# The Impact of Land Use Regulations on the Welfare of High- and Low-Skilled Workers

Camilo Acosta\*

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#### VERY PRELIMINARY WORK, PLEASE DO NOT CITE

#### Abstract

Given the large heterogeneity among residents of a city, policies that regulate the use of land should affect different types of workers differently. In this paper, I develop a quantitative model of a city with low and high-skilled workers, and two possible land use regulations: zoning and height restrictions. Welfare effects can be decomposed into three terms: a floor price effect, a relative income effect, and a sorting effect. My model predicts that residential zoning has an ambiguous effect on the welfare of both types of workers since it lowers both residential rents and wages. However, height restrictions could benefit high-skilled workers through an increase in their non-labor income. Using data for Chicago, we find some preliminary correlations that are in line with our model predictions.

**JEL:** O18, R12, R23, R54.

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<sup>\*</sup>Email: c.acostamejia14@rotman.utoronto.ca; Economic Analysis and Policy, Rotman School of Management.

## 1 Introduction

Regulations on the use of land are prevalent in (almost) any city in the modern world.<sup>1</sup> However, there is very little research on the effects of the different types of land use regulations (LUR, hereafter) beyond its consequences on the housing market.<sup>2</sup> However, if different LUR such as zoning, height or lot-size restrictions affect residential and commercial real estate prices, they should also affect other variables. For instance, given the interdependence between land and labor markets, these regulations could affect the wages that firms offer, as well as the households' and firms' location decisions. This paper will try to expand our knowledge of the effects of LUR by studying the effects of zoning and height restrictions on the welfare of high and low skilled workers. In particular, I will consider the effects of these two types of regulations on the location choices and the distribution of income and rents across locations, and ultimately, on the welfare of high-and low-skilled workers.

Given the large heterogeneity among residents of a city, one would expect LUR to affect different types of workers differently. For instance, welfare effects might be different for rich relative to poor, or for high- relative to low-skill workers. In this paper, I focus on this last distinction. In particular, the urban economics literature highlights the following three differences between these two types of workers. First, there are significant differences in real wages between high- and low-skill workers inside urban areas, with this gap increasing with city size (Baum-Snow and Pavan, 2013; Moretti, 2013). Moreover, there is a large evidence starting with Mincer (1974) showing that, low skilled workers have a lower probability of earning a high income. Second, low-income households spend a higher share of their income in housing (Notowidigdo, 2013). This would make low-skill workers more susceptible to increases in housing prices. Third, high-skill / high-income workers seem to have a higher opportunity cost of commuting (Wheaton, 1977). These differences could cause different types of workers to have different residential and work location preferences. Thus, not including worker heterogeneity in welfare calculations of the effects of LUR would lead to an incomplete picture of its effects.

I will tackle these issues by developing a quantitative theoretical model of internal city structure that builds on Ahlfeldt et al. (2015). Contrary to their model and similar to Tsivanidis (2018), I introduce two types of workers that differ in their skill level. Both types of workers differ not only in their income, but also in some of their fundamentals. My model includes a slightly more realistic housing sector in which real estate developers supply both residential and commercial floor space subject to zoning and/or height restrictions. This constitutes an important contribution of my model. Moreover, city blocks in my model differ in terms of their location, size, skill specific productivity and amenities. These latter two act as agglomeration forces. The model also displays

<sup>&</sup>lt;sup>1</sup>With the exception of Houston, I do not know of any city that does not use different tools to regulate the use of its land.

<sup>&</sup>lt;sup>2</sup>This fact becomes evident after reading the latest volume of the Handbook of Regional and Urban Economics (Duranton and Puga, 2015; Gyourko and Molloy, 2015).

dispersion forces generated by commuting costs that increase in travel time and differ by type of worker, also by the inelastic supply of land used for construction.

Overall the model remains very tractable; however, the effect of LUR on the main outcomes of the model are not trivial. For this reason, I do comparative statics that analyze the effects of a new or changes in land use restriction in a given block. In particular, the model predicts that more "residential only" restrictions, would lead to a decrease in residential floor prices, an increase in commercial floor prices, and a decrease in wages. It also could lead to a reorganization of economic activity in a way such that some agglomeration externalities are internalized. These effects lead to an ambiguous effect on the welfare of both types of workers since it lowers both rents and wages. On the other hand, the imposition of a height restriction, would lead to higher residential and commercial real estate prices, and to a decrease in wages, implying that low-skilled workers would be worse off, while high-skilled workers could be better off through an increase in their non-labor income. In general, I show that the total relative effect on welfare can be decompose in three terms: a floor price effect, a relative wages/income effect, and a sorting effect.

Afterward, I show how this model can be applied to real world data. In particular, I show how from the equilibrium equations of my model, there is a unique mapping from observed data and values for different parameters, to (measures of) income, skill specific productivities and amenities. Just as in Ahlfeldt et al. (2015), this mapping is unique regardless of whether the model has a single or multiple equilibria. Using data for the City of Chicago, I show descriptive facts indicating some correlations in favor of what my theory predicts. After implementing the model, potential applications include the assessment of different zoning policies across Chicago such as the high-rise residential developments in the Gold Coast district, or the high-tech manufacturing developments planned in the Near West End. This paper will attempt to contribute to the debate on whether zoning is a regressive measure, and if it possible to identify a group of the population that benefits from it.

Endogeneity of the regulatory laws could be a thread for the identification of causal effects mainly by three reasons. First, low-density zoning is more likely to happen in neighborhoods with a larger share of high income population. Since richer households are more likely to own properties, if LUR leads to higher real estate prices, they might want to strengthen the policies and have more expensive properties. This phenomenon is known as the Home-voter Hypothesis (Fischel, 2001). Second, locations with more desirable amenities (such as parks, lakes or historical buildings) might have a more stringent LUR in order to prevent more development (Hilber and Robert-Nicoud, 2013). Third, in some parts of the city, LUR could be modified in order to coincide with the market solution instead of modifying it; this is known as the question to whether zoning follows the market (Wallace, 1988). Since we have a panel data of all the census blocks within the city, we can control for the second issue using block fixed effects. For the two other, I am going to rely on historical land prices and satellite soil characteristics as instrumental variables for changes in land use regulations.

This paper is related to the literature studying the consequences of LUR on different outcomes.<sup>3</sup> In terms of theory, Helsley and Strange (1995), Turner (2005), and Rossi-Hansberg (2004) analyze some of the general consequences of different types of land use regulation. In particular, the latter shows that the optimal zoning restrictions concentrate business areas near the CBD, even if firms would like to move to cheaper areas. Empirically, most of the literature agrees that: i) LUR leads to higher land and housing values (both in levels and in growth rates), and to a more inelastic supply of floor space (McMillen and McDonald, 2002; Glaeser et al., 2005; Glaeser and Ward, 2009); and that, ii) these effects lead to decreases in total welfare, with potentially large distributional impacts (Cheshire and Sheppard, 2002; Turner et al., 2014; Hsieh and Moretti, 2015). Saks (2008) presents evidence indicating that places with tighter LUR, when facing a shock that leads to an increase in the demand for housing, will have larger reductions in long term employment and wages relative to places with more relaxed regulations.

This paper is also related to the articles that explore how LUR affects different types of people. In particular, Kahn et al. (2010) and Levine (1999) find that areas with more low density zoning or growth regulations experience more gentrification, being minorities and low-income households the most affected groups. Muehlegger and Shoag (2015) suggest that LUR are associated with increases in commuting time and that this burden is most heavily borne by more educated and wealthier individuals. My paper complements Ganong and Shoag (2017) who study to what extent LUR differences explain the dramatic decline in income convergence across states and the flow of population to wealthy places. They show that the returns to living in productive places net of housing costs have fallen for unskilled workers but have remained similar for skilled workers, caused partially by stringent LUR in productive cities. This has lead to a sharp sorting of high-skilled workers into high rents-high wages-high productivity states, and viceversa. I am contributing to this literature in two ways. First, by analyzing the impact of LUR on the distribution of skills within a city. Second, by disentangling the effects that zoning and height restrictions have on rents, wages, welfare, and sorting of agents within a city, in a general equilibrium setting.

My paper is part of a recent literature using quantitative models of urban economics such as Ahlfeldt et al. (2015); Arkolakis et al. (2015); Tsivanidis (2018); Baum-Snow and Han (2019), among others. Most notably, Ahlfeldt et al. (2015) build a quantitative model of internal city structure and used Berlin's division and reunification as a natural experiment in order to analyze density patterns across the city, and structurally estimate production and residential agglomeration economies. Arkolakis et al. (2015) show that there exists opportunities for a city planner to increase the welfare of the people of a city by restricting some uses of land in certain locations. The main departures of my model with respect to these models are: i) similar to Tsivanidis (2018),

<sup>&</sup>lt;sup>3</sup>Two superb summaries about the recent state of the literature are Fischel (2015) and Gyourko and Molloy (2015). Shertzer et al. (2016) concludes that in Chicago zoning has been more important than geography or transportation networks in determining the distribution of economic activity.

<sup>&</sup>lt;sup>4</sup>The sorting of workers by skill across cities has been studied by Diamond (2016), Notowidigdo (2013), among others

my model includes different types of workers (e.g., by skill); ii) I also include a more complete model of space supply into this quantitative setting—in the spirit of Muth (1969)—that allows me to incorporate LUR and study their effects; iii) even though Arkolakis et al. (2015) estimates the block specific elasticities of welfare of increasing commercial or residential space, they do not analyze the underlying mechanisms leading to such changes. I am able to decompose these welfare effects in changes in rent, wages and sorting across locations.

Finally, given my analysis of the distribution of jobs and different types of workers within the city, this paper also contributes to the literature on the spatial mismatch hypothesis (Kain, 1968; Gobillon et al., 2007).<sup>5</sup> Recent research has found an inversion in the spatial mismatch patterns: A centralization of jobs but a suburbanization of the low income population.(Baum-Snow and Hartley, 2016; Couture and Handbury, 2015; Florida et al., 2014; Ehrenhalt, 2012). I contribute to this literature by studying the role of regulations in the use of land on the spatial mismatch between jobs and residential locations.

The paper proceeds as follows. In the following section, I describe my data and present some facts about LUR and its relation with real estate prices and the distribution of high- and low-skill workers within the City of Chicago. In Section 3, I present the theoretical model and analyze the comparative statics of imposing or changing zoning districts and height limits. These comparative statics are the heart of this paper. Section 4 exploits the recursive structure of my model in order to show what variables can be recovered, given some data and values for different parameters. The final section concludes and highlights current challenges and directions for possible future research.

# 2 Chicago: Data and Some Facts

As will be evident in Section 4, in order to implement my model I need data on workplace and residence employment, commuting times between locations, price and stock of floor space, and zoning and height designations of all locations in the city. In particular, I use data on workplace and residence employment by skill from the Origin-Destination Employment Statistics from the US Census Bureau. Regarding real estate prices and stocks, I use data from Zillow Economic Research, which contains the universe of housing transactions and assessment records for the United States. These two previous data sources are available for any city in the country. However, the main challenge comes when trying to find a city with detailed data on land use regulations. Chicago is one of these cities. In particular, the Data Portal of the City of Chicago contains a geographic database of all the zoning districts within city boundaries. In the rest of this section, I will describe these data and present some facts documenting the relationship between LUR, housing prices and the distribution of high- and low-skill workers within Chicago.

<sup>&</sup>lt;sup>5</sup>The spatial mismatch hypothesis refers to the mismatch between residence and workplace locations. In particular, regarding the suburbanization of employment experienced in the US in the second half of the 20th century (Kain, 1968).

## 2.1 Real Estate: Prices and Stock

In order to build measures of the price and stock of floor space at a detailed geographic level, I use data from Zillow Economic Research. In particular, we use their ZTRAX transactions, current assessment and historical assessment data files to build quality-adjusted house price indices between 2008 and 2016 for each census block group in Cook County, using both hedonic and repeated-sales approaches. We use the same data to construct two measures of stock: count of the total number of units and floor space.

The Zillow ZTRAX data sets contains real estate transactions for around 3000 counties in the United States. It is constructed from information in local deed transfers and mortgages for mostly residential properties, but also includes some commercial ones.(Zillow, 2017).<sup>6</sup> Among other things, these data contains information includes the sale price, transaction date, location coordinates and the type of property. We merge this transactions dataset with other registers containing the most current and historical assessments. Zillow collects property characteristics, geographic information, current (the last one before 2017) and prior valuations for approximately 150 million parcels in more than 3,000 counties. From all these data sources, I drop governmental, institutional, historical, communication, recreational, miscellaneous and transportation properties, and group quarters, trailer parks and parking garages. The previous restrictions mean that we include most of all residential and commercial units.

For building the price indexes, I use only use arm's length transactions and drop those with a sales price below 10,000 USD. Moreover, I drop houses that reported more than 10 transactions between 2008 and 2016, and census block groups with fewer than 5 transactions each year. Even within a census block group, houses and commercial properties could be quite heterogenous in both observed and unobserved characteristics. Therefore, we construct four different quality adjusted measures for the price of real estate in each census block group.

First, using both the transaction and assessment data, we run a hedonic regression of the log sales price of a house  $(ln(P_{himt}))$  on property characteristics  $(X_{himt})$  and census block group-year fixed effects  $(ln(r_{it}^{HI}))$ :

$$ln(P_{himt,T}) = X_{himt,T}\beta^{HI} + ln(r_{it,T}^{HI}) + \delta_m^{HI} + \varepsilon_{himt,T}^{HI}, \tag{1}$$

where h denotes a property of type T (residential R or commercial C), in a census block group i,

<sup>&</sup>lt;sup>6</sup>Data provided by Zillow through the Zillow Transaction and Assessment Dataset (ZTRAX). More information on accessing the data can be found at http://www.zillow.com/ztrax. The results and opinions are those of the author and do not reflect the position of Zillow Group.

