DPpack: An R Package for Differentially Private Statistical Analysis and Machine Learning

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Abstract

Differential privacy (DP) is the state-of-the-art framework for guaranteeing privacy for individuals when releasing aggregated statistics or building statistical/machine learning models from data. We develop the open-source R package DPpack that provides a large toolkit of differentially private analysis. The current version of DPpack implements three popular mechanisms for ensuring DP: Laplace, Gaussian, and exponential. Beyond that, *DPpack* provides a large toolkit of easily accessible privacy-preserving descriptive statistics functions. These include mean, variance, covariance, and quantiles, as well as histograms and contingency tables. Finally, *DPpack* provides user-friendly implementation of privacy-preserving versions of logistic regression, SVM, and linear regression, as well as differentially private hyperparameter tuning for each of these models. This extensive collection of implemented differentially private statistics and models permits has le-free utilization of differential privacy principles in commonly performed statistical analysis. We plan to continue developing *DPpack* and make it more comprehensive by including more differentially private machine learning techniques, statistical modeling and inference in the future.

Keywords: differential privacy, empirical risk minimization, support vector machines, privacy-preserving, R, randomized mechanism, regression

1 Introduction

Data is an invaluable resource harnessed to inform impactful technology development and guide decision-making. However, utilizing data that contain personally sensitive information (e.g., medical or financial records) poses privacy challenges. Anonymized datasets, as well as statistics and models derived from sensitive datasets are susceptible to attacks that may result in the leakage of private information (Narayanan and Shmatikov, 2008; Ahn, 2015; Sweeney, 2015; Shokri et al., 2017; Zhao et al., 2021). As technology continues to evolve to become more data-reliant, privacy issues will become increasingly more prevalent, necessitating easy access to tools that provide privacy guarantees when releasing information from sensitive datasets.

Differential privacy (DP) (Dwork et al., 2006b) is a popular state-of-the-art framework for providing provable guarantees of privacy for outputs from a statistical or machine learning (ML) procedure. A variety of randomized procedures and mechanisms exist to achieve DP guarantees for a wide range of analyses. These include, to list some examples, summary statistics (Dwork et al., 2006b; Smith, 2011), empirical risk minimization (Chaudhuri et al., 2011; Kifer et al., 2012), classifiers (Chaudhuri and Monteleoni, 2009; Vaidya et al., 2013), deep learning (Abadi et al., 2016; Bu et al., 2020), Bayesian networks (Zhang et al., 2017a), Bayesian procedures (Dimitrakakis et al., 2014; Wang et al., 2015b), statistical hypothesis testing (Gaboardi et al., 2016; Couch et al., 2019; Barrientos et al., 2019), confidence interval construction (Karwa and Vadhan, 2018; Wang et al., 2019), and synthetic data generation (Zhang et al., 2017b; Torkzadehmahani et al., 2019; Bowen and Liu, 2020). Privacy-preserving analysis has also been adopted by many companies in the technology sector, including Google (Guevara et al., 2020), Apple (Apple, 2017), Meta (Nayak, 2020), as well as government agencies like the U.S. Census Bureau (Bureau, 2021).

Given the popularity of DP, many open-source projects have been devoted to developing tools and code for privacy-preserving analysis with DP. The *OpenDP Library* (The OpenDP Team), Google's DP libraries (Google) (with accompanying OpenMined Python wrapper *PyDP* (OpenMined)), the *TensorFlow Privacy* library (TensorFlow), and the IBM's DP library diffprivlib (Holohan et al., 2019) collectively provide tools for DP analysis for the Rust, Python, C++, Go, and Java programming languages.

This paper presents the *DPpackR* package (https://github.com/sgiddens/DP pack) (Giddens and Liu, 2023), which provides convenient implementations of common DP procedures. R is arguably the most popular languages among statisticians. Prior to *DPpack*, R packages diffpriv (Rubinstein and Aldà, 2017) and *PrivateLR* (Vinterbo, 2018) were the only available DP R packages. Both of these packages are limited in scope compared to *DPpack*; *PrivateLR*, in fact, implements only a single function for DP logistic regression. Additionally, neither package has seen an update in the last five years. Meanwhile, *DPpack* has been downloaded from CRAN by R users $\sim 4,000$ times as of September 2023, averaging 242 downloads per complete month of being available, overtaking diffpriv and PrivateLR to become the most downloaded DP-focused R package in the past 10 months. While diffpriv does implement several randomized mechanisms for DP, DPpack goes well beyond these basic mechanisms by specifically implementing privacy-preserving versions of various commonly used descriptive statistics, as well as statistical analysis and machine learning procedures. The implemented functions are accessible even to individuals without a strong background in DP because sensitivity calculations are handled internally based on proven theoretical results and user-provided bounds on the input data. This makes DPpack more user-friendly than diffpriv for non-expert users. Even for DP experts, DPpack is attractive due to its scope. No other R package implements as extensive a collection of privacy-preserving functions. We plan to continue to develop and update the package by adding more privacy-preserving analysis procedures in the future.

2 Capabilities

2.1 Randomized Mechanisms

DPpack provides the LaplaceMechanism, GaussianMechanism, and ExponentialMechanism functions for implementing general mechanisms for ensuring DP for a desired output.

The LaplaceMechanism function implements the Laplace mechanism (Dwork et al., 2006b) for ensuring ϵ -DP for a statistical analysis or function by adding to the output Laplacian noise with a scale parameter dependent on the function's ℓ_1 -global sensitivity and the privacy budget ϵ . The function generalizes using DP composition to multidimensional function inputs, in which case it allows the user to specify the allocation of the privacy budget across the multiple computations.

The GaussianMechanism function implements the Gaussian mechanism (Dwork and Roth, 2014). It can be used to ensure either approximate (ϵ, δ) -DP (Dwork et al., 2006a), or probabilistic (ϵ, δ) -DP (Machanavajjhala et al., 2008; Liu, 2019a), depending on user input. It adds Gaussian noise with a variance dependent on ϵ, δ , and the function's ℓ_2 -global sensitivity, and can be generalized to multidimensional inputs.

The ExponentialMechanism function implements the exponential mechanism (McSherry and Talwar, 2007), which guarantees ϵ -DP and returns a result randomly from a set of possible candidates, with probability proportional to its "utility." This allows for DP releases of non-numeric information, to which adding numerical noise would be nonsensical.

2.2 Privacy-preserving Descriptive Statistics

One of the unique aspects of DPpack compared to the other DP R packages is that it provides direct support for DP-satisfying versions of many common descriptive statistics. The meanDP, varDP, covDP, and sdDP functions of DPpack provide DP counterparts to the analogously named R functions for calculating mean, variance, covariance, and standard deviation of a data vector. Pooled variances and covariances are also available with pooledVarDP and pooledCovDP. Through function arguments, a user specifies whether the output should satisfy ϵ -DP via the Laplace mechanism or (ϵ, δ) -DP via the Gaussian mechanism and global bounds on the data, from which appropriate ℓ_p -global sensitivities are computed internally based on known theoretical results (Liu, 2019b).

The histogramDP and tableDP functions compute DP histograms and contingency tables. Similar to previously described statistics, users may specify which mechanism and type of DP are used for the output, and additional arguments help format the output. Global bounds on the data are unnecessary as the global sensitivity is a fixed constant for frequency output.

DPpack implements differentially private quantiles and medians using the quantileDP and medianDP functions, respectively. By again only requiring the user to input global bounds on the data, these release ϵ -DP values via the exponential mechanism using the private quantile algorithm (Smith, 2011; Gillenwater et al., 2021).

2.3 Privacy-preserving Statistical Models and Machine Learning

Empirical risk minimization (ERM) is a statistical learning principle to find the best model from a given set of models. The goal of ERM is to minimize the empirical risk that measures the goodness of fit of a model to the training data. We implemented privacy-preserving procedures for a few ERM problems in supervised learning. Specifically, for binary classification, we create the EmpiricalRiskMinimizationDP.CMS class by employing the methods from Chaudhuri et al. (2011) for guaranteeing ϵ -DP for the output of training via ERM under necessary regularity conditions. Either the output or objective perturbation methods can be used. For linear regression, we employ the methods from Kifer et al. (2012) to create the EmpiricalRiskMinimizationDP.KST class for guaranteeing either ϵ -DP or (ϵ, δ) -DP under necessary regularity conditions. The intent is that these classes are used through an inheritance structure to implement binary classifiers or regressors as instances of ERM.

Specifically, logistic regression and support vector machine (SVM) models with ε-DP guarantees (Chaudhuri and Monteleoni, 2009; Chaudhuri et al., 2011) are implemented via the LogisticRegressionDP and symDP classes, respectively. Each of these classes inherits from EmpiricalRiskMinimizationDP.CMS. Released trained model coefficients or predictions made on new data satisfy ϵ -DP. Linear regression of either ϵ -DP or (ϵ, δ) -DP (Kifer et al., 2012) is implemented in *DPpack* via the LinearRegressionDP class, which inherits from the EmpiricalRiskMinimizationDP.KST class. Released trained model coefficients from those classes or predictions made on new data using these coefficients also satisfy user-specified DP guarantees. The symDP class currently supports ϵ -DP training via the linear and radial (Gaussian) kernels, with the radial kernel method being based on an approximation technique from Rahimi and Recht (2007, 2008); Chaudhuri et al. (2011). Training with individually weighted loss function contributions with ϵ -DP guarantees is also supported (Giddens et al., 2023). Each of these methods is user-friendly, even to those without a strong DP background, as they only require the user to specify certain hyperparameters (such as ϵ , δ , and γ) and global bounds on each feature contained in \mathbf{x}_i (and y_i , in the case of linear regression). Sensitivity calculations and scaling necessary to satisfy regularity conditions for DP guarantees are handled internally.

When the selection of hyperparameter values (e.g., the regularization constant in the ERM loss function) uses information from the sensitive dataset itself, the incurred privacy loss needs to be accounted for. DPpack provides the tune_classification_model function for privacy-preserving hyperparameter tuning for binary classifiers based on the exponential mechanism (Chaudhuri et al., 2011) and the tune_linear_regression_model function for hyperparameter tuning for linear regression.

