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CENTRO CIENTÍFICO – TECNOLÓGICO DE VALPARAÍSO

Densimeter Project Report

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1. Introduction

During this document we will report what has been done in order to calibrate and test the Nuclear Densimeter proposed by Professor Sergey Kuleshov.

Gamma interactions with matter consist mainly in 3 processes:

1. Photoelectric Effect
2. Compton Scattering
3. Pair Production

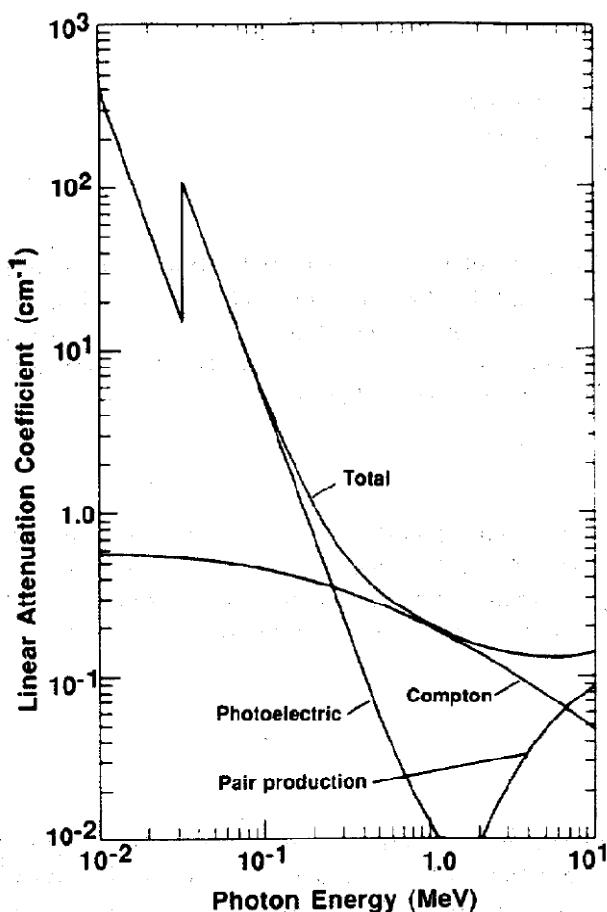


Figure 1: Linear Attenuation Coefficient of NaI showing contributions from Photoelectric Absorption, Compton Scattering and Pair Production. (We will discuss the meaning of this later, image from [3]).

For low energy processes (up to a few keV), photoelectric effect is the most probable process. On the other hand, pair production has a threshold energy of 1.022 [MeV]. This means that for Gamma Rays of a few hundreds keV the preferred way of interaction is Compton Scattering.

2. Interaction Probabilities

Photoelectric effect cross section is proportional to the total number of atoms in the sample, the same as the pair production does. Instead of this Compton Scattering Cross Section is proportional to the whole amount of electrons in the sample, that is Z times the number of atoms of the sample (where Z is the atomic number).

The reason of this lies on the form of the total cross section. If we write the individual cross section per atom, as appears on [1]:

$$\sigma = \sigma_{Photo} + Z\sigma_{Compton} + \sigma_{Pair} \quad (1)$$

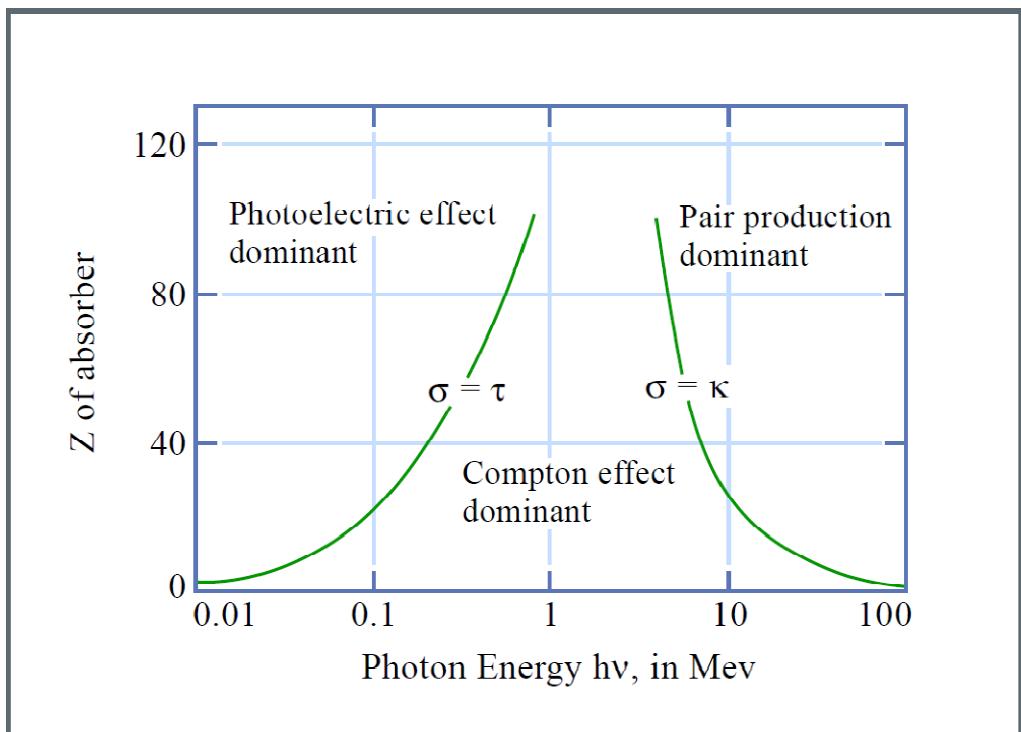


Figure 2: The relative importance of the three major types of gamma-ray interaction. The lines show the values of Z and $h\nu$ for which the two neighboring effects are just equal (from [2]).

