3D Cuboid Labeling

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1 Introduction

Given an image, we want to recover the inherent 3D structure behind it. For simplicity, we often treat the scene as a cuboid. Such approximation is quite enough for applications such as environment map generation. To label each face of the cuboid, we should firstly set up the 3 axes, as Figure 1 shows. p_0 is the intersection of the 3 axes and the vector $\overrightarrow{p_0p_1}$, $\overrightarrow{p_0p_2}$ and $\overrightarrow{p_0p_3}$ indicate the direction of x, y and z. So how do we get the 3D positions of these points based on those projected coordinates in 2D?

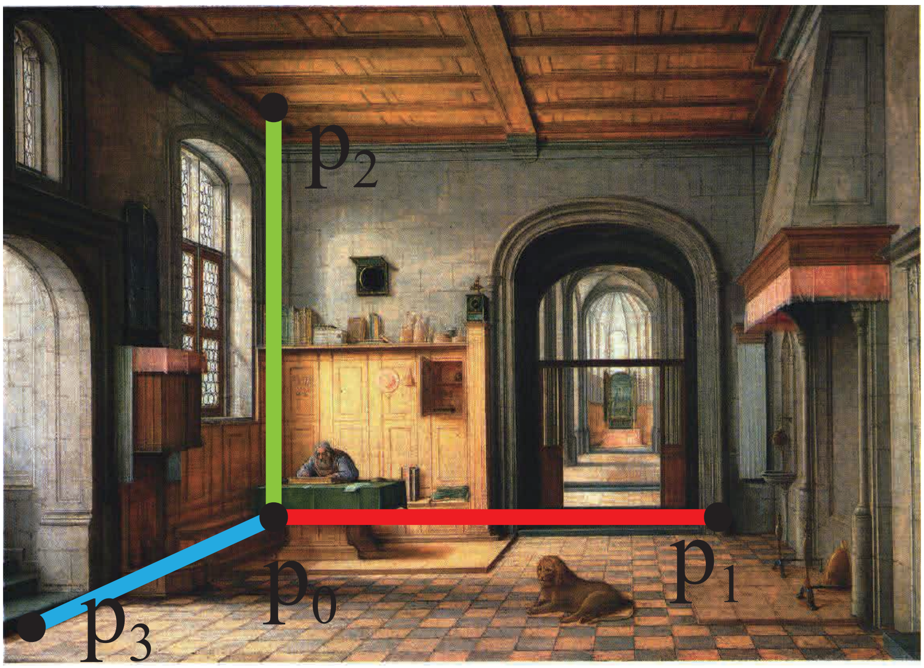

Figure 1: A image with labeled axes.

2 Inference

Assume the width and height of the image are w and h respectively. The focal length of the camera is f. Applying the projection transformation to a 3D point

$[X, Y, Z]$, we get

$$\begin{bmatrix} 2f/w & 0 & 0 & 0 \\ 0 & 2f/h & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 2fX/w \\ 2fY/h \\ aZ+b \\ -Z \end{bmatrix} \sim \begin{bmatrix} -2fX/(wZ) \\ -2fY/(hZ) \\ -(aZ+b)/Z \\ 1 \end{bmatrix}, \tag{1}$$

where $a = -\frac{F+N}{F-N}$, $b = -\frac{2FN}{F-N}$, F and N are the near and far plane. The 2D position $[u, v] = [-2fX/(wZ), -2fY/(hZ)]$ is the projected coordinate of $[X, Y, Z]$. The labeled 2D position $[u, v]$ is known, so we have

$$X = (-\frac{w}{2f})uZ \tag{2}$$

$$Y = (-\frac{h}{2f})vZ. \tag{3}$$

Back to Figure 1, assume the 2D position of the labeled point is $p_k = [u_k, v_k]$ and the corresponding 3D position is $P_K = [X_k, Y_k, Z_k]$. With $\overrightarrow{p_0p_1} \perp \overrightarrow{p_0p_2}$, we have

$$(X_1 - X_0)(X_2 - X_0) + (Y_1 - Y_0)(Y_2 - Y_0) + (Z_1 - Z_0)(Z_2 - Z_0) = 0. \tag{4}$$

Substituting Equation 2 and 3 into 4:

$$\begin{aligned} (\frac{w}{2f})^2(u_1Z_1 - u_0Z_0)(u_2Z_2 - u_0Z_0) + \\ (\frac{h}{2f})^2(v_1Z_1 - v_0Z_0)(v_2Z_2 - v_0Z_0) + \\ (Z_1 - Z_0)(Z_2 - Z_0) = 0, \end{aligned} \tag{5}$$

or

$$\begin{aligned} (\frac{w}{2f})^2(u_1u_2Z_1Z_2 - u_0u_2Z_0Z_2 - u_0u_1Z_0Z_1 + u_0^2Z_0^2) + \\ (\frac{h}{2f})^2(v_1v_2Z_1Z_2 - v_0v_2Z_0Z_2 - v_0v_1Z_0Z_1 + v_0^2Z_0^2) + \\ (Z_1Z_2 - Z_0Z_2 - Z_0Z_1 + Z_0^2) = 0. \end{aligned} \tag{6}$$

Define $C_{mn} = (\frac{w}{2f})^2u_mu_n + (\frac{h}{2f})^2v_mv_n + 1$ and substitute it to Equation 6:

$$C_{12}Z_1Z_2 - C_{02}Z_0Z_2 - C_{01}Z_0Z_1 + C_{00}Z_0^2 = 0, \tag{7}$$

i.e.

$$(C_{12}Z_2 - C_{01}Z_0)Z_1 = (C_{02}Z_2 - C_{00}Z_0)Z_0. \tag{8}$$

Symmetrically, we also have

$$(C_{23}Z_3 - C_{02}Z_0)Z_2 = (C_{03}Z_3 - C_{00}Z_0)Z_0 \tag{9}$$

$$(C_{31}Z_1 - C_{03}Z_0)Z_3 = (C_{01}Z_1 - C_{00}Z_0)Z_0. \tag{10}$$

According to Equation 10, $Z_3 = \frac{(C_{01}Z_1 - C_{00}Z_0)Z_0}{C_{31}Z_1 - C_{03}Z_0}$. Substituting it back to Equation 9, we can express Z_2 using Z_0 and Z_1. Then Substituting Z_2 into Equation 8

(I use Matlab for such tedious work), we get

$$
\begin{aligned}
(C_{01}^2 C_{23} - C_{01}C_{03}C_{12} - C_{01}C_{02}C_{31} + C_{00}C_{12}C_{31})Z_1^2 + \\
(C_{01}C_{02}C_{03} - C_{00}C_{01}C_{23} + C_{01}C_{02}C_{03} - C_{00}C_{01}C_{23})Z_1 Z_0 + \\
C_{00}^2 C_{23} - C_{00}C_{02}C_{03})Z_0^2 = 0.
\end{aligned} \tag{11}
$$

Solving such a quadratic equation, we get Z_1. Finally there are 2 sets of solution for Equation 8 - 10. We can choose either one.