3 Summary and Future Work

The *DPpack* package implements three general mechanisms for DP (Laplace, Gaussian, and exponential), a variety of DP descriptive statistics, and some privacy-preserving regression and classification methods. Making these functions accessible independent of the mechanisms they are based on permits code simplicity and ease-of-use (since users do not need to know how to compute sensitivities for their desired statistics, but only need to give global bounds on the data as inputs). Compared with other options for DP in R, *DPpack* offers a more complete set of privacy-preserving functions and models in a user-friendly manner that makes them easily accessible even to those without a strong background in DP.

We plan to keep developing the package and make it more comprehensive. For example, for ML techniques, we may include functionality for DP principal component analysis (Dwork et al., 2014; Chaudhuri et al., 2013), Bayesian networks (Zhang et al., 2016), and stochastic gradient descent based on the concepts of moment accountant (Abadi et al., 2016) and Gaussian DP (Bu et al., 2020), to name a few. For statistical analysis, we plan to include functionality for differently private z-tests (Gaboardi et al., 2019), t-tests (Ding et al., 2018), and some nonparametric tests (e.g. Wilcoxon rank sum test) (Couch et al., 2019), as well as hypothesis testing for linear regression (Barrientos et al., 2019; Chen et al., 2016) and confidence interval construction for certain problems (Karwa and Vadhan, 2018; Wang et al., 2019). We note that the list above is not comprehensive nor are the cited references the only existing work on each respective topic.

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Appendix A. Differential Privacy (DP)

DP protects the information of each individual whose information is contained in a dataset by ensuring that the results of a mechanism acting on the dataset would be almost identical to the results had their information not been present in the dataset. To formalize the notion of DP, we first define neighboring datasets: D_1 and D_2 are neighboring datasets if they differ in at most one observation. There are two equally valid methods by which a neighboring dataset D_2 may be constructed from a given dataset D_1 , depending on if the number of elements of each dataset must remain the same (i.e. is bounded), or if the number is allowed to vary (i.e. is unbounded) (Kifer and Machanavajjhala, 2011).

Definition 1 (Bounded neighboring datasets) We consider D_1 and D_2 to be bounded neighboring datasets if they are neighboring datasets and D_1 can be obtained from D_2 by modifying at most one observation.

Definition 2 (Unbounded neighboring datasets) We consider D_1 and D_2 to be unbounded neighboring datasets if they are neighboring datasets and D_1 can be obtained from D_2 by adding or removing at most one observation.

The two definitions of neighboring datasets may necessitate different amounts of calibrated noise to achieve the same level of privacy guarantees when releasing the same statistics (e.g, histograms). When the sample size is large, the difference between the two is largely ignorable.

We can now formally define a few different types of differential privacy.

Definition 3 (Differential privacy) (Dwork et al., 2006b,a) A randomized mechanism \mathcal{M} satisfies (ϵ, δ) -differential privacy if for all $S \subseteq \text{Range}(\mathcal{M})$,

$$P(\mathcal{M}(D_1) \in S) \le e^{\epsilon} P(\mathcal{M}(D_2) \in S) + \delta \tag{1}$$

for any neighboring datasets D_1 and D_2 , where $\epsilon > 0$ and $\delta \geq 0$ are privacy loss parameters. It is common to refer to $(\epsilon, 0)$ -DP (or ϵ -DP) as "pure" DP, and (ϵ, δ) -DP $(\delta > 0)$ as "approximate" DP.

Definition 4 (Probabilistic differential privacy) (Machanavajjhala et al., 2008) A randomized mechanism \mathcal{M} satisfies (ϵ, δ) probabilistic differential privacy if for all $S \subseteq Range(\mathcal{M})$,

$$P\left(\left|\log\left(\frac{P(\mathcal{M}(D_1) \in S)}{P(\mathcal{M}(D_2) \in S)}\right)\right| > \epsilon\right) \le \delta \tag{2}$$

for any neighboring datasets D_1 and D_2 , $\epsilon > 0$ and $\delta \geq 0$.

Intuitively, DP guarantees that the distributions of outputs from a randomized mechanism operating on neighboring datasets are similar. Thus, information gained from the mechanism will be essentially the same (within tunable bounds given by ϵ and δ) whether a given individual's data is used in the dataset or not. The individual that differs between datasets is arbitrary, meaning that differential privacy provides these individual-level privacy guarantees for all members of the dataset simultaneously.

DP has several nice properties to which its popularity in research and applications is attributed. We will briefly mention three here that are relevant to the package, and refer the interested reader to Dwork and Roth (2014) for other properties and additional information. The first two are composition theorems, which provide differential privacy bounds for the use of multiple randomized mechanisms on the same dataset.

Theorem 5 (Basic sequential composition) (McSherry, 2009) Let $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_n$ be n randomized mechanisms such that each \mathcal{M}_i satisfies (ϵ_i, δ_i) -differential privacy. $\mathcal{M}(D) = (\mathcal{M}_1(D), \ldots, \mathcal{M}_n(D))$ satisfies $(\sum_{i=1}^n \epsilon_i, \sum_{i=1}^n \delta_i)$ -differential privacy.

Theorem 6 (Parallel composition) (McSherry, 2009) Let $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_n$ be n randomized mechanisms such that each \mathcal{M}_i satisfies (ϵ_i, δ_i) -differential privacy, and let D_1, D_2, \ldots, D_n be n disjoint datasets such that their union is D. Then we have $\mathcal{M}(D) = (\mathcal{M}_1(D_1), \ldots, \mathcal{M}_n(D_n))$ satisfies $(\max_i \{\epsilon_i\}, \max_i \{\delta_i\})$ -differential privacy.

The second is immunity to post-processing, which ensures that there is no manipulation (not relying on the data itself) that can be performed on the results of a differentially private mechanism to weaken the privacy guarantees.

Theorem 7 (Immunity to post-processing) (Dwork and Roth, 2014) Let \mathcal{M} be a randomized mechanism satisfying (ϵ, δ) -differential privacy. $f \circ \mathcal{M}$ satisfies (ϵ, δ) -differential privacy for any arbitrary function f.

We conclude this section by defining the global sensitivity of statistics, which is used in some of the most general randomized mechanisms for achieving DP. Global sensitivity was originally defined using the ℓ_1 norm by Dwork et al. (2006b). Here, we use a more general definition.

Definition 8 (ℓ_p -global sensitivity) (Liu, 2019a) Let the distance between two datasets (denoted $d(D_1, D_2)$) be defined to be the number of observations that differ between the datasets. Note that $d(D_1, D_2) = 1$ if D_1 and D_2 are neighboring datasets. The ℓ_p -global sensitivity of a function f is defined to be

$$\Delta_{p,f} = \max_{\substack{D_1, D_2 \\ d(D_1, D_2) = 1}} ||f(D_1) - f(D_2)||_p, \tag{3}$$

where $||\cdot||_p$ is the ℓ_p norm.

The global sensitivity may be different depending on if the bounded or the unbounded neighboring dataset definition is used. For example, consider the function that outputs a histogram (a list of counts for each bin) from a given dataset. If the bounded neighboring dataset definition is used, the ℓ_1 -global sensitivity of the histogram function is 2 since modifying a dataset observation can at most change the count of two bins. However, under the unbounded neighboring dataset definition, the ℓ_1 -global sensitivity of the histogram function is 1 since adding or removing a dataset observation can at most change the count of one bin. For functions where there may be a difference between the two definitions of neighboring datasets, DPpack allows the user to choose which one to use.

The global sensitivity sets a bound on the amount the statistics can change in the worst-case scenario between two neighboring data sets. The higher the sensitivity for a statistic is, the larger the amount of noise that will be injected to the original observed statistics to achieve the pre-specified level of privacy guarantees defined by ϵ and δ .

For many statistics (e.g., mean, quantiles, regression coefficients), the global sensitivity is dependent on the global range of values that can occur in the dataset. In these cases, DPpack assumes the existence of known or reasonably inferred global or public bounds on the dataset, from which the global sensitivity is computed.

Appendix B. DP Mechanisms

There exist many general mechanisms for ensuring DP for a given analysis procedure or output. We introduce in this section three popular mechanisms: the Laplace mechanism (Dwork et al., 2006b), the Gaussian mechanism (Dwork et al., 2006a), and the exponential mechanism (McSherry and Talwar, 2007). We also provide examples of implementing these mechanisms using DPpack.

B.1 Laplace Mechanism

Definition 9 (Laplace mechanism) (Dwork et al., 2006b) Let D be a sensitive database. Let f be a given function with ℓ_1 -global sensitivity $\Delta_{1,f}$ and range \mathbb{R}^n . The Laplace mechanism of ϵ -differential privacy is defined to be

$$\mathcal{M}_L(D, f, \epsilon) = f(D) + \mathbf{e},$$
 (4)

where $\mathbf{e} = (e_1, \dots, e_n)^T$ and e_i is drawn independently from distribution $Lap(0, \Delta_{1,f}/\epsilon)$.

The LaplaceMechanism function in DPpack implements the Laplace mechanism. For a given scalar or a vector of observed statistic(s), the corresponding ℓ_1 -global sensitivity, and ϵ , it releases a real number or numeric vector of values satisfying ϵ -DP. Global sensitivity calculated based either on bounded or unbounded neighboring datasets can be used.