But in order to have the total cross section, we need to multiply this by the atom volumetric density, $N_{atoms/vol}$:

$$N_{atoms/vol}\sigma = N_{atoms/vol}\sigma_{Photo} + N_{atoms/vol}Z\sigma_{Compton} + N_{atoms/vol}\sigma_{Pair} \quad (2)$$

If A is Mass Number, then for a certain material mass M , the number of moles is $n_{moles} = \frac{M}{A}$, then the amount of atoms is the Avogadro Number, N_a , times this number of moles. So the number of Atoms is:

$$\text{number of atoms} = N_a \frac{M}{A} \quad (3)$$

Then:

$$\frac{\text{number of atoms}}{\text{volume}} = N_a \frac{M}{AV} \quad (4)$$

If ρ is the material density, finally we have:

$$N_{atoms/vol} = \frac{N_a}{A} \rho \quad (5)$$

Then, taking to account the fact that we are using a Na-22 source with positron emission, we are focusing on measure 511 [keV] gammas, then:

$$\sigma_{Total} = N_{atoms/vol} \sigma_{Photo} + N_{atoms/vol} Z \sigma_{Compton} \quad (6)$$

Also, since in this region Compton is the predominant interaction:

$$\sigma_{Total} \approx N_{atoms/vol} Z \sigma_{Compton} = \frac{N_a Z}{A} \rho \sigma_{Compton} \quad (7)$$

Now, let's make some definitions of Cross Section, as seen on [1]:

N_s = Mean Number of Scattered Particles over a $d\Omega$ solid angle per unit time

F = Flux of incident particles per unit time per unit area

Then, the differential Cross Section is defined as:

$$\frac{d\sigma(E, \Omega)}{d\Omega} = \frac{1}{F} \frac{dN_s}{d\Omega} \quad (8)$$

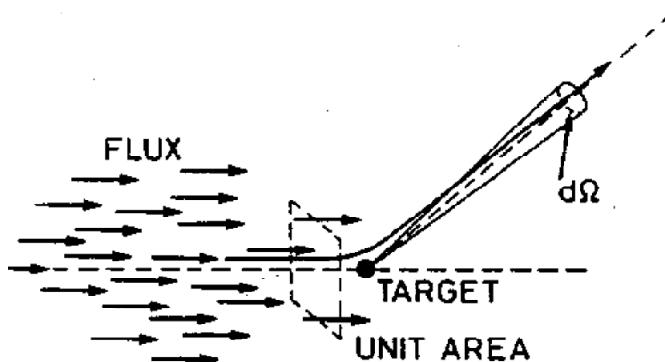


Figure 3: Definition of the scattering cross section (from [1]).

So now we can calculate the Total Cross Section:

For a certain energy of the incident particles Flux, we can compute the total cross section for all solid angles:

$$\sigma(E) = \int d\Omega \frac{d\sigma}{d\Omega} \quad (9)$$

S = Material area interacting with beam

$$\emptyset = \frac{\text{Number of Incident Particles}}{\text{Unit Time}} = FS$$

$N_{target/vol}$ = Target Density per Volumetric Unit

dx = Material Thickness along the beam direction

$$N_s(\Omega) = \frac{\phi}{S} SN_{target/vol} dx \frac{d\sigma}{d\Omega} \quad (10)$$

$$\text{Total Mean Number of Scattered Particles over All Solid Angles} = \frac{\phi}{S} SN_{target/vol} dx \sigma \quad (11)$$

For an incoming flux of particles, we can define the differential of the incoming flux ϕ_0 in the following way:

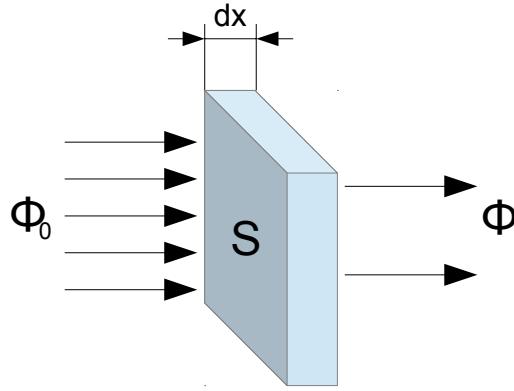


Figure 4: Differential of the Incoming Flux of Gamma due to Scattering.

$$\phi = \phi_0 + d\phi \Rightarrow d\phi = \phi - \phi_0 \quad (12)$$

Then, taking into account the previously calculated Cross Section, the number of interactions can be calculated in the following way, using (11) and (7), which equals the differential of the incoming flux (12):

$$-d\phi = \frac{\phi N_a Z}{S A} \rho \sigma_C S dx \Rightarrow -d\phi = \phi \frac{N_a Z}{A} \rho \sigma_C dx \quad (13)$$

Then:

$$\begin{aligned} \frac{d\phi}{\phi} &= -\frac{N_a Z}{A} \rho \sigma_C dx \\ \int_{\phi_0}^{\phi} \frac{d\phi}{\phi} &= -\frac{N_a Z}{A} \rho \sigma_C \int_0^x dx \\ \ln(\phi) - \ln(\phi_0) &= -\frac{N_a Z}{A} \rho \sigma_C x \\ \phi &= \phi_0 e^{-\frac{N_a Z}{A} \rho \sigma_C x} \end{aligned} \quad (14)$$

Where σ_C is the Compton Cross Section per individual electron, written as $\sigma_{Compton}$ on equation (1). So with this basis, we can start to do our own definitions.

