Partitioning



Goal: Partition *cells* into *blocks* \Longrightarrow Graph partitioning. Given a graph G(V, E), with |V| nodes and |E| edges. Each node has area s(v) and each edge has cost/weight w(e). Divide graph *G* into *k* disjoint subgraphs. (NP-hard) Optimization goals:

- Minimum number of connections between partitions
- Design constraints (size, external connections)
- Balanced partition

Kernighan-Lin Algorithm.

Given: Graph with 2n nodes, same weight. Goal: Partition into *A* and *B*; min. cost, |A| = |B| = n. *Cost D(v) of moving a node v

$$D(v) = |E_c(v)| - |E_{nc}(v)|$$

 $E_c(v)/E_{rc}(v) = \text{edges of } v \text{ that are (not) cut}$

*Gain of swapping a pair of nodes a and b

$$\Delta g = D(a) + D(b) - 2 \cdot c(a, b)$$

$$D(a), D(b) = costs of nodes a, b$$

c(a, b) =connection weight between a and b

*Maximum positive gain G_m of a pass

$$G_m = \sum_{i=1}^m \Delta g_i$$

Algorithm: $O(n^3)$ per pass

repeat // a single pass

repeat

calculate D(v) for all nodes find max. Δg_i and swap until each node swapped over select max. positive gain $G_m = \sum_{i=1}^m \Delta g_i$ perform *m* moves

until no improvement ($G_m < 0$)

Extensions:

- unequal partition size
- unequal node weights
- *k*-way partitioning (recursive)

Fiduccia-Mattheyses Algorithm.

- swapping becomes moving nodes
- slight imbalance allowed
- edges become nets (w/2 + pins)
- Node = cell; subgraph = block

Given: Hypergraph G(V, H), weighted hyperedges Goal: Partition int *k*, min. weight cost *Gain $\Delta g(c)$ for cell c

$$\Delta g(c) = FS(c) - TE(c)$$

FS(c) = moving force, i.e. cut nets only connected to c

TE(c) = retention force, i.e. uncet nets connected to c

*Maximum positive gain G_m of a pass

$$G_m = \sum_{i=1}^m \Delta g_i$$

*Ratio factor, i.e. relative balance between two partitions

$$r = \frac{area(A)}{area(A) + area(B)}$$

*Balance criterion

$$area(V) \in [r \cdot area(V) \pm area_{max}(V)]$$

*Base cell: Cell c with max. cell gain $\Delta g(c)$ among all free cells and does not violate balance criterion

Algorithm: O(n) per pass (sorted node list)

repeat // a single pass

compute balance criterion repeat

find max. Δg_i and fix cell c_i

until all cells are fixed

select max. positive gain $G_m = \sum_{i=1}^m \Delta g_i$ perform *m* moves

until no improvement

♦ Multilevel Partitioning.

Clustering: (⇒ Simulated Annealing)

- group tighlty-connected nodes into clusters
- partition clusters
- refine partition by splitting clusters

Floor Planning



 $\left\{ \begin{array}{c} \text{Set of} \\ \text{modules} \end{array} \right\} \Rightarrow \begin{array}{c} -\text{Layout} \\ -\text{Network} \\ -\text{Network} \end{array}$

Optimization goals:

- minimize area of global bounding box
- minimize total wire length
- combination of the above $(\alpha \beta)$
- signal delays (critical path)

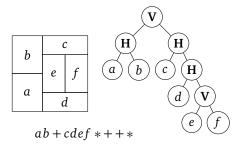
*Rectangular dissection, division of the chip area into a set of blocks (non-overlapping rectangles).

*Slicing floorplan, rectangular dissection

- Obtained by repeatedly dividing each rectangle
- Horizontal or vertical cut line

*Slicing tree, binary tree with k leaves, k-1 internal nodes

- Each leaf represents a block
- Each internal node represents a h/v cut line
- Polish expression $(V \rightarrow *, H \rightarrow +, W \rightarrow \text{wheel})$



*Non-slicing floorplan (wheels)

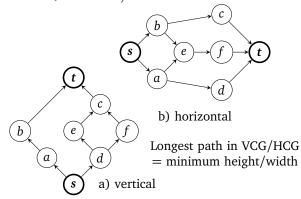
*Vertical constraint graph (VCG)

Edge from node v_i to v_i if block m_i under m_i .

*Horizontal constraint graph (HCG)

Edge from node v_i to v_i

if block m_i is left of m_i .



*Sequence pair (S_{\perp}, S_{\perp})

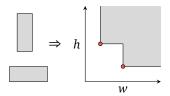
Two permutations represent geometric relations between every pair of blocks, e.g. (baced f, abcdec f)

> $(\dots A \dots B \dots \dots A \dots B \dots) \rightarrow A$ is left of B $(...A...B...,...B...A...) \rightarrow A$ is above of B $(...B...A....A...B...) \rightarrow A$ is below of B $(...B...A....B...A...) \rightarrow A$ is right of B

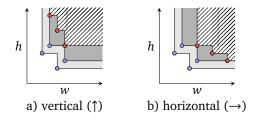
♦ Floorplan Sizing.

Goals: Minimize wire length and area *Shape function,*Corner points

Step 1: Construct the shape functions of the blocks



Step 2: Determine the shape fun. of the top-level floorplan Choose minium bounding box $(w \times h)$



Step 3: Find individual block's dimensions and locations

\diamond Cluster Growth.

- Iteratively add blocks to the cluster until all are assigned
- Only the different orientations of the blocks are taken into account
- Linear ordering to minimize total wire length

Continue..?

Placement