<sup>&</sup>lt;sup>7</sup>Baum-Snow and Han (2019) use these data for the whole United States and compare their tract-level price indices and stock measures with similar measures built using US Census data. They find that, even though the correlation between prices indices from both sources is quite low, they generate similar estimates of housing supply elasticities. On the other hand, the average number of new construction and stock of units generated by both data are very similar and very highly correlated.

that was sold in month m and year t. The vector of house characteristics includes a polynomial of degree two of age and size, and other discrete categories, such as bedrooms, bathrooms, flooring, roofing, fireplace, condition and type of property (e.g., single-detached, townhouse, etc.). Month of sale fixed effects  $\rho_m^{HI}$  account for seasonality in the real-estate market. Due to our sample restrictions, the hedonic residential price index  $(r_{itR}^{HI})$  covers between 80% to 90% of the full block group sample between 2009 and 2016. For commercial real estate  $(r_{it,F}^{HI})$ , we only have data for the latest assessment (2014-2016), with a coverage of 27%.

Second, I use the transactions data to build a repeated sales index at the block group-year level. Following Baum-Snow and Han (2019), for this index I exclude homes that sold only once and any sales within six months after the previous sale, and run the following regression:

$$ln(P_{himt,T}) = ln(r_{it,T}^{RS}) + \alpha_{hi,T}^{RS} + \delta_m^{RS} + \varepsilon_{himt,T}^{RS}, \qquad (2)$$

where  $\alpha_{hi,T}^{RS}$  denotes property-level fixed effects, which take into account individual heterogeneity within properties that do not change over time. Due to our sample restrictions, the residential repeated sales index  $(r_{it,R}^{RS})$  covers between 60% to 70% of the full block group sample between 2009 and 2016. Since the data for commercial transactions is quite limited, it is impossible to compute a repeated sales price index for commercial real estate.

Third, we compute similar measures to the one from equation (1), but using the assessed value of the properties instead of the sales price. We denote this measure with the superscript HA, denoting hedonic assessment. Since assessment values are not usually updated every year, and are typically smaller than the transaction values, the HA measure might not reflect the real price dynamics observed in the real estate market. However, since I can use all the properties to compute it, the residential HA measure  $(r_{it,R}^{HA})$  covers around 99% of the full block group sample, while the commercial HA for the latest assessment has a coverage of 92%. This high coverage constitutes an advantage over their transaction counterparts.

Table 1 presents descriptive statistics for these measures and correlations between them for both residential (Panel A) and commercial properties (Panel B), separately. Panel A shows that even though the HA index seems to be larger than the HI index, the correlation coefficient between both is 0.86. Moreover, both measures exhibit a correlation around 0.45 with the median price by square foot in their census block groups. On the other hand, the correlation of the repeated sales index with both hedonic measures and the median price is poor, and its coverage lower. Panel B shows similar results for the commercial hedonic measures. Based on this discussion, we are going to use the HI and HA indices as our main measures of real estate prices.

[Table 1 about here.]

In Figures 1, I map the distribution of real estate prices for each block group in the City of

Chicago and their respective changes. As can be seen in the left map of Panel A from of Figure 1, the highest residential real estate in Chicago in 2015-2016 was found in the blocks surrounding the CBD, as well as the area north of downtown. On the other hand, the cheapest areas are located in the south part of the city. For commercial real estate, the patterns are similar (right map from Panel A). Panel B shows how residential housing prices changed between the period 2009-2011 and 2015-2016. The figure shows that the largest appreciation of housing prices took place in the west part of the city (specially, northwest), while some blocks in the south experience a decline in prices.

## [Figure 1 about here.]

In Figure 2, I map the distribution of built square footage for each block group in the City of Chicago and their respective changes. The left panel shows the average levels between 2015 and 2016, while the right panel shows the percentage changes with respect to the period 2009-2011. Note three things. First, the largest amount of built space is located in the downtown area and along the Gold Coast district. Second, most of the city experienced an increase in the stock of units in this period. Third, the area west of downtown was the area with the highest increase in built area.

[Figure 2 about here.]

# 2.2 Land Use Regulation

The Data Portal of the City of Chicago contains a geographic database of all the zoning districts within city boundaries.<sup>8</sup> These data contain coordinates and the specific zone class category, and are available for 2012 and 2016. Specific regulations for every zone class are available in the Chicago Zoning and Land Use Ordinances.<sup>9</sup>. An example of zone class is **RS-3**, which corresponds to a detached, single family home in a Residential Single-Unit District, with a maximum FAR of 0.9, a maximum height of 30 feet and a minimum lot size of 2500 square feet, with no commercial activity allowed. In total, there are 66 zone classes (including parks and open spaces (POS), and transportation (T) districts), 15 Planned Manufacturing Developments (PMD) districts, and some Planned Developments (PD) throughout the city.<sup>10</sup>

In this database, around half of the city area is currently categorized as residential district (R), while manufacturing (M), commercial (C) and business (B) districts take approximately 11%, 6%

<sup>&</sup>lt;sup>8</sup>The Data Portal also has geographic database at the building level with information of the square footage and number of stories of approximately 400,000 buildings (half of the total). For more information, visit https://data.cityofchicago.org/.

<sup>&</sup>lt;sup>9</sup>Titles 16 and 17 of the Municipal Code of Chicago. Available at: https://chicagocode.org/

<sup>&</sup>lt;sup>10</sup>This last category includes to tall buildings, campuses, and other large developments in which developers must work with the City to ensure that the project integrates with surrounding neighborhoods.

and 3% of the total area, respectively. Locations categorized as downtown districts (D) take about 1% of the area. PMD or PD cover 6% and 12% of the area, respectively. Using these data, I I categorize all these classes into four categories: those designated for residential purposes only, those for commercial only (M, C and B), those with mixed uses, and the rest (including PD, POS, and T). In the upper left panel of Figure 3, I present a map of the city using this categorization. Note that the land that can be used by firms only (in red) is spread along and around the main highways and waterways, while the areas designated for mixed uses (in purple) is located mainly in the downtown area and along the main streets of the city. The areas reserved for residential purposes only (in blue) are located mainly within areas of mixed use.

### [Figure 3 about here.]

In the upper right panel of Figure 3, I map the maximum FAR allowed for every block in the city. I take this measure from the Municipal Code of Chicago, which specifies the maximum FAR allowed for every zone class. The map shows a clear sorting pattern of allowed heights. First, higher density (reds) is allowed in the downtown area, in some parts along the Gold Coast district and in other coastal blocks. Second, medium density (greens) is allowed in almost every location along the coast of Lake Michigan, the two branches of the Chicago River, the main highways and street roads, and in most of the areas where only firms are allowed. Finally, low density development (blues) is allowed in the rest of the city. In particular, comparing both maps there seems to be a high correlation between the blocks were only residential uses are allowed and the areas where only low density development is allowed. Even though there have not been big changes in the Chicago Zoning Ordinance recently, there have been some amendments in different locations throughout the city between 2012 and 2016. The bottom panel of Figure 3 shows the blocks where the zoning class was modified during this period. These changes will be useful for identification when estimating the model's parameters.

## 2.3 Spatial Distribution of Skills

For data on workplace and residence employment, I use version 7.2 of the Longitudinal Employer-Household Dynamics (LEHD) Origin-Destination Employment Statistics (LODES) from the US Census Bureau for 2009 and 2014. The LODES data is built from administrative records, census and survey data. In particular, information about the location of jobs and residences are provided by state unemployment insurance reporting and account information, and federal worker earning records. The LODES data contains counts for the total number of people living and working (stocks), for every census block in the US between 2002 and 2016. This information can be further split by age category, monthly income categories, economic sector, race, gender, and educational attainment. However, the latter three categories are only available for years 2009 and later. The LODES database also contains the number of commuters between every pair of blocks (flows), but

this information can only be separated by age or monthly income categories, but not by educational attainment.

In 2014, of those workers of age 30 or older in Chicago MSA, 12% had less than a high school degree, 24% had a high school degree only, 30% had a college or an associate degree, and 34% had a bachelors or a higher degree. I categorize as low skilled those workers within the former three categories. Table 2 presents the mean and median of the number of people by skill, by block of residence and block of work, for both the City of Chicago and the whole MSA. Four things to note. First, note that blocks of residence in the city are very similar in terms of population compared to the MSA ones. Second, on average blocks of work in the city are more populated and probably more dense as well, since city blocks are smaller. Third, the average block of residence in the city has less people (22) than the average block of work (73), while this pattern changes when we look at the median (13 and 7, respectively). These numbers imply that the distributions of population across blocks of residence and work are highly skewed, in particular across blocks of work. Fourth, these patterns are very similar if take into account low skilled or high-skilled workers. In particular, the average block of residence in Chicago has 15 low- and 7 high-skilled workers, while the average block of work has 47 low- and 26 high-skilled workers.

## [Table 2 about here.]

To see how both types of workers are spatially distributed, Panel A of Figure 4 maps the number of share of high-skilled workers for every block in the city. I do this by block of residence (left) and by block of work (right). The most striking pattern is the high centralization of high-skilled workers in both maps. By block of residence, the high-skilled workers seem to be highly concentrated in the downtown area and in the northern part of the city along the coast, while the low-skilled seem to be concentrated in the south and west ends of the city. Although weaker, the same pattern follows by block of work. Panel B of Figure 4, shows a histogram of the change between 2009 and 2014 in the share of high-skilled workers by type of block. These figures show important variation in skill composition across blocks. In particular, these distributions seem to be symmetric around zero, which could imply a reallocation workers within the city, rather than a general concentration of a particular type of worker.

[Figure 4 about here.]

# 2.4 LUR and the Spatial Distribution of Skills

To further motivate the model and the relationship between LUR and the distribution of highand low-skilled workers in Chicago, Table 3 presents correlations between the share of low-skilled workers in each block and the two measures of land use regulations presented in Figure 3. For these regressions, I computed for each type of block the share of the block's area were only residential, commercial and mixed uses were allowed, and I built a measure of the (weighted) average floor-to-area ratio for each block. Columns (1) and (3) from the table show a negative correlation between the shares of residential or mixed land (relative to commercial only) and the share of low skilled residents and workers, respectively. Moreover, Columns (2) and (4) show a negative correlation between the FAR measure and the share of low-skilled residents and workers, respectively. This correlation implies that blocks with higher buildings allowed are associated with a lower share of low skilled people, either working or living.

[Table 3 about here.]

To be completed.

## 3 The Model

My model corresponds to a spatial equilibrium model of a city in which households choose where to live and work and how much goods and floor space to consume. Firms choose the amount of high and low skilled labor and floor space to use in production. Moreover, real estate developers might face land use regulations and, subject to them, use land and capital to provide floor space. The model builds mainly upon the model presented in Ahlfeldt et al. (2015) but also draws some elements from Diamond (2016), Arkolakis et al. (2015) and Tsivanidis (2018). I will use this model to examine how land use regulations affect rents, income and the sorting of workers by skill across blocks within the city.

Start by considering a closed city consisting of a set of  $\Lambda = \{1, 2, ..., L\}$  blocks, each with an area of  $L_i$ . This assumption implies that population of each skill group is exogenous, but the expected utility will be endogenously determined. Blocks differ in terms of final good productivity and residential amenities. We will start by assuming they are exogenous, but relax such assumption later on. Finally, blocks also differ in terms of their access to the rest of the city.

#### 3.1 Workers

There are two types of workers: high-skilled (s) or low skilled (u); the type will be indexed by e. Low skilled workers receive only income from their labor, while high-skilled workers receive income both from their labor and from the rents of land paid by all the agents in the city. Throughout the section I will index residence locations with i or m, and (most of the time) work locations with j or n.

Workers, who are indexed by o, are endowed with one unit of labor that is supplied inelastically. Every worker o of type  $e \in \{s, u\}$  that lives in location  $i \in \Lambda$  and works in location  $j \in \Lambda$  faces a commuting cost  $d_{ije} \in [1, \infty]$  and has a Cobb-Douglas utility function of the form:

$$u_{ijeo} = B_i \left(\frac{c_{io}}{\beta_e}\right)^{\beta_e} \left(\frac{h_{io}^R}{1 - \beta_e}\right)^{1 - \beta_e} v_{ijeo},\tag{3}$$

where  $B_i$  denotes the amenities at the place of residence;  $h_{io}^R$  represents the consumption of residential floor space (amount of housing) agent o consumes;  $c_{io}$  represents the consumption of the single final good of worker o living in block i;  $v_{ijeo}$  denotes an individual specific idiosyncratic shock that models heterogeneity in the utility that workers derive from living in location i and working in location j.

Note also that the Cobb-Douglas parameter is indexed by skill type. In principle, I will assume  $\beta_s \geq \beta_u \Leftrightarrow (1-\beta_s) \leq (1-\beta_u)$  following recent literature that shows that low-skilled workers spend a higher share of their income on housing than high-skilled workers (Notowidigdo, 2013; Ganong and Shoag, 2017). The budget constraint each worker faces can be written as:

$$c_{io} + r_i^R h_{io}^R \le I_{ije} = \begin{cases} \frac{w_{ju}}{d_{iju}} & \text{if } e = u\\ \frac{w_{js}}{d_{ijs}} + \varphi_i R & \text{if } e = s \end{cases},$$

where the price of one unit of final good is the same across locations and has been normalized to one;  $r_i^R$  is the price of housing in location i. Moreover,  $I_{ije}$  is the total income of a type e worker working in j and living in i. For low-skilled workers  $I_{ije}$  equals their wage in j net of commuting costs, while for high-skilled workers it corresponds to their wage net commuting costs plus some share  $(\varphi_i)$  of the total land rents collected in the city,  $R = \sum_{i=m}^{L} (p_m L_m)$ , where  $p_i$  is the price of the land in i. The shares  $\varphi_i$  are the same for every type s worker in i and satisfy:  $\sum_{i=1}^{L} \varphi_i N_{Ris} = 1$ . Also, commuting costs act as a dispersion force, reducing the effective units of labor. I assume iceberg commuting costs of the form  $d_{ije} = e^{\kappa_e \tau_{ij}} \geq 1$ , where  $\tau_{ij} \in [0, \infty)$  represents travel time between two locations and  $\kappa_e$ , represents the opportunity costs faced by a worker of type e. Based on Wheaton (1977), I assume  $\kappa_s > \kappa_u$ . From the solution of this problem, the indirect utility function can be written as

$$u_{ijeo} = B_i I_{ije} (r_i^R)^{\beta_e - 1} v_{ijeo}. \tag{4}$$

I assume that the individual specific idiosyncratic shock  $v_{ijeo}$  is drawn from an independent Fréchet distribution with cdf  $F(v_{ijeo}) = Pr[V \le v_{ijeo}] = \exp\{-T_{ie}E_{je}v_{ijeo}^{-\theta}\}$ , where  $T_{ie}$  and  $E_{je}$  denote the average utility faced by a worker of type e from living in i and working in j, respectively, and  $\theta > 1$  is the shape parameter and controls the dispersion of these shocks: A higher  $\theta$  implies less degree of unobserved heterogeneity of workers. Note that the location parameters of this distribution are indexed by the skill type e, implying that workers of different type perceive different average utilities from living and working in a given location.