3. Calculating Density

Commonly the previous result is written in the following way, where I is the measured flux, I_0 is the incoming flux and $\frac{1}{\lambda}$ is the absorption coefficient:

$$I = I_0 e^{-\frac{x}{\lambda}} \quad (15)$$

From this, it is clear that, from our previous result and what we are able to measure:

$$\frac{N_a Z}{A} \rho \sigma_C = \frac{1}{\lambda} \quad (16)$$

$$I = I_0 e^{-\frac{x}{\lambda}} \Rightarrow \ln\left(\frac{I}{I_0}\right) = -\frac{x}{\lambda} \Rightarrow \frac{1}{\lambda} = \frac{\ln\left(\frac{I_0}{I}\right)}{x} \quad (17)$$

This means that if we know the incoming flux, the material thickness and we are able to measure the out coming flux, we can compute the linear attenuation coefficient.

So, if we can measure this amount for 2 different (pure) materials, with the same incoming flux, and with the same energy for gammas:

For material 1:

$$\frac{1}{\lambda_1} = \frac{\ln\left(\frac{I_0}{I_1}\right)}{x_1} \quad (18)$$

For material 2:

$$\frac{1}{\lambda_2} = \frac{\ln\left(\frac{I_0}{I_2}\right)}{x_2} \quad (19)$$

So, we can compute the ratio in this way, using our previous definitions:

$$\frac{\frac{1}{\lambda_1}}{\frac{1}{\lambda_2}} = \frac{\frac{N_a Z_1}{A_1} \rho_1 \sigma_C}{\frac{N_a Z_2}{A_2} \rho_2 \sigma_C} \quad (20)$$

Since we are using the same gamma energy, the Compton Cross Section per electron is the same (and Avogadro Number is a constant):

$$\frac{\frac{1}{\lambda_1}}{\frac{1}{\lambda_2}} = \frac{\frac{Z_1}{A_1} \rho_1}{\frac{Z_2}{A_2} \rho_2} \quad (21)$$

If we know $\rho_1, \frac{Z_1}{A_1}, \frac{Z_2}{A_2}$ and we want to know ρ_2 :

$$\rho_2 = \frac{\frac{Z_1}{A_1} \rho_1}{\frac{Z_2}{A_2} \frac{1}{\lambda_1}} \quad (22)$$

But here comes the magic. Usually the number of Neutrons, N , is very close to the number of electrons, Z . Since, for a certain Isotope:

$$A = Z + N \quad (23)$$

For low Z elements:

$$Z \approx N$$

This leads to:

$$A = Z + N \approx 2Z$$

This means that for low Z elements, this ratio is almost constant and equal to:

$$\frac{Z}{A} = \frac{Z}{Z+N} \approx \frac{Z}{2Z} = 0.5 \quad (24)$$

So, if Z/A ratio is almost the same for materials 1 and 2 the calculated density results:

$$\rho_2 = \frac{\frac{Z_1}{A_1} \lambda_2 \rho_1}{\frac{Z_2}{A_2} \lambda_1} \cong \frac{\frac{1}{\lambda_2} \rho_1}{\frac{1}{\lambda_1}} \quad (25)$$

But, as we can see, this shouldn't be true, just by looking at the Segre Chart (Chart of the Nuclides):

