The Role of Criticality in Information Processing: A Simulation Study of Neuronal Network Dynamics

Introduction:

The concept of criticality has emerged as an influential framework in neuroscience for understanding the variability and functional implications of brain activity. Critical systems are characterized by a metastable state, which optimizes dynamic range, storage capacity, and computational efficiency. Power-law scaling, a hallmark of criticality, has been observed across different organizational levels in the brain, such as neuronal avalanche distributions both in vitro and in vivo, as well as in the decay of temporal correlations in behavioral performance and ongoing oscillations in humans.[1]

Despite these observations, a critical unresolved issue is whether the power-law scaling seen at various organizational levels in the brain, and potentially in other hierarchically organized systems, can be interconnected. Our research addresses this by demonstrating that the critical-state dynamics of avalanches and oscillations can simultaneously emerge in a neuronal network model when excitation and inhibition are balanced. This model's oscillatory activity closely mirrors the patterns typically observed in human resting-state (MEG) recordings.[2]

In cortical slices, neuronal avalanches in local field potential activity exhibit a power-law distribution of event sizes with an exponent of -1.5, consistent with a critical branching process. Similar findings have been observed in vivo.[2] These observations have inspired the development of computational models that generate power-law-distributed neuronal avalanches through self-organization. Scale-free dynamics have been detected at various levels of neuronal organization, with power-law scaling identified in the decay of temporal correlations in behavioral performance and phase-locking intervals between brain regions.[2]

This project addresses a significant computational challenge: modeling and understanding how criticality can arise and be maintained in complex neuronal networks. By simulating a network with balanced excitatory and inhibitory dynamics, we explore the emergence of critical behavior and its implications for brain function.

Methods:

Step I :

We modeled networks of excitatory (75%) and inhibitory (25%) neurons (n = 2500) arranged in an open grid, with local functional connectivity. To define the local connectivity, each neuron was assigned a square area (width = 7 neurons), within which they could connect to 48 other neurons. The excitatory connectivity was set between 10– 70%, and the inhibitory connectivity was set between 30–90% of the total number of neurons within this local range. Neurons were arranged on a 2D open grid, resulting in border neurons having fewer connections due to a reduced number of neighboring neurons within their local range.[2]

In neocortical tissue, local functional connectivity between neurons decays with distance. Given the limited range of connectivity in our model, we implemented the connectivity with an exponential decay function. We created a network for a combination of excitatory and inhibitory connectivity, and the network was simulated for 2000 seconds. The weights (Wji) of these connections depended on the type (excitatory or inhibitory) of the presynaptic (i) and postsynaptic (j) neurons. Randomness was introduced at the level of assigning whether a neuron was excitatory or inhibitory, and in determining whether neurons would connect to each other.

Neurons were modeled using a synaptic model integrating received spikes, and a probabilistic spiking model.[2] Each time step (*dt*) of 1 ms starts with each neuron, i, updating I_i with received input from any of the set of connected neurons (J), together with an exponential synaptic decay, which can either be excitatory or inhibitory.[2]

Step II :

In addition we replicated The Izhikevich neuron model using Matlab,this model developed by Eugene M. Izhikevich and aims to bridge the gap between the biological plausibility of the Hodgkin-Huxley model and the computational simplicity of the integrate-and-fire model. This model is capable of replicating a wide range of spiking and bursting behaviors observed in cortical neurons while being efficient enough to simulate large-scale networks of spiking neurons in real-time. This model represents a different network architecture which shows the ability of deep learning as a multi-agent network.

Results and Discussions :

Step I and II of the project aimed to demonstrate the criticality of a computationally simulated brain networks characterized mainly with spiking excitatory and inhibitory neurons and to draw meaningful conclusions from the observed dynamics.