Since there is a monotonic relationship between the indirect utility function and the idiosyncratic shock, the distribution of utility for a worker of type e living in block i and working at j is also Fréchet distributed with scale parameter  $\Phi_{ije} \equiv T_{ie}E_{je}(r_i^R)^{-\theta(1-\beta_e)}(B_iI_{je})^{\theta}$  and shape parameter  $\theta$ . Each worker chooses the bilateral commute that offers him the maximum utility,  $u_{ijeo} = \max_{m,n} \{u_{mneo}\}$ . Given the distribution of utility, the expected utility of a worker of type e from moving to the city is:

$$\mathbb{E}[u_e] = \gamma \Phi_e^{1/\theta} = \gamma \left[ \sum_{m=1}^L \sum_{n=1}^L T_{me} E_{ne}(r_m^R)^{-\theta(1-\beta_e)} (B_m I_{mne})^{\theta} \right]^{1/\theta}, \tag{5}$$

where  $\gamma = \Gamma(1 - 1/\theta)$  is the Gamma function evaluated at point  $1 - 1/\theta$ . This equation states that the expected utility of a worker of type e living in the city depends on the average of all wages he could get given his type, the average price of housing, the commuting times and his type's average utility for every (residence and work) location. For simplicity, define this expected utility as  $\bar{u}_e = \mathbb{E}[u_e]/\gamma$ . The spatial sorting generated by the model depends on each type's elasticities of the expected utility with respect to marginal commuting costs and with respect to housing prices  $(\beta_e, \kappa_e)$  and some general equilibrium terms relating the non-labor income).

From equation (??), I derive the probability that a worker of type e chooses to live in i and work in j, out of all possible block-pairs within the city:

$$\pi_{ije} = Pr[u_{ije} \ge \max\{u_{mne}\}, \forall m, n]$$

$$= \frac{\Phi_{ije}}{\Phi_e} = \frac{T_{ie}E_{je}(r_i^R)^{-\theta(1-\beta_e)} (B_iI_{ije})^{\theta}}{\sum_{m=1}^{L} \sum_{n=1}^{L} T_{me}E_{ne}(r_m^R)^{-\theta(1-\beta_e)} (B_mI_{mne})^{\theta}} = \frac{N_{ije}}{N_e}.$$
(6)

This expression suggests that this probability depends positively on the wage she receives relative to all other locations, on the amenities of her residence location, and the average utilities she would get from choosing those two locations. It depends negatively on the rents she pays for housing relative to what she would pay in all other blocks, and the commuting time between both blocks. Summing across all possible employment locations, I obtain the probability that a worker of type e chooses to live in block i:

$$\pi_{Rie} = \frac{\sum_{n=1}^{L} T_{ie} E_{ne}(r_i^R)^{-\theta(1-\beta_e)} (B_i I_{ine})^{\theta}}{\sum_{m=1}^{L} \sum_{n=1}^{L} T_{me} E_{ne}(r_m^R)^{-\theta(1-\beta_e)} (B_m I_{mne})^{\theta}} \equiv \frac{\Phi_{ie}}{\Phi_e} = \frac{N_{Rie}}{N_e}.$$
 (7)

Since there are a very large number of locations, this probability also corresponds to the number of type e workers residing in location i relative to the total type e workers in the city. Similarly, summing across all possible residence locations m, I obtain the probability that a worker of type e chooses to work in block j out of all possible other locations within the city. This also corresponds

to the number of type e people working in location j relative to the total type e workers in the city:

$$\pi_{Fje} = \frac{\sum_{m=1}^{L} T_{me} E_{je}(r_m^R)^{-\theta(1-\beta_e)} \left(B_m I_{mje}\right)^{\theta}}{\sum_{m=1}^{L} \sum_{n=1}^{L} T_{me} E_{ne}(r_m^R)^{-\theta(1-\beta_e)} \left(B_m I_{mne}\right)^{\theta}} \equiv \frac{\Phi_{je}}{\Phi_e} = \frac{N_{Fje}}{N_e}.$$
 (8)

I can also write expressions for the conditional probabilities. First, conditional on having chosen to work in block j, the probability for a worker of type e that they commute from block i depends mainly on the rents, amenities and tastes for that particular location relative to all other residence locations:

$$\pi_{ije|j} = Pr[u_{ijeo} \ge \max\{u_{nje}\}, \forall n] = \frac{T_{ie}(r_i^R)^{-\theta(1-\beta_e)}(B_i I_{ije})^{\theta}}{\sum_{m=1}^{L} T_{me}(r_m^R)^{-\theta(1-\beta_e)}(B_m I_{mje})^{\theta}}.$$
 (9)

Similarly, conditional on having chosen to live in block i, the probability for a worker of type e that they commute to block j depends mainly in the income and average utility the worker gets when working in j relative to all other workplace locations:

$$\pi_{ije|i} = Pr[u_{ijeo} \ge \max\{u_{ime}\}, \forall m] = \frac{E_{je}I_{ije}^{\theta}}{\sum_{n=1}^{L} E_{ne}I_{ine}^{\theta}}.$$
 (10)

Commuting market clearing requires that the measure of workers of each type employed at each location j equals the sum across all locations i of their measures of residents of that particular type times their conditional probability of commuting from i to j. Define  $N_{Fje}$  as the measure of workers of type e employed in each location j, and  $N_{Rie}$  as the measure of residents of type e living in each location i. The commuting market clearing condition is:

$$N_{Fje} = \sum_{i=1}^{L} \pi_{ije|i} N_{Rie} = \sum_{i=1}^{L} \frac{E_{je} I_{ije}^{\theta}}{\sum_{n=1}^{L} E_{ne} I_{ine}^{\theta}} N_{Rie},$$

$$N_{Fj} \equiv N_{Fju} + N_{Fjs} = \sum_{i=1}^{L} \left( \pi_{iju|i} N_{Riu} + \pi_{ijs|i} N_{Ris} \right).$$
(11)

The expected income that a worker of type e gets in the city conditional on living in block i can be written as the sum of the total income in all possible employment locations weighted by the conditional probabilities of commuting to those locations:

$$\bar{I}_{ie} \equiv \mathbb{E}_{j}[I_{ije}|i] = \sum_{j=1}^{L} \pi_{ije|i} \cdot I_{ije} = \sum_{j=1}^{L} \frac{E_{je}I_{ije}^{\theta}}{\sum_{n=1}^{L} E_{ne}I_{ine}^{\theta}} \cdot I_{ije},$$
(12)

where  $I_{ije}$  was defined in the budget constraint. Finally, using the solution for each worker's demand for floor space, I can write the total demand of housing services by type e workers and the total

demand in location i as:

$$H_{ie}^{R} \equiv \mathbb{E}[h_{ie}^{R}] N_{Rie} = (1 - \beta_{e}) \frac{N_{Rie}}{r_{i}^{R}} \bar{I}_{ie},$$

$$H_{i}^{R} = H_{iu}^{R} + H_{is}^{R} = \frac{1}{r_{i}^{R}} \left\{ (1 - \beta_{u}) N_{Riu} \bar{I}_{iu} + (1 - \beta_{s}) N_{Ris} \bar{I}_{is} \right\}.$$
(13)

As expected, these demands depends positively on the mass of residents and their respective expected income and negatively on the price of residential floor space in that block.

## 3.1.1 Endogenous Amenities

So far I have treated the block specific residential amenities  $B_i$  as exogenous. However, amenities are an important factor when people determine their place of residence. For instance, Bayer et al. (2007) show that households have higher willingness to pay to live in a location densely populated by high-skilled workers. On the other hand, people living in locations with parks, lakes or historical buildings, might experience a disutility from high population density. Moreover, different types of workers have different valuation for different types of amenities (Albouy, 2016; Couture and Handbury, 2015).

Therefore, I assume that neighborhood amenities have two components: residential fundamentals and residential externalities. The first one, captures features of physical geography that affects the willingness to live in a location. The latter, represents how population density of both types of residents of surrounding neighborhoods affects the amenities of a given location. In particular, assume amenities in location i are given by:

$$B_i = b_i \Omega_i^{\eta}, \tag{14}$$

where  $b_i$  represents the residential fundamentals and  $\Omega_i$  the residential externalities:

$$\Omega_i \equiv \sum_{m=1}^{L} \left[ e^{-\sigma_u \cdot \tau_{im}} \cdot \left( \frac{N_{Rmu}}{L_m} \right) + e^{-\sigma_s \cdot \tau_{im}} \cdot \left( \frac{N_{Rms}}{L_m} \right) \right]. \tag{15}$$

Here,  $N_{Rmu}/L_m$  and  $N_{Rms}/L_m$  correspond to the residence density per unit of land area  $L_m$  of low- and high-skilled workers in location m, respectively. In this expression,  $\eta$  controls the relative importance of the externalities in overall residential amenities, while the  $\sigma_s$  and  $\sigma_u$  determine their rate of spatial decay. I expect  $\sigma_s < \sigma_u$ , this is, the externalities created by skilled workers decay slower that those created by unskilled workers. However, their sign will depend on whether the workers get utility or disutility from each type of population density. For example, if consumers associate higher density with more consumption amenities, or with more travel times and higher crime rates.

## 3.2 Firms

Firms produce the same final good that is costlessly traded within the city and with the outside world. The good is produced under perfect competitions and constant returns to scale. Therefore, firm level input demand translates directly to block level aggregate labor demand. Firms produce the final good using floor space and both types of labor according to the following production function:

$$Y_j = A_j(\tilde{N}_j^F)^{\alpha} (H_j^F)^{1-\alpha},$$

where  $H_j^F$  corresponds to office floor space,  $\tilde{N}_j^F$  corresponds to the total effective units of labor force used by firms in location j and  $A_j$  is block-specific productivity term. Finally,  $\tilde{N}_j^F$  is the CES aggregator between low and high-skilled workers with a constant elasticity of substitution of  $\frac{1}{1-\rho}$ :

$$\tilde{N}_j^F = \left[ (N_{ju}^F)^\rho + (N_{js}^F)^\rho \right]^{\frac{1}{\rho}}.$$

Notice that  $\tilde{N}_j^F$  is different from the  $N_j^F$  defined in the previous section: while the former is an aggregator of effective units of both types of labor, the latter corresponds to the sum of both types of workers in a given block.

Firms in location j choose the quantity of low and high-skilled labor, and office floor space that maximizes their profit function taking the respective input prices  $w_{ju}$ ,  $w_{js}$  and  $r_j^F$  in that location as given. From the first order conditions:

$$N_{ju}^{F} = \left(\frac{\alpha A_{j}}{w_{ju}}\right)^{\frac{1}{1-\rho}} (\tilde{N}_{j}^{F})^{\frac{\alpha-\rho}{1-\rho}} (H_{j}^{F})^{\frac{1-\alpha}{1-\rho}}, \tag{16}$$

$$N_{js}^{F} = \left(\frac{\alpha A_{j}}{w_{is}}\right)^{\frac{1}{1-\rho}} (\tilde{N}_{j}^{F})^{\frac{\alpha-\rho}{1-\rho}} (H_{j}^{F})^{\frac{1-\alpha}{1-\rho}}, \tag{17}$$

$$H_j^F = \left[\frac{(1-\alpha)A_j}{r_j^F}\right]^{\frac{1}{\alpha}} \tilde{N}_j^F. \tag{18}$$

Dividing equation (16) by (17), it is possible to see how firms substitute between both types of labor:

$$w_{ju}(N_{ju}^F)^{1-\rho} = w_{js}(N_{js}^F)^{1-\rho}, (19)$$

which, together with the first order conditions, yield to:

$$\tilde{N}_j^F = (\alpha A_j)^{\frac{1}{1-\alpha}} \left[ w_{ju}^{\frac{-\rho}{1-\rho}} + w_{js}^{\frac{-\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho(1-\alpha)}} H_j^F. \tag{20}$$

Therefore, effective total employment in block j increases when the productivity of the firm in block

j increases, when the wages paid to either type of worker decrease or when the amount of floor space available in that block increases. Using the zero profit condition, I also get an expression for the equilibrium price of office floor space in location j:

$$r_j^F = \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) A_j^{\frac{1}{1-\alpha}} \left[ w_{ju}^{\frac{-\rho}{1-\rho}} + w_{js}^{\frac{-\rho}{1-\rho}} \right]^{\frac{\alpha(1-\rho)}{\rho(1-\alpha)}}.$$
 (21)

This equation says that firms are able to pay higher rents for commercial floor space in blocks with higher productivity and/or lower wages, and still make zero profits.

## 3.2.1 Endogenous Productivity

In order to endogeneize productivity, I write an expression for  $A_j$  based on the standard approach in the urban economics literature (e.g., Fujita (1989), Ahlfeldt et al. (2015)), and similar to the expression presented above for the amenities. In particular, define:

$$A_j = a_j \Upsilon_j^{\nu}, \tag{22}$$

where  $a_j$  represents production fundamentals from location j. Production externalities are captured by  $\Upsilon_j$  in the following way::

$$\Upsilon_{j} \equiv \sum_{n=1}^{L} \left[ e^{-\delta_{u} \cdot \tau_{jn}} \cdot \left( \frac{N_{nu}^{F}}{L_{n}} \right) + e^{-\delta_{s} \cdot \tau_{jn}} \cdot \left( \frac{N_{ns}^{F}}{L_{n}} \right) \right], \tag{23}$$

where  $N_{nu}^F/L_n$  and  $N_{ns}^F/L_n$  correspond to the workplace employment density per unit of land area  $L_n$  of low- and high-skilled workers in location n, respectively. The standard interpretation of these externalities could be that density facilitates knowledge spillovers, input sharing or market pooling (Rosenthal and Strange, 2004). However, I allow for the density of high and low-skilled workers to have a different impact on the productivity. In these expressions,  $\nu$  controls the relative importance of externalities on determining the productivity, while the  $\delta_s$  and  $\delta_u$  determine their rate of spatial decay. I expect  $\delta_s < \delta_u$ , this is, the externalities created by high-skilled workers decay slower that those created by low-skilled.

