivers (NCD):** Also known as background traffic, these agents—non-EV drivers or EV drivers not requiring charging—choose routes to minimise travel time and congestion. The number of NCDs for O-D pair $\omega$ is $\gamma^0_\omega$, where superscript 0 denotes NCD-specific parameters. Their strategy is the mixed strategy $\mathbf{q}^0_\omega = (q^0_{\omega,r})_{\forall r}$, with $q^0_{\omega,r}$ as the probability of selecting route $r \in R_\omega$.
3. **EV Drivers with En-route Charging:** Travelling between O-D pairs $\omega$, these agents select a path and charging station (i.e., choose an extended path) based on factors like travel time, queuing time, and charging fees. For simplicity, we refer to these agents as EV drivers and the number of such drivers for O-D pair $\omega$ is denoted by $\gamma_\omega$. The strategy of EV drivers for O-D pair $\omega$ is represented by the mixed strategy $\mathbf{q}_\omega = (q_{\omega,p})_{\forall p}$, where $q_{\omega,p}$ is the probability of choosing an extended path $p \in P_\omega$.

**Extended paths:** To capture EV charging events and chosen locations, we introduce *star nodes*, $i^*$, as virtual nodes, each linked exclusively to its real counterpart $i$. These nodes track charging decisions and extend paths to include charging stations. Bidirectional links connect star nodes to their originals, assuming negligible link distance. Figure 1 illustrates a sample network with virtual nodes. For example, if a user travelling from node 1 to 2 ($\omega = (1, 2)$) selects route $(1, 4, 2)$ and charges at node 4, the path extends to $(1, 4, 4^*, 4, 2)$. The set of feasible extended paths for $\omega = (1, 2)$ consists: $(1, 1^*, 1, 4, 2), (1, 4, 4^*, 4, 2), (1, 4, 2, 2^*, 2), (1, 1^*, 1, 3, 2), (1, 3, 3^*, 3, 2), (1, 3, 2, 2^*, 2)$.

We define $s(p)$ to identify the charging node in an extended path $p$, enabling precise tracking and analysis of charging behaviour in the TN.

$$s(p) = i^*, \quad \forall p \in P_\omega. \tag{2.1}$$

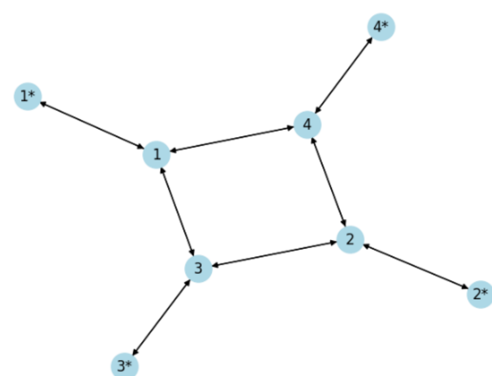


**Figure 1.** Sample network with virtual (star) nodes representing EV charging events

## 2.2. Factors Influencing Driver Behaviour

As discussed in Section 2.1, EV users consider travel time, CS congestion, and charging cost when choosing an extended path. Longer distances, higher traffic, longer queues, and higher costs make routes less appealing. EV users assign a cost to each factor and aim to minimise a weighted sum of these costs, with weights reflecting the importance of each factor. Thus, travel cost, queuing cost, and financial cost are key in modelling EV drivers' decisions. NCDs, however, are influenced only by travel costs, as they are unaffected by charging fees or queuing times. The mathematical model of agent behaviour is discussed in the following subsections.

**Expected Travel Cost:** A key factor influencing the choice of a route is the travel cost, which represents the time a driver spends reaching their destination. Travel time depends on both the distance and congestion level of the links along the selected route. Congestion on a link is determined by two factors: the number of vehicles and the link's capacity ($c_l$). The congestion level on link $l$ can be expressed as $(\gamma^0_l + \gamma_l)/c_l$, where

$$\gamma^0_l = \sum_{\omega, r: r \in R_\omega, l \in r} \gamma^0_\omega q^0_{\omega,r}, \quad \text{and} \quad \gamma_l = \sum_{\omega, p: p \in P_\omega, l \in p} \gamma_\omega q_{\omega,p}, \tag{2.2}$$

represent the total number of NCD and EV drivers using link $l$, respectively [25]. Congestion is inversely related to link capacity; higher capacities lead to lower congestion. The travel cost for link $l$ is defined as $f_l = \frac{d_l(\gamma_l^0+\gamma_l)}{c_l}$. The total travel cost for a driver (EV or non-EV) is the sum of travel costs across all links in their chosen route or extended path from origin to destination. For an EV driver and an NCD with O-D pair $\omega$, the total travel costs along an extended path $p$ and route $r$ are respectively given by:

$$F_{\omega,p} = \sum_{l\in p} f_l, \quad F_{\omega,r} = \sum_{l\in r} f_l. \tag{2.3}$$

