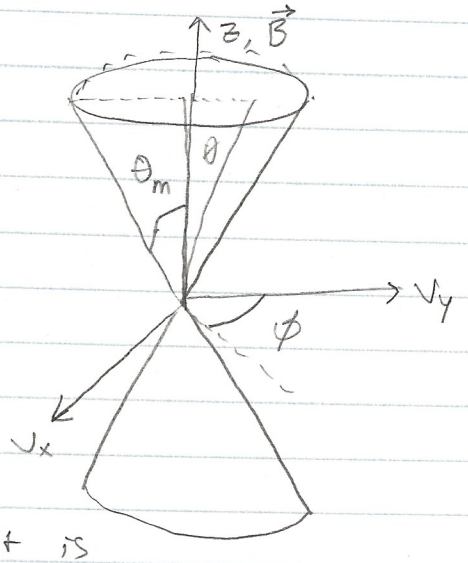
Chen 2.11) Plasma w/ isotropic v distrib., placed in trap w/ $R_m = 4$. Particles in loss cone escape, particles outside are trapped. What fraction is trapped?

We have a mirror ratio $R_m = 4$, and from the text, we have

$$R_m = \frac{1}{\sin^2\theta_m}, \text{ so let's find the angle corresponding;}$$

$$\sin^2\theta_m = \frac{1}{R_m} = \frac{1}{4}, \quad \sin\theta_m = \frac{1}{2}, \text{ so } \theta_m = \sin^{-1}\left(\frac{1}{2}\right),$$

and $\theta_m = \pi/6$



We are dealing with a cone, whose total solid angle is 4π (look @ google for a picture).
So the boundary through which the particles will be lost is a differential on a sphere; $d\Omega = \sin\theta\, d\theta\, d\phi$, integrate to find solid angle loss cone:

$$\Omega_{\text{loss cone}} = 2\int_0^{\pi/6}\sin\theta\, d\theta \int_0^{2\pi} d\phi = 2(2\pi)\left(+\left(\cos\pi/6 + \cos(0)\right)\right)$$

(2 ends of cone; $\cos(0) \to 1$)

$$= 4\pi\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\rightarrow \Omega_{\text{total}} - \Omega_{\text{loss cone}} = \Omega_{\text{trapped}} = \cancel{4\pi} - \cancel{4\pi} + 2\pi\sqrt{3} = 2\pi\sqrt{3}.$$

Then the fraction is: $\frac{\Omega_{\text{trapped}}}{\Omega_{\text{total}}} = \frac{\cancel{2}\cancel{\pi}\sqrt{3}}{\cancel{2}\cancel{\pi}} \Rightarrow \frac{\Omega_{\text{trapped}}}{\Omega_{\text{total}}} = \frac{\sqrt{3}}{2}.$

$\sim 13\%$ is lost.