## 3.3 Real Estate Market

At every location j, the demand for residential and commercial floor space is given by the solutions of the households' and firms' maximization problems, respectively. Assume for simplicity that all the land is used either by firms or households. For the supply side, assume that the production of floor space takes place in a competitive market. This is not a strong assumption: Gyourko and Molloy (2015) mention that there is no evidence of market power by firms in the residential

construction market; instead, there are over 100,000 companies in the single family construction business.

Based on the models of housing proposed by Muth (1969) and Fujita (1989), I present a simple model of real estate developers that face restrictions when developing new buildings. Assuming that the floor space demanded by households has the same characteristics than the one demanded by firms, developers only need to worry about producing one type of floor space. Given these assumptions, and in a world with no zoning restrictions, developers build floor space for the group of agents (workers or firms) with the highest willingness to pay for it, until the rents paid by both equalize. In order to produce floor space, developers require land and capital according to a production function  $H(L_i, K_i)$  that exhibits constant returns to scale. Finally, assume that the owners of capital are absent and that its price  $\varrho$  is the same at every location.

#### 3.3.1 No Restrictions

Consider first the case where there are no restrictions on the supply of floor space. Since the land area of a given block is fixed, a representative developer chooses the level of capital that maximizes his profits, given the price of land in block i ( $p_i$ ) and the price for capital  $\varrho$ . Given constant returns to scale, the developer solves:

$$\max_{k_i} r_i h(k_i) - p_i - \varrho k_i,$$

where  $k_i \equiv \frac{K_i}{L_i}$  represents capital per unit of land area used for construction in block i, and  $h(k_i)$  represents the total amount of floor space by unit of area built in that plot as a function of the capital. This function approximates the floor-to-area ratio (FAR). Assume a Cobb-Douglas production function for space,  $H(L_i, K_i) = (L_i)^{\mu} K_i^{1-\mu}$ . From, the first order condition and the zero profit condition, the total supply of floor space in a given block i and the price of land in block i can be written as:

$$H^{S}(L_{i}, K_{i}^{*}, r_{i}) = L_{i} \left[ \frac{(1-\mu)r_{i}}{\varrho} \right]^{\frac{1-\mu}{\mu}}, \tag{24}$$

$$p_i = \left(\frac{1-\mu}{\varrho}\right)^{\frac{1-\mu}{\mu}} \mu r_i^{\frac{1}{\mu}}. \tag{25}$$

Let  $\omega_i$  be the share of floor space built in block *i* used for commercial purposes. Hence,  $1 - \omega_i$  is the share used for residential purposes. Residential and commercial floor space market clearing in a block *i* must satisfy, respectively:

$$H_i^R = \frac{1}{r_i^R} \left\{ (1 - \beta_u) N_{Riu} \bar{I}_{iu} + (1 - \beta_s) N_{Ris} \bar{I}_{is} \right\} = (1 - \omega_i) H_i^S = (1 - \omega_i) L_i \left[ \frac{1 - \mu}{\varrho} r_i^R \right]^{\frac{1 - \mu}{\mu}} (26)$$

$$H_i^F = \left[\frac{1-\alpha}{r_i^F}\right]^{\frac{1}{\alpha}} \tilde{N}_i^F = \omega_i H_i^S = \omega_i L_i \left[\frac{1-\mu}{\varrho} r_i^F\right]^{\frac{1-\mu}{\mu}}$$
(27)

From these two conditions, I derive the optimal price of residential and commercial floor space, respectively, which must be equal in the equilibrium:

$$r_i^{R*} = \left(\frac{\rho}{1-\mu}\right)^{1-\mu} \left\{ \frac{\sum_{e \in \{u,s\}} (1-\beta_e) N_{Rie} \bar{I}_{ie}}{(1-\omega_i) L_i} \right\}^{\mu}$$
 (28)

$$r_i^{F*} = \left\{ (1 - \alpha)^{\mu} \left( \frac{\rho}{1 - \mu} \right)^{\alpha(1 - \mu)} \left[ \frac{\tilde{N}_i^F}{\omega_i L_i} \right]^{\alpha \mu} \right\}^{\frac{1}{\mu(1 - \alpha) + \alpha}}$$
(29)

$$r_i^{R*} = r_i^{F*} \tag{30}$$

This last equality determines the optimal level of  $\omega_i$  for every block, which depends on the living and working population density of each type of worker, their skill specific productivities and income. I can plug  $\omega_i^*$  back into either equation (28) or (29) to get an expression for  $r_i^*$ . Finally, note that from equation (25), I could also back out the price of the land  $p_i$  and the total rents  $R = \sum_{i=1}^{L} p_i L_i$ , which is an element of the income of the high-skilled workers.

## 3.3.2 Equilibrium without any floor space restrictions

Given the model's parameters  $\{\{\beta_e, \kappa_e\}_{e \in \{u,s\}}, \theta, \alpha, \rho, \mu\}$ , the exogenous price for capital  $\varrho$  and exogenous location-specific characteristics  $\{\{\mathbf{T_e}, \mathbf{E_e}\}_{e \in \{u,s\}}, \mathbf{A}, \mathbf{B}, \boldsymbol{\tau}, \boldsymbol{L}\}^{1}$  the general equilibrium of the model with no agglomeration externalities and no LUR is given by the expected utility of both types of workers  $\{\bar{u}_u, \bar{u}_s\}$  and the vectors  $\{\{\boldsymbol{\pi_{Re}}, \boldsymbol{\pi_{Fe}}, \mathbf{w_e}\}_{e \in \{u,s\}}, \mathbf{r^R}, \mathbf{r^F}, \boldsymbol{\omega}\}$  that satisfy equations (5), (7) and (8) for both types of workers, and (19), (21), (26), (27) and (30):

$$\bar{u}_e = \left[ \sum_{m=1}^{L} \sum_{n=1}^{L} T_{me} E_{ne}(r_m^R)^{-\theta(1-\beta_e)} (B_m I_{mne})^{\theta} \right]^{1/\theta}, \quad \forall e \in \{u, s\},$$

$$\pi_{Rie} = \bar{u}_e^{-\theta} \sum_{n=1}^{L} T_{ie} E_{ne}(r_i^R)^{-\theta(1-\beta_e)} (B_i I_{ine})^{\theta}, \quad \forall e \in \{u, s\},$$

$$\pi_{Fje} = \bar{u}_e^{-\theta} \sum_{m=1}^{L} T_{me} E_{je}(r_m^R)^{-\theta(1-\beta_e)} (B_m I_{mje})^{\theta}, \quad \forall e \in \{u, s\},$$

$$w_{ju}(N_{ju}^F)^{1-\rho} = w_{js}(N_{js}^F)^{1-\rho},$$

<sup>&</sup>lt;sup>11</sup>Throughout all this section, bold letters will denote vectors or matrices.

$$r_j^F = \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) A_j^{\frac{1}{1-\alpha}} \left[ w_{ju}^{\frac{-\rho}{1-\rho}} + w_{js}^{\frac{-\rho}{1-\rho}} \right]^{\frac{\alpha(1-\rho)}{\rho(1-\alpha)}},$$

$$H_i^R = \frac{1}{r_i^R} \left[ (1 - \beta_u) N_{Riu} \sum_{j=1}^L \frac{E_{ju} I_{iju}^{\theta}}{\sum_{n=1}^L E_{nu} I_{inu}^{\theta}} I_{iju} + (1 - \beta_s) N_{Ris} \sum_{j=1}^L \frac{E_{js} I_{ijs}^{\theta}}{\sum_{n=1}^L E_{ns} I_{ins}^{\theta}} I_{ijs} \right] = (1 - \omega_i) H_i^S,$$

$$H_i^F = \left[\frac{1-\alpha}{r_i^F}\right]^{\frac{1}{\alpha}} \tilde{N}_i^F = \omega_i H_i^S,$$

$$r_i^{R*} = r_i^{F*},$$

with  $(1 - \omega_i)H_i + \omega_i H_i = H_i = L_i \left[\frac{1-\mu}{\varrho}r_i^R\right]^{\frac{1-\mu}{\mu}}$  for every block i; and where,  $I_{ju} = w_{ju}$  and  $I_{js} = w_{js} + \varphi_i \sum_{i=1}^L p_i L_i$ , and  $p_i$  is determined by (25). By the First Welfare Theorem, this competitive solution is efficient. However, when amenities and productivity are endogenous, the competitive equilibrium will not necessarily be efficient. in this case, the vector of parameters would be  $\{\{\beta_e, \kappa_e, \sigma_e, \delta_e\}_{e \in \{u,s\}}, \theta, \alpha, \rho, \mu, \eta, \nu\}$  and the vector of exogenous location-specific characteristics would be  $\{\{\mathbf{T_e}, \mathbf{E_e}\}_{e \in \{u,s\}}, \mathbf{a}, \mathbf{b}, \tau, L\}$ .

## 3.4 Zoning Restrictions

Consider the case where the city authorities can restrict what can be built in a given block. In this case, the developer's profit maximization problem described above still holds, as well as the floor space market clearing conditions. However, now  $\omega_i$  is determined exogenously and there would be three possible cases.

Case 1: Block i can be used for both purposes but the proportions are fixed.

In this case,  $\omega_i = \bar{\omega}_i \in (0,1)$ . The equilibrium in the housing market is still given by equations (26) to (29), but now  $\bar{\omega}_i$  is treated as an exogenous variable. Moreover,  $r_i^{R*} = r_i^{F*}$  does not necessarily hold.

Case 2: Block i must be used for residential units only.

In this case,  $\omega_i = \bar{\omega}_i = 0$  and the price of floor space in block i is given by (28):

$$r_{i}^{*} = r_{i}^{R*} = \left(\frac{\rho}{1-\mu}\right)^{1-\mu} \left\{ \frac{\sum_{e \in \{u,s\}} (1-\beta_{e}) N_{Rie} \bar{I}_{ie}}{(1-\omega_{i}) L_{i}} \right\}^{\mu},$$

$$H_{i}^{R*} = H_{i}^{S} = L_{i} \left[ \frac{1-\mu}{\varrho} r_{i}^{*} \right]^{\frac{1-\mu}{\mu}}, \qquad \bar{H}_{i}^{F} = 0.$$
(31)

Case 3: Block i must be used for commercial space only.

In this case,  $\omega_i = \bar{\omega}_i = 1$  and the price of floor space in block i is given by (29):

$$r_{i}^{*} = r_{i}^{F*} = \left\{ (1 - \alpha)^{\mu} \left( \frac{\rho}{1 - \mu} \right)^{\alpha(1 - \mu)} \left[ \frac{\tilde{N}_{i}^{F}}{\omega_{i} L_{i}} \right]^{\alpha \mu} \right\}^{\frac{1}{\mu(1 - \alpha) + \alpha}},$$

$$H_{i}^{F*} = H_{i}^{S} = L_{i} \left[ \frac{1 - \mu}{\varrho} r_{i}^{*} \right]^{\frac{1 - \mu}{\mu}}, \quad \bar{H}_{i}^{R} = 0.$$
(32)

In either case, we could then determine  $p_i$  using a modified version of equation (25):

$$p_{i} = \left(\frac{1-\mu}{\varrho}\right)^{\frac{1-\mu}{\mu}} \mu \left[\bar{\omega}_{i}(r_{i}^{F})^{\frac{1}{\mu}} + (1-\bar{\omega}_{i})(r_{i}^{R})^{\frac{1}{\mu}}\right].$$

## 3.4.1 Equilibrium with Zoning

Given the model's parameters  $\{\{\beta_e, \kappa_e\}_{e \in \{u,s\}}, \theta, \alpha, \rho, \mu\}$ , the exogenous price for capital  $\varrho$ , exogenous location-specific characteristics  $\{\{\mathbf{T_e}, \mathbf{E_e}\}_{e \in \{u,s\}}, \mathbf{A}, \mathbf{B}, \boldsymbol{\tau}, \boldsymbol{L}\}$ , and the zoning restrictions for some blocks  $\bar{\omega}_i$ , the general equilibrium of the model with zoning and no agglomeration externalities is given by the expected utility of both types of workers  $\{\bar{u}_u, \bar{u}_s\}$  and the vectors  $\{\{\boldsymbol{\pi_{Re}}, \boldsymbol{\pi_{Fe}}, \mathbf{w_e}\}_{e \in \{u,s\}}, \mathbf{r^R}, \mathbf{r^F}, \boldsymbol{\omega}\}$  that satisfies equations (5), (7) and (8) for both types of workers, and (19), (21), (26) and (27) for every block. Also, equation (30) must hold for those blocks without any zoning restriction, determining  $\omega_i \in (0, 1)$ .

#### 3.4.2 Comparative Statics

In this subsection, I analyze how the different equilibrium variables change when zoning policies are implemented or modified. I do this for the case without agglomeration externalities and at the end of the whole theoretical section, I discuss briefly what happens when they are present. Assume that the city's Department of Zoning decides to make block i more residential, i.e., to lower  $\omega_i$  ( $\Delta\omega_i < 0$ ) from its equilibrium value. The following figure shows the effect on rents in block i given this zoning policy:

Notice first that rent equalization no longer holds in that block. In particular, residential rents in block i decrease  $(\frac{\partial r_i^R}{\partial \omega_i} \geq 0)$  because there is more space that can be use for housing purposes, while commercial rents will increase  $(\frac{\partial r_i^F}{\partial \omega_i} \leq 0)$ . Moreover, equation (21) indicates that wages for both types of workers decrease,  $\frac{\partial w_{ie}}{\partial \omega_i} \geq 0$ , for  $e \in \{u, s\}$ . The magnitude of the wage reduction or the

relative reduction between both groups are not trivial. However, from the equilibrium definition, it is easy to see that it will depend on the initial composition of types within the block and their respective productivity.