**Expected Queuing Cost**: This cost is related to the queuing time that drivers experience at CS and depends on several factors: the total number of EVs queuing to charge at a station $s(p)$, denoted by $\sum_{\omega,p':s(p')=s(p)} \gamma_\omega q_{\omega,p'}$ (for simplicity, we denote it by $\gamma_{s(p)}$), the service capacity of a single charger per hour, denoted by $\mu$, and the number of charging points available at $s(p)$, represented by $x_{s(p)}$. The queuing cost at the station $s(p)$ for each EV driver would be:

$$G_p = \frac{\gamma_{s(p)}}{\mu x_{s(p)}}. \tag{2.4}$$

**Financial Cost**: EV drivers are charged based on electricity consumed at each CS, with fees constituting the sole income for the CSO. For simplicity, all vehicles are assumed to have identical energy demands and charging times. Future work may consider heterogeneous charging needs. The financial cost for an EV driver choosing path $p$ and charging at station $s(p)$ is:

$$Y_p = y_{s(p)}, \tag{2.5}$$

where $y_{s(p)}$ is the service price at station $s(p)$, covering electricity fees ($e_{s(p)}$), maintenance and rental costs ($T_{s(p)}$), and ensuring a marginal profit $\pi > 1$ (e.g., $\pi = 1.2$ ensures 20% profit) for the CSO, resulting in prices that vary by location. Additional insights into pricing are discussed in Section 2.3.

**Drivers' Cost Functions:** In this section, we consolidate the discussion from Section 2.2 to derive the expected cost function that drivers aim to minimise.

**EV user:** Let $\lambda_1$, $\lambda_2$, and $\lambda_3$ represent the importance weights for travel, queuing, and financial costs, respectively. The expected cost for an EV driver with O-D pair $\omega$ can be expressed as:

$$C_\omega(\mathbf{Q}) = \sum_{p\in P_\omega} q_{\omega,p}[\lambda_1 F_{\omega,p} + \lambda_2 G_p + \lambda_3 Y_p], \tag{2.6}$$

**NCD:** This agent cares only about the travel cost; hence, the expected cost of an NCD with O-D $\omega$ will be

$$C_\omega^0(\mathbf{Q}) = \sum_{r\in R_\omega} q_{\omega,r}^0 F_{\omega,r}. \tag{2.7}$$

where $\mathbf{Q} = (\mathbf{q}_\omega, \mathbf{q}_\omega^0)_{\forall\omega}$ is the vector of mixed strategies for all EV drivers and NCDs across O-D pairs $\omega$.

## 2.3. Charging Station Owner's Objective

As discussed in 2.2, drivers act strategically, focusing solely on minimising their individual costs. In contrast, the CSO, viewed from the perspective of government authority, aims to minimise the overall social cost $C(\mathbf{Q}) = \sum_\omega(\gamma_\omega C_\omega(\mathbf{Q}) + \gamma_\omega^0 C_\omega^0(\mathbf{Q}))$, subject to a set of constraints as follows:

$$\begin{aligned}
&\min_{x,y,\mathbf{q}} C(\mathbf{Q}) && \textbf{CSO Problem}\\
s.t: &\sum_{i\in N} x_i \leq B, &&\\
&\pi[\sum_{\omega,p:s(p)=i}(\gamma_\omega q_{\omega,p} e_i) + x_i T_i] \leq \sum_{\omega,p:s(p)=i}(\gamma_\omega q_{\omega,p} y_i) && \forall i,\\
&\text{Feasibility Constraints, and} &&\\
&\text{Driver Behaviour Compliance Constraints.} &&
\end{aligned} \tag{2.8}$$

In this problem, the first constraint limits the total number of CSs based on budget, while the second ensures profitability by requiring revenue from charging fees $y_i$ to cover costs $e_i$ and $T_i$, with a profit margin $\pi$. Feasibility constraints define variable conditions: $x_i$ (integer CS count), $y_i$ (non-negative prices), and probabilities $q_{\omega,p}, q^0_{\omega,r}$ (between 0 and 1, summing to 1 per O-D pair $\omega$). Since CS placement and pricing create a competitive game, the driver behaviour compliance constraint ensures the mixed strategy profile $\mathbf{Q}$ follows NE, where no driver benefits from unilateral deviation. The CSO must analyse strategic interactions to optimise placement and pricing. Further details on these dynamics follow in the next section.