The following example uses the Laplace mechanism to release the sample mean with ϵ -DP guarantees. Consider a sensitive dataset of n=100 observations with one attribute, the (public) global range of which is $[c_0, c_1] = [5, 10]$. For the sample mean, the ℓ_1 -global sensitivity is the same for both bounded and unbounded DP and equals $(c_1 - c_0)/n = 0.05$ (Liu, 2019b).

```
library(DPpack)
set.seed(42) # For reproducibility
# Simulate a dataset
n <- 100
c0 <- 5
c1 <- 10
D <- runif(n, c0, c1)</pre>
```

```
epsilon <- 1 # Privacy budget
sensitivity <- (c1-c0)/n

private.mean <- LaplaceMechanism(mean(D), epsilon, sensitivity)
cat("Privacy preserving mean: ", private.mean, "\nTrue mean: ", mean(D))
#> Privacy preserving mean: 7.636944
#> True mean: 7.622394
```

The LaplaceMechanism function can also be used to release privacy-preserving multi-dimensional statistics which are the composition of scalar statistics, each with their own ℓ_1 sensitivity. For example, let $\mathbf{f}(D) = (f_1(D), \ldots, f_n(D))$, where each f_i has ℓ_1 -global sensitivity Δ_{1,f_i} . By default, the LaplaceMechanism function sanitizes \mathbf{f} by drawing Laplace noise from Lap $(0, \Delta_{1,\mathbf{f}}/\epsilon)$, where $\Delta_{1,\mathbf{f}} = \sum_{i=1}^n \Delta_{1,f_i}$. This approach corresponds to allocating a privacy budget of $\epsilon \Delta_{1,f_i}/\Delta_{1,\mathbf{f}}$ to sanitizing each scalar function f_i . If desired, users may specify how to divide the total budget ϵ among the elements in \mathbf{f} by passing a vector of proportions to the alloc.proportions argument instead of using the default allocation. The following example demonstrates this functionality for the same situation as the previous example, but with an additional variance computation. The ℓ_1 -global sensitivity of the variance is also the same for both bounded and unbounded DP and equals $(c_1 - c_0)^2/n = 0.25$ (Liu, 2019b).

```
# Simulate a dataset
n <- 100
c0 <- 5
c1 <- 10
D <- runif(n, c0, c1)
f <- function(D) c(mean(D), var(D))</pre>
sensitivities <-c((c1-c0)/n, (c1-c0)^2/n)
epsilon <- 1 # Total privacy budget for f
# Here, privacy budget is split relative to the individual sensitivities
# of the sample mean and sample variance. Collectively, the computation
# satisfies 1-differential privacy.
private.vals <- LaplaceMechanism(f(D), epsilon, sensitivities)</pre>
cat("Privacy preserving values: ", private.vals, "\nTrue values: ", f(D))
#> Privacy preserving values: 7.623156 2.401604
#> True values: 7.61271 2.036525
# Here, privacy budget is split so that 25% is given to the mean
# and 75% is given to the variance
private.vals <- LaplaceMechanism(f(D), epsilon, sensitivities,</pre>
                                  alloc.proportions = c(0.25, 0.75))
cat("Privacy preserving values: ", private.vals, "\nTrue values: ", f(D))
```

#> Privacy preserving values: 7.58841 1.652268

#> True values: 7.61271 2.036525

B.2 Gaussian Mechanism

Another popular mechanism for DP implemented in DPpack is the Gaussian mechanism. This mechanism can be used to provide either (ϵ, δ) approximate DP (Dwork et al., 2006a) or (ϵ, δ) probabilistic DP (Machanavajjhala et al., 2008).

Definition 10 (Gaussian mechanism) (Dwork et al., 2006a) Let D be a sensitive database. Let f be a given function with ℓ_2 -global sensitivity $\Delta_{2,f}$ and range \mathbb{R}^n . The Gaussian mechanism is defined to be

$$\mathcal{M}_G(D, f, \epsilon, \delta) = f(D) + \mathbf{e},$$
 (5)

where $\mathbf{e} = (e_1, \dots, e_n)^T$ and e_i is drawn independently from $\mathcal{N}(0, \sigma^2)$. In the case that $\epsilon \in (0, 1)$ and

$$\sigma \ge c\Delta_{2,f}/\epsilon \tag{6}$$

for a constant c such that $c^2 > 2\log(1.25/\delta)$, this mechanism was proven to satisfy approximate (ϵ, δ) -DP (Dwork et al., 2006a). Additionally, when

$$\sigma \ge (2\epsilon)^{-1} \Delta_{2,f} \left(\sqrt{(\Phi^{-1}(\delta/2))^2 + 2\epsilon} - \Phi^{-1}(\delta/2) \right), \tag{7}$$

where Φ is the CDF of the standard normal distribution, this mechanism was proven to satisfy (ϵ, δ) probabilistic DP (Liu, 2019a).

Note the requirement that $\epsilon < 1$ for approximate DP, which is not required for the Gaussian mechanism to satisfy probabilistic DP. It is also worth highlighting that the Laplace mechanism requires ℓ_1 -sensitivity, while the Gaussian mechanism requires ℓ_2 -sensitivity. If f is scalar-valued, $\Delta_{1,f} = \Delta_{2,f}$, but they are generally different for vector-valued f except in some special cases.

The GaussianMechanism function in DPpack implements the Gaussian mechanism by adding Gaussian noise to a given scalar (or vector) of observed statistic(s) according to specified values of ϵ , δ , and ℓ_2 -global sensitivity. It releases a scalar (or vector) satisfying either (ϵ, δ) approximate DP if the type.DP argument is 'aDP', or (ϵ, δ) probabilistic DP if the type.DP argument is 'pDP'. Global sensitivity calculated based either on bounded or unbounded neighboring datasets can be used.

We use the same example as for the Laplace mechanism to demonstrate the Gaussian mechanism for (ϵ, δ) approximate DP and (ϵ, δ) probabilistic DP for a sample mean. Consider again a sensitive dataset of n = 100 elements drawn uniformly from the range $[c_0, c_1] = [5, 10]$. Since the mean is a scalar in this case, the ℓ_2 -global sensitivity is equal to the ℓ_1 -global sensitivity, which is $(c_1 - c_0)/n = 0.05$ (Liu, 2019b).

```
# Simulate a dataset
n <- 100
c0 <- 5
c1 <- 10
D \leftarrow runif(n, c0, c1)
# Privacy budget
epsilon <- 0.9 # eps must be in (0, 1) for approximate DP
delta <- 0.01
sensitivity <- (c1-c0)/n
# Approximate differential privacy
private.approx <- GaussianMechanism(mean(D), epsilon,</pre>
                                     delta, sensitivity)
cat("Privacy-preserving mean (approximate): ", private.approx,
    "\nTrue mean: ", mean(D))
#> Privacy preserving mean (approximate): 7.426412
#> True mean: 7.170852
# Probabilistic differential privacy
private.prob <- GaussianMechanism(mean(D), epsilon, delta,</pre>
                                   sensitivity, type.DP = 'pDP')
cat("Privacy preserving mean (probabilistic): ", private.prob,
    "\nTrue mean: ", mean(D))
#> Privacy-preserving mean (probabilistic): 7.018747
#> True mean: 7.170852
```

The GaussianMechanism function can also be used to release privacy-preserving multi-dimensional statistics analogously to the LaplaceMechanism function with only one difference. If we again consider $\mathbf{f}(D) = (f_1(D), \dots, f_n(D))$ to be the multi-dimensional statistics of interest, then $\Delta_{2,\mathbf{f}}$ for the Gaussian mechanism is computed as $\Delta_{2,\mathbf{f}} = \sqrt{\sum_{i=1}^n \Delta_{2,f_i}^2}$ by default. If desired, users can specify their own privacy budget allocation (which applies to both ϵ and δ) using the alloc.proportions argument.

B.3 Exponential Mechanism

The third privacy-preserving mechanism implemented in DPpack is the exponential mechanism, developed in McSherry and Talwar (2007). This mechanism is preferred for situations where it is not possible to inject numerical noise (such as when the function output is categorical) or not appropriate to add noise directly to the result of a given function or algorithm. The exponential mechanism resolves this issue by assigning real-valued utilities to data/output pairs by specifying a utility function u.

An output is chosen and released with probability proportional to its corresponding utility.

Definition 11 (Exponential mechanism) (McSherry and Talwar, 2007) Let D be a sensitive database, f be a given function with range \mathcal{R} , and u be a utility function mapping data/output pairs to \mathbb{R} with ℓ_1 -global sensitivity $\Delta_{1,u}$. For output values $r \in \mathcal{R}$, the exponential mechanism achieving ϵ -DP is

$$\mathcal{M}_E(D, u, \mathcal{R}, \epsilon) = r \text{ with probability } \propto \exp\left(\frac{\epsilon u(D, r)}{2\Delta_{1, u}}\right).$$
 (8)

The Exponential Mechanism function in DPpack implements the exponential mechanism for differential privacy for a given sensitive dataset D and for finite \mathcal{R} . It takes as input a numeric vector utility representing the values of the utility function u for each $r \in \mathcal{R}$, as well as a privacy budget ϵ and the ℓ_1 -global sensitivity of u. It releases the index corresponding to the value $r \in \mathcal{R}$ randomly selected according to (8). Global sensitivity of u calculated based either on bounded or unbounded neighboring datasets can be used.

The ExponentialMechanism function also has two optional arguments: measure and candidates. Each of these arguments, if provided, should be of the same length as utility. If measure is given, the probabilities of selecting each value r are weighted according to the numeric values in measure before the value r is randomly chosen. If candidates is provided, ExponentialMechanism returns the value in candidates at the randomly chosen index rather than the index itself.

We demonstrate the ExponentialMechanism function with a toy example. Assume that a function f has range $\mathcal{R} = \{\text{`a', `b', `c', `d', `e'}\}$. Numerical noise cannot be added directly to the output of f due to the non-numeric nature of its range. Instead, we define a utility function u that yields the following values when applied to the sensitive dataset D and each element of \mathcal{R} , respectively: (0,1,2,1,0). Finally, assume the ℓ_1 -sensitivity of u is 1. We can use the ExponentialMechanism function to release an element of \mathcal{R} as follows.

```
candidates <- c('a', 'b', 'c', 'd', 'e') # Range of f
# Utility function values in same order as corresponding candidates
utility <- c(0, 1, 2, 1, 0)
epsilon <- 1 # Privacy budget
sensitivity <- 1

# Release privacy-preserving index of chosen candidate
idx <- ExponentialMechanism(utility, epsilon, sensitivity)
candidates[idx]
#> 'b'
```

Appendix C. Implementation of DP Descriptive Statistics

Descriptive statistics are popular and effective ways to summarize data. However, if these statistics are computed from a sensitive dataset and released directly, they could be susceptible to attacks that reveal private information about the individuals in the data, even if the dataset itself is not breached. Many of these statistics can be made differentially private through the application of one or more of the mechanisms discussed in the previous section. For ease of use, DPpack implements privacy-preserving versions of many descriptive statistics directly, utilizing the previously defined mechanisms under the hood.

