John Arbuthnot's Sexy Big Data: Reexamining *An Argument for Divine Providence*

James P. Ascher

27 Mar 2017

The most up-to-date version of this paper is stored `https://cacology.github.io/ASECS-2017-Arbuthnot-talk/` where you can also see the slides

It's a pleasure to present on such a varied panel and I'm particularly eager for your feedback. This talk represents some, very little bit, of the content of my dissertation but–largely–an experiment in a way of talking about reading matter in the early part of the eighteenth century. I'm going to try to tell you a three-century old joke based on mathematics: let's have at it.

It's a commonplace that the supposed 'age of reason' might be better characterized as an aspiration, rather than an accomplishment. Bonamy Dobrée, rightly, calls the period beginning roughly with the Glorious Revolution and ending in the 1740s with the rise of the middle class, an "age of attempted clarity."[1] This clarity finds its way into poetry, a faltering of drama, and by the mid-century–novels. We know that everyone was appreciating the clarity of Locke because everyone *talks* about reading Locke, but along with his empirical project, I think there were other important sources of reading material influencing how writing, literary and otherwise, came to be done and read.

Indeed, I think many writers of the epoch worried about clearly visualizing and representing data in terms of tables, engravings, charts, apparatuses, and especially language. Numbers, after all, means numbers as well as measure.

As a part of this, I'm trying to understand how one might read the longest-running natural philosophy journal, *The Philosophical Transactions*. Begun in 1665, it published with some breaks, the undertakings of the

---

[1] Bonamy Dobrée, *English Literature in the Early Eighteenth Century 1700–1740*, The Oxford History of English Literature (Oxford: Clarendon Press, 1959), p. 16

curious for Britain and the world to see. Too often seen primarily as a precursor to the heroic accomplishments of science under William Whewell and the present day, I think there is value in returning to the old texts not as benighted attempts to achieve modern science, but examples of thinkers deploying Augustinian prose and language that draw on an international tradition of "classic prose" discussed by Francis-Noël Thomas and Mark Turner. If we want to understand how the clarity of Blaise Pascal ends up in the mouth of Thomas Jefferson, we need to trace the journal that introduced this mode of writing and thinking to an English audience.

Collation slide

In the process of working towards a descriptive bibliography of the *Philosophical Transactions*, I'm proceeding systematically through roughly a century of issues, which has yielded a number of interesting articles. Here is the collation from volume twenty-seven, number three-hundred twenty-eight, for the end of 1710. It's useful to note here that the journal has taken a break from it's usual half-sheet beginning and ending, where a shorter article is tacked onto the back of the table of contents to fill out the issue. Here pi-one verso begins the real star of this issue "An Anatomical Description of the Heart of Land Tortoises from America. By Mr. Paul Bussiere" which also gets double-star treatment by getting an engraving.

Typically, the second article of the number is the longer one and I think they're breaking the pattern because this issue for October, November, and December 1710 came out around April 1711: four months late. After the star article, these numbers often turn to a little bit of math or data and this one doesn't disappoint.

Article slide

We get "An Argument for Divine Providence, taken from the constant Regularity observ'd in the Births of both Sexes. By Dr. John Arbuthnott." A careful viewer, or perhaps one with better eyesight than me, will note that we have "Arbuthnott' with two 't's here, which is not the spelling he favored in print but the one he favored

in manuscript. We know that William Burnet, son of Gilbert Burnet, read the paper at the Royal Society meeting on April 19, 1711.[2] This was something of a formality, as nothing could be published in the journal without first being read at the Society.[3] It's possible that Arbuthnot was at the meeting, had someone else read the paper, and the unedited manuscript was handed off to the printer, but I think it far more likely that he sent his manuscript to be read without attending.

Not attending and having something read in was fairly common. Arbuthnot's only other article in the *Transactions* was included in just such a way: Edward Berkeley sent him observations by letter on Mount Vesuvius and told him "I doubt there is nothing in this worth shewing the Society: as to that, you will use your discretion"[4]

So, we have an article that is in a sort-of second place, read by the author's assignee, talking about human reproduction and proving the existence of God. Let me walk you through the argument.

M+F

He represents one birth as being either male or female by addition. M plus F is two terms, one M, one F, so the probability of either M or F is one out of two.