Before analyzing what happens with the probability of working or living in block i, or with the expected utility, it is important to analyze what happens with wages and rents in the other blocks. Assume that all other blocks are already fully zoned and therefore  $\omega_j = \bar{\omega}_j \in [0,1] \quad \forall j$ . Given that more residences are allowed in block i, the demand for housing in block j shifts towards the origin and residential rents decrease. The opposite effect takes place in the commercial real estate sector, leading to an increase in commercial rents. This implies that wages in block j decrease; effect that is reinforced by an increase in labor supply from workers from all over given that more people will be commuting to block j instead of block i. The magnitude of these effects will depend on the distance between blocks i and j. Summarizing, we have  $\frac{\partial r_j^R}{\partial \omega_i} \geq 0$ ,  $\frac{\partial r_j^F}{\partial \omega_i} \leq 0$  and  $\frac{\partial w_{je}}{\partial \omega_i} \geq 0$ .

Taking into account these effects, from equation (7) I can analyze the impact of this zoning restriction on the probability that a worker of type e chooses to live in block i. After some math, the resulting elasticity for each type e is:

$$\begin{split} \varepsilon_{\pi_{Riu},\omega_i} &= \theta \left[ -(1-\beta_u)\varepsilon_{r_i,\omega_i} + \sum_{n=1}^L \pi_{inu|i} \cdot \varepsilon_{w_{nu},\omega_i} \right] \geqslant 0, \\ \varepsilon_{\pi_{Ris},\omega_i} &= \theta \left\{ -(1-\beta_s)\varepsilon_{r_i,\omega_i} + \sum_{n=1}^L \pi_{ins|i} \left[ \left( \frac{w_{ns}}{I_{ins}} \right) \varepsilon_{w_{ns},\omega_i} + \left( \frac{\varphi_i R}{I_{ins}} \right) \varepsilon_{R,\omega_i} \right] \right\} \geqslant 0, \end{split}$$

where  $\pi_{ine|i}$  corresponds to the probability of a worker of type e to commute to block n conditional om living in block i, and given by equation (10). Also,  $\varepsilon_{I_{inu},\omega_i} = \varepsilon_{w_{nu},\omega_i}$ , and  $\varepsilon_{I_{ins},\omega_i} = \left(\frac{w_{ns}}{I_{ins}}\right)\varepsilon_{w_{ns},\omega_i} + \left(\frac{\varphi_i R}{I_{ins}}\right)\varepsilon_{R,\omega_i}$ . The previous expressions are quite intuitive. There are two effect going in contrary directions. On one hand, due to the reduction in residential rents, more people would like to live in block i. This willingness to move is scaled by the share of income individuals spend on housing. On the other hand, wages in some locations close by decrease and, conditional on living in i, making that daily commute might become less desirable.

There are two other things to point out. First, for low-skilled workers, the change in income corresponds to the change in wages, while for high-skilled workers, the change in income corresponds to a weighted average between the change in wages and the change in non labor income coming from the total rents of land. Second, it is not clear whether this non labor income increases or decreases: It depends on the relative change between commercial and residential rents, and the shares  $\omega_i$  in every block. Therefore, the income effect high-skilled workers face might be positive, which would imply that they are more likely to move into that location.

Comparing the two previous equations, the change in the relative probability of living in block i, which corresponds to an approximation for the relative change of block i's composition of the

population, can be written as:

$$\varepsilon_{\pi_{Riu},\omega_i} - \varepsilon_{\pi_{Ris},\omega_i} = \theta \left[ (\beta_u - \beta_s) \varepsilon_{r_i,\omega_i} + \sum_{n=1}^{L} \pi_{ins|i} (\varepsilon_{I_{inu},\omega_i} - \varepsilon_{I_{ins},\omega_i}) + \sum_{n=1}^{L} (\pi_{inu|i} - \pi_{ins|i}) \varepsilon_{I_{inu},\omega_i} \right].$$
(33)

This expression depends on three terms. First, on the change in housing rents, which affects both types of workers differently times the differences in the share of housing expenditures. In particular, if the policy causes residential rents in block i to go down, both types of people would like to move to that block, but the effect would be larger for low-skilled workers. Second, it also depends on the changes of the relative income received by both types of workers in all the blocks of the city. Third, it depends on the sorting of both types of workers across all possible workplaces.

Similarly, I can find the impact of this zoning restriction on the probability that a worker of type e chooses to work in block i:

$$\begin{split} \varepsilon_{\pi_{Fiu},\omega_i} &= \theta \left[ \varepsilon_{w_{iu},\omega_i} - (1 - \beta_u) \sum_{n=1}^L \pi_{niu|i} \cdot \varepsilon_{r_n,\omega_i} \right] \geqslant 0, \\ \varepsilon_{\pi_{Fis},\omega_i} &= \theta \sum_{n=1}^L \pi_{nis|i} \left[ \left( \frac{w_{is}}{I_{nis}} \right) \varepsilon_{w_{is},\omega_i} + \left( \frac{\varphi_n R}{I_{nis}} \right) \varepsilon_{R,\omega_i} - (1 - \beta_s) \varepsilon_{r_n,\omega_i} \right] \geqslant 0, \end{split}$$

where  $\pi_{nie|i}$  corresponds to the probability of a worker of type e to commute from block n conditional on working in block i, given by equation (9). Just as before, there are two effects going in opposite directions. On one hand, due to the increase in commercial rents, firms have to reduce their wages and people would be less willing to commute to that block to work. On the other hand, residential rents in some other blocks close by decrease and, conditional on working in i, making that daily commute might become more desirable. Comparing the two previous equations, the change in the relative probability of working in block i, which corresponds to an approximation for the relative change of block i's composition of the working population, can be written as:

$$\varepsilon_{\pi_{Fiu},\omega_{i}} - \varepsilon_{\pi_{Fis},\omega_{i}} = \theta \sum_{n=1}^{L} \left\{ \pi_{nis|i} \left[ (\beta_{u} - \beta_{s}) \varepsilon_{r_{n},\omega_{i}} + (\varepsilon_{I_{niu},\omega_{i}} - \varepsilon_{I_{nis},\omega_{i}}) \right] + (\pi_{niu|i} - \pi_{nis|i}) (\varepsilon_{I_{niu},\omega_{i}} - (1 - \beta_{u}) \varepsilon_{r_{n},\omega_{i}}) \right\}.$$
(34)

This expression can also be decompose in the same three parts. First, it depends on the change of housing rents in all the blocks of the city, which affects low-skilled workers more since  $\beta_u < \beta_s$ . Second, it depends on the changes of the relative income received by both types of workers in block i. Finally, it depends on how the sorting of both types of workers across blocks of residence changes.

Finally, consider what happens with changes in the expected utility of a particular type of

worker when a zoning restriction is imposed in one block. Differentiating equation (5) with respect to  $\omega_i$ , and writing it in elasticity terms, I get:

$$\varepsilon_{\bar{u}_e,\omega_i} = \sum_{m=1}^{L} \left[ \left( \sum_{n=1}^{L} \pi_{mne} \cdot \varepsilon_{I_{mne},\omega_i} \right) - (1 - \beta_e) \pi_{Rme} \cdot \varepsilon_{r_m,\omega_i} \right] \geq 0.$$

This expression corresponds to the average effect on both wages and rents perceived in the different blocks throughout the city weighted by the probability of working or living in such block, respectively. The expected value of a type e worker is a result of two effects. First, if more space is devoted to residences in block i, residential rents go down in several blocks because there is more land available for housing supply and as a result workers are better off. Second, since commercial rents are going up, labor demand shifts down resulting in a decrease in wages in several blocks, implying a lower utility. In order to see which type of worker is more affected by the zoning policy, subtract the previous equation for both types of workers:

$$\varepsilon_{\bar{u}_{u},\omega_{i}} - \varepsilon_{\bar{u}_{s},\omega_{i}} = \sum_{m=1}^{L} \left[ (\beta_{u} - \beta_{s}) \pi_{Rms} \cdot \varepsilon_{r_{n},\omega_{i}} + \sum_{n=1}^{L} \pi_{mns} \left( \varepsilon_{I_{mnu},\omega_{i}} - \varepsilon_{I_{mns},\omega_{i}} \right) + \sum_{n=1}^{L} \left( \pi_{mnu} - \pi_{mns} \right) \varepsilon_{I_{mnu},\omega_{i}} + \left( 1 - \beta_{u} \right) \left( \pi_{Rms} - \pi_{Rmu} \right) \varepsilon_{r_{n},\omega_{i}} \right]$$
(35)

Note that the total relative effect can be decompose as the sum of three terms. First, changes in residential rents throughout the city. Second, changes in relative incomes and wages. Third, sorting of both types of workers by workplace and by residence location, respectively.

#### 3.5 FAR Restrictions

Consider now that the city's Department of Zoning imposes a FAR restriction of  $\bar{h}_i$  on block i. The developer now solves:

$$\max_{k_i} r_i h(k_i) - p_i - \varrho k_i \quad \text{s.t.} \quad h(k_i) \le \bar{h}_i.$$

Denoting the lagrangean multiplier of this problem as  $\chi_i$ , the first order condition and the complementary slackness condition of this problem are, respectively:

$$(r_i - \chi_i) \frac{\partial h(k_i)}{\partial k_i} = \varrho, \qquad \chi_i [h(k_i) - \bar{h}_i] = 0.$$

If the restriction does not bind, the solution would be given by the no restriction case presented in Section 3.3. However, if the restriction is binding, the total supply of floor space in block i, the

shadow price of this restriction and the price of land in block i are given by:

$$H_{i}^{S} = L_{i}\bar{h}_{i},$$

$$\chi_{i}^{*} = r_{i} - \frac{\varrho}{1-\mu}\bar{h}_{i}^{\frac{\mu}{1-\mu}},$$

$$p_{i} = r_{i}\bar{h}_{i} - \varrho\bar{h}_{i}^{\frac{1}{1-\mu}}.$$
(36)

Residential and commercial floor space market clearing in a block i must satisfy:

$$H_i^R = \frac{1}{r_i^R} \left\{ (1 - \beta_u) N_{Riu} \bar{I}_{iu} + (1 - \beta_s) N_{Ris} \bar{I}_{is} \right\} = (1 - \omega_i) H_i^S = (1 - \omega_i) L_i \bar{h}_i, \tag{37}$$

$$H_i^F = \left[\frac{1-\alpha}{r_i^F}\right]^{\frac{1}{\alpha}} \tilde{N}_i^F = \omega_i H_i^S = \omega_i L_i \bar{h}_i. \tag{38}$$

From these conditions, the optimal price of residential and commercial floor space is determined by:

$$r_i^{R*}(\bar{h}_i) = \frac{1}{(1-\omega_i)\bar{h}_i} \left\{ \sum_{e \in \{u,s\}} (1-\beta_e) \left(\frac{N_{Rie}}{L_i}\right) \bar{I}_{ie} \right\}, \tag{39}$$

$$r_i^{F*}(\bar{h}_i) = (1-\alpha) \left(\frac{1}{\omega_i \bar{h}_i}\right)^{\alpha} \left(\frac{\tilde{N}_i^F}{L_i}\right)^{\alpha}.$$
 (40)

In this restricted equilibrium, the developer will provide floor space to both households and firms until  $r_i^{R*} = r_i^{F*}$ , determining the shares of floor space built for residential and commercial purposes,  $\omega_i^*$ . These shares will depend on the living and working population density of each type of worker, their incomes and productivities, and the FAR rule. With these shares, I can back out the price of the land, total rents R, and the shadow price of this restriction from (36).

#### 3.5.1 Equilibrium with FAR restrictions

Given the model's parameters  $\{\{\beta_e, \kappa_e\}_{e \in \{u,s\}}, \theta, \alpha, \rho, \mu\}$ , the exogenous price for capital  $\varrho$ , exogenous location-specific characteristics  $\{\{\mathbf{T_e}, \mathbf{E_e}\}_{e \in \{u,s\}}, \mathbf{A}, \mathbf{B}, \boldsymbol{\tau}, \boldsymbol{L}\}$ , and the FAR restrictions for some blocks  $\bar{h}_i$ , the general equilibrium of the model with FAR restrictions and no agglomeration externalities is given by the expected utility of both types of workers  $\{\bar{u}_u, \bar{u}_s\}$  and the vectors  $\{\{\boldsymbol{\pi_{Re}}, \boldsymbol{\pi_{Fe}}, \mathbf{w_e}\}_{e \in \{u,s\}}, \mathbf{r^R}, \mathbf{r^F}, \boldsymbol{\omega}, \boldsymbol{\chi}\}$  that satisfy equations (5), (7) and (8) for both types of workers, and (19) and (21) for every block. Regarding the real estate market:

• Equations (26) and (27) have to be satisfied with  $H_i^S = L_i \left[ \frac{1-\mu}{\varrho} r_i^R \right]^{\frac{1-\mu}{\mu}}$  for every block i without a binding FAR restriction, and with  $H_i^S = L_i \bar{h}_i$  for every block with a binding FAR

restriction given by  $\bar{h}_i$ .

- Equation (30) has to hold for every block i.
- The shadow cost of the FAR restriction is given by  $\chi_i = 0$  for those blocks without a binding FAR restriction and by equation (36) for those blocks with a binding FAR restriction.