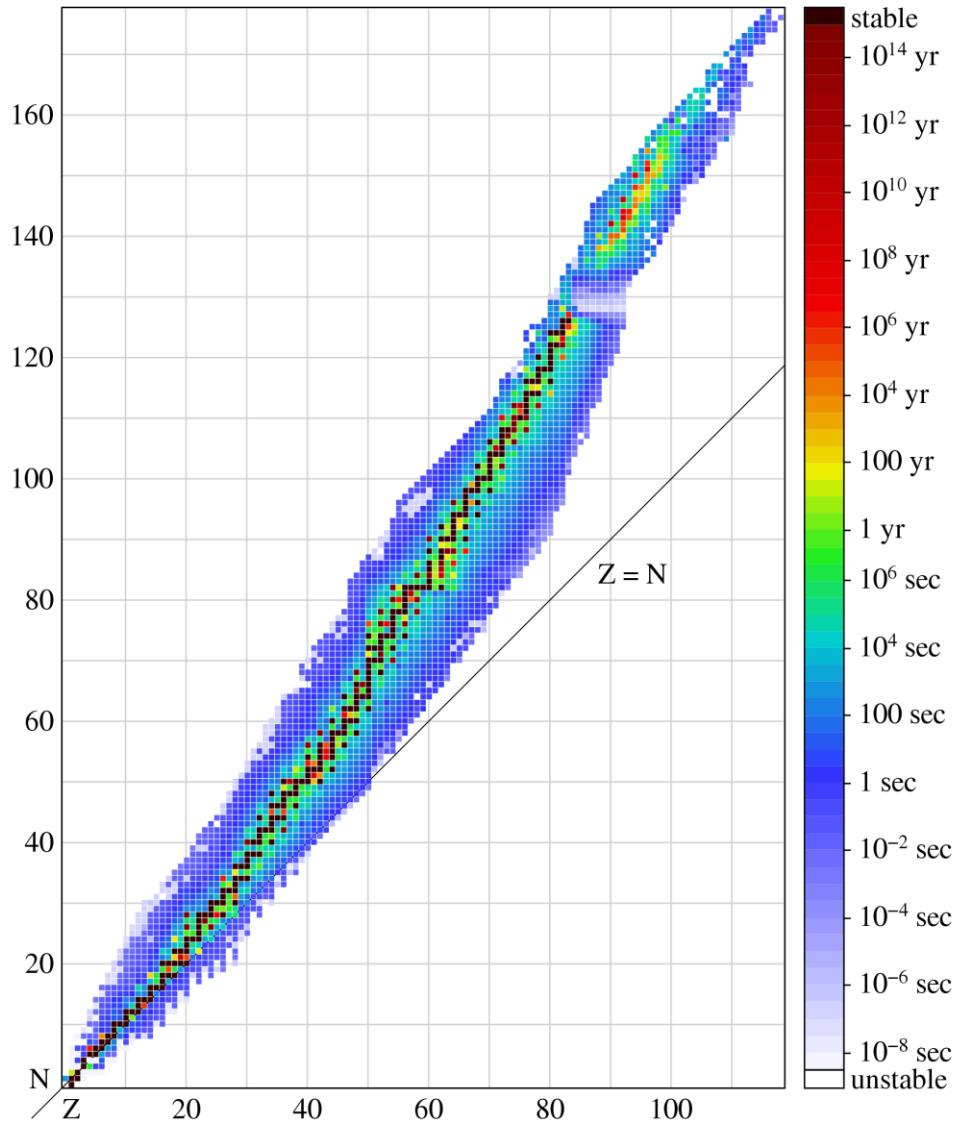


Figure 5: The Nuclide Chart. Y axis is Neutron number N , X axis is Atomic Number Z , which is the amount of protons for a certain nuclide. The colors shows the stability of the nuclide.

Image from Wikipedia.

What happens if the material is a compound?

Let's first think about layers of different materials, one next to the other. As we can see, the solution is rather simple:

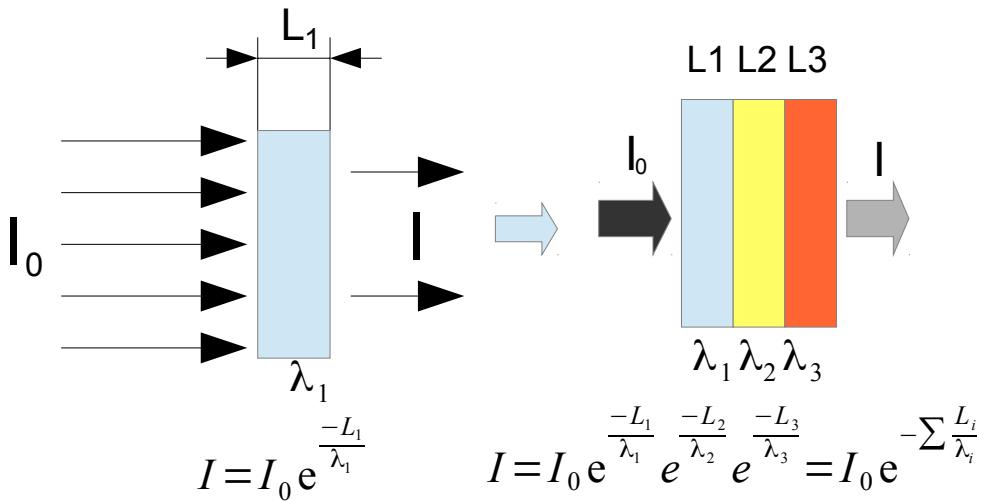


Figure 6: Linear attenuation due to layers of different materials.

So for a set of layer, one next to the other, the out coming flux can be calculated as:

$$I = I_0 e^{-\sum_{i=1}^{n_{layers}} \frac{L_i}{\lambda_i}} \quad (26)$$

And this implies that, for n_{layers} of materials, of total length $L_{layers} = \sum_{i=1}^{n_{layers}} L_i$, the coefficient:

$$\frac{L_{layers}}{\lambda_{layers}} = \sum_{i=1}^{n_{layers}} \frac{L_i}{\lambda_i} \Rightarrow \frac{1}{\lambda_{layers}} = \frac{1}{L_{layers}} \sum_{i=1}^{n_{layers}} \frac{L_i}{\lambda_i} \quad (27)$$

That makes us think about a compound.

If we have a compound of two elements, let's say $X_m Y_n$ with Atomic Mass and Atomic Numbers A_X, A_Y, Z_X, Z_Y .

The molar mass A_{mol} of the compound will be:

$$\text{Molar Mass} = A_{mol} = mA_X + nA_Y \quad (28)$$

Then, the Mole Fraction k , by weight, will be:

$$k_X = \frac{m A_X}{m A_X + n A_Y} \quad (29) \text{ and } k_Y = \frac{n A_Y}{m A_X + n A_Y} \quad (30)$$

So, it is obvious that $k_X + k_Y = 1$ (31).

Let's suppose we have a certain amount of mass M of this compound. Then the amount of Moles of that compound is:

$$n_{moles} = \frac{M}{A_{mol}} \quad (32)$$

So this means:

$$n_{moles} = \frac{M}{A_{mol}} = \frac{k_X(mA_X+nA_Y)n_{moles}+k_Y(mA_X+nA_Y)n_{moles}}{A_{mol}} \quad (33)$$

Besides that, if we know the volume V of the compound with density ρ_{comp} :

$$\frac{n_{moles}}{V} = \frac{M}{VA_{mol}} = \frac{\rho_{comp}}{A_{mol}} \quad (34)$$

So, in this way, the number of molecules per volumetric unit in the sample is:

$$\frac{n_{moles}}{V} N_a = \frac{\rho_{comp}}{A_{mol}} N_a \quad (35)$$

If we remember what we defined before:

$$n_{moles} = \frac{M}{A_{mol}} = \frac{\frac{k_X n_{moles}(mA_X+nA_Y)}{M} M + \frac{k_Y n_{moles}(mA_X+nA_Y)}{M} M}{A_{mol}} = \frac{w_X M + w_Y M}{A_{mol}} \quad (36)$$

Where $w_{X,Y} = \frac{k_{x,y} K n_{moles}}{M}$ (37) is the weight fraction.