C.1 Mean, Standard Deviation, Variance, and Covariance

The meanDP, sdDP, and varDP, functions can be used to release differentially private means, standard deviations, and variances respectively, calculated from a sensitive dataset. These functions all share the same set of arguments: a dataset x, a privacy budget eps (and possibly delta), as well as bounds on the attributes in the dataset lower.bound and upper.bound. Any values of x that happen to fall outside the bounds are clipped to the bounds before the mean is computed. These bounds are used to compute the global sensitivity of the desired statistic function based on proven values (Liu, 2019b).

By default, each function releases sanitized values satisfying eps-DP via the Laplace mechanism. The mechanism argument defaults to 'Laplace', indicating to use the Laplace mechanism. However, the output can be changed by modifying the value of some additional arguments and setting mechanism to 'Gaussian'. In this case, the delta argument must be positive. The type.DP argument can be either 'aDP' (default) or 'pDP' for satisfying (eps, delta) approximate DP and (eps, delta) probabilistic DP, respectively, and indicates the type of DP provided when the Gaussian mechanism is used. The which.sensitivity argument can be one of 'bounded' (default), 'unbounded', or 'both', indicating whether to release results satisfying bounded and/or unbounded DP. The following example demonstrates how these functions can be used.

```
# Simulate a dataset
D <- rnorm(500, mean=3, sd=2)
lower.bound <- -3 # 3 standard deviations below mean
upper.bound <- 9 # 3 standard deviations above mean</pre>
```

```
# Get mean satisfying bounded 1-differential privacy
private.mean <- meanDP(D, 1, lower.bound, upper.bound)</pre>
cat("Privacy preserving mean: ", private.mean, "\nTrue mean: ", mean(D))
#> Privacy preserving mean: 2.872637
#> True mean: 2.857334
# Get variance satisfying unbounded approximate (0.5, 0.01)-DP
private.var <- varDP(D, 0.5, lower.bound, upper.bound,</pre>
                     which.sensitivity = 'unbounded',
                     mechanism = 'Gaussian', delta = 0.01)
cat("Privacy preserving variance: ", private.var,
"\nTrue variance: ", var(D))
#> Privacy preserving variance: 3.276551
#> True variance: 4.380399
# Get std dev satisfying bounded probabilistic (0.5, 0.01)-DP
private.sd <- sdDP(D, 0.5, lower.bound, upper.bound,</pre>
                   mechanism='Gaussian', delta=0.01, type.DP='pDP')
cat("Privacy preserving standard deviation: ", private.sd,
    "\nTrue standard deviation: ", sd(D))
#> Privacy preserving standard deviation: 1.978296
#> True standard deviation: 2.09294
```

The pooledVarDP function in DPpack can be used to compute a differentially private pooled variance for multiple groups of data. The inputs are similar to those of meanDP, varDP, and sdDP with a few differences. First, the function accepts multiple numeric vectors representing different data groups, rather than a single dataset \mathbf{x} . The function uses provided lower and upper bounds on the entire collection of data to compute the sensitivity of the function based on the derived formulas in Liu (2019b), then releases a privacy preserving pooled variance of the entire collection of data based on provided privacy budget parameters. The formulas to compute the function's sensitivity require a value n_{max} representing the size of the largest provided dataset vector. If the value itself is sensitive, it can be approximated by setting the approximax argument to TRUE. The following examples demonstrate this function's use.

```
# Simulate three datasets from the same distribution
D1 <- rnorm(500, mean=3, sd=2)
D2 <- rnorm(200, mean=3, sd=2)
D3 <- rnorm(100, mean=3, sd=2)
lower.bound <- -3 # 3 standard deviations below mean
upper.bound <- 9 # 3 standard deviations above mean</pre>
```

DPpack also implements functions for privacy-preserving covariance and pooled covariance: covDP and pooledCovDP, which have similar arguments to the previously described functions. The covDP function accepts two numeric vector datasets x1 and x2, as well as upper and lower bounds on each of these two datasets individually. The function then returns the sanitized covariance between x1 and x2, based on provided privacy budget values and sensitivity computed using the bounds via the proven formula from Liu (2019b). The pooledCovDP function accepts any number of matrices. These matrices can have a variable number of rows, but must each have two columns. Two sets of bounds for the entire collection of data from each column must also be provided. The function releases a sanitized pooled covariance between the columns of the provided matrices based on the privacy budget, bounds, and the sensitivity computed according to the formula from Liu (2019b). Finally, pooledCovDP utilizes the value n_{max} in the computation of the sensitivity similar to the pooledVarDP function, so the approx.n.max argument is also present in this function and indicates the same thing. The following examples show the use of both of these functions.

```
# Simulate datasets
D1 <- sort(rnorm(500, mean=3, sd=2))
D2 <- sort(rnorm(500, mean=-1, sd=0.5))
lb1 <- -3 # 3 std devs below mean
lb2 <- -2.5 # 3 std devs below mean
ub1 <- 9 # 3 std devs above mean
ub2 <- .5 # 3 std devs above mean
# Covariance satisfying 1-differential privacy
private.cov <- covDP(D1, D2, 1, lb1, ub1, lb2, ub2)
cat("Privacy preserving covariance: ", private.cov,</pre>
```

C.2 Counting Functions

DPpack supports differentially private histograms and contingency tables via the functions histogramDP and tableDP, respectively. The functions release privacypreserving results based on given sensitive input data (in the same form required by the standard hist and table functions) and privacy budget parameters. Bounds on the dataset are not necessary as the global sensitivity for both functions is a constant independent of the data. As with many of the previously described functions, the guaranteed DP for both of these functions can be bounded or unbounded, as well as pure, approximate, or probabilistic depending on the values given for the which.sensitivity, mechanism, and type.DP arguments. Due to noise added to the typical output by both the Laplace and Gaussian mechanisms, it is possible that some counts obtained directly from the chosen mechanism are negative. By default, both of these functions coerce any such values to 0. However, if in a particular application it is preferred that negative counts be allowed, this can be done by setting the allow.negative argument to TRUE. The histogramDP function has two additional arguments: breaks and normalize. The breaks argument is equivalent to the argument with the same name in the standard hist function, while the normalize argument indicates whether the outputs should correspond to frequencies (if set to FALSE) or if they should be normalized so that the total area under the histogram is 1 (if set to TRUE).

The following examples demonstrate the proper use of the histogramDP and tableDP functions. Note that histogramDP returns an object similar to that returned by the standard hist function, but does not plot the histogram by default. Plotting the result is as easy as calling the plot function on the object released from histogramDP. The results are shown in Figure 1.

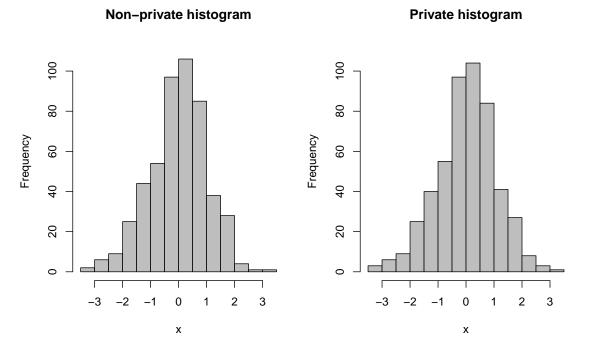


Figure 1: Original and privacy-preserving histograms from the histogram example.

We use a subset of variables from the Cars93 dataset in the MASS R package to demonstrate the generation of privacy-preserving contingency table. The results are shown in Table 1.

```
x <- MASS::Cars93$Type
y <- MASS::Cars93$Origin
z <- MASS::Cars93$AirBags
table(x, y, z) # Non-private contingency table
tableDP(x, y, z, eps=1) # Private contingency table</pre>
```

C.3 Quantiles

DPpack also implements differentially private quantiles and medians using the quantileDP and medianDP functions, respectively. The quantileDP function accepts a sensitive dataset as a numeric vector, a real number between 0 and 1 indicating the desired quantile, a single privacy budget parameter eps, and global bounds on the values in

Table 1: The outputs from the contingency table example. The privacy-preserving cell counts are listed with the original (non-private) values in parentheses.

Airbag	Origin	Type					
		Compact	Large	Midsize	Small	Sporty	Van
Driver	USA	0 (1)	4 (4)	7 (2)	0 (0)	1 (2)	0 (0)
& Passenger	non-USA	1 (1)	0 (0)	5(5)	0 (0)	1(1)	0 (0)
Driver only	USA	0 (2)	5 (7)	6 (5)	2(2)	11 (5)	3 (2)
	non-USA	7(7)	4(0)	4(6)	2(3)	3(3)	1(1)
None	USA	4 (4)	0 (0)	6 (3)	4 (5)	0 (1)	2 (3)
	non-USA	0 (1)	1 (0)	3 (1)	16 (11)	2(2)	7(3)

the dataset. It implements the private quantile algorithm from Smith (2011), which defines a utility function, and utilizes the exponential mechanism to release a quantile satisfying ϵ -DP based on the proven ℓ_1 -global sensitivity of the utility function (Smith, 2011; Gillenwater et al., 2021). The algorithm from Smith (2011) used in quantileDP uses the exponential mechanism to select a specific dataset value from the given dataset but releases a value drawn uniformly from the interval between the selected value and the subsequent value in ascending order. This means that the released value may not necessarily be a value present in the original dataset. If this behavior is not desirable for a certain application, the uniform.sampling argument can be set to FALSE, in which case the function releases the result of the exponential mechanism step directly without the uniform sampling step. The medianDP function is present in DPpack for convenience, and works identically to the quantileDP function with the quantile argument set to 0.5.

Both functions accept two additional arguments. The which.sensitivity argument operates analogously to the identically named argument in the other functions described in this section. The mechanism argument indicates which mechanism should be used to satisfy DP when running the function. Currently, only the exponential mechanism (the default for this argument) is supported for quantiileDP, but this argument was still included for symmetry with the other descriptive statistic functions, as well as for robustness in future versions of DPpack. The following examples show the use of both of these functions.

```
# Simulate a dataset
D <- rnorm(500)
lower.bound <- -3 # 3 standard deviations below mean
upper.bound <- 3 # 3 standard deviations above mean

quant <- 0.25
eps <- 1
# Get 25th quantile satisfying 1-differential privacy
private.quantile <- quantileDP(D, quant, eps, lower.bound, upper.bound)</pre>
```

Appendix D. Implementation of DP Statistical and ML Methods

DPpack implements privacy-preserving versions of some commonly used classification and regression models. Many such models can be formulated as empirical risk minimization (ERM) problems, which have been generally shown to have privacy-preserving counterparts under certain assumptions (Chaudhuri et al., 2011; Kifer et al., 2012). This section first provides a brief introduction to differentially private ERM algorithms and their necessary assumptions, then discusses the specific implementation in DPpack of logistic regression, support vector machines (SVM) and their extension to outcome weighted learning (OWL), and linear regression.