## 3.5.2 Comparative Statics

In this subsection, I analyze how different equilibrium variables change when a FAR restriction is implemented. Just as in the previous case, I do so for the case without agglomeration externalities. Assume that the city's Department of Zoning decides to impose a height or a Floor-to-Area restriction on block i's constructions and that this restriction is binding:  $\bar{h}_i < h_i^* \Leftrightarrow \bar{H}_i < H_i^*$ . The figure below shows the effect of this restriction on the residential and commercial real estate markets:

Several things to notice. First, the equilibrium rents increase for both residential and commercial floor space  $(\frac{\partial r_i^*}{\partial h_i} < 0)$  and remain equal. Second, since rents paid by firms are higher, wages must go down to satisfy the zero profit condition:  $\frac{\partial w_{ie}^*}{\partial h_i} > 0$ . Third, assuming that all the other blocks are already fully zoned, this height restriction causes more competition for space everywhere in the city, leading to higher rents and lower wages in every block. Fourth, total rents in the city (R) increase, leading to a higher non-labor income for high-skilled workers. Fifth, following Section 3.4, it is easy to obtain expressions for the elasticities of people living and working in block i, and the expected utility for both types of workers, with respect to changes in FAR:

$$\begin{split} \varepsilon_{\pi_{Rie},\omega_{i}} &= \theta \left[ -(1-\beta_{e})\varepsilon_{r_{i},\bar{h}_{i}} + \sum_{n=1}^{L} \pi_{inu|i} \cdot \varepsilon_{I_{ine},\bar{h}_{i}} \right], \\ \varepsilon_{\pi_{Fie},\bar{h}_{i}} &= \theta \sum_{n=1}^{L} \pi_{nis|i} \left[ \varepsilon_{I_{nie},\bar{h}_{i}} - (1-\beta_{e})\varepsilon_{r_{n},\bar{h}_{i}} \right] \\ \varepsilon_{\bar{u}_{e},\bar{h}_{i}} &= \sum_{m=1}^{L} \left[ \left( \sum_{n=1}^{L} \pi_{mne} \cdot \varepsilon_{I_{mne},\bar{h}_{i}} \right) - (1-\beta_{e})\pi_{Rme} \cdot \varepsilon_{r_{m},\bar{h}_{i}} \right], \end{split}$$

where  $\varepsilon_{I_{inu},\bar{h}_i} = \varepsilon_{w_{nu},\bar{h}_i}$ , and  $\varepsilon_{I_{ins},\bar{h}_i} = \left(\frac{w_{ns}}{I_{ins}}\right) \varepsilon_{w_{ns},\bar{h}_i} + \left(\frac{\varphi_i R}{I_{ins}}\right) \varepsilon_{R,\bar{h}_i}$ . Since rents increase and wages decrease as the result of this policy, every block in the city is (weakly) less desirable for low-skilled workers to both work and live. Naturally, this implies that the expected utility of these workers is lower. Since total rents are higher, high-skilled workers could potentially be better off as the result of the FAR restriction. Finally, the expressions describing the relative changes in the probability of living and working in a block, and the expected utility can also be decompose into three effect coming from changes in rents, changes in relative income and sorting.

## 3.6 Zoning and FAR restrictions

What we actually see most of the time in the zoning ordinances is a combination between zoning regulations and height restrictions. For instance, zoning designation **RM-5** allows only for a maximum floor-to-area ratio of 2 ( $\bar{h} \approx 2$ ) and does not allow any commercial activity ( $\bar{\omega} = 0$ ). Therefore, in this section I analyze what happens if both zoning and FAR restrictions are included in the model. Three cases can happen:

Case 1: Block i can be used for both purposes but the proportions are fixed.

In this case,  $\omega_i = \bar{\omega}_i \in (0,1)$ . The equilibrium in the housing market is still given by equations (37) to (40), but now  $\bar{\omega}_i$  is treated as an exogenous variable. Moreover,  $r_i^{R*} = r_i^{F*}$  does not necessarily hold.

Case 2: Block i must be used for residential units only.

In this case,  $\omega_i = \bar{\omega}_i = 0$  and the price of floor space in block i is given by (28):

$$r_{i}^{*}(\bar{h}_{i}) = r_{i}^{R*} = \frac{1}{\bar{h}_{i}} \left\{ \sum_{e \in \{u,s\}} (1 - \beta_{e}) \left( \frac{N_{Rie}}{L_{i}} \right) \bar{I}_{ie} \right\},$$

$$H_{i}^{R*} = \bar{H}_{i}^{S} = L_{i}\bar{h}_{i}, \qquad \bar{H}_{i}^{F} = 0.$$

$$(41)$$

Case 3: Block i must be used for commercial space only.

In this case,  $\omega_i = \bar{\omega}_i = 1$  and the price of floor space in block i is given by (29):

$$r_i^*(\bar{h}_i) = r_i^{F*} = \frac{1-\alpha}{\bar{h}_i^{\alpha}} \left(\frac{\tilde{N}_i^F}{L_i}\right)^{\alpha},$$

$$H_i^{F*} = \bar{H}_i^S = L_i \bar{h}_i, \quad \bar{H}_i^R = 0.$$
(42)

In either case, we could then determine  $p_i^*(\bar{h}_i)$  and  $\chi_i^*(\bar{h}_i)$ . In particular,  $p_i^*(\bar{h}_i)$  can be found from a modified version of equation (36):

$$p_i = \left[\bar{\omega}_i r_i^F + (1 - \bar{\omega}_i) r_i^R\right] \bar{h}_i - \varrho \bar{h}_i^{\frac{1}{1 - \mu}}.$$

#### 3.6.1 Equilibrium with Zoning and FAR restrictions

Given the model's parameters  $\{\{\beta_e, \kappa_e\}_{e \in \{u,s\}}, \theta, \alpha, \rho, \mu\}$ , the exogenous price for capital  $\varrho$ , exogenous location-specific characteristics  $\{\{\mathbf{T_e}, \mathbf{E_e}\}_{e \in \{u,s\}}, \mathbf{A}, \mathbf{B}, \boldsymbol{\tau}, \boldsymbol{L}\}$ , the zoning and FAR restrictions

<sup>&</sup>lt;sup>12</sup>The RM-5 class corresponds to Residential Multi-Unit District: Medium to high-density apartment buildings; Two-flats, townhouses, and single family homes are also allowed.

for some blocks  $(\bar{\omega}_i, \bar{h}_i)$ , the general equilibrium of the model with zoning and FAR restrictions and no agglomeration externalities is given by the expected utility of both types of workers  $\{\bar{u}_u, \bar{u}_s\}$  and the vectors  $\{\{\boldsymbol{\pi}_{Re}, \boldsymbol{\pi}_{Fe}, \mathbf{w}_e\}_{e \in \{u,s\}}, \mathbf{r}^R, \mathbf{r}^F, \boldsymbol{\omega}, \boldsymbol{\chi}\}$  that satisfy equations (5), (7) and (8) for both types of workers, and (19) and (21) for every block. Regarding the real estate market:

- Equations (26) and (27) must satisfy with  $H_i^S = L_i \left[ \frac{1-\mu}{\varrho} r_i^R \right]^{\frac{1-\mu}{\mu}}$  for every block i that does not have a binding FAR restriction or with  $H_i^S = L_i \bar{h}_i$  for every block with a binding FAR restriction given by  $\bar{h}_i$ .
- Equation (30) must hold for those blocks without any zoning restriction determining  $\omega_i \in (0,1)$ ; while for those blocks with zoning restrictions,  $\omega_i = \bar{\omega}_i \in [0,1]$ .
- The shadow cost of the FAR restriction is given by  $\chi_i = 0$  for those blocks without a binding FAR restriction and by equation (36) for those blocks with a binding FAR restriction.

In this scenario, the total effect of this new restriction is a combination of both effects explored in Sections 3.4 and 3.5. For intuition, in the Appendix I present a particular example in which the total effect of this policy can be decomposed into these two components.

## 3.6.2 The Case with Agglomeration Externalities

Because density generates agglomeration externalities, in the presence of endogenous productivity and amenities the effect could depend heavily on how firms and both type of workers value density. Even though I expect firms to always value density positively (because it increases their productivity), it is not clear how households should value it. For instance, households might value density positively if it generates more consumption amenities, but they might value it negatively if it causes congestion of roads or natural amenities, or higher crime rates.

In the production sector, I always expect firms to value externalities positively. Therefore, if firms are not being allowed in one specific block, causing people to leave some workplace locations, the total effect of a zoning policy would be stronger than what I predicted in Section 3.4. This is, there would be a stronger contraction in the demand for commercial real estate—not only in that block but also in all the surrounding blocks—since there would be less benefits from agglomeration economies.

This is also the case if LUR causes people to leave certain residence locations and households consider higher density to be a good externality. In this case, the demand curve for housing will shift inwards causing a further reduction in residential rents, and reinforcing the effects derived in Sections 3.4 and/or 3.5. However, if households value density negatively, the demand curve for residential floor space would shift outwards after the policy, leading to increases in the residential

rents, softening the effects Sections 3.4 and/or 3.5. Depending on the size of these externalities, the effect could more than compensate the initial effect and lead to higher residential prices.

# 4 Calibration - Recovering Location Characteristics

Following Ahlfeldt et al. (2015) closely, I show in this section that there is a unique mapping from the observed variables to unobserved values of location characteristics. In particular, given observed data on the number of people living and working in every block, together with commuting times between every pair of blocks, I can recover measures of skill-specific income, amenities and productivities.

In order to recover these characteristics, I assume that I know the value for the model's parameters. In a future draft, I will cover how do I recover these values. I will use a combination between parameters that have already been properly estimated in other articles (e.g.,  $\theta$  from Ahlfeldt et al. (2015), or the  $\beta$ 's from Notowidigdo (2013)), and moments from the data, together with an instrumental variable strategy, to identify the rest. For example, for the commuting cost parameters ( $\kappa$ 's), I could use the gravity equations from my model to estimate them. The general goal of the calibration is to solve for the equilibrium and fundamentals of my model and see if its theoretical predictions hold given the observed changes in zoning restrictions in Chicago. In a future draft, I will perform different counterfactuals.

Since some of these variables enter the model isomorphically, define the following adjusted variables for both types  $e \in \{u, s\}$ :

$$\tilde{B}_{ie} = T_{ie}^{1/\theta} B_i, \qquad \tilde{b}_{ie} = T_{ie}^{1/\theta} b_i 
\tilde{A}_{je} = E_{je}^{\alpha/\theta} A_j, \qquad \tilde{a}_{je} = E_{je}^{\alpha/\theta} a_j 
\tilde{I}_{ije} = E_{je}^{1/\theta} I_{ije}, \qquad \tilde{w}_{je} = E_{je}^{1/\theta} w_{je},$$
(43)

These clusters of variables cannot be separately identified. Consider first the adjusted income for a type e worker,  $\tilde{I}_{ije}$ : It captures the wage that a type e worker would get in a certain location j and the average utility that he would get for that employment location  $(E_{je}^{1/\theta})$ , because both affect the relative attractiveness of block j as a workplace. For high-skilled workers, it also includes the share of the total rents he is receiving in his block of residence i. On the production side, the skill-specific adjusted productivity  $(\tilde{A}_{je})$  captures the Fréchet scale parameter for that employment location  $(E_{je}^{\alpha/\theta})$  and the TFP term  $(A_{je})$ , as they both play the same role inside the firms' zero profit condition. Finally, on the consumption side, adjusted amenities  $(\tilde{B}_{ie})$  capture block i's general level of amenities  $(B_i)$  and a skill specific term given by the average utility that a type e worker gets from living in block i  $(T_{ie}^{1/\theta})$ ; both of these affect the relative attractiveness of block i to live in by a type e worker. Adjusted production and amenities fundamentals  $(\tilde{a}_{je}, \tilde{b}_{ie})$  are interpreted analogously.

Now, I will proceed to show how the model can be calibrated to recover recursively these variables.

## 4.1 Adjusted Income

Given the parameters  $\{\theta, \kappa_u, \kappa_s\}$  and observed data  $\{\mathbf{N_{Fu}}, \mathbf{N_{Fs}}, \mathbf{N_{Ru}}, \mathbf{N_{Rs}}, \boldsymbol{\tau}\}$ , commuting market clearing condition (11) provides a system of equations for both types of workers in observed workplace and residence employment that determines a unique adjusted income vector for both types of workers  $\{\tilde{\mathbf{I}}_{\mathbf{u}}, \tilde{\mathbf{I}}_{\mathbf{s}}\}$ :

$$N_{Fje} = \sum_{i=1}^{L} \frac{\left(E_{je}^{1/\theta} I_{ije}\right)^{\theta}}{\sum_{n=1}^{L} \left(E_{ne}^{1/\theta} I_{ine}\right)^{\theta}} N_{Rie} = \sum_{i=1}^{L} \frac{\left(\tilde{I}_{ije}\right)^{\theta}}{\sum_{n=1}^{L} \left(\tilde{I}_{ine}\right)^{\theta}} N_{Rie}, \quad \text{for } e \in \{u, s\},$$

Adjusted income  $(\tilde{I}_{ije} = E_{je}^{1/\theta} I_{ije})$  for both types of worker captures (i) wages  $(w_{je})$  in employment location j and (ii) the Frechet scale parameter that determines the average utility from working there, as well as (iii) the share of total rents determined by residence block i for high-skilled workers. Note that since  $E_{je}^{1/\theta}$  enters the equation isomorphically to  $I_{ije}$ , I cannot separately identify both vectors. The proof that the vectors  $\{\tilde{\mathbf{I}}_{\mathbf{u}}, \tilde{\mathbf{I}}_{\mathbf{s}}\}$  are uniquely identified resembles to that one presented in Ahlfeldt et al. (2015). Finally, homogeneity of degree zero in  $\{\tilde{\mathbf{I}}_{\mathbf{e}}\}$  of the previous system of equations implies that the equilibrium is unique up to a normalization.