If we do the same trick again:

$$\frac{n_{moles}}{V} = \frac{\rho_{comp}}{A_{mol}} = \frac{w_X \frac{M}{V} + w_Y \frac{M}{V}}{A_{mol}} = \frac{w_X \rho_{comp} + w_Y \rho_{comp}}{A_{mol}} = \frac{w_X \rho_{comp} + w_Y \rho_{comp}}{mA_X + nA_Y} \quad (38)$$

So, again, the number of molecules is:

$$\text{Number of Molecules} = \frac{n_{moles}}{V} N_a = \frac{\rho_{comp}}{A_{mol}} N_a = \frac{w_X \rho_{comp} + w_Y \rho_{comp}}{mA_X + nA_Y} N_a \quad (39)$$

The number of electrons per molecule is:

$$\text{Number of Electrons per Molecule} = Z_{mol} = mZ_X + nZ_Y \quad (40)$$

Then, the number of electrons per volumetric unit in the compound is:

$$\frac{n_{moles}}{V} N_a Z_{mol} = \frac{\rho_{comp}}{A_{mol}} N_a Z_{mol} = \frac{w_X + w_Y}{mA_X + nA_Y} \rho_{comp} N_a Z_{mol} = \frac{(w_X + w_Y)(mZ_X + nZ_Y)}{mA_X + nA_Y} N_a \rho_{comp} \quad (41)$$

This means that the linear attenuation coefficient can be computed as:

$$\frac{1}{\lambda_{compound}} = \frac{(w_X + w_Y)(mZ_X + nZ_Y)}{mA_X + nA_Y} N_a \rho_{comp} \sigma_C \quad (42)$$

Since $w_X + w_Y = 1$ (43):

$$\frac{1}{\lambda_{compound}} = \frac{mZ_X + nZ_Y}{mA_X + nA_Y} N_a \rho_{comp} \sigma_C \quad (44)$$

If we generalize this result:

$$\frac{1}{\lambda_{compound}} = \frac{Z_{ef}}{A_{ef}} N_a \rho_{comp} \sigma_C \quad (45)$$

Then for a compound made of n_{el} elements, where a_i is the number of atoms of the i_{th} element on the compound formulae, Z_{ef} (**Z effective**) is:

$$Z_{ef} = \sum_{i=1}^{n_{el}} a_i Z_i \quad (46)$$

In the same way, A_{ef} (**A effective**) is:

$$A_{ef} = \sum_{i=1}^{n_{el}} a_i A_i \quad (47)$$

Now, let's think about a mixture of elements or compounds.

How is the density of a mixture or alloy calculated?

Let's suppose that we have n_{el} original substances of masses m_i occupying certain volumes v_i each.

Then the mixture mass is:

$$m_{mix} = \sum_{i=1}^{n_{el}} m_i \quad (48)$$

And so the volume is:

$$v_{mix} = \sum_{i=1}^{n_{el}} v_i \quad (49)$$

So this means that the mixture density is:

$$\rho_{mix} = \frac{m_{mix}}{v_{mix}} = \frac{\sum_{i=1}^{n_{el}} m_i}{\sum_{i=1}^{n_{el}} v_i} \quad (50)$$

On the other hand, if we know the densities ρ_i of each substance:

$$\rho_{mix} = \frac{m_{mix}}{v_{mix}} = \frac{\sum_{i=1}^{n_{el}} \rho_i v_i}{v_{mix}} \quad (51)$$

Now, let's define the weight fraction w_i :

$$w_i = \frac{m_i}{m_{mix}} = \frac{\rho_i v_i}{\rho_{mix} v_{mix}} \quad (52)$$

So, if again, we define the density of the mixture:

$$\rho_{mix} = \frac{\sum_{i=1}^{n_{el}} w_i m_{mix}}{v_{mix}} \quad (53)$$

From this is clear that $\sum_{i=1}^{n_{el}} w_i = 1$ (54).

How we can calculate the amount of electrons of a mixture?

So, again, let's try to calculate the amount of electrons in the mixture.

Let's suppose that we know Z_i and A_i for each element of the mixture and, for the sake of simplicity, each substance of the mixture is made of a single element.

First, let's relate the Weight Fraction w_i with the Mole Fraction k_i .

We know that the mass of the mixture must be, with A_{mix} defined as the Molar Mass of the mixture:

$$m_{mix} = n_{moles} \sum_{i=1}^{n_{el}} k_i A_i = n_{moles} A_{mix} \quad (55)$$

Then the Molar Mass:

$$A_{mix} = \frac{m_{mix}}{n_{moles}} = \sum_{i=1}^{n_{el}} k_i A_i \quad (56)$$

If we remember the definition of Weight Fraction:

$$\frac{\sum_{i=1}^{n_{el}} w_i m_{mix}}{n_{moles}} = \sum_{i=1}^{n_{el}} k_i A_i \Rightarrow \frac{m_{mix} \sum_{i=1}^{n_{el}} w_i}{n_{moles}} = A_{mix} \sum_{i=1}^{n_{el}} w_i = \sum_{i=1}^{n_{el}} k_i A_i \quad (57)$$

So since the sums have the same index numbers:

$$A_{mix} \sum_{i=1}^{n_{el}} w_i = \sum_{i=1}^{n_{el}} k_i A_i \Rightarrow A_{mix} w_i = k_i A_i \Rightarrow w_i = \frac{k_i A_i}{A_{mix}} \quad (58)$$

Now, let's calculate the number of electrons per volumetric unit.