### 2.4. The Game Interpretation

In the non-cooperative game studied, drivers act selfishly to minimise their own costs, often leading to greater overall congestion than in coordinated systems. As shown in (2.6) and (2.7), each driver optimises their own cost—EV drivers consider both road and CS congestion. This yields two interdependent congestion games: one over roads (all drivers) and one over CSs (EV drivers), linked via EV decisions. The next section addresses the challenges of analysing these coupled games and outlines our methodology.

## 3. Proposed Methodology

Analysing both congestion games is complex due to the large number of agents. To simplify, we adopt a sequential two-stage approach, separating EV drivers and NCDs. Given low EV adoption, NCD behaviour is largely unaffected by EVs, so Stage 1 analyses the NCD game to find NE strategies. These are then used as background traffic in Stage 2, which examines the intertwined road and CS congestion games among EV drivers. This reduces complexity and enables effective analysis of both groups. For clarity, we present Stage 2 first, as Stage 1 is a simpler subset. With higher EV adoption, an iterative method, alternating between groups, may be needed, and studying its convergence is a key area for future work.

### 3.1. Nash Equilibrium in Intertwined EV Games

Atomic congestion games provide a suitable framework for modelling strategic driver behaviour, particularly with a finite number of agents. This is a type of model that considers how individual drivers' choices contribute to overall traffic and station use. We use the concept of a mixed-strategy NE, where drivers assign probabilities to different route choices such that all selected options result in the same expected cost, none offers a clear advantage. Since drivers with the same origin and destination behave similarly, it is standard to assume they adopt the same strategy. This simplifies analysis and is justified, as all equilibria in such settings lead to the same overall congestion and system cost [26, 27].

Derived from (2.6), the cost for an EV user with O-D $\omega$ and path $p$ can be expressed as:

$$C_{\omega,p}(\mathbf{Q}) = \lambda_1 \sum_{l \in p} \frac{d_l}{c_l}(\gamma^0_l + \gamma^-_l + 1) + \lambda_2 \frac{\gamma^-_{s(p)} + 1}{\mu x_{s(p)}} + \lambda_3 y_{s(p)}, \tag{3.1}$$

where $\gamma^-_l$ denotes the number of EV users on link $l$ excluding one user, and $\gamma^-_{s(p)}$ represents the number of EV users at station $s(p)$ excluding the same user. Also, based on (2.2), we can present $\gamma^-_l$ and $\gamma^-_{s(p)}$ as:

$$\gamma^-_l = \sum_{\omega',p':p' \in P_{\omega'}, l \in p', \omega' \neq \omega} \gamma_{\omega'} q_{\omega',p'} + \sum_{p \in P_\omega : l \in p} (\gamma_\omega - 1) q_{\omega,p}, \tag{3.2}$$

$$\gamma^-_{s(p)} = \sum_{\omega',p':s(p)=s(p'), \omega' \neq \omega} \gamma_{\omega'} q_{\omega',p'} + \sum_{p' \in P_\omega : s(p)=s(p')} (\gamma_\omega - 1) q_{\omega,p'}. \tag{3.3}$$

At an NE, no player benefits from unilaterally deviating from their chosen strategy. In the mixed-strategy setting, this implies that for any O-D pair $\omega$, all extended paths $p \in P_\omega$ assigned positive probability ($q_{\omega,p} > 0$) must yield equal expected costs $C_{\omega,p}(\mathbf{Q}^{NE})$. Otherwise, reallocating probability to a lower-cost path would violate the equilibrium condition [26]. Hence, a mixed strategy profile $\mathbf{Q}^{NE}$ constitutes an NE if and only if it satisfies:

$$q_{\omega,p} C_{\omega,p}(\mathbf{Q}^{NE}) \leq q_{\omega,p} C_{\omega,p'}(\mathbf{Q}^{NE}), \quad \forall \omega, \forall p, p' \in P_\omega. \tag{3.4}$$

For paths with $q_{\omega,p} > 0$, the term cancels, yielding the minimal cost condition; for paths with $q_{\omega,p} = 0$, the inequality holds trivially without constraining costs.