$(M+F)^2$

This can be extended to two births, both either M or F, but multiplied together. If you remember foil, from algebra, you'll see the expansion here is M squared, 2 MF, F squared. This makes sense because there is only one

---

[2] Eddie Shoesmith, "The Continental Controversy over Arbuthnot's Argument for Divine Providence," *Historia Mathematica* 14 (1987): 133–46, p.134

[3] Arbuthnot's response to Justice Chamberlayne from 30 April 1711, thanking him for his comments, suggests that even if Arbuthnot was in attendance at the meeting, his paper's exposure to an audience was in the journal., John Arbuthnot, *The Correspondence of Dr John Arbuthnot*, ed. Angus Ross, Monographs on Eighteenth-Century English Literature and Culture (Müchen: Wilhelm Fink Verlag, 2006), 140–41

[4] ibid., , p.233; PT 30 pp. 708

way to get two Ms, male birth followed by male birth; there's only one way to get two Fs, female followed by female; and, there's two ways to get MF: a male birth followed by a female birth, or a female birth followed by a male one.

$(M+F)^n$

Now, Arbuthnot extends this to an arbitrary large number, n, say the number of births in a year. He spends a great deal of time explaining how the binomial expansion works, but basically, we have–still–only one way to get all male or all female. But n ways to get one male among all other female births–any one birth can deviate. It's a straightforward matter to see that the coefficients of each of the terms are mirrored across the middle and he spends some time explaining how to calculate this.

Thus–and this is the critical bit–in the left half of the formula, Ms dominate; there are more Ms than Fs for half the possibilities. So, half the terms had more M than F, well, nearly, so half is a good estimate, certainly for large n.

Data slides

So he actually has data for 82 years. For each of which there are more male than female births. He calculates the probability that this happens as one over two raised to the power of eighty two. That is, 1 over nearly 484 sextillion, a very small number representing something very unlikely. He writes: "But if A wager with B, not only that the Number of Males shall exceed that of Females, every Year, but that this Excess shall happen in a constant Proportion, and the Difference lye within fix'd limits; and this not only for 82 Years, but for Ages of Ages, and not only at London, but all over the World; … then A's Chance will be near an infinitely small Quantity." Thus, "it is Art, not Chance, that governs."

Arbuthnot continues, "From hence it follows, that Polygamy is contrary **to the Law** of Nature and Justice, and to the Propagation of Human Race; for where Males and Females are in equal number, if one Man takes Twenty Wives, Nineteen Men must live in Celibacy, which is repugnant to the Design of Nature; nor is it probably that

Twenty Women will be so well impregnated by one Man as by Twenty." Thus, it's near impossible to have more male births, men die easily, polygamy is bad, God exists, QED.

The problem is that the math is wrong. It assumes that there is a 50/50 chance of producing a male child, not an unreasonable assumption, but this particular table demonstrates it to be false. But, maybe Arbuthnot didn't notice. Well, this data matches the data produced by John Graunt in 1662; they both swapped 1641 and 42 and the only other data available didn't.[5] Arbuthnot, thus, most likely used Graunt's tables, but along with his tables Graunt notices the difference in birth rates between women and men, estimating slightly more than half would be born male, not 50/50.[6] But, perhaps Arbuthnot just missed it when he copied the data.

Arbuthnot's *Essay on the Usefulness of Mathematical Learning* also mentions the "encrease and the decrease of the people" as an appropriate topic for a gentleman to consider when learning mathematics and suggests that Arbuthnot knew of the possibility of an unfair die.[7] Since he imagines a fair die in birth rates, perhaps he's left this as a puzzle for the gentlemen of the Royal Society?[8]

It's generally agreed that Arbuthnot translated Hugenius into English with his *Of The Laws of Chance or a Method of the Calculation of the Hazards of Game* which puts it more explicitly in the preface: "if a Woman is with Child, but it shall be a Boy; and if you would know the just odds, you must consider the Proportion in the Bills that the Males bear to the Females."[9] The preface furthermore suggests that the calculations might be improved

---

[5] The great fire of 1666 had destroyed all the bills of mortality for the years before 1658; the only known check on this table is John Bell's *London's Remembrancer* (1665) which matches this and Graunts except for 1641 and 1642 which both Arbuthnot and Graunt swapped., Lester M. Beattie, *John Arbuthnot: Mathematician and Satirist* (Cambridge: Harvard University Press, 1935), p. 341

[6] John Graunt, *Natural and Political Observations Mentioned in a Following Index and Made Upon the Bills of Mortality* (London: Printed by Tho: Roycroft, for John Martin, James Allestry,; Tho: Dicas at the Sign of the Bell in St. Paul's Church-yard, 1662), pp. 48–49, 64

[7] John Arbuthnot, *An Essay on the Usefulness of Mathematical Learning in a Letter from a Gentleman in the City to His Friend in Oxford* (Oxford: Printed for A. Peisley, 1701), p. 55

[8] In his *Essay on the Usefulness of Mathematical Learning* Arbuthnot notes that he learned probability from the first treatise on the topic by Hugenius., ibid., , p. 33 Hugenius includes the possibility of an unfair die in the same treatise, which would suggest that Arbuthnot was not reading closely.

