

# 3D Cuboid Labeling

Leo

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## 1 Introduction

Given an image, we want to recover the inherent 3D structure behind it. For simplicity, we often treat the scene as a cuboid. Such approximation is quite enough for applications such as environment map generation. To label each face of the cuboid, we should firstly set up the 3 axes, as Figure 1 shows.  $p_0$  is the intersection of the 3 axes and the vector  $\overrightarrow{p_0p_1}$ ,  $\overrightarrow{p_0p_2}$  and  $\overrightarrow{p_0p_3}$  indicate the direction of  $x$ ,  $y$  and  $z$ . So how do we get the 3D positions of these points based on those projected coordinates in 2D?

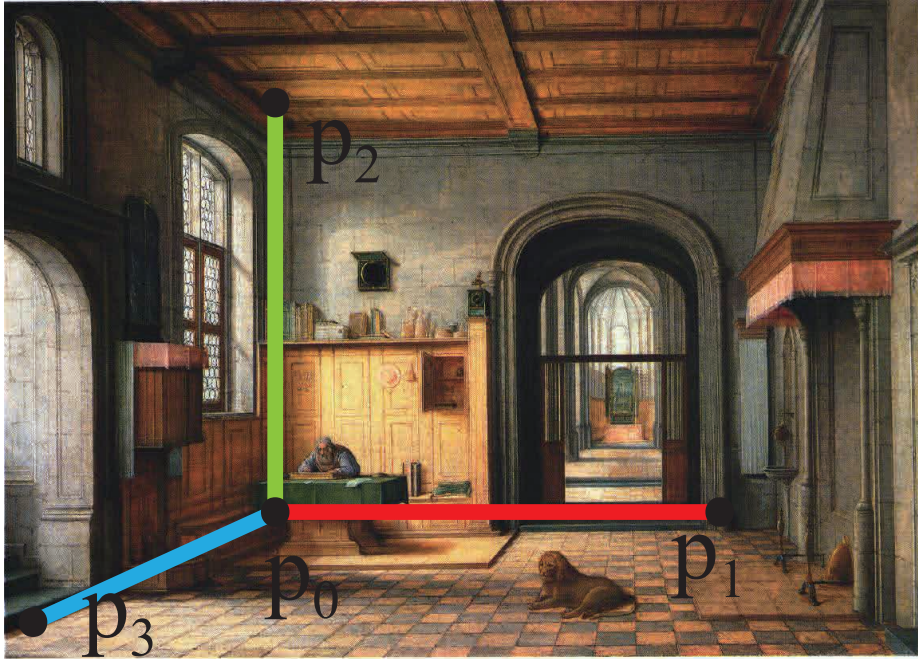


Figure 1: A image with labeled axes.

## 2 Inference

Assume the width and height of the image are  $w$  and  $h$  respectively. The focal length of the camera is  $f$ . Applying the projection transformation to a 3D point

$[X, Y, Z]$ , we get

$$\begin{bmatrix} 2f/w & 0 & 0 & 0 \\ 0 & 2f/h & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 2fX/w \\ 2fY/h \\ aZ + b \\ -Z \end{bmatrix} \sim \begin{bmatrix} -2fX/(wZ) \\ -2fY/(hZ) \\ -(aZ + b)/Z \\ 1 \end{bmatrix}, \quad (1)$$

where  $a = -\frac{F+N}{F-N}$ ,  $b = -\frac{2FN}{F-N}$ ,  $F$  and  $N$  are the near and far plane. The 2D position  $[u, v] = [-2fX/(wZ), -2fY/(hZ)]$  is the projected coordinate of  $[X, Y, Z]$ . The labeled 2D position  $[u, v]$  is known, so we have

$$X = (-\frac{w}{2f})uZ \quad (2)$$

$$Y = (-\frac{h}{2f})vZ. \quad (3)$$

Back to Figure 1, assume the 2D position of the labeled point is  $p_k = [u_k, v_k]$  and the corresponding 3D position is  $P_K = [X_k, Y_k, Z_k]$ . With  $\overrightarrow{p_0 p_1} \perp \overrightarrow{p_0 p_2}$ , we have

$$(X_1 - X_0)(X_2 - X_0) + (Y_1 - Y_0)(Y_2 - Y_0) + (Z_1 - Z_0)(Z_2 - Z_0) = 0. \quad (4)$$

Substituting Equation 2 and 3 into 4:

$$\begin{aligned} & (\frac{w}{2f})^2(u_1 Z_1 - u_0 Z_0)(u_2 Z_2 - u_0 Z_0) + \\ & (\frac{h}{2f})^2(v_1 Z_1 - v_0 Z_0)(v_2 Z_2 - v_0 Z_0) + \\ & (Z_1 - Z_0)(Z_2 - Z_0) = 0, \end{aligned} \quad (5)$$

or

$$\begin{aligned} & (\frac{w}{2f})^2(u_1 u_2 Z_1 Z_2 - u_0 u_2 Z_0 Z_2 - u_0 u_1 Z_0 Z_1 + u_0^2 Z_0^2) + \\ & (\frac{h}{2f})^2(v_1 v_2 Z_1 Z_2 - v_0 v_2 Z_0 Z_2 - v_0 v_1 Z_0 Z_1 + v_0^2 Z_0^2) + \\ & (Z_1 Z_2 - Z_0 Z_2 - Z_0 Z_1 + Z_0^2) = 0. \end{aligned} \quad (6)$$

Define  $C_{mn} = (\frac{w}{2f})^2 u_m u_n + (\frac{h}{2f})^2 v_m v_n + 1$  and substitute it to Equation 6:

$$C_{12} Z_1 Z_2 - C_{02} Z_0 Z_2 - C_{01} Z_0 Z_1 + C_{00} Z_0^2 = 0, \quad (7)$$

i.e.

$$(C_{12} Z_2 - C_{01} Z_0) Z_1 = (C_{02} Z_2 - C_{00} Z_0) Z_0. \quad (8)$$

Symmetrically, we also have

$$(C_{23} Z_3 - C_{02} Z_0) Z_2 = (C_{03} Z_3 - C_{00} Z_0) Z_0 \quad (9)$$

$$(C_{31} Z_1 - C_{03} Z_0) Z_3 = (C_{01} Z_1 - C_{00} Z_0) Z_0. \quad (10)$$

According to Equation 10,  $Z_3 = \frac{(C_{01} Z_1 - C_{00} Z_0) Z_0}{C_{31} Z_1 - C_{03} Z_0}$ . Substituting it to Equation 9, we can express  $Z_2$  using  $Z_0$  and  $Z_1$ . Then Substituting  $Z_2$  into Equation 8 (I

use Matlab for such tedious work), we get

$$\begin{aligned}
& (C_{01}^2 C_{23} - C_{01} C_{03} C_{12} - C_{01} C_{02} C_{31} + C_{00} C_{12} C_{31}) Z_1^2 + \\
& (C_{01} C_{02} C_{03} - C_{00} C_{01} C_{23} + C_{01} C_{02} C_{03} - C_{00} C_{01} C_{23}) Z_1 Z_0 + \\
& C_{00}^2 C_{23} - C_{00} C_{02} C_{03}) Z_0^2 = 0. \quad (11)
\end{aligned}$$

Solving such a quadratic equation, we get  $Z_1$ . Finally there are 2 sets of solution for Equation 8 - 10. We can choose either one.