First, the number of atoms, as we know from equation (3):

$$\text{Number of Atoms} = N_a \frac{m_{mix}}{\sum_{i=1}^{n_{el}} k_i A_i} \quad (59)$$

Then, it is straightforward that:

$$\frac{\text{Number of Atoms}}{\text{Volumetric Unit}} = N_a \frac{m_{mix}}{v_{mix} \sum_{i=1}^{n_{el}} k_i A_i} = N_a \frac{\rho_{mix}}{\sum_{i=1}^{n_{el}} k_i A_i} \quad (60)$$

But this case is different from a molecular compound. The amount of electrons is an amount proportional to each element, so we need to know what is the amount of atoms that corresponds to each one of the elements. We can get this number it using the mole fraction:

$$\text{Number of Atoms of the } i^{\text{th}} \text{ element} = k_i N_a \frac{m_{mix}}{\sum_{i=1}^{n_{el}} k_i A_i} \quad (61)$$

Then the amount of electrons due to the specific i_{th} element:

$$\text{Number of electrons due to the } i_{th} \text{ element} = k_i N_a \frac{m_{mix}}{\sum_{i=1}^{n_{el}} k_i A_i} Z_i \quad (62)$$

Finally, the total amount of electrons in the material is:

$$\text{Number of electrons} = \sum_{i=1}^{n_{el}} k_i N_a \frac{m_{mix}}{\sum_{i=1}^{n_{el}} k_i A_i} Z_i \quad (63)$$

Then it is straightforward:

$$\begin{aligned} \frac{\text{Number of electrons}}{\text{Volumetric Unit}} &= \sum_{i=1}^{n_{el}} k_i N_a \frac{m_{mix}}{v_{mix} \sum_{i=1}^{n_{el}} k_i A_i} Z_i = \sum_{i=1}^{n_{el}} k_i N_a \frac{\rho_{mix}}{\sum_{i=1}^{n_{el}} k_i A_i} Z_i \Rightarrow \\ \frac{\text{Number of electrons}}{\text{Volumetric Unit}} &= N_a \rho_{mix} \sum_{i=1}^{n_{el}} \frac{k_i Z_i}{\sum_{i=1}^{n_{el}} k_i A_i} \end{aligned} \quad (64)$$

Now, let's try to use the weight fraction instead of the mole fraction. From equation (58):

$$w_i = \frac{k_i A_i}{\sum_{i=1}^{n_{el}} k_i A_i} \Rightarrow k_i = w_i \frac{\sum_{i=1}^{n_{el}} k_i A_i}{A_i} \quad (65)$$

Then replacing (65) on (64):

$$\frac{\text{Number of electrons}}{\text{Volumetric Unit}} = N_a \rho_{mix} \sum_{i=1}^{n_{el}} \frac{w_i \frac{\sum_{i=1}^{n_{el}} k_i A_i}{A_i} Z_i}{\sum_{i=1}^{n_{el}} k_i A_i} = N_a \rho_{mix} \sum_{i=1}^{n_{el}} \frac{w_i Z_i \sum_{i=1}^{n_{el}} k_i A_i}{A_i \sum_{i=1}^{n_{el}} k_i A_i} = N_a \rho_{mix} \sum_{i=1}^{n_{el}} \frac{w_i Z_i}{A_i} \quad (66)$$

Finally, from equation (16) we can conclude that:

$$\frac{1}{\lambda_{mix}} = N_a \rho_{mix} \sigma_C \sum_{i=1}^{n_{el}} \frac{w_i Z_i}{A_i} \quad (67)$$

Let's check this result by doing the derivation in the opposite direction.

Suppose that if we do a mixture of elements, each one will add certain amount of electrons:

$$\text{Number of electrons added by the } i_{th} \text{ element} = N_a \frac{\rho_i v_i}{A_i} Z_i \quad (68)$$

Therefore,

$$\text{Number of electrons} = N_a \sum_{i=1}^{n_{el}} \frac{\rho_i v_i}{A_i} Z_i \quad (69)$$

Afterwards, what left to do is to divide by the total volume:

$$\frac{\text{Number of electrons}}{\text{Volumetric Unit}} = N_a \sum_{i=1}^{n_{el}} \frac{\rho_i v_i}{A_i \sum_{i=1}^{n_{el}} v_i} Z_i \quad (70)$$

Now, let's play a little with the equations. From equation (52):

$$w_i = \frac{\rho_i v_i}{\rho_{mix} v_{mix}} \Rightarrow w_i \rho_{mix} v_{mix} = \rho_i v_i \quad (71)$$

Replacing (71) on equation (70):

$$\begin{aligned} N_a \sum_{i=1}^{n_{el}} \frac{\rho_i v_i}{A_i \sum_{i=1}^{n_{el}} v_i} Z_i &= N_a \sum_{i=1}^{n_{el}} \frac{w_i \rho_{mix} v_{mix}}{A_i \sum_{i=1}^{n_{el}} v_i} Z_i = N_a \sum_{i=1}^{n_{el}} \frac{w_i \rho_{mix} v_{mix}}{A_i v_{mix}} Z_i \Rightarrow \\ \frac{\text{Number of electrons}}{\text{Volumetric Unit}} &= N_a \rho_{mix} \sum_{i=1}^{n_{el}} \frac{w_i Z_i}{A_i} \end{aligned} \quad (72)$$

Which is the same result from equation (66), and of course we will get again (67) as the linear attenuation coefficient for a mixture of elements.