Each of the ERM-based methods implemented in *DPpack* requires the selection of various hyperparameter values that can impact model performance. A variety of techniques exist to tune these parameters, but many of these techniques threaten to leak private database information themselves. Thus, privacy-preserving hyperparameter tuning methods for both the classification and regression models are implemented in *DPpack*. These are also described in this section.

D.1 Empirical Risk Minimization

Assume that we have a set of n input-output pairs $(\mathbf{x}_i, y_i) \in (\mathcal{X}, \mathcal{Y})$ representing a sensitive training dataset \mathcal{D} . Additionally, define $\ell : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$ to be a loss function over pairs of values from the output space. In general, ERM attempts to produce an effective predictor function $f : \mathcal{X} \to \mathcal{Y}$ by minimizing the empirical risk

$$\frac{1}{n}\sum_{i=1}^{n}\ell(f_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i) = \frac{1}{n}\sum_{i=1}^{n}\ell_i(\boldsymbol{\theta}). \tag{9}$$

For the algorithms implemented in DPpack, we assume there exists a one-to-one mapping from a p-dimensional vector θ to f, where p is the length of \mathbf{x}_i (i.e. the

number of predictors). In order to mitigate overfitting, it is also common to introduce a regularizer function R. This produces the regularized ERM model

$$\frac{1}{n} \sum_{i=1}^{n} \ell(f_{\theta}(\mathbf{x}_i), y_i) + \frac{\gamma}{n} R(\boldsymbol{\theta}) = \ell_i(\boldsymbol{\theta}) + \frac{\gamma}{n} R(\boldsymbol{\theta}), \tag{10}$$

where $\ell_i(\boldsymbol{\theta}) = \ell(f_{\boldsymbol{\theta}}(\mathbf{x}_i), y_i)$ and γ is a tunable hyperparameter known as the regularization constant.

For binary classification problems, Chaudhuri et al. (2011) proved ϵ -DP can be satisfied for regularized ERM by two different algorithms if certain assumptions are met. The first algorithm is an *output* perturbation method, and the second is an *objective* perturbation method. We briefly mention the assumptions here and refer the interested reader to Chaudhuri et al. (2011) for more information and for the proofs. Both algorithms assume $\|\mathbf{x}_i\|_2 \leq 1$ for all i, that the regularizer R is differentiable and 1-strongly convex, and that the loss function ℓ is differentiable and convex with $\left|\frac{\partial}{\partial f}\ell(f,y)\right| \leq 1$ for all f and g. The objective perturbation algorithm has additional assumptions that R and ℓ are doubly differentiable and that $\left|\frac{\partial^2}{\partial f^2}\ell(f,y)\right| \leq c$ for some constant c.

The output perturbation method first solves Eqn (10) and then perturbs the output $\hat{\boldsymbol{\theta}}$ by adding noise determined by the values of n, ϵ , and γ . The objective function perturbation method adds random noise determined by n, ϵ , γ , and c directly to the objective function, then finds $\hat{\boldsymbol{\theta}}$ minimizing the perturbed function. This amounts to the privacy-preserving regularized ERM model

$$\frac{1}{n} \sum_{i=1}^{n} \ell_i(\boldsymbol{\theta}) + \frac{\gamma}{n} R(\boldsymbol{\theta}) + \frac{\Delta}{2n} ||\boldsymbol{\theta}||_2^2 + \frac{\mathbf{b}^T \boldsymbol{\theta}}{n}, \tag{11}$$

where **b** is the injected random noise and $\frac{\Delta}{2n} \|\boldsymbol{\theta}\|_2^2$ is an additional slack term necessary for DP via objective perturbation. Chaudhuri et al. (2011) show that objective perturbation generally provides better utility guarantees than output perturbation for the same privacy budget.

DPpack implements both algorithms using the EmpiricalRiskMinimizationDP.CMS R6 class. This class provides a general framework for running these algorithms, but is not intended to be utilized directly. Rather, it should be used as the parent class in an inheritance structure where the child class implements a specific realization of ERM for binary classification (i.e. logistic regression). Examples of this will be discussed in the subsequent sections.

For regression problems, Kifer et al. (2012) proposed a slightly different algorithm that satisfies DP for regularized ERM in Eqn (11). Assume that $\ell_i(\boldsymbol{\theta})$ is convex with a continuous Hessian, R is convex, and the following conditions hold for all \mathbf{x}_i, y_i and for all $\boldsymbol{\theta} \in \mathbb{F}$ (\mathbb{F} is a closed convex subset of \mathbb{R}^p): $\|\nabla_{\boldsymbol{\theta}}\ell_i(\boldsymbol{\theta})\|_2 \leq \zeta$ for some constant ζ , the eigenvalues of $\nabla^2_{\boldsymbol{\theta}}\ell_i(\boldsymbol{\theta})$ are bounded above by some constant λ , and the rank

of $\nabla^2_{\boldsymbol{\theta}}\ell_i(\boldsymbol{\theta})$ is at most one. Then the solutions $\hat{\boldsymbol{\theta}} \in \mathbb{F}$ from minimizing the perturbed objective Eqn (11) satisfy DP¹. The algorithm can be used to satisfy either ϵ -DP or approximate (ϵ, δ) -DP. If ϵ -DP is desired, the noise vector is drawn from a Gamma distribution depending on the values of ϵ and ζ , while if approximate DP is desired, the noise vector is drawn from a Gaussian distribution depending on the values of ϵ , and ζ . We emphasize that for this algorithm, the resulting value $\hat{\boldsymbol{\theta}}$ is restricted to the set \mathbb{F} .

DPpack implements this algorithm using the EmpiricalRiskMinimizationDP.KST R6 class. Similar to the EmpiricalRiskMinimizationDP.CMS class, this class provides a general framework for using the algorithm, but is not intended to be used directly. Child classes inheriting from this class and implementing a specific realization of the ERM for regression algorithm should be used instead. DPpack implements linear regression in this way, which will be discussed in the subsequent section on regression methods.

D.2 Logistic Regression

The two algorithms described in the previous section for privacy preserving ERM for binary classification can be applied to logistic regression. The loss function given a single observation is the cross entropy loss (or the negative log-likelihood)

$$\ell_i(\boldsymbol{\theta}) = -(y_i \log(f_{\boldsymbol{\theta}}(\mathbf{x}_i)) + (1 - y_i) \log(1 - f_{\boldsymbol{\theta}}(\mathbf{x}_i))), \tag{12}$$

where $f_{\theta}(\mathbf{x}_i) = (1 + e^{-\mathbf{x}_i \theta})^{-1}$ is the predicted value of y. The regularized objective function given data $\mathcal{D} = (\mathbf{x}, \mathbf{y})$ is

$$\frac{1}{n} \sum_{i=1}^{n} \left(y_i \log(1 + e^{-\mathbf{x}_i \boldsymbol{\theta}}) + (1 - y_i) \log(1 + e^{\mathbf{x}_i \boldsymbol{\theta}}) \right) + \frac{\gamma}{n} R(\boldsymbol{\theta}). \tag{13}$$

The loss function in Eqn (13) meets all of the regularity conditions necessary for both the output perturbation and the objective perturbation algorithms to satisfy DP^2 .

DPpack uses the LogisticRegressionDP R6 class to implement differentially private logistic regression in three steps using the EmpiricalRiskMinimizationDP.CMS framework that is based on the algorithm from Chaudhuri et al. (2011) ³.

^{1.} Though the objective perturbation algorithms from Chaudhuri et al. (2011) and Kifer et al. (2012) can both be written in the general form of Eqn (11), it is worth emphasizing that the former requires R to be 1-strongly convex, while the latter only requires R to be convex. The popular ℓ_1 regularizer is an example of a convex, but not 1-strongly convex regularizer.

^{2.} with c=1/4 for the objective perturbation algorithm (Chaudhuri et al., 2011). Also noted is that differentially private logistic regression was first proved outside of the ERM setting in Chaudhuri and Monteleoni (2009).

^{3.} Logistic regression can also be implemented using the algorithm from Kifer et al. (2012) implemented in EmpiricalRiskMinimizationDP.KST. While *DPpack* does not currently provide such an application in its current version, users are able to develop their own using the inheritance structure if desired.

The first step is to construct a LogisticRegressionDP object. The constructor for this class accepts a callable function regularizer for the regularizer function, a privacy budget parameter eps, a regularization constant gamma, and a string perturbation.method indicating whether to use the output or the objective perturbation algorithm. If the argument perturbation.method is set to 'output', the output perturbation algorithm is run. The user must ensure in this case that the regularizer meets the necessary requirements, namely that it is differentiable and 1-strongly convex. If perturbation.method is set to 'objective', the objective perturbation algorithm is run. In this case, the user must ensure that the regularizer is doubly differentiable and 1-strongly convex. One popular regularization function is the ℓ_2 regularizer $R(\theta) = \frac{1}{2} \|\theta\|_2^2$. For convenience, this regularization function (and its gradient) can be used by simply setting regularizer to '12'. An optional callable function regularizer.gr representing the gradient of the regularizer can also be provided.

After constructing a LogisticRegressionDP object, the second step is to train the model with a dataset. To do this, the user should call the \$fit method of the constructed object. This method accepts as arguments a sensitive dataset X and corresponding sensitive labels for each row y. It also accepts numeric vectors giving the global or public bounds on the data in each column of X. There are several points to note regarding \$fit. First, the method assumes that the binary labels provided by y are either 0 or 1. Second, both the output and objective perturbation algorithms assume that for each row \mathbf{x}_i of the input dataset we have $\|\mathbf{x}_i\|_2 \leq 1$. Given that this requirement is not met by most practical datasets, to allow for more realistic datasets to train the model, the \$fit method utilizes the provided upper and lower bounds on the columns of X to pre-process and scale the values of X in such a way that this constraint is met. The privacy-preserving algorithm is then run, producing differentially private coefficients for the scaled dataset. After the private coefficients are generated, these are then post-processed and un-scaled before being stored as the object attribute \$coeff, so that the stored coefficients correspond to the original data. Because both the pre-processing and the post-processing steps rely solely on the global or public bounds, DP is maintained by the post-processing theorem.