## 4.2 Price of Land and Adjusted Wages

In the previous subsection, I show how to recover vectors of adjusted income for both types of workers  $\{\tilde{\mathbf{I}}_{\mathbf{u}}, \tilde{\mathbf{I}}_{\mathbf{s}}\}$ . However, it is important to recover a measure of wages because firms make their decisions based on these. Since low-skilled workers only receive income from their labor, recovering  $\{\tilde{\mathbf{I}}_{\mathbf{u}}\}$  is equivalent to recovering  $\{\tilde{\mathbf{w}}_{\mathbf{u}}\}$  provided data  $\{\tau\}$ . This is not the case for the high-skilled workers since they also receive income from land ownership and therefore we need another equation to determine this second source of income. In particular, notice that:

$$\tilde{w}_{js} = \left[\tilde{I}_{ijs} - E_{js}^{1/\theta} \varphi \sum_{i=m}^{L} (p_m L_m)\right] e^{\kappa_s \tau_{ij}}.$$

To be completed

## 4.3 Residential Amenities

Given the parameters  $\{\theta, \kappa_u, \kappa_s, \beta_u, \beta_s\}$ , observed data  $\{\mathbf{N_{Fu}}, \mathbf{N_{Fs}}, \mathbf{N_{Ru}}, \mathbf{N_{Rs}}, \boldsymbol{\tau}, \mathbf{r^R}\}$  and the adjusted incomes recovered above  $\{\tilde{\mathbf{I}}_{\mathbf{u}}, \tilde{\mathbf{I}}_{\mathbf{s}}\}$ , residential choice probabilities (7) determine unique vectors of adjusted skill specific residential amenities  $\{\tilde{\mathbf{B}}_{\mathbf{u}}, \tilde{\mathbf{B}}_{\mathbf{s}}\}$  up to a normalization as a function of expected utility, as well as the ratio  $\{\frac{\mathbf{T}_{\mathbf{s}}}{\mathbf{T}_{\mathbf{u}}}\}$ , from:

$$\tilde{B}_{ie} = \bar{u}_e \left(\frac{N_{Rie}}{N_e}\right)^{1/\theta} \frac{(r_i^R)^{1-\beta_e}}{W_{ie}^{1/\theta}},\tag{44}$$

where  $W_{ie} = \sum_{n=1}^{L} \left( \tilde{I}_{ine} \right)^{\theta}$ . Notice that skill specific adjusted residential amenities  $\tilde{B}_{ie}$  include the block specific residential amenities and the Fréchet scale parameter that determines the average utility of a worker of type e of living in block i. Moreover, since  $T_{ie}^{1/\theta}$  enters the equation isomorphically to  $B_i$ , I cannot separately identify these terms. I have chosen to impose the normalization that the geometric mean of the adjusted amenities is equal to one for the low-skilled workers, and equal to  $x_b \in \Re^+$  for the high-skilled. Therefore, dividing equation (44) by the respective geometric means (denoted with a bar above the respective variable), I can determine skill specific adjusted residential amenities from:

$$\tilde{B}_{iu} = \left(\frac{N_{Riu}}{\bar{N}_{Ru}}\right)^{1/\theta} \left(\frac{r_i^R}{\bar{r}^R}\right)^{1-\beta_u} \left(\frac{W_{iu}}{\bar{W}_u}\right)^{-1/\theta};$$

$$\tilde{B}_{is} = x_b \left(\frac{N_{Ris}}{\bar{N}_{Rs}}\right)^{1/\theta} \left(\frac{r_i^R}{\bar{r}^R}\right)^{1-\beta_s} \left(\frac{W_{is}}{\bar{W}_s}\right)^{-1/\theta}.$$
(45)

## 4.4 Final Goods (Skill-Specific) Productivity

Starting with equation (21), it is possible to derive an expression that relates block i's firm productivity to commercial floor space rents and wages:

$$A_i = \left(\frac{r_i^F}{1-\alpha}\right)^{1-\alpha} \left[\frac{1}{\alpha} \left(w_{iu}^{\frac{\rho}{\rho-1}} + w_{is}^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}\right]^{\alpha}.$$

The previous equation implies that, given a value for the parameters  $\{\alpha, \rho\}$  and observed data  $\{\mathbf{w_u}, \mathbf{w_s}, \mathbf{r^F}\}$ , I could identify vectors  $\{\mathbf{A_u}, \mathbf{A_s}\}$ , this is total factor productivity in every block. However, notice that in reality I have a vector of adjusted wages recovered earlier  $(\{\tilde{\mathbf{w_u}}, \tilde{\mathbf{w_s}}\})$  instead of data on actual wages. Therefore, using equation (19) and defining  $\tilde{A}_{ie}$  and  $\tilde{w}_{ie}$  as stated above, I can get the following equation for both types of workers:

$$\tilde{A}_{ie} = \left(\frac{r_i^F}{1-\alpha}\right)^{1-\alpha} \left(\frac{\tilde{w}_{ie}}{\alpha}\right)^{\alpha} \left(\frac{N_{Fei}^{\rho}}{N_{Fui}^{\rho} + N_{Fsi}^{\rho}}\right)^{\frac{\alpha(1-\rho)}{\rho}}.$$
(46)

With this equation for both type of workers, and given values for the parameters  $\{\alpha, \rho\}$ , observed data  $\{N_{Fu}, N_{Fs}, r^F\}$ , and adjusted wages  $\{\tilde{\mathbf{w}}_{\mathbf{u}}, \tilde{\mathbf{w}}_{\mathbf{s}}\}$ , I can determine unique vectors of skill specific adjusted productivity  $\{\tilde{\mathbf{A}}_{\mathbf{u}}, \tilde{\mathbf{A}}_{\mathbf{s}}\}$  up to a normalization, as well as the ratio  $\{\frac{\mathbf{E}_{\mathbf{s}}}{\mathbf{E}_{\mathbf{u}}}\}$ . Note that the skill-specific adjusted productivity terms include the block-specific productivity and the Fréchet scale parameter that determines the average utility of a worker of type e of working in block i. Moreover, since  $E_{ie}^{\alpha/\theta}$  enters the equation isomorphically to  $A_i$ , I cannot separately identify these terms. Additionally, for normalization, I impose that the geometric mean of the skill-specific adjusted productivity is equal to one for the low-skilled workers, and equal to  $x_a \in \Re^+$  for the high-skilled. Therefore, dividing equation (46) by the respective geometric means, I can determine skill-specific adjusted productivity from:

$$\tilde{A}_{iu} = \left(\frac{r_i^F}{\bar{r}^F}\right)^{1-\alpha} \left(\frac{\tilde{w}_{iu}}{\bar{w}_u}\right)^{\alpha} \left(\frac{\vartheta_{iu}}{\bar{\vartheta}_u}\right)^{\alpha(1-\rho)}, 
\tilde{A}_{is} = x_a \left(\frac{r_i^F}{\bar{r}^F}\right)^{1-\alpha} \left(\frac{\tilde{w}_{is}}{\bar{w}_s}\right)^{\alpha} \left(\frac{\vartheta_{is}}{\bar{\vartheta}_s}\right)^{\alpha(1-\rho)},$$
(47)

where  $\vartheta_{ie} = \left(\frac{N_{Fei}^{\rho}}{N_{Fui}^{\rho} + N_{Fsi}^{\rho}}\right)^{\frac{1}{\rho}}$ , corresponds to a measure of the share of workers of type e in block i. Equations (45) and (47) are very similar to those determining the spatial equilibrium in the Rosen-Roback type of models (Roback, 1982), this is, amenities and productivity are determined from the wages earned (or paid), and the floor space rent paid in a given location.

## 4.5 Expected Utility

Divide residential and commercial floor space prices by their geometric mean. These normalizations provide me with a choice of units in which to measure utility. In particular, from the population mobility condition given by equation (5), given values for the parameters  $\{\beta_u, \beta_s, \theta\}$  and data (either observed or calibrated)  $\{\mathbf{r}^{\mathbf{R}}, \tilde{\mathbf{I}}_{\mathbf{u}}, \tilde{\mathbf{I}}_{\mathbf{s}}, \tilde{\mathbf{B}}_{\mathbf{u}}, \tilde{\mathbf{B}}_{\mathbf{s}}\}$ , I can recover the expected utility that a type  $e \in \{u, s\}$  worker would get if she moves into the city:

$$\bar{u}_e = \left[ \sum_{m=1}^L \sum_{n=1}^L T_{me} E_{ne}(r_m^R)^{-\theta(1-\beta_e)} \left( B_m I_{mne} \right)^{\theta} \right]^{1/\theta} = \left[ \sum_{m=1}^L \sum_{n=1}^L (r_m^R)^{-\theta(1-\beta_e)} \left( \tilde{B}_{me} \tilde{I}_{ne} \right)^{\theta} \right]^{1/\theta}.$$

## 4.6 Production and Residential Fundamentals

All of the previous relationships hold regardless of whether the amenities and the productivity terms are exogenous or endogenous: given the parameters  $\{\kappa_u, \kappa_s, \beta_u, \beta_s, \theta, \alpha, \rho\}$  and observed data, the recursive structure of the model allows me to recover  $\{\tilde{\mathbf{B}}_{\mathbf{u}}, \tilde{\mathbf{B}}_{\mathbf{s}}, \tilde{\mathbf{A}}_{\mathbf{u}}, \tilde{\mathbf{A}}_{\mathbf{s}}\}$ . Therefore, just as in Ahlfeldt et al. (2015), this one-to-one mapping holds irrespective of the relative importance

 $<sup>^{13}</sup> In$  the special case when  $\mathbf{E_s} = \mathbf{E_u} = \mathbf{E},$  I could recover one single adjusted total factor productivity vector  $\{\mathbf{\tilde{A}}\}.$ 

of the fundamentals and the externalities determining productivity and amenities, and irrespective of whether the model has a single or multiple equilibria.

It is also possible to recover production and residential fundamentals  $\{\tilde{\mathbf{b}}_{\mathbf{u}}, \tilde{\mathbf{b}}_{\mathbf{s}}, \tilde{\mathbf{a}}_{\mathbf{u}}, \tilde{\mathbf{a}}_{\mathbf{s}}\}$  from equations (14), (15), (22), and (23), together with the results from the previous subsection, and given the extra set of parameters  $\{\eta, \sigma_u, \sigma_s, \nu, \delta_u, \delta_s\}$  and data on the area of each location  $\{\mathbf{L}\}$ . These fundamentals are consistent with the data being an equilibrium of the model. Given the variable definitions in (43), and the definitions of  $\Omega_i$  and  $\Upsilon_i$  from equations (15) and (23):

$$\tilde{b}_{iu} = \tilde{B}_{iu}\Omega_i^{-\eta}, \qquad \tilde{b}_{is} = \tilde{B}_{is}\Omega_i^{-\eta}$$
(48)

$$\tilde{a}_{iu} = \tilde{A}_{iu} \Upsilon_i^{-\nu} \qquad \tilde{a}_{is} = \tilde{A}_{is} \Upsilon_i^{-\nu}$$

$$\tag{49}$$

This completes my characterization of the one-to-one mapping from the known parameters  $\{\kappa_u, \kappa_s, \beta_u, \beta_s, \theta, \alpha, \rho, \eta, \sigma_u, \sigma_s, \nu, \delta_u, \delta_s\}$  and the observed data  $\{\mathbf{N_{Fu}, N_{Fs}, N_{Ru}, N_{Rs}, \tau, \mathbf{r^R}, \mathbf{r^F}}\}$  to the unobserved location characteristics  $\{\tilde{\mathbf{I}}_{\mathbf{u}}, \tilde{\mathbf{I}}_{\mathbf{s}}, \tilde{\mathbf{b}}_{\mathbf{u}}, \tilde{\mathbf{b}}_{\mathbf{s}}, \tilde{\mathbf{a}}_{\mathbf{u}}, \tilde{\mathbf{a}}_{\mathbf{s}}\}$ .

## 5 Results

To be completed

# 6 Conclusions

Given the large heterogeneity among residents of a city, policies that regulate the use of land should affect different types of workers differently. In this paper, I tackle the question of the effects of land use regulations on the welfare of high- and low-skilled workers. I do so by developing a quantitative model of a city that builds on Ahlfeldt et al. (2015). However, in my model I include two types of workers and two possible regulations in the use of land: zoning and height restrictions. The distinction between high- and low-skilled workers is interesting for a large number of reasons. For instance, there is a widening gap in real wages between both groups in larges urban areas. Moreover, high-skilled households spend a lower share of their income in housing and have a higher opportunity cost of commuting.

I use comparative statics to study the different effects that these policies have on both type of workers. In particular, in the absence of agglomeration economies and when considering the effect of a zoning policy that allows more residential space on a certain block, the model predicts a decrease in residential floor prices, an increase in commercial rents and a decrease in wages. These effects lead to an ambiguous effect on the welfare of both types of workers. On the other hand, the imposition of a height restriction leads to higher residential and commercial real estate prices and

to a decrease in wages. These effects imply that low-skilled workers are worse off, while high-skilled workers could be better off through an increase in their non-labor income. In general, the total relative effects on welfare can be decompose into three terms: a floor price effect, a relative income effect, and a sorting effect.

Because density generates agglomeration externalities, in the presence of endogenous productivity and amenities, the effect would depend heavily on how do firms and households value density. For instance, households might value density positively if it generates more consumption amenities, but they might value it negatively if it causes congestion or other disamenities.

After presenting my theoretical model, I show how this model can be applied to real world data. In particular, I show how from the equilibrium equations of my model, there is a unique mapping from observed data and values for different parameters, to (measures of) income, skill specific productivity and amenities. Using data for the City of Chicago, I also show some descriptive facts indicating some correlations in favor of what my theory predicts. Potential applications of the model include the assessment of different zoning policies across Chicago such as the high-rise residential developments in the Gold Coast district, or the high-tech manufacturing developments planned in the Near West End. This paper will attempt to contribute to the debate on whether zoning is a regressive measure, and if it possible to identify a group of the population that benefits from it.

Lastly, this model is also excluding other possible forces of sorting and urban development that have been widely explore in the literature. For instance, I am not taking into account a wide range of fiscal issues such as Tiebout sorting, in which there are small jurisdictions with their own fiscal policy and local public good (usually, school districts). This would be important if some places within Chicago control different land uses with tax policy, both in a general equilibrium or in a strategic setting. I also exclude from my analysis issues like durable housing and purely exclusionary zoning in which some homeowners collude to exclude low income or minorities households. All of these issues are left for potential future research.