[9] John Arbuthnot, *Of the Laws of Chance, or a Method of Calculation of the Hazards of Game Plainly Demonstrated and Applied to Games at Present Most in Use* (London: Printed by Benj. Motte,; sold by Randall Taylor near Stationers-Hall, 1692), A9^{v}

with "the Series's and Logarithms"[10] which he does in the article. If you believe–as the consensus seems to since the late 19th-century–that he indeed translated and wrote the preface to *Of The Laws of Chance* then his paper in the *Transactions* seems stranger and stranger. Why would he ignore a fact he knew? Perhaps to leave the way open for someone else?

Well, there were a series of responses and recalculations.

Estimates Arbuthnot

William Burnet apparently also sent Arbuthnot's paper to Dutch mathematician Willem 'sGravesande (Ssssafga-sunda) who thought that the argument wasn't strong enough. Arbuthnot had merely considered the probability that there would be *more* males each year, not the likelihood that there would be more males born within a certain range. Restricting the possible outcomes further to more males born within the observed range and using the same binomial expansion we've already seen, he found a different probability.

Estimates 'sGravesande

So, the likelihood of observing that pattern of male births was even less likely. We go from 26 zeros to 45. Much smaller. The math itself is very interesting, and those curious should look at Eddie Shoesmith's paper in *Historia Mathematica*.

The problem here is still that the likelihood of a male birth is considered 50/50, which from the observed data we know it's not. Sometime in 1712 Niklaus Bernoulli, of the prominent family of mathematicians, came to know of Arbuthnot's work while he was visiting England. 'sGravesande (Ssssafga-sunda) broached the subject and Bernoulli took it up with other members of the Royal Society. Disagreeing with the results, Bernoulli points out what Arbuthnot seems to have known all along, that the likelihood of male births was not 50/50, but higher.

---

[10] ibid., , A11^{r}

He takes the likelihood of a particular birth as 18/35, which is close to what Graunt gives. Again, after some clever mathematics that further develop what comes to be called binomial distributions, he comes up with a high likelihood that he publishes in 1713.

All estimates

He calculates the probability of achieving the average number of surplus male births within a small range and, furthermore, the probability that the birth rate might deviate the few times it did.[11] So, it turns out, divine providence is merely one of the absolutely most likely outcomes given the greater frequency of male births.

This puts Arbuthnot's paper in a strange position. He never published a follow-up and the only correspondence that has surfaced is his response to Justice Chamberlayne about the problem where he suggests not the hand of God, but "physical Causes of the production of the different sexes. The Most probable is that they exists [originally in Semine Masculo]."[12]

I think there's a clue, though, in the opening phrase of Arbuthnot's paper: "Among innumerable Footsteps of Divine Providence to be found in the Works of Nature." The late Robert Boyle had endowed "an Annual Salary for some Divine or Preaching Minister, … to preach eight sermons in the year for proving the Christian Religion against notorious infidels"[13], connecting natural philosophy with proper religion. Richard Bentley gave the first eight sermons over the year 1692, which focused on what we would now call intelligent design, where he talks about seeing "visible footsteps" or "divine footsteps." And, this word "footsteps" comes to be associated with traces of divine: a peculiar topic for the *Transactions*. I can find no other paper arguing directly for the existence of God, just

---

[11] In particular, he estimates that the likelihood of male births being no more than 163 people different than the average over those years as 0.978, but observing that for 11 years out of the 82, the number fell outside that limit, he calculated that the probability of "falling outside the prescribed limits no more than 10 times in a run of 82 years" as 0.9999.

[12] Arbuthnot, *The Correspondence of Dr John Arbuthnot*, p. 140–141

[13] Richard Bentley, *The Folly of Atheism, and (What Is Now Called) Deism, Even with Respect to the Present Life: A Sermon Preached in the Church of St. Martin in the Fields, March the Vii 1691/2 Being the First of the Lecture Founded by the Honourable Robert Boyle* (London: Printed for Tho. Parkhurst at the Bible; THree Crowns in Cheapside,; H. Mortlock at the Phœnix in St. Paul's Church-yard, 1692) A2^{r}

using God as a sort of topos for appreciating nature or beauty. (though, there's a lot to read, so I'd welcome any listeners who know of one)

More to the point of this panel, Arbuthnot is using representations of data to argue for the existence of a deity. Since the math turns out to be flawed in ways that he most likely knew, we have to wonder what he might have been doing. I think of the *Dunciad*, which he contributed to:

Dunciad

[No read] the *Dunciad in Four Books* glosses this passage with Mathesis as "the strange Conclusions some Mathematicians have deduced from their principles" and "the action of men who look about with full assurance of seeing what does not exist, such as those who expect to find *Space* a real being."[14] Arbuthnot was the most informed of the Scriblerians regarding mathematics, so the knowledge for this satire of the over-reach of mathematics seems most likely to have come from him. The objects ridiculed here are mathematicians who ignore physical reality in favor of metaphysical-like musings on abstract possibilities. Somewhat like a proof of God that ignores a portion of the data in favor of its own conclusion.