Specifically, X is pre-processed as follows. First, the largest in absolute value of the upper and lower bounds on each column are used to scale each column individually such that the largest value in each column is at most 1 in absolute value. Second, each value in X is divided by \sqrt{p} , the square root of the number of predictors of X. These two scalings ensure that each row of X satisfies the necessary constraints for DP. After training, the post-processing of the private coefficients is then accomplished by dividing each element of the trained vector by the same value used to scale the corresponding column individually in the pre-processing step, then dividing the entire vector by \sqrt{p} .

The original privacy-preserving ERM algorithms assume there is no bias term present in the predictor function. If a bias term is necessary, this issue can be partially

circumvented by prepending a column of 1s to X before fitting the model. In this case, the first element of the fitted vector \$coeff is essentially the bias term. The \$fit method does this when the add.bias argument is set to TRUE. We caution that adding a column of 1s to X results in an additional column that must be scaled in the pre-processing step, and we recommend not using a bias term if at all possible.

After training the model, the third and final step is to release the trained coefficients or to use them to predict the labels of new datapoints. The privacy-preserving coefficients are stored in the attribute \$coeff, which can be directly released without violating privacy guarantees. Alternatively, the \$predict method can be used. This method accepts a set of data X of the same form (i.e. dimensions, variable order, etc.) as the one provided to the \$fit method, as well as boolean add.bias and boolean raw.value arguments. The method then returns a matrix of predicted values corresponding to each row of X based on the logistic regression predictor function f_{θ} and the trained and stored coefficients \$coeff. The add.bias argument should be set to the same value as the identically named argument was when the \$fit method was called. The raw.value argument is used to indicate whether the returned matrix should consist of the raw scores from the logistic regression predictor function (i.e. real numbers between 0 and 1), or whether it should consist of predicted labels for the rows (i.e. 0 or 1 values) obtained by rounding the scores.

The following example shows the usage of the LogisticRegressionDP class on a 2-dimensional toy dataset.

```
# Simulate train dataset X and y, and test dataset Xtest and ytest
N <- 200
K <- 2
X <- data.frame()</pre>
y <- data.frame()</pre>
for (j in (1:K)){
  t \leftarrow seq(-.25, .25, length.out = N)
  if (j==1) m <- rnorm(N, -.2, .1)
  if (j==2) m <- rnorm(N, .2, .1)
  Xtemp \leftarrow data.frame(x1 = 3*t , x2 = m - t)
  ytemp <- data.frame(matrix(j-1, N, 1))</pre>
  X <- rbind(X, Xtemp)
  y <- rbind(y, ytemp)
}
# Bounds for X based on construction
upper.bounds <- c( 1, 1)
lower.bounds <-c(-1,-1)
# Train-test split
Xtest <- X[seq(1,(N*K),10),]
ytest \leftarrow y[seq(1,(N*K),10),,drop=FALSE]
```

```
X \leftarrow X[-seq(1,(N*K),10),]
y \leftarrow y[-seq(1,(N*K),10),,drop=FALSE]
# Construct object for logistic regression
regularizer <- function(coeff) coeff%*%coeff/2
regularizer.gr <- function(coeff) coeff</pre>
eps <- 1
gamma <- 0.1
lrdp <- LogisticRegressionDP$new(regularizer, eps, gamma,</pre>
                                    regularizer.gr = regularizer.gr)
# Fit with data
lrdp$fit(X, y, upper.bounds, lower.bounds) # No bias term
lrdp$coeff # Gets private coefficients
#> 1.449110 5.562798
# Predict new data points
predicted.y <- lrdp$predict(Xtest)</pre>
n.errors <- sum(predicted.y!=ytest)</pre>
```

D.3 Support Vector Machine (SVM)

The privacy-preserving binary classification ERM algorithms can also be applied to linear and nonlinear SVM. For notational simplicity, we let $\{-1,1\}$ be the binary labels for y when defining loss functions in SVM; for the implementation, for consistency with the LogisticRegressionDP class, we require y in the input dataset to be coded in $\{0,1\}$.

For linear SVM, the loss function given a single observation is the hinge loss

$$\ell_i(\boldsymbol{\theta}) = \max(0, 1 - y_i f_{\boldsymbol{\theta}}(\mathbf{x}_i)), \tag{14}$$

where $f_{\theta}(\mathbf{x}_i) = \mathbf{x}_i \boldsymbol{\theta}$ is the predicted value of y. The regularized objective function given data $\mathcal{D} = (\mathbf{x}, \mathbf{y})$ is

$$\frac{1}{n}\sum_{i=1}^{n}\max(0, 1 - y_i \mathbf{x}_i \boldsymbol{\theta}) + \frac{\gamma}{n}R(\boldsymbol{\theta}). \tag{15}$$

Unfortunately, the hinge loss is not differentiable everywhere and therefore does not satisfy the requirements for privacy-preserving ERM. One solution to this (used by DPpack) is to use the smooth Huber loss approximation to the hinge loss (Chapelle, 2007) defined by

$$\ell_{\text{Huber}}(z) = \begin{cases} 0, & \text{if } z > 1 + h \\ \frac{1}{4h}(1 + h - z)^2, & \text{if } |1 - z| \le h \\ 1 - z, & \text{if } z < 1 - h \end{cases}$$
 (16)

for a given Huber loss parameter h. Figure 2 shows a comparison between the Huber loss and the hinge loss for various values of h. For linear SVM, the described predictor function and the Huber loss meet all of the requirements necessary for both the output perturbation and the objective perturbation algorithms for DP^4 .

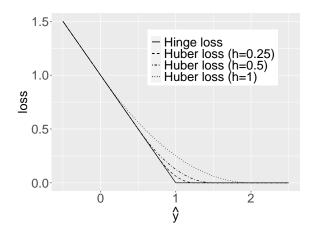


Figure 2: Comparison between Huber and hinge loss assuming y = 1.

Linear SVM implicitly assumes that the given dataset is (at least approximately) linearly separable. When this is not the case, nonlinear SVM is a better choice. Intuitively, nonlinear SVM first maps the potentially linearly non-separable input data in the original space to a higher dimension in which the data is linearly separable, then uses linear SVM in the higher-dimensional space. Performing this mapping directly suffers from the curse of dimensionality, and computations on the higher-dimensional dataset quickly become prohibitively expensive. For that reason, the kernel trick is used so that SVM can be applied easily in practice. Briefly, rather than explicitly transforming the data to the higher-dimensional space, the kernel trick utilizes a kernel function k to produce a similarity score between two datapoints in the original dimension. This is more computationally efficient than performing computations in the higher-dimensional space. One popular kernel function is the Gaussian or radial kernel

$$k(\mathbf{x}, \mathbf{x}') = \exp\left(-\beta \|\mathbf{x} - \mathbf{x}'\|_{2}^{2}\right),\tag{17}$$

where β is a Gaussian kernel hyperparameter and equals to p^{-1} by default. The optimized predictor function at \mathbf{x} is a linear combination of kernel functions

$$\hat{y} = f(\mathbf{x}) = \sum_{i=1}^{n} a_i k(\mathbf{x}_i, \mathbf{x}), \tag{18}$$

where \mathbf{x}_i is the input of the original dataset.

When there is no privacy concern, one may release estimated a_i and the observed input \mathbf{x}_i , which can be plugged in Eqn (18) to predict the label of a given data point

^{4.} with c = 1/2h for the objective perturbation algorithm (Chaudhuri et al., 2011)

 \mathbf{x} . For privacy-preserving analysis, this practice poses problems due to the direct release of \mathbf{x}_i . Chaudhuri et al. (2011) avoids this issue by using random projections to approximate the desired kernel function. Specifically, the algorithm first randomly samples D vectors \mathbf{z}_j based on the desired kernel function according to the approximation technique from Rahimi and Recht (2007, 2008). The algorithm then produces D-dimensional data \mathbf{v}_i (using \mathbf{z}_j) for each \mathbf{x}_i , representing an approximate projection of each of the original \mathbf{x}_i onto the kernel space. Finally, the differentially private ERM algorithm for linear SVM is run on the new dataset (\mathbf{v}_i, y_i) . The vectors \mathbf{z}_j are not functions of the observed dataset, meaning the privacy-preserving linear SVM algorithm satisfies ϵ - DP for a given ϵ . Therefore, this algorithm also satisfies ϵ -DP when releasing the sampled vectors \mathbf{z}_j and the estimated coefficients from the linear SVM.

DPpack implements differentially private SVM via the symDP R6 class using the framework provided by EmpiricalRiskMinimizationDP.CMS. Like the logistic regression model, using the SVM model requires three steps. The first step is to construct an symDP object. The constructor for this class accepts a callable function regularizer for the regularizer function, a privacy budget parameter eps, a regularization constant gamma, a string perturbation.method indicating whether to use the output or the objective perturbation algorithm, a string kernel for the kernel used in SVM, and a constant huber.h defining the h value in the Huber loss in Eqn (16). Setting regularizer to '12' uses the ℓ_2 regularization function and its gradient. The perturbation.method argument operates identically to the argument of the same name used in constructing a LogisticRegressionDP object, and expects the user to verify the same requirements for the regularizer and regularizer.gr functions. The kernel argument can be set to either 'linear' or 'Gaussian'. In the former case, linear SVM is run using the specified predictor function and the Huber loss; for the latter, the Gaussian kernel approximation algorithm is run, where the constructor also requires the specification of two additional arguments: D to indicate the dimensionality of the projection dataset \mathbf{v}_i and kernel.param to indicate the value of β in Eqn (17).