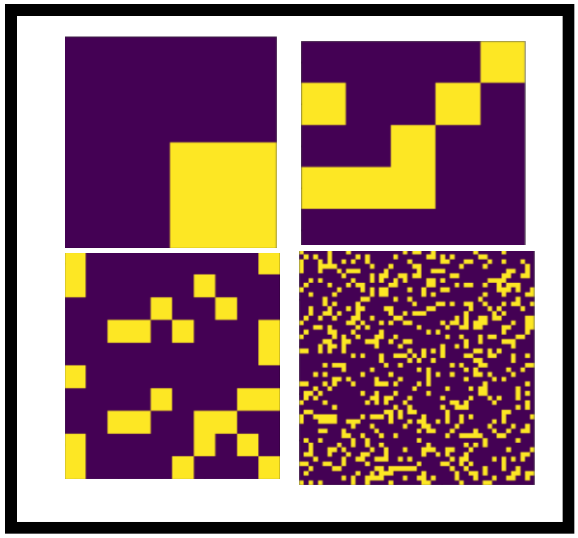


Fig. 1 – simulation on a changing neuronal grid size

The observation of scale invariance across various grid sizes, as depicted in Figure 1, lends robust support to the theory that neuronal networks adhere to the concept of criticality.

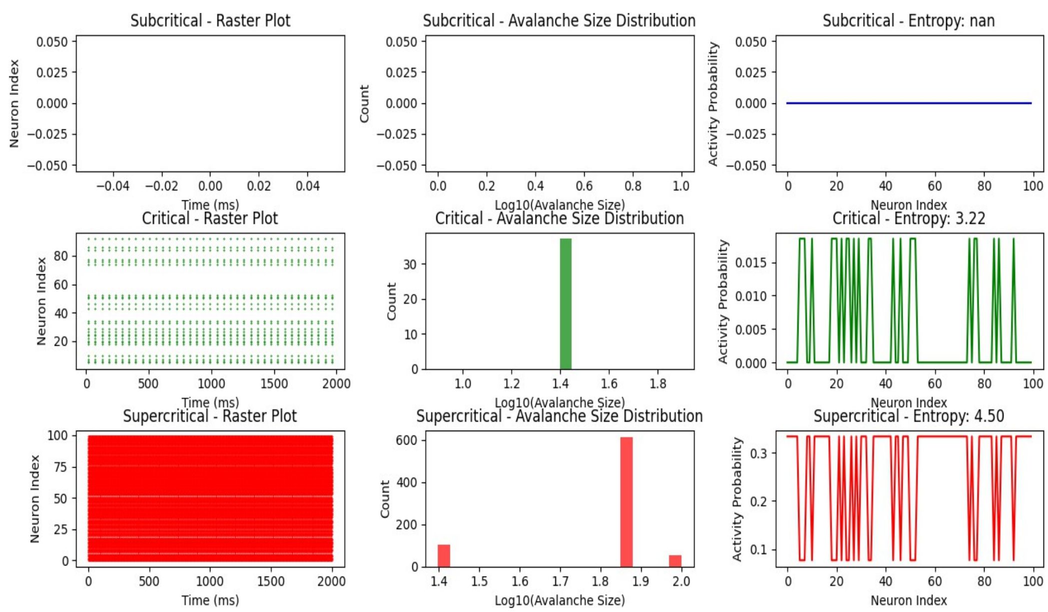


Fig. 2 – Raster plot, Avalanche size distribution and Entropy for a subcritical, Critical and supercritical dynamics.

As depicted in Figure 2, the subcritical network exhibits insufficient neuronal activity, characterized by the absence of avalanches and a constant entropy of zero, indicating an inactive network. In contrast, the supercritical network shows excessive neuronal activity, which can result in diminished information transfer and a lack of avalanche distribution, potentially causing delays in the signaling process. Meanwhile, the critical network demonstrates a balanced and moderate level of neuronal activity with an even distribution of avalanches. Additionally, we observe a reduction in entropy compared to the supercritical state, reflecting a more organized network.

Step II: Izhikevich neuron shows signs of criticality

The Izhikevich neuron model, developed by Eugene M. Izhikevich, aims to bridge the gap between the biological plausibility of the Hodgkin-Huxley model and the computational simplicity of the integrate-and-fire model. This model is capable of replicating a wide range of spiking and bursting behaviors observed in cortical neurons while being efficient enough to simulate large-scale networks of spiking neurons in real-time.

Mathematical Formulation

The Izhikevich neuron model is defined by two coupled differential equations:

1. Membrane Potential Dynamics:

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I$$

Here, v represents the membrane potential of the neuron, and I is the input current.