### 3.2. Game-Theoretic Joint Optimisation of Placement and Pricing (G-T JOPP)

Using (3.4), we complete our G-T JOPP by substituting the driver behaviour compliance constraints in (2.8) with the finite set of constraints in (3.4). This allows us to proceed with the primary objective of the study: solving the CSO problem (2.8) to determine the optimal placement and pricing scheme.

### 3.3. Solving the model

The CSO problem (2.8) includes integer variables ($x_i$ for CS count) and real-valued variables ($y_i$, $\mathbf{q}$). As TN size increases, MINLP solvers become inefficient. To address this, we relax $x_i$ to continuous values, solve the relaxed CSO problem with an NLP solver, and then apply our Adjustment Algorithm (AA) (Algorithm 1) to round $x_i^*$ to integers. AA gives an integer solution for the number of stations $x_i^{'}$ at each node, which may differ from the relaxed values. Consequently, the optimal prices and drivers' responses must be adjusted. We re-run the CSO problem with fixed placement decisions $x_i^{'}$ to obtain the optimised strategies $q_{\omega,p}$ and prices $y_i$ at each node.

**Algorithm 1** Adjustment Algorithm (AA)

1. **Sort** fractional parts: $x_i^* - \lfloor x_i^* \rfloor$
2. **Floor** each value: $x_i^{*'} = \lfloor x_i^* \rfloor$
3. **Initial sum**: $S_{\text{initial}} = \sum_i x_i^{*'}$
4. **Adjustment**: $\Delta = \sum_i x_i^* - S_{\text{initial}}$
5. **for** $i = 1$ to $\Delta$ **do**
6. Increment largest fractions: $x_i^{*'} = x_i^{*'} + 1$
7. **end for**

### 3.4. Deriving NCD Background Traffic

Before evaluating the G-T JOPP model, note that Sections 3.1 to 3.3 analyse Stage 2—intertwined EV games with fixed NCD background traffic. To complete Stage 1 and derive this background, we determine the NE strategy for NCDs in their congestion game. Since EV charging does not affect NCD decisions, we exclude charging parameters ($x_i$, $y_i$) and budget constraints, focusing solely on minimising NCD social costs under feasibility and driver behaviour constraints. From (2.6), this is achieved by setting queuing and financial cost weights ($\lambda_2$, $\lambda_3$) to zero. As in (3.4), for NCD users, the expected travel cost $C^0_{\omega,r}(\mathbf{Q})$ is minimised across all routes used with nonzero probability ($q^0_{\omega,r} > 0$): $q^0_{\omega,r} C^0_{\omega,r}(\mathbf{Q}) \leq q^0_{\omega,r} C^0_{\omega,r'}(\mathbf{Q}), \quad \forall \omega, \forall r, r' \in R_\omega$.

## 4. Experiments and Evaluation

In this section, we validate the G-T JOPP model on the Nguyen-Dupuis TN [28] (Fig. 2), a benchmark widely used in the literature [9, 29, 30]. The model was implemented in Python using Pyomo, and the NLP component was solved with 'ipopt' [31] [1]. This section is organised as follows: Section 4.1 introduces the Nguyen-Dupuis network and testing parameters. Section 4.2 validates our approach on the benchmark, while Section 4.3 compares our joint optimisation with state-of-the-art methods. Section 4.4 evaluates the error from relaxing integer variables, and Section 4.5 examines model robustness and proposes a method to improve scalability and reduce computational complexity.

### 4.1. Benchmark Network Overview

The Nguyen-Dupuis network consists of 13 nodes, 19 links, and 4 O-D pairs: $W = \{(1, 2), (1, 3), (4, 2), (4, 3)\}$, as shown in Fig. 2. In [9], SUMO was used to simulate driver behaviour within this network and identified

[1]For reproducibility, the executed code is available at: GitHub Repository

the most frequently used routes, listed in Table 1. There are 10 feasible routes and 54 extended paths when incorporating star nodes. The network parameters can generally be chosen arbitrarily, and the model maintains high accuracy as long as the EV adoption rate remains moderate[2]. In our experiment, we assume that for each O-D pair $\omega$, there are 100 NCDs and 15 EV drivers. Other parameters are: $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$, $c_l = 200$ for all $l \in A$, and $T_i = 10$ for all $i \in N$. With uniform maintenance costs, electricity prices $e_i$ are randomly set between 5 and 7 for non-O-D nodes and between 11 and 15 for O-D nodes, reflecting higher demand in densely populated areas.