After constructing the svmDP object, the second step is to train the model on a dataset. Users should call the \$fit method of the constructed object. This method accepts input data X with labels y, numeric vector global or public bounds for each column of X, and a boolean add.bias indicating whether to add a column of 1s to X to act as a bias variable⁵. If kernel is set to 'linear' when the object is constructed in the first step, the method finds $\hat{\boldsymbol{\theta}}$ satisfying eps-DP, where eps is the privacy budget provided when the object is initialized. If kernel is set to 'Gaussian', the method first converts X to the D-dimensional new dataset V, then finds $\hat{\boldsymbol{\theta}}$ corresponding to V satisfying eps-DP.

^{5.} The add.bias argument functions analogously to the respective argument for the LogisticRegressionDP class, and we again recommend not using a bias term if at all possible.

For linear SVM, the same pre-processing of X using the provided bounds on its columns and subsequent post-processing of the private coefficients is performed. The results are again stored in the \$coeff attribute. For Gaussian kernel nonlinear SVM, the mapping from X to V ensures that each row \mathbf{v}_i satisfies $\|\mathbf{v}_i\|_2 \leq 1$ regardless of the values of \mathbf{x}_i . For this reason, no pre-processing of X is needed. In fact, providing bounds on the columns of X when calling the \$fit method is unnecessary for the Gaussian kernel approximation.

The third and final step is to release the estimated coefficients \$coeff⁶ or use them to predict the labels given a set of new datapoints. For the latter, the \$predict method can be used, which accepts input X of the same form (i.e. dimensions, variable order, etc.) as the one provided to the \$fit method, as well as boolean add.bias and boolean raw.value arguments⁷. It returns a matrix of predicted values corresponding to each row of X.

The following example shows how to use the symDP class.

```
# Simulate training dataset X and y, and testing dataset Xtest and ytest
N < -400
X <- data.frame()</pre>
y <- data.frame()</pre>
for (i in (1:N)){
  if (sum(Xtemp^2)<.15) ytemp <- data.frame(y=0)</pre>
  else ytemp <- data.frame(y=1)</pre>
  X <- rbind(X, Xtemp)</pre>
  y <- rbind(y, ytemp)
# Train-test split
Xtest \leftarrow X[seq(1,N,10),]
ytest <- y[seq(1,N,10),,drop=FALSE]</pre>
X \leftarrow X[-seq(1,N,10),]
y \leftarrow y[-seq(1,N,10),,drop=FALSE]
# Construct object for SVM
regularizer <- '12'
eps <- 1
gamma <- 0.1
kernel <- 'Gaussian'
D <- 20
```

^{6.} For Gaussian kernel SVM, the dimension conversion function, \$XtoV, can also be released in conjunction with \$coeff.

^{7.} The add.bias and raw.value arguments operate analogously to the respective arguments for LogisticRegressionDP.

```
svmdp <- svmDP$new(regularizer, eps, gamma, kernel=kernel, D=D)

# Fit with data (note no bounds necessary because kernel='Gaussian')
svmdp$fit(X, y) # No bias term

# Predict new data points
predicted.y <- svmdp$predict(Xtest)
n.errors <- sum(predicted.y!=ytest)</pre>
```

D.4 Outcome Weighted Learning

Outcome weighted learning (OWL) (Zhao et al., 2012) is a technique used for determining individualized treatment rules (ITRs), and can be categorized broadly as a method for causal inference ML. The primary goal of ITR is to derive a treatment assignment function that maps an individual's set of characteristics to a treatment that maximizes the expected benefit to that individual. A significant strength of OWL is its ability to tailor treatment assignments in response to individual characteristics, rather than using a one-size-fits-all approach. Its development was motivated by developing techniques for precision medicine (Council et al., 2011; Collins and Varmus, 2015) in randomized clinical trials, though other potential applications include personalized advertising (Wang et al., 2015a; Sun et al., 2015), and recommender systems (Schnabel et al., 2016; Lada et al., 2019). The original ITR problem considered in Zhao et al. (2012) is to find the treatment assignment function T by maximizing the expected treatment benefit $E[\frac{B}{P(A|\mathbf{x})}\mathbb{I}(A=T(\mathbf{x}))]$, where A and B are random variables representing the randomly assigned treatment and observed benefit, respectively, P is the conditional probability function, and \mathbb{I} is the indicator function.

The key insight in Zhao et al. (2012) that produces the OWL framework is that the expected benefit problem can be reformulated as the weighted SVM problem

$$\frac{1}{n} \sum_{i=1}^{n} \frac{B_i}{P(A_i | \mathbf{x}_i)} \max(0, 1 - A_i \mathbf{x}_i \boldsymbol{\theta}) + \frac{\gamma}{n} \|\boldsymbol{\theta}\|_2,$$
(19)

where $\mathcal{D}=(\mathbf{x},\mathbf{A},\mathbf{B})$. It is straightforward to see that this is a generalization of the standard SVM case to the case where individual observations are unevenly weighted according to the weights $w_i=\frac{B_i}{P(A_i|\mathbf{x}_i)}$. Giddens et al. (2023) showed that weighted ERM in general can be made to satisfy ϵ -DP via output perturbation, as long as a global bound on the weights is provided. In order to incorporate OWL into DPpack, we generally implement DP weighted ERM through the general WeightedERMDP.CMS class. The svmDP class described in the previous section inherits from WeightedERMDP.CMS, which permit users to provide unequal observation weights such as those found in OWL.

The following example shows how to use the svmDP class with weighted observations.

```
# Simulate train dataset X and y, and test dataset Xtest and ytest
N < -200
K <- 2
X <- data.frame()</pre>
y <- data.frame()</pre>
for (j in (1:K)){
     t \leftarrow seq(-.25, .25, length.out = N)
     if (j==1) m <- rnorm(N, -.2, .1)
     if (j==2) m <- rnorm(N, .2, .1)
     X \leftarrow x^2 = x^2 + x^2 = 
     ytemp <- data.frame(matrix(j-1, N, 1))</pre>
     X <- rbind(X, Xtemp)</pre>
     y <- rbind(y, ytemp)
# Bounds for X based on construction
upper.bounds <- c(1, 1)
lower.bounds \leftarrow c(-1,-1)
# Train-test split
X = X[seq(1,(N*K),10),]
ytest <- y[seq(1,(N*K),10),,drop=FALSE]</pre>
X \leftarrow X[-seq(1,(N*K),10),]
y \leftarrow y[-seq(1,(N*K),10),,drop=FALSE]
# Weights
weights <- rep(1, nrow(y)) # Uniform weighting</pre>
weights[nrow(y)] <- 0.5 # Half weight for last observation</pre>
wub <- 1 # Upper bound on weights</pre>
# Construct object for logistic regression
regularizer <- function(coeff) coeff%*%coeff/2
regularizer.gr <- function(coeff) coeff</pre>
eps <- 1
gamma <- 0.1
perturbation.method <- 'output'</pre>
svmdp <- svmDP$new(regularizer, eps, gamma, perturbation.method,</pre>
                                                   regularizer.gr = regularizer.gr)
# Fit with data
svmdp$fit(X, y, upper.bounds, lower.bounds, weights=weights,
                            weights.upper.bound=wub)
svmdp$coeff # Gets private coefficients
```

#> 1.547518 13.456029

```
# Predict new data points
predicted.y <- svmdp$predict(Xtest)
n.errors <- sum(predicted.y!=ytest)</pre>
```

D.5 Linear Regression

The differentially private ERM algorithm for regression problems can be applied to linear regression. The loss function given a single observation is the squared error

$$\ell_i(\boldsymbol{\theta}) = \frac{(f_{\boldsymbol{\theta}}(\mathbf{x}_i) - y_i)^2}{2},\tag{20}$$

where the $f_{\theta}(\mathbf{x}_i) = \mathbf{x}_i \boldsymbol{\theta}$ is the predicted value of y. The regularized objective function given data $\mathcal{D} = (\mathbf{x}, \mathbf{y})$ is

$$\frac{1}{n} \sum_{i=1}^{n} \frac{(\mathbf{x}_i \boldsymbol{\theta} - y_i)^2}{2} + \frac{\gamma}{n} R(\boldsymbol{\theta}). \tag{21}$$

In order to satisfy all of the assumptions needed to ensure privacy for the ERM algorithm, we must assume that each \mathbf{x}_i , as well as the coefficient vector $\boldsymbol{\theta}$ have a bounded ℓ_2 norm. For the purposes of DPpack, we choose to bound \mathbf{x}_i by $\|\mathbf{x}_i\|_2 \leq \sqrt{p}$ and the coefficient vector by $\|\boldsymbol{\theta}\|_2 \leq \sqrt{p}$, where p is the number of predictors, following Kifer et al. (2012). This implies that each value of the output y is contained in [-p, p] automatically. With these assumptions, the conditions for the differentially private regression ERM algorithm are satisfied for linear regression with parameters $\mathbb{F} = \{\boldsymbol{\theta} \in \mathbb{R}^p : \|\boldsymbol{\theta}\|_2 \leq \sqrt{p}\}, \zeta = 2p^{3/2}$, and $\lambda = p$.

DPpack implements differentially private linear regression via the LinearRegressionDP R6 class using the framework of EmpiricalRiskMinimizationDP.KST. Similar to the classification models, this is done in three steps: constructing a LinearRegressionDP object, training the model by calling the \$fit method of the constructed object, and releasing the trained coefficients \$coeff or using them for prediction via \$predict. The arguments and specification of the construction and prediction steps are similar to the those for LogisticRegressionDP and svmDP, so we refer the reader to those sections for explanations of the arguments.

There are a few minor differences in the training step via the fit method when compared to LogisticRegressionDP and svmDP. First, the arguments lower.bounds and upper.bounds should be vectors representing the global or public bounds on both the columns of X and the values of y. If X has n columns, then each vector of bounds should be of length n+1. The first n elements of the vectors correspond to the bounds on the n columns of X, and are in the same order as the respective columns. The last element of the vectors corresponds to the bounds on the values in y. Similar to the training step for LogisticRegressionDP and svmDP, these bounds are used to

pre-process X and y so that they satisfy the necessary constraints for privacy. The pre-processing/post-processing is essentially the same for LinearRegressionDP as it is for the classification methods, except that y is also shifted (and the resulting coefficients unshifted) to be centered at 0 if add.bias is set to TRUE.