2. Recovery Variable:

$$\frac{du}{dt} = a(bv - u)$$

The variable u represents the membrane recovery, which provides negative feedback to v.

3. Reset Mechanism:
After the membrane potential v reaches a threshold (e.g., 30 mV), the following reset condition is applied:

$$\text{if } v \geq 30 \text{ mV, then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$$

The parameters a, b, c, and d are dimensionless and define the specific spiking and bursting behavior of the neuron.

Parameter Explanation

- a: Determines the time scale of the recovery variable u. Typical value is 0.02.
- b: Controls the sensitivity of u to the fluctuations of v . Typical value is 0.2.
- c: Represents the after-spike reset value of v . Typical value is -65 mV.
- d: Determines the after-spike reset of u . Typical value is 2.

Types of Neuronal Dynamics

Different combinations of these parameters can model various types of neurons:
- Regular Spiking (RS): a = 0.02 , b = 0.2 , c = -65 , d = 8
- Intrinsically Bursting (IB): a = 0.02 , b = 0.2 , c = -55 , d = 4
- Chattering (CH): a = 0.02 , b = 0.2 , c = -50 , d = 2
- Fast Spiking (FS): a = 0.1 , b = 0.2 , c = -65 , d = 2
- Low-Threshold Spiking (LTS): a = 0.02 , b = 0.25 , c = -65 , d = 2

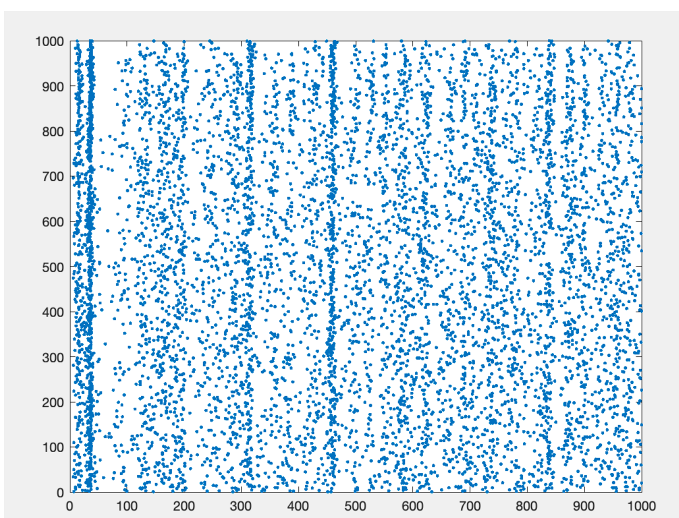
The criticality paper by Poil et al. (2012) investigates how balanced excitation and inhibition in neuronal networks lead to the emergence of critical dynamics. The Izhikevich neuron model can be used to simulate these networks due to its ability to replicate realistic neuronal behaviors efficiently.

In the context of criticality:

- Avalanche Dynamics: The Izhikevich model can simulate networks where neuronal avalanches exhibit power-law distributions, indicative of critical states.
- Oscillatory Behavior: The model's ability to generate various spiking and bursting patterns allows for the study of how neuronal oscillations emerge from network interactions. These oscillations can show long-range temporal correlations, another hallmark of criticality.

By adjusting the parameters of the Izhikevich neurons to reflect different types of cortical neurons, researchers can explore how variations in excitation and inhibition balance affect network dynamics and critical behavior, similar to the investigations in the criticality paper. The simplicity and versatility of the Izhikevich model make it a powerful tool for large-scale simulations that probe the complex dynamics of neuronal networks.

There are visible neuronal avalanches from the neuronal model of Izhikevich neuron in Matlab.

