R: Simultations and Basic Math

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Objectives

- Review normal and t distributions
- Simulate and plot these distrubtions
- Practice plotting data in R
- Be able to complete problem set 1

Normal Distrubtion

Parameters of the Normal Distrubtion

- mean: μ
- Variance: σ^2
- Standard Normal($\mu = 0$, $\sigma^2 = 1$) PDF:

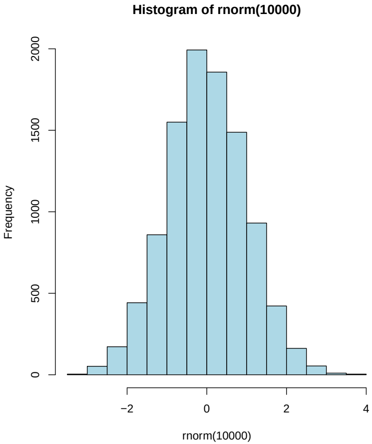
$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$$

Sample of Normal Distrubtion in R

```
summary(rnorm(10000))
```

```
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-4.58080 -0.69847 -0.01011 -0.01447  0.66496  4.08319
```

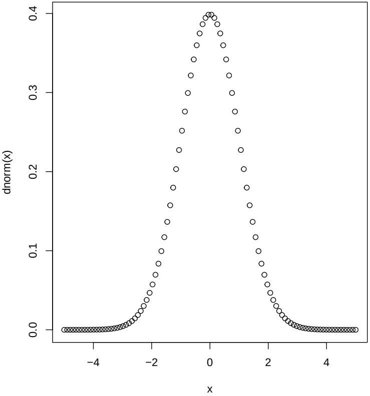
```
hist(rnorm(10000),col="lightblue")
```



Normal Distrubtion in R

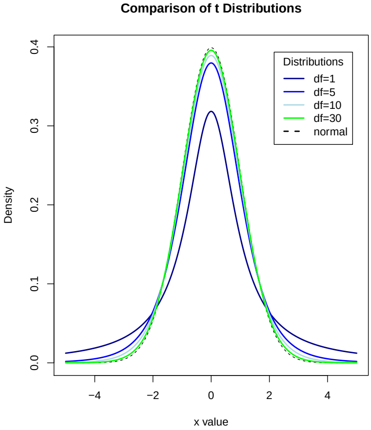
Plotting a standard normal

```
x<- seq(-5,5, length= 100)
plot(x, dnorm(x))
```



Compare Normal to t-Distrubtion

Reminder t-distrubtion is similar to normal with larger tails. The parameters is the degrees of freedom



Compare Normal to t-Distrubtion

The code:

```r
x <- seq(-5, 5, length=1000)
normal <- dnorm(x)

degf <- c(1, 5, 10, 30)
colors <- c("darkblue" , "blue", "lightblue", "green", "black")
labels <- c("df=1", "df=5", "df=10", "df=30", "normal")

plot(x, normal, type="l", lty=2, xlab="x value",
ylab="Density", main="Comparison of t Distributions")

for (i in 1:4){
  lines(x, dt(x,degf[i]), lwd=2, col=colors[i])
}
legend("topright", inset=.05, title="Distributions",
  labels, lwd=2, lty=c(1, 1, 1, 1, 2), col=colors)
```

Sample Mean and Variance

For n observations: Sample Mean:

$$\bar{x} = \sum_{i=1}^{n} x_i$$

Sample Variance:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}.$$

Sample mean in code:

As $n \to \infty$, the sample mean converges to population mean.s

```
sample_mean<-function(x){
  return(sum(x)/length(x))
}
sample_mean(rnorm(10))
```

```
[1] -0.3110993
```

```
sample_mean(rnorm(100))
```

```
[1] -0.02862144
```

```
sample_mean(rnorm(1000))
```