The following example shows how to use the LinearRegressionDP class.

```
# Simulate an example dataset
n <- 500
X <- data.frame(X=seq(-1,1,length.out = n))</pre>
true.theta <-c(-.3,.5) # First element is bias term
p <- length(true.theta)</pre>
y <- true.theta[1] + as.matrix(X)%*%true.theta[2:p] + rnorm(n=n,sd=.1)
# Bounds based on construction. We assume y has values between -p and p
upper.bounds <- c(1, p) # Bounds for X and y
lower.bounds <- c(-1, -p) # Bounds for X and y
# Construct object for linear regression
regularizer <- '12'
eps <- 1
delta <- 0.01 # Indicates to use approximate (1,0.01)-DP
gamma <- 1
lrdp <- LinearRegressionDP$new('12', eps, delta, gamma)</pre>
# Fit with data
lrdp$fit(X, y, upper.bounds, lower.bounds, add.bias=TRUE)
lrdp$coeff # Gets private coefficients
#> -0.3812353 0.3704237
# Predict new data points
Xtest \leftarrow data.frame(X=c(-.5, -.25, .1, .4))
predicted.y <- lrdp$predict(Xtest, add.bias=TRUE)</pre>
```

D.6 Hyperparameter Tuning

Model training often involves the selection of hyperparameter values such as, for example, the constant γ for the regularizer in Eqn (10) or (11). Poorly selected values for these hyperparameters can result in models with poor performance. Often, hyperparameter selection relies on the observed dataset itself, resulting in privacy costs in the setting of privacy-preserving analysis. Chaudhuri et al. (2011) presents an algorithm for privacy-preserving hyperparameter tuning based on the exponential mechanism, which is implemented in DPpack.

For binary classification models, differentially private hyperparameter tuning is realized in DPpack via the tune_classification_model function. It accepts as inputs a

list of model objects models of the same type⁸, each constructed with a different value from the set of potential hyperparameter values, observed input X, labels y, vectors representing global or public bounds on the columns of X, and a boolean add.bias argument. The function splits X and y into m+1 equally sized sub-datasets, where m is the number of candidate models, and trains each model on one of the sub-datasets. The negative of the misclassification frequency by each model on the labels of the final sub-dataset is used as the utility function u for the exponential mechanism. It can be easily seen that the ℓ_1 -global sensitivity of u is $\Delta_{1,u}=1$. The exponential mechanism is used to select and return one of the trained models provided with ϵ -DP.

For example, assume one wishes to select a constant for the l_2 regularizer from the set $\{100, 1, 0.0001\}$ for privacy-preserving logistic regression. To do this, three objects from the LogisticRegressionDP class are constructed with the same privacy budget parameter eps and initialized with one of the three constant values. The three model objects are then passed into the tuning function, and the exponential mechanism returns one of them. The remaining arguments for the tuning function, X, y, upper.bounds, lower.bounds, and add.bias, should be given values according to their respective descriptions in the \$fit method of the corresponding R6 class being used. An example of this situation follows.

```
# Simulate a training dataset (X, y), and testing dataset (Xtest, ytest)
N <- 200
K <- 2
X <- data.frame()</pre>
y <- data.frame()</pre>
for (j in (1:K)){
          t \leftarrow seq(-.25,.25,length.out = N)
          if (j==1) m <- rnorm(N,-.2,.1)
          if (j==2) m <- rnorm(N, .2,.1)
          X \leftarrow x^2 = x^2 + x^2 = 
          ytemp <- data.frame(matrix(j-1, N, 1))</pre>
          X <- rbind(X, Xtemp)</pre>
          y <- rbind(y, ytemp)
}
# Bounds for X based on construction
upper.bounds <- c(1, 1)
lower.bounds <-c(-1,-1)
# Train-test split
Xtest <- X[seq(1,(N*K),10),]
ytest <- y[seq(1,(N*K),10),,drop=FALSE]</pre>
```

^{8.} Such as a list of LogisticRegressionDP objects. Each model object must have the same privacy budget parameters.

```
X \leftarrow X[-seq(1,(N*K),10),]
y \leftarrow y[-seq(1,(N*K),10),,drop=FALSE]
y <- as.matrix(y)</pre>
# Grid of gamma values for tuning logistic regression model
grid.search <- c(100, 1, .0001)
# Construct objects for logistic regression parameter tuning
eps <- 1 # Privacy budget should be the same for all models
lrdp1 <- LogisticRegressionDP$new("12", eps, grid.search[1])</pre>
lrdp2 <- LogisticRegressionDP$new("12", eps, grid.search[2])</pre>
lrdp3 <- LogisticRegressionDP$new("12", eps, grid.search[3])</pre>
models <- c(lrdp1, lrdp2, lrdp3)
# Tune using data and bounds for X based on its construction
tuned.model <- tune_classification_model(models, X, y,</pre>
                                            upper.bounds, lower.bounds)
tuned.model$gamma # Gives resulting selected hyperparameter
#> 0.0001
# tuned.model can be used in the same way as any
# LogisticRegressionDP model
predicted.y <- tuned.model$predict(Xtest)</pre>
n.errors <- sum(predicted.y!=ytest)</pre>
```

DPpack also implements differentially private hyperparameter tuning for linear regression via the tune_linear_regression_model function. This function was inspired by the binary classification hyperparameter tuning algorithm from Chaudhuri et al. (2011) as well as the feature selection algorithm for high-dimensional regression from Kifer et al. (2012). This function accepts the same input arguments as the tune_classification_model function, except that the models argument should be a list of constructed LinearRegressionDP objects with the same privacy budget parameters eps and delta. The function then splits the provided data X and y into m+1 equally sized sub-datasets, where m is the number of provided models, and trains each model on one of the sub-datasets. The negative of the square of the Euclidean distance between the predicted values and the true values for the remaining sub-dataset is defined to be the utility function u for each of the models, the ℓ_1 -global sensitivity for which is given in Theorem 12. Finally, the exponential mechanism is used to select and return one of the trained models provided with (eps, delta)-DP.

Theorem 12 Let c_0 and c_1 be the global or public lower and upper bounds, respectively, on the possible values of y_i . Let g be the linear regression model with coefficient parameters $\boldsymbol{\theta}$. For a dataset $\mathcal{D} = (\mathbf{x}_i, y_i)$ with n rows, define $u(\mathcal{D}, g) = \mathbf{x}_i$

 $-\sum_{i=1}^{n}(g(x_i)-y_i)^2$. The ℓ_1 -global sensitivity of u is given by

$$\Delta_{1,u} = (c_1 - c_0)^2. \tag{22}$$

Proof Let \mathcal{D}_1 and \mathcal{D}_2 be (bounded) neighboring datasets. Without loss of generality, assume they differ only in their first element and define $(x_1, y_1) \in \mathcal{D}_1$ and $(x'_1, y'_1) \in \mathcal{D}_2$. Then

$$\Delta_{1,u} = \max_{g} \max_{\mathcal{D}_1, \mathcal{D}_2} |u(\mathcal{D}_1, g) - u(\mathcal{D}_2, g)|$$

= $\max_{g} \max_{\mathcal{D}_1, \mathcal{D}_2} |(g(x_1) - y_1)^2 - (g(x_1') - y_1')^2|.$

Given that $(g(x_1) - y_1)^2 \ge 0$ and $(g(x_1') - y_1')^2 \ge 0$ for all g and for all \mathcal{D}_1 and \mathcal{D}_2 with $d(\mathcal{D}_1, \mathcal{D}_2) = 1$,

$$\Delta_{1,u} \le \max_{x_1,y_1} (g(x_1) - y_1)^2 = \max_{x_1,y_1} (x_1 \boldsymbol{\theta} - y_1)^2 = (c_1 - c_0)^2,$$

where we note the last step is a result of the assumptions made on the bounds of $||x_1||_2$, $||\boldsymbol{\theta}||_2$, and $|y_1|$ in order to ensure DP for linear regression. For unbounded DP, $\Delta_{1,u} = \max_g \max_{\mathcal{D}_1,\mathcal{D}_2} (g(x_1) - y_1)^2 = (c_1 - c_0)^2$, the same as in the bounded case.

Similar to the tune_classification_model function, the list of models provided to tune_linear_regression_model should be a list of objects constructed using the R6 class LinearRegressionDP with a different hyperparameter value and the same privacy budget parameters provided to each model. The remaining arguments for the tuning function, X, y, upper.bounds, lower.bounds, and add.bias, should be given values according to their respective descriptions in the \$fit method of the LinearRegressionDP class. An example of using the tuning function for the regularization constant for linear regression follows.

```
# Simulate an example dataset
n <- 500
X <- data.frame(X=seq(-1,1,length.out = n))
true.theta <- c(-.3,.5) # First element is bias term
p <- length(true.theta)
y <- true.theta[1] + as.matrix(X)%*%true.theta[2:p] + rnorm(n=n,sd=.1)
# Bounds for X and y based on their construction
upper.bounds <- c( 1, 2) # Bounds for X and y
lower.bounds <- c(-1,-2) # Bounds for X and y</pre>
# Grid of possible gamma values for tuning linear regression model
grid.search <- c(100, 1, .0001)
# Construct objects for logistic regression parameter tuning</pre>
```

```
# Privacy budget should be the same for all models
eps <- 1
delta <- 0.01
linrdp1 <- LinearRegressionDP$new("12", eps, delta, grid.search[1])</pre>
linrdp2 <- LinearRegressionDP$new("12", eps, delta, grid.search[2])</pre>
linrdp3 <- LinearRegressionDP$new("12", eps, delta, grid.search[3])</pre>
models <- c(linrdp1, linrdp2, linrdp3)</pre>
tuned.model <- tune_linear_regression_model(models, X, y, upper.bounds,</pre>
                                              lower.bounds, add.bias=TRUE)
tuned.model$gamma # Gives resulting selected hyperparameter
#> 100
# tuned.model result can be used the same as a trained
# LogisticRegressionDP model
tuned.model$coeff # Gives coefficients for tuned model
#> -0.5038190 0.2589978
# Simulate a test dataset for prediction
Xtest < -data.frame(X=c(-.5, -.25, .1, .4))
predicted.y <- tuned.model$predict(Xtest, add.bias=TRUE)</pre>
```

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