**Table 1.** Feasible Routes for O-D Pairs [9]

| O-D Pair | Feasible Routes ($R_\omega$) |
|---|---|
| $\omega_1$: (1, 2) | $r_1$: [1, 12, 8, 2] |
| $\omega_2$: (1, 3) | $r_2$-$r_6$: [1, 5, 6, 7, 11, 3], [1, 12, 6, 10, 11, 3], [1, 12, 6, 7, 11, 3], [1, 5, 9, 13, 3], [1, 5, 6, 10, 11, 3] |
| $\omega_3$: (4, 2) | $r_7$, $r_8$: [4, 5, 6, 7, 8, 2], [4, 9, 10, 11, 2] |
| $\omega_4$: (4, 3) | $r_9$, $r_{10}$: [4, 5, 6, 7, 11, 3], [4, 9, 13, 3] |

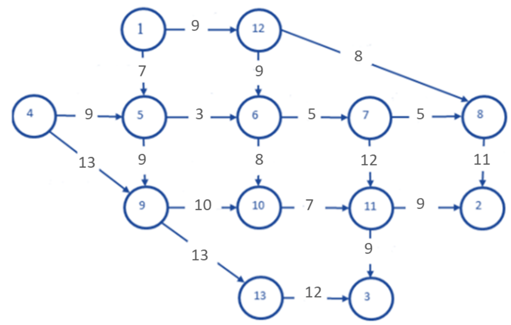

**Figure 2.** Nguyen-Dupuis Network

## 4.2. Technique Application and Validation

As discussed in Section 3.4, to determine optimal charger placement and pricing, we first simplify the CSO problem by analysing the road congestion game among NCDs, assuming no EV drivers or CS, so we can derive the NCDs' optimal route distribution ($q^0_{\omega,r}$), which becomes the background traffic for EV drivers. With this background traffic, we run the G-T JOPP model to optimise charger placement, pricing, and EV driver responses. The model reveals that as the budget ($B$) increases, the social cost drops until $B = 7$, after which it stabilises (see Fig. 3). Thus, for $B \geq 7$, the optimal solution assigns 1 charger to nodes 11 & 7 (priced at $y^*_{11} = 6.71$, $y^*_7 = 6.39$ ), 2 chargers to node 12 (priced at $y^*_{12} = 7.60$), 3 chargers to node 9 (priced at $y^*_9 = 6.38$) with no chargers at other nodes. For $B < 7$, chargers are still placed within same nodes, but with a higher social cost due to charger shortage.

## 4.3. Comparison with State-of-the-Art Methods

In this section, we evaluate our Joint Optimisation (JO) approach against two benchmarks that focus separately on pricing or placement (see Section 1.2): (1) Price Optimisation (PrO): Optimises prices with evenly

[2]As noted in Section 3, our method requires iterating between its two stages to handle high EV adoption effectively.

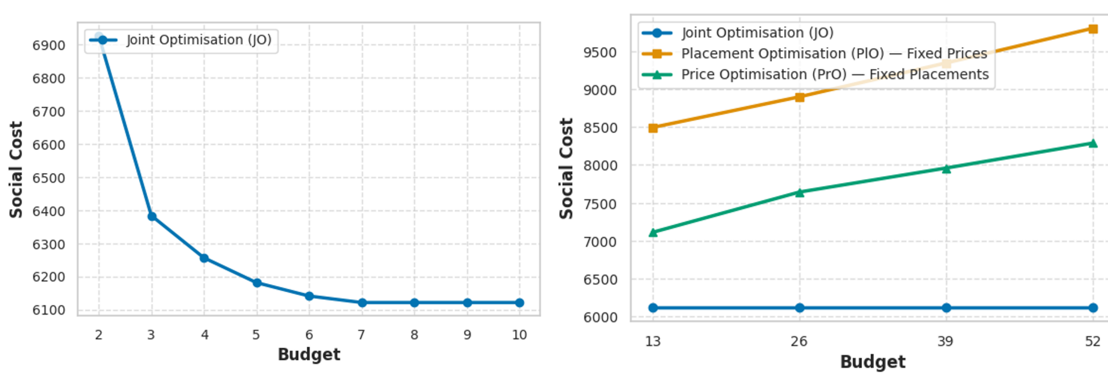


**Figure 3.** Social Cost Comparison: **Left:** JO model shows optimal charger count at $B = 7$, with no gains beyond. **Right:** JO outperforms single-focus methods, reducing social costs by at least 14% across all budgets.