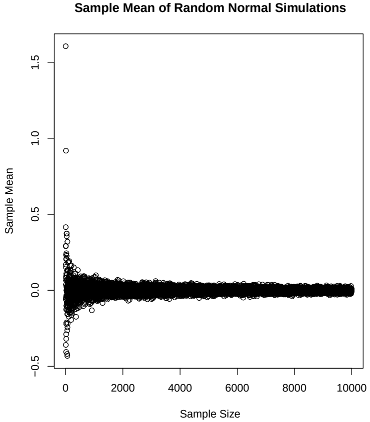
```
[1] 0.04750065
```

```
sample_mean(rnorm(1000000))
```

```
[1] 0.0007805178
```

Sample mean in code:

```
x<- c(1:10000)
y<- c()
for(val in x){
  temp = sample_mean(rnorm(val))
  y = append(y, temp)
}
plot(x,y , xlab=" Sample Size",
ylab="Sample Mean", main=" Sample Mean of Random Normal Simulations")
```



Bernoulli Distribution

A Bernoulli random variable$\sim X$ with success probability p has probability mass function

$$f(x) = p^x(1-p)^{1-x} \qquad\qquad x = 0, 1$$

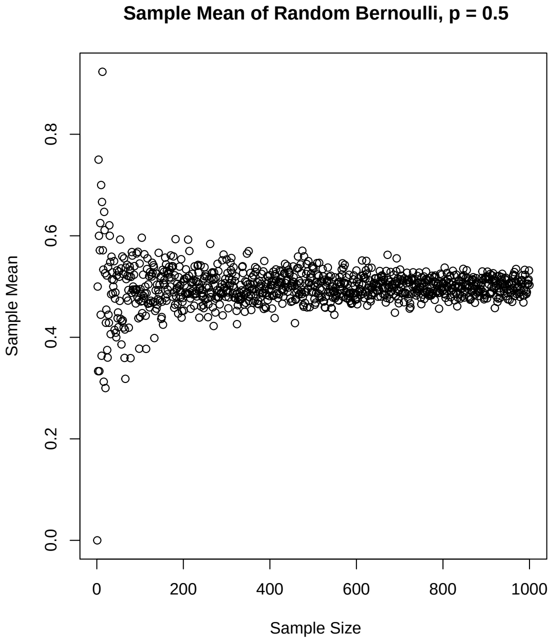
for $0 < p <1$.

The expected value $E(x)$ of a Bernoulli r.v. is p

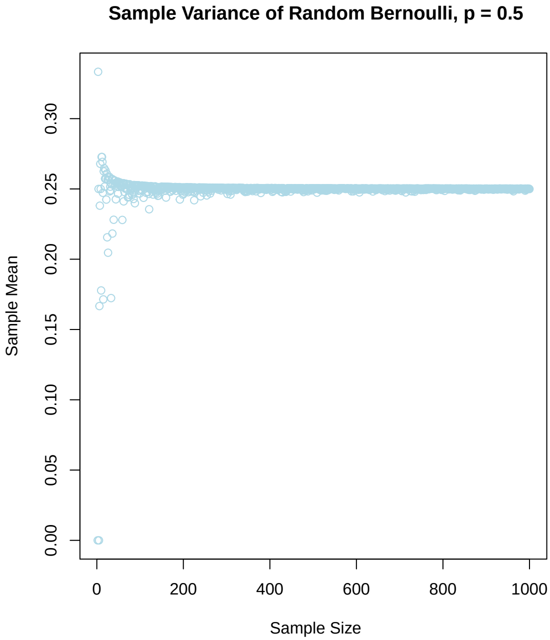
The varaince: $Var(X) = p(1-p)$.

Example: Flipping a fair coin, then $E(x) = 0.5$ and $Var(X) = 0.25$

Bernoulli Simultation



Bernoulli Simulation



Other Distributions

You'll get to review sonme of these in your homework

– Binomial (Bernoulli is Binomial where $n = 1$)

– Uniform

– Poisson

– F

Plotting Functions

Code for $y = x^2$

```
x<- seq(-50, 50, length= 1000)
y<- x^2
plot(x ,y, col = 'blue', main= expression(x^2), type = "l",  yaxs="i")
abline(v = 0) # adds a vertical line at zero
```