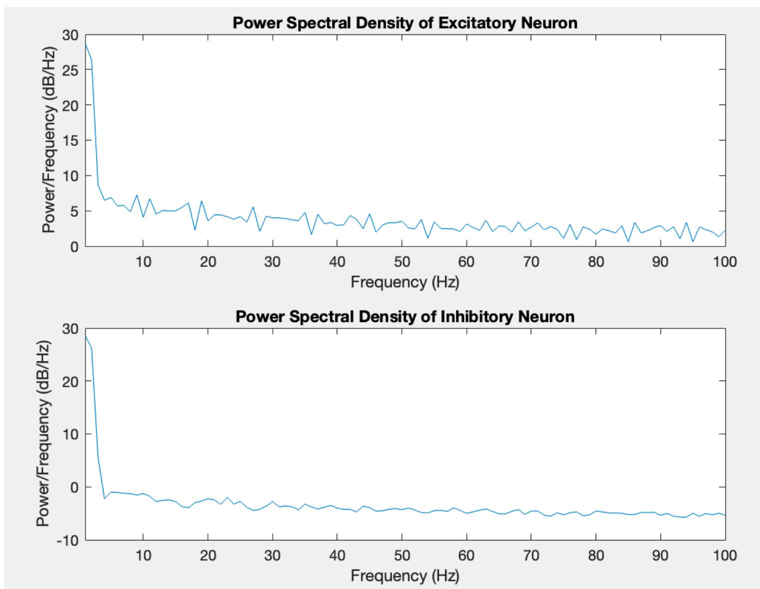


One of the hallmarks of critical systems is the presence of power laws, which describe the relationship between the frequency and size of events. In neural systems, this is observed in the distribution of neuronal avalanches – bursts of activity that propagate through the network. The size S and duration T of these avalanches follow power laws:

$$P(S) \sim S^{-\alpha}$$
$$P(T) \sim T^{-\beta}$$

where α and β are scaling exponents. These power-law distributions indicate that the brain operates at a critical state, balancing excitation and inhibition to maintain this optimal functioning.

We can see clear power law distribution as for excitatory and for inhibitory Izhikevich neurons in frequency distributions, good signs of criticality



Global Workspace Theory (GWT)

Global Workspace Theory (GWT) is a cognitive architecture and theoretical framework proposed by Bernard Baars in the 1980s to explain the nature of consciousness. GWT suggests that the brain functions somewhat like a theater, where the "global workspace" is a stage on which information is made available to a broad audience of unconscious processes. Key aspects of GWT include:

1. Conscious Access: Information becomes conscious when it is broadcasted to the global workspace, where it can influence various cognitive processes such as decision-making, memory, and motor functions.
2. Competition and Cooperation: Multiple processes compete for access to the global workspace. Once information gains access, it can be integrated and influence behavior and cognition broadly.
3. Integration of Information: The theory emphasizes the integration of diverse types of information (sensory, emotional, mnemonic) to form a coherent, unified conscious experience.

Dehaene-Changeux Model (DCM)

The Dehaene-Changeux Model (DCM), developed by Stanislas Dehaene and Jean-Pierre Changeux, builds upon GWT and provides a neural basis for the theory. It specifically models how conscious access emerges from neuronal activity. Key features include:

1. Neuronal Workspace: DCM proposes a network of neurons, particularly in the prefrontal and parietal cortices, that form a "global neuronal workspace." These neurons can sustain and broadcast information across the brain.
2. Sustained Activity and Ignition: Conscious perception is associated with a sustained increase in neural activity, referred to as "ignition." This occurs when neural assemblies in the workspace synchronize and amplify a specific signal.

3. Broadcasting Information: Once a signal is ignited, it is broadcasted throughout the brain, enabling various cognitive modules to access and process this information.

Conscious Processing and the Global Neuronal Workspace Hypothesis

The Global Neuronal Workspace Hypothesis further elaborates on the mechanisms of conscious processing:

1. Widespread Availability: Conscious processing involves making specific information widely available across different brain regions. This enables diverse neural processes to access and utilize the information for various functions.
2. Top-Down Attentional Amplification: Attention plays a critical role in selecting which information gains access to the global workspace. Top-down signals from the prefrontal cortex can amplify relevant sensory inputs, facilitating their entry into conscious awareness.
3. Synchronized Neural Activity: Conscious processing is associated with the synchronization of neural activity across distant brain areas. This synchronization facilitates the integration of information and the formation of a unified conscious experience.

Connection Between GWT, DCM, and Conscious Processing

The Global Workspace Theory provides the foundational framework, suggesting that consciousness arises from the broadcasting of information across a global workspace. The Dehaene-Changeux Model provides a detailed neural implementation of this theory, describing how specific neural mechanisms support the global workspace and conscious processing. The Global Neuronal Workspace Hypothesis expands on these ideas, emphasizing the roles of attention, synchronization, and integration in conscious experience.