## References

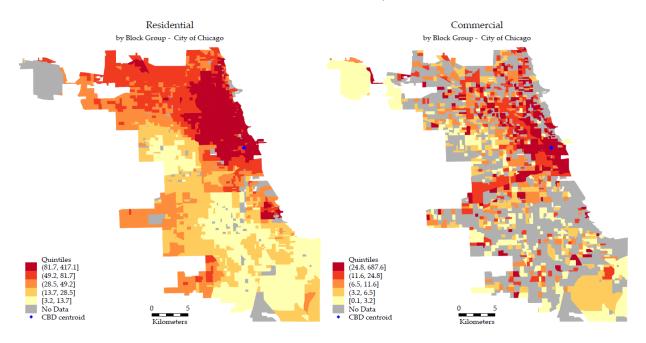
- Ahlfeldt, G., Redding, S. J., Sturm, D. M., and Wolf, N. (2015). The Economics of Density: Evidence from the Berlin Wall. *Econometrica*, 83(6):2127–2189.
- Albouy, D. (2016). What are cities worth? land rents, local productivity and the total value of amenities. *Review of Economics and Statistics*, 98(3):477–487.
- Arkolakis, C., Allen, T., and Li, X. (2015). Optimal City Structure. Work in progress, Yale University.
- Baum-Snow, N. and Han, L. (2019). The microgeography of housing supply. Technical report.
- Baum-Snow, N. and Hartley, D. (2016). Accounting for central neighborhood change, 1980-2010. Work in progress, University of Toronto.
- Baum-Snow, N. and Pavan, R. (2013). Inequality and City Size. The Review of Economics and Statistics, 95(5):1535–1548.
- Bayer, P., Ferreira, F., and McMillan, R. (2007). A Unified Framework for Measuring Preferences for Schools and Neighborhoods. *Journal of Political Economy*, 115(4):588–638.
- Cheshire, P. and Sheppard, S. (2002). The welfare economics of land use planning. *Journal of Urban Economics*, 52(2):242–269.
- Couture, V. and Handbury, J. (2015). Urban revival in america, 2000 to 2010. Work in progress.
- Diamond, R. (2016). The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980-2000. *American Economic Review*, 106(3):479–524.
- Duranton, G. and Puga, D. (2015). Urban Land Use. In Gilles Duranton, J. H. and Strange, W., editors, *Handbook of Regional and Urban Economics*, volume 5 of *Handbook of Regional and Urban Economics*, chapter 8, pages 467–560. Elsevier.
- Ehrenhalt, A. (2012). The Great Inversion and The Future of the American City. Alfred A. Knopf.
- Fischel, W. (2001). The Homevoter Hypothesis: How Home Values Influence Local Government Taxation, School Finance, and Land-Use Policies. Harvard University Press.
- Fischel, W. A. (2015). Zoning Rules!: The Economics of Land Use Regulation. Lincoln Institute of Land Policy.
- Florida, R., Matheson, Z., Adler, P., and Brydges, T. (2014). The divided city and the shape of the new metropolis. Working paper, Martin Prosperity Institute, University of Toronto.
- Fujita, M. (1989). Urban Economic Theory. Cambridge Books. Cambridge University Press.

- Ganong, P. and Shoag, D. (2017). Why has regional income convergence in the US declined? Journal of Urban Economics, 102:76–90.
- Glaeser, E. L., Gyourko, J., and Saks, R. (2005). Why Is Manhattan So Expensive? Regulation and the Rise in Housing Prices. *Journal of Law and Economics*, 48(2):331–69.
- Glaeser, E. L. and Ward, B. A. (2009). The causes and consequences of land use regulation: Evidence from Greater Boston. *Journal of Urban Economics*, 65(3):265–278.
- Gobillon, L., Selod, H., and Zenou, Y. (2007). The mechanisms of spatial mismatch. *Urban Studies*, 44(12):2401–2427.
- Gyourko, J. and Molloy, R. (2015). Regulation and Housing Supply. In Gilles Duranton, J. H. and Strange, W., editors, *Handbook of Regional and Urban Economics*, volume 5 of *Handbook of Regional and Urban Economics*, chapter 19, pages 1289–1337. Elsevier.
- Helsley, R. W. and Strange, W. C. (1995). Strategic growth controls. *Regional Science and Urban Economics*, 25(4):435–460.
- Hilber, C. A. and Robert-Nicoud, F. (2013). On the origins of land use regulations: Theory and evidence from US metro areas. *Journal of Urban Economics*, 75(C):29–43.
- Hsieh, C.-T. and Moretti, E. (2015). Why do cities matter? local growth and aggregate growth. Working Paper 21154, National Bureau of Economic Research.
- Kahn, M. E., Vaughn, R., and Zasloff, J. (2010). The housing market effects of discrete land use regulations: Evidence from the California coastal boundary zone. *Journal of Housing Economics*, 19(4):269–279.
- Kain, J. (1968). Housing segregation, negro employment, and metropolitan decentralization. *Quarterly Journal of Economics*, 82(2):175–197.
- Levine, N. (1999). The Effects of Local Growth Controls on Regional Housing Production and Population Redistribution in California. *Urban Studies*, 36(12):2047–2068.
- McMillen, D. P. and McDonald, J. F. (2002). Land Values In A Newly Zoned City. *The Review of Economics and Statistics*, 84(1):62–72.
- Mincer, J. (1974). Schooling, Experience, and Earnings. National Bureau of Economic Research.
- Moretti, E. (2013). Real Wage Inequality. American Economic Journal: Applied Economics, 5(1):65–103.
- Muehlegger, E. and Shoag, D. (2015). Commuting times and land use regulations. *Procedia Engineering*, 107:488 493.

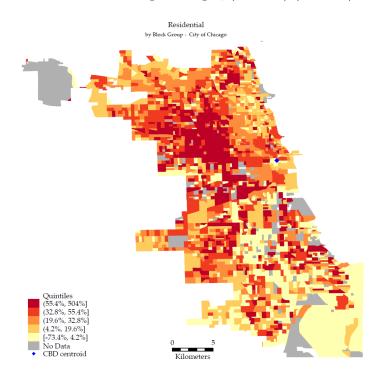
- Muth, R. F. (1969). Cities and Housing: The Spatial Pattern of Urban Residential Land Use. University of Chicago Press.
- Notowidigdo, M. J. (2013). The Incidence of Local Labor Demand Shocks. Working paper, Northwestern University.
- Roback, J. (1982). Wages, Rents and the Quality of Life. *Journal of Political Economy*, 90(6):1257–78.
- Rosenthal, S. S. and Strange, W. C. (2004). Evidence on the nature and sources of agglomeration economies. In Henderson, J. V. and Thisse, J. F., editors, *Handbook of Regional and Urban Economics*, volume 4 of *Handbook of Regional and Urban Economics*, chapter 49, pages 2119–2171. Elsevier.
- Rossi-Hansberg, E. (2004). Optimal Urban Land Use and Zoning. Review of Economic Dynamics, 7(1):69–106.
- Saks, R. E. (2008). Job creation and housing construction: Constraints on metropolitan area employment growth. *Journal of Urban Economics*, 64(1):178–195.
- Shertzer, A., Twinam, T., and Walsh, R. P. (2016). Zoning and the Economic Geography of Cities. NBER Working Papers 22658, National Bureau of Economic Research, Inc.
- Tsivanidis, N. (2018). The aggregate and distributional effects of urban transit infrastructure: Evidence from bogotá's transmilenio. Technical report.
- Turner, M. A. (2005). Landscape preferences and patterns of residential development. *Journal of Urban Economics*, 57(1):19–54.
- Turner, M. A., Haughwout, A., and van der Klaauw, W. (2014). Land Use Regulation and Welfare. *Econometrica*, 82(4):1341–1403.
- Wallace, N. E. (1988). The market effects of zoning undeveloped land: Does zoning follow the market? *Journal of Urban Economics*, 23(3):307–326.
- Wheaton, W. C. (1977). A bid rent approach to urban housing demand. *Journal of Urban Economics*, 4(2):15–32.
- Zillow (2017). ZTRAX: Zillow Transaction and Assessor Dataset, 2017-Q4.

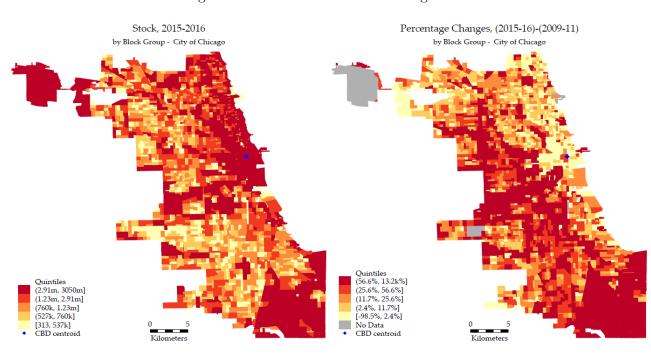
Figure 1: Real Estate Prices in Chicago

# Panel A: Hedonic Price Index, 2015-2016



Panel B: Percentage Changes, (2015-16)-(2009-11)

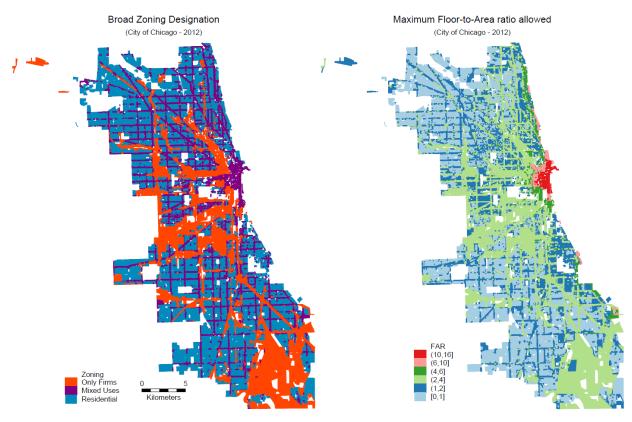




Kilometers

Figure 2: Real Estate Stock in Chicago

Figure 3: Zoning in Chicago



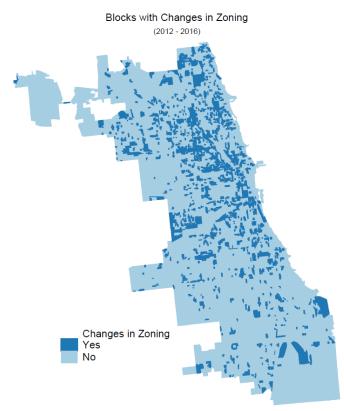
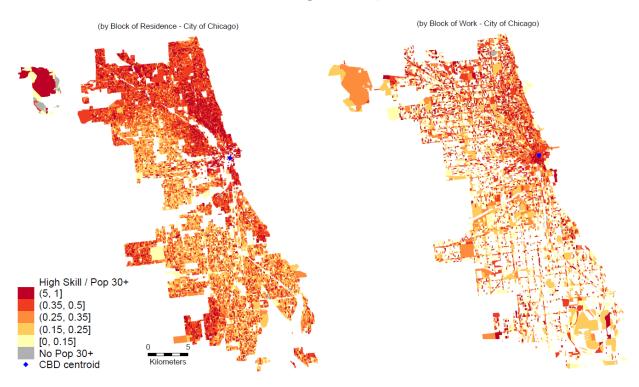


Figure 4: Distribution of Skills

Panel A: Map of Levels, 2014



Panel B: Histogram of Changes, 2009-2014

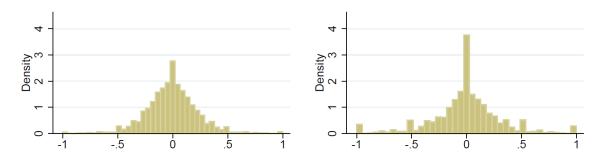


Figure 5: Effects of More Residential Zoning on the Real Estate Market

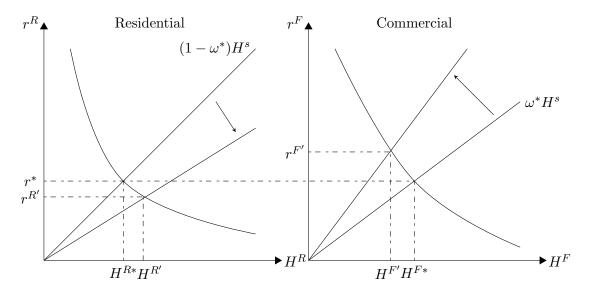


Figure 6: Effects of a Height Restriction on the Real Estate Market

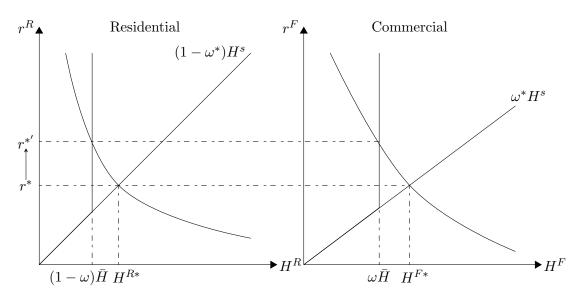


Table 1: Real Estate Price Indices

Panel A: Residential (2009-2016)					
	Hedonic Transactions	Hedonic Assessment	Repeated Sales Transactions		
N. Block Groups	16,681	16,702	12,916		
Mean	45.35	518.4	111.0		
Min	2.10	0.28	0.83		
Median	32.79	365.3	92.22		
Max	581.7	$13,\!653$	2,084		
Corr with HI	1.0	0.86	-0.06		
Corr with HA	0.86	1.0	-0.09		
Corr with RS	-0.06	-0.09	1.0		
Corr median price	0.41	0.47	0.04		
Corr median value	0.10	0.33	-0.01		

Panel B: Commercial (2014-2016)

	Hedonic Transactions	Hedonic Assessment	
N. Block Groups	1,916	6,012	
Mean	21.83	53.68	
Min	0.13	0.48	
Median	8.41	40.33	
Max	903.8	855.3	
Corr with HI	1.0	0.47	
Corr with HA	0.47	1.0	
Corr median price	0.69	0.44	
Corr median value	0.43	0.83	

Table 2: Average and Median Block

## By Block of Residence

		Total 30+	Low-Skilled	High-Skilled
3.50.4	3.5	22.24	17.07	
MSA	Mean	23.26	15.25	8.00
	Median	12	8	4
City	Mean	21.92	15.06	6.85
	Median	13	9	3
By Block of Work				
Total 30+ Low-Skilled High-Skilled				

		Total 30+	Low-Skilled	High-Skilled
MSA	Mean	60.98	39.88	21.10
	Median	6	4	2
City	Mean	73.20	46.94	26.26
	Median	7	5	2

Table 3: Distribution of Skills and Land Use Regulation

# Dependent Variable: Block Share of Low Skilled Workers

	(1)	(2)	(3)	(4)
Share of residential only	-0.0183***		-0.0438***	
	(0.0050)		(0.0056)	
Share of mixed uses	-0.0287***		-0.0413***	
	(0.0054)		(0.0059)	
Mean FAR allowed		-0.0181***		-0.0095***
		(0.0006)		(0.0007)
Mean distance	to work	to work	from home	from home
	0.0026***	0.0021***	-0.0027***	-0.0023***
	(0.0001)	(0.0001)	(0.0002)	(0.0002)
By Block of	Residence	Residence	Work	Work
Observations	74,988	$74,\!538$	$31,\!451$	31,000

Standard errors in parentheses; \*\*\* p<0.01, \*\* p<0.05, \* p<0.1**Note:** The omitted category is the share of the area for only firms