The *Memoirs of Scriblerus*–yet more closely associated with Arbuthnot's writing–play on the same image of mathematics as engaged in political arithmetic with one of Scriblerus's publications

Scribelerius

[read] This fictional title links data collection to over-reaching calculations about a city.[15] Scriblerus, having been brought-up as Blaise Pascal–instructed in mathematics from an early age–produces execrable scholarship later on. The object of the satire here would seem to fall fairly closely to what Arbuthnot himself was doing. Yet,

---

[14] Alexander Pope, *The Dunciad in Four Books*, ed. Valerie Rumbold, 2nd ed. (London: Routledge, 2009), p. 277

[15] George A. Aitken, *The Life and Works of John Arbuthnot* (Oxford: Clarendon Press, 1892), p. 357

his satire works in subtle ways. Analyzing Arbuthnot's *The Art of Political Lying* Conal Condren observes that it's not a party tract; it's not a Tory accusation of bad Whiggish thought, but accuses everyone of manipulating truth to their own ends. He says "Arbuthnot's satire exposed … [the] common belief [that lying, cheating, and misinformation played their part in creating group identity] by pretending to take it with deadly literalness."[16] Following Pascal in his *Provincial Letters*, Arbuthnot seems present an earnest explanation to someone who may not understand the subtlety of the sorts of political arguments going on.[17]

Article again

As I see it, there are two possibilities then for Arbuthnot's error in his article and the strange topic. First, that in his enthusiasm to encourage the gentlemen of the Royal Society to take-up mathematics and the doctrine of series in particular, he overlooked the obvious flaw in his study, which he should have been well aware. In this case, while the strategy itself is not satirical, he selected birth rates and divinity as appealing topics *because* of their significance. Later thinking about the over-reach of data into overly-significant topics makes this a somewhat funny self-satire in retrospect; Arbuthnot derides the very approach he sees as critical for convincing people, which is totally in keeping with his understanding of political lying. It's also possible that his "overlooking" was knowing–maybe he knew that the data didn't support the claim he wanted to make, but meant to give an interesting example to encourage further research. In this case, we have a sort of boosterism of mathematics at the expense of reality for purposes of education–another political lie.

---

[16] Conal Condren, *Satire, Lies and Politics* (New York: St. Martin's Press, Inc., 1997), p. 17

[17] In analyzing Arbuthnot's *The Art of Political Lying* Conal Condren observes that it's not quite so simple as taking the *Art* as a Tory tract, that points out inconsistencies in Whiggish thought. But, that it "draws on and ironically generalizes from the way in which mutual accusations about lying, cheating, and misinformation played their part in maintaining, or even creating group identity. It was almost a presupposition of debate that opposing parties were always dishonest, dishonesty helped explain why they existed. Most generally characterized, Arbuthnot's satire exposed this common belief by pretending to take it with deadly literalness.", ibid., , p. 17 That is, Arbuthnot's *Art* takes the technique of manipulating truth and lays it on all of humanity, not a certain crowd, by following the same sort of approach used by Pascal in his *Provincial Letters*, a sort of earnest explanation to someone who may not understand the subtlety of the sorts of political arguments going on.

Second, perhaps he *meant* the article to be an elaborate straight-faced joke about the limitations of mathematics. Before we decide this is implausible as being too obscure, remember that Samuel Johnson noted that Arbuthnot's *Memoirs* didn't succeed because the philosophy was too obscure to be of broad interest. Arbuthnot's response to Justice Chamberlayne, that he should think of semen, not look to God, suggests that he intended the error to be uncovered and noticed. Since he does no further work in this area, it seems possible that he was playing some sort of joke on the idea of how eagerly people like Richard Bentley were world-building under the guise of linking theology to natural philosophy.

Then again, these two options aren't really that different. I think plenty of people have noted that the news these days matches the jokes a bit too closely to be comfortable. Sometimes a text doesn't have to be intended to be satire to satirize, even before we declared irony dead.