Together, these theories and models offer a comprehensive account of how conscious processing emerges from neural activity, highlighting the interplay between local neuronal circuits and global brain networks in generating consciousness.

The Dehaene-Changeux Model (DCM) is a computational model designed to explain how consciousness arises in the brain. It builds on the Global Workspace Theory (GWT), which suggests that consciousness involves the widespread sharing of information across different brain regions. The DCM uses mathematical equations to describe how neurons interact to produce this global sharing of information.

The Dehaene-Changeux Model (DCM) incorporates criticality through its mathematical framework, which simulates how conscious access and cognitive functions emerge from neural activity. Here’s how criticality is mathematically represented and understood within the DCM

Mathematical Representation of Criticality in DCM

Neural Dynamics and Network Architecture

1. Neural Activation and Synchronization:

The DCM posits that conscious access involves the synchronization of neural assemblies across different brain regions. The model uses differential equations to describe the activity of neurons and their interactions. These equations capture the excitatory and inhibitory influences that lead to neural synchronization.

The basic dynamics of neuron i in a population can be described by:

$$\tau \frac{dv_i}{dt} = -v_i + \sum_j W_{ij} \cdot S(v_j) + I_i$$

where:
- v_i is the membrane potential of neuron i ,
- tau is the membrane time constant,
- W_ij represents the synaptic weight from neuron j to neuron i ,
- Sv_j is a nonlinear sigmoidal function representing the firing rate of neuron j ,
- I_i is the external input to neuron i .

2. Balanced Excitation and Inhibition:

Criticality arises from a balance between excitation and inhibition. In the DCM, this balance is achieved through the proper tuning of synaptic weights W_ij and the inclusion of both excitatory and inhibitory neurons. The network is designed such that the overall excitation roughly equals inhibition, maintaining the system near a critical point.

Mathematically, this balance can be represented as:

$$\sum_j W_{ij}^{\text{exc}} \approx \sum_j W_{ij}^{\text{inh}}$$

where W_ij_exc and W_ij-inh are the weights for excitatory and inhibitory connections, respectively.

Critical Dynamics in the Global Neuronal Workspace

Conscious access is modeled as the ignition of neural assemblies, a process where a sufficient level of synchronized activity leads to a sudden and sustained increase in firing rates across the network. This is described by:

$$\frac{dA_i}{dt} = -A_i + f\left(\sum_j W_{ij} \cdot S(v_j) + I_i\right)$$

where Ai represents the activity of assembly i and f is a function that captures the nonlinear thresholding behavior characteristic of ignition.

Global Workspace Dynamics:

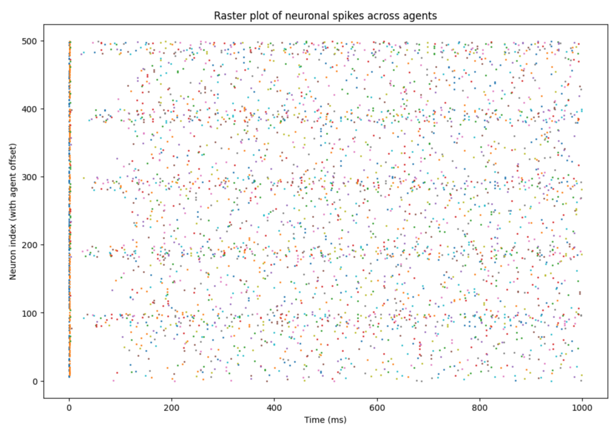
Once ignition occurs, the information is broadcasted across the global workspace, making it available for various cognitive processes. The workspace dynamics can be captured by network-wide synchronization measures, such as the coherence between different brain regions.

The overall synchronization rho in the network can be measured as:

$$\rho = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i}\right|$$

where theta_i is the phase of the oscillatory activity of neuron i , and N is the total number of neurons.

For our model, we decided to do classical DCM with 5 agents each with similar policy
We did simple statistics to show just the criticality patterns of the model, which are, of course, only tiny parts of the whole GWT



We can notice the avalanches appearing on the raster plot. If we look at the distributions of the avalanches, we notice the power law emerging. For 5 neurons-agents we can conclude a pretty good hit on the fitness to the sigma