**Table 2.** Comparison of Objective Values (OV)

| G-T JOPP Scenario | OV (AA) | OV (RX) | Difference (%) |
|---|---|---|---|
| $\lambda_2 = 0.5$, $B \geq 7$ | 5887 | 5760 | 2.20 |
| $\lambda_2 = 2$, $B \geq 7$ | 6121 | 6101 | 0.32 |
| $\lambda_2 = 4$, $B \geq 7$ | 6442 | 6429 | 0.20 |
| $\lambda_2 = 2$, $B = 3$ | 6383 | 6279 | 1.65 |
| $\lambda_2 = 2$, $B = 6$ | 6140 | 6107 | 0.54 |

**Table 3.** Sensitivity Analysis over $\lambda_2$

| Alternative Parameter | Required Chargers | Placement | | | |
|---|---|---|---|---|---|
| | | $x_7^*$ | $x_9^*$ | $x_{11}^*$ | $x_{12}^*$ |
| $\lambda_2 = 0.5$ | 3 | 0 | 1 | 1 | 1 |
| $\lambda_2 = 1$ | 5 | 1 | 2 | 1 | 1 |
| $\lambda_2 = 2$ | 7 | 1 | 3 | 1 | 2 |
| $\lambda_2 = 4$ | 10 | 1 | 5 | 2 | 2 |

distributed chargers, following [13–16]; (2) Placement Optimisation (PlO): Optimises charger locations with uniform pricing to cover costs and yield marginal profit, following [7–11].

Figure 3 shows that our JO approach consistently reduces social costs compared to the benchmarks. JO improves performance by 14% with limited budgets, reaching 37.5% at $B = 52$ and continuing to increase as the budget grows. However, excessive charger installation due to higher budgets raises rental and maintenance costs for the CSO, forcing them to set higher prices, which in turn increases social cost. By optimising both placement and pricing simultaneously, JO allows the CSO to allocate the budget based on demand rather than depleting it inefficiently, preventing the over-installation often seen in single-parameter methods. This results in more efficient resource use and a more effective charging infrastructure.

### 4.4. Error Evaluation of Adjustment Algorithm

Before Section 4.5, where we propose methods to reduce computational complexity and improve scalability, we evaluate our approach's effectiveness in avoiding mixed-integer optimisation. As detailed in Section 3.3, we relax integer variables, solve using an NLP solver, and then apply the AA to obtain integer solutions. The difference between the relaxed (RX) and AA-adjusted solutions is minimal (maximum error 2.20% at $\lambda_2 = 0.5$ and $B \geq 7$, see Table 2), confirming near-optimal performance.

### 4.5. Generalisability & Sensitivity Analysis

Our findings allow the G-T JOPP model to scale to larger TNs. Experiments show that optimal solutions concentrate chargers at a few key nodes, avoiding widespread distribution and addressing scalability issues from exponential growth in paths and variables. To improve scalability, we use an iterative process: limiting charger locations to candidates, solving the model, and replacing zero-charger nodes with new candidates until all are evaluated. This makes the model practical for larger networks. Additional experiments on larger networks, including Sioux Falls and synthetic networks, confirmed the method's scalability.

We conducted sensitivity analysis on parameters $B$, $T$, $e$, and $\mu$, varying each while holding others constant. Figure 3 covers budget $B$; this section focuses on $\lambda_2$, the queuing cost weight. A buffer of $B = 20$ avoided shortages without changing optimal node choices. Higher $\lambda_2$ led to more chargers to reduce queuing, raising maintenance costs and prices (Table 3). Lower $\lambda_2$ kept charger numbers and prices down.

## 5. Conclusion & Future Direction

This study presents a joint optimisation model for EV charging station placement and pricing, two critical elements in the UK's shift to cleaner transport. Unlike most existing work that treats these decisions separately, our model integrates them to reflect how drivers make strategic choices under both road and charging congestion. Results show that this integrated approach reduces social costs by at least 14% compared to methods that optimise either placement or pricing alone. The model is robust across scenarios and sensitive parameters like budget and queuing costs. While the current model assumes uniform demand and duration, future work will consider diverse vehicle needs, dynamic pricing, grid limits, and competition. The framework can also extend to other congestion-sensitive systems like shared mobility and energy networks. Integration with AI methods, such as multi-agent learning, could enable real-time, data-driven decision-making under uncertainty.

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