The Bragg's Rule

On the reference literature, specifically on [1], there is an approximation to compute the energy loss of particles, $\frac{dE}{dx}$, passing through mixtures or compounds of density ρ_i . This is called the Bragg's Rule (equation (2.38) of [1]), which stands the following:

$$\frac{1}{\rho_{mix}} \frac{dE}{dx} = \sum_{i=1}^{n_{el}} \frac{w_i}{\rho_i} \left(\frac{dE}{dx} \right)_i \quad (73)$$

Where w_i is again the weight fraction, as defined on (58), and $\left(\frac{dE}{dx} \right)_i$ is the energy loss for particles due to the specific i_{th} element. Since the attenuation length is proportional to the energy loss of the particles, the Bragg's Rule can be applied, which leads to the following, as shown in equation (2.129) of [1]:

$$\frac{1}{\rho_{mix}} \frac{1}{\lambda_{mix}} = \sum_{i=1}^{n_{el}} \frac{w_i}{\rho_i} \frac{1}{\lambda_i} \quad (74)$$

If we remember, we can compute $\frac{1}{\lambda_i}$ from equation (16) and (68):

$$\frac{1}{\lambda_i} = N_a \frac{\rho_i}{A_i} Z_i \sigma_C \quad (75)$$

Then, replacing (75) on (74):

$$\frac{1}{\rho_{mix}} \frac{1}{\lambda_{mix}} = \sum_{i=1}^{n_{el}} \frac{w_i}{\rho_i} N_a \frac{\rho_i}{A_i} Z_i \sigma_C \Rightarrow \frac{1}{\lambda_{mix}} = \rho_{mix} N_a \sigma_C \sum_{i=1}^{n_{el}} w_i \frac{Z_i}{A_i} \quad (76)$$

Which, again, is the same result obtained in (67).

4. The Experimental Setup

The Mechanical Setup

Basically the setup consist in the following:

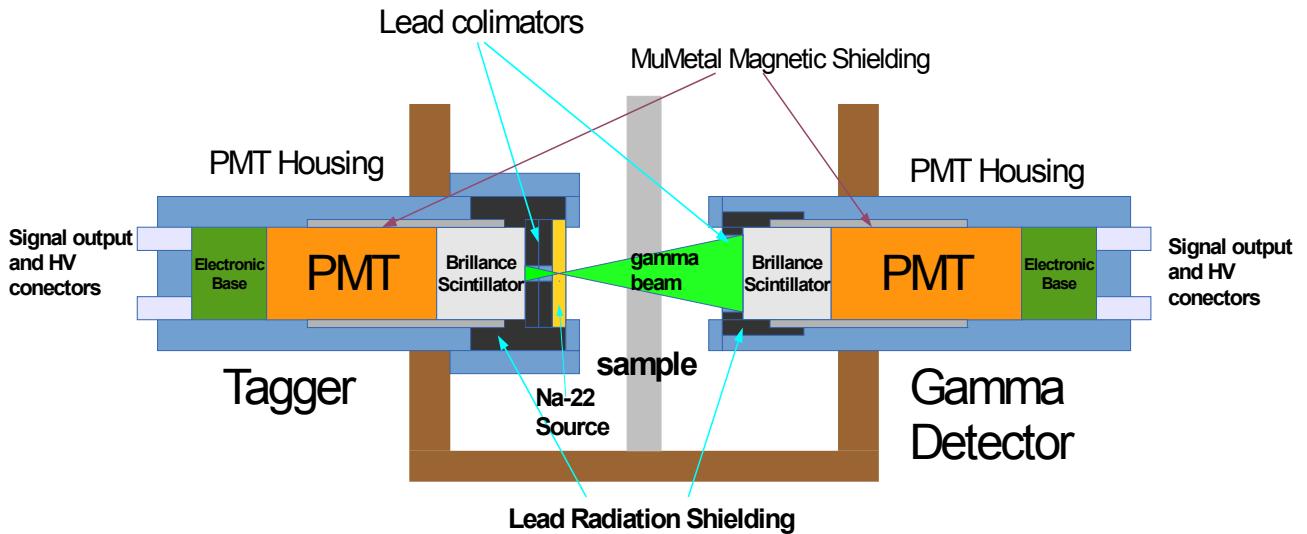


Figure 7: The Experimental Setup.

As we can see in the figure (7), the setup consist in 2 Gamma Spectrometers and a Gamma Source. Each spectrometer is a PMT coupled to a Fast Scintillator, in this case a Saint-Gobain Brillance 380 Crystal ($LaBr_3(Ce)$). Each crystal is cylindrical, with more or less 33 [mm] of length and 29 [mm] of diameter¹. Besides that, one of the PMT, called “Tagger”, is coupled through a 2 lead collimators, one next to the other, to a Disc Shaped 1 [μCi] (September 2011) Na-22 Source. The collimators are the following:

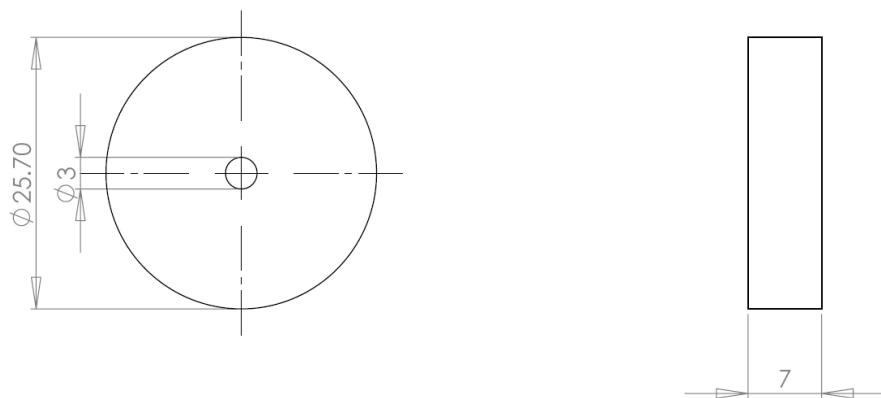


Figure 8: The Lead Collimators. All dimensions are in millimeters.

¹for instance, one of the crystals has a measured length of 32.8 [mm] and a diameter of 28.6 [mm].

The reason to use 2 collimators, one next to the other, lies in the mass attenuation coefficient of lead, as we can see in the following figure:

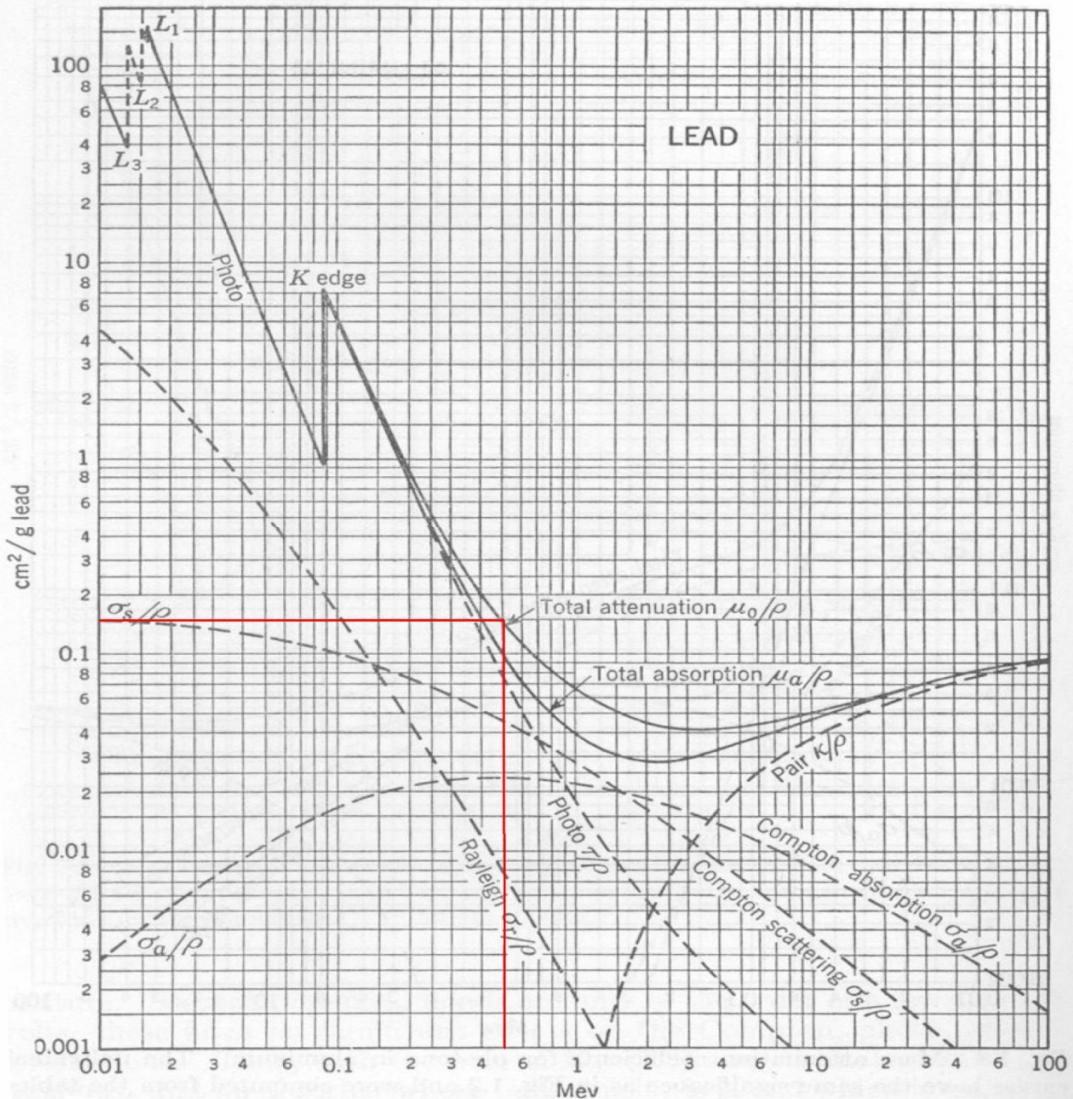


Figure 9: Mass Attenuation Coefficient for Gamma Rays in Lead. Image from [5].

From the Graph in figure (9), we can get the linear attenuation coefficient for the 511 [keV] in Lead, which is close to $\mu_{Pb} = 0.15 \text{ [cm}^2/\text{g]}$. Since $\rho_{Pb} = 11.34 \text{ [g/cm}^3]$, we get:

$$\frac{\mu_{Pb}}{\rho_{Pb}} \cong 0.15 \left[\frac{\text{cm}^2}{\text{g}} \right] \Rightarrow \frac{1}{\lambda_{Pb}} = \mu_{Pb} \cong 1.7 \text{ [cm}^{-1}] \Rightarrow \lambda_{Pb} \cong 0.59 \text{ [cm]} \quad (77)$$

Then, for 1 collimator of 7 [mm] thickness, the attenuation is:

$$e^{-1.7[\text{cm}^{-1}] \times 0.7[\text{cm}]} = e^{-1.19} \cong 0.30 \equiv 30\% \Rightarrow 70\% \text{ of attenuation} \quad (78)$$

Which maybe is not enough, for 2 collimators:

$$e^{-1.7[\text{cm}^{-1}] \times 1.4[\text{cm}]} = e^{-2.38} \cong 0.09 \equiv 9\% \Rightarrow 91\% \text{ of attenuation} \quad (79)$$

Which is a good attenuation in order to have a narrow beam.

Afterwards, the Tagger and Detector are assembled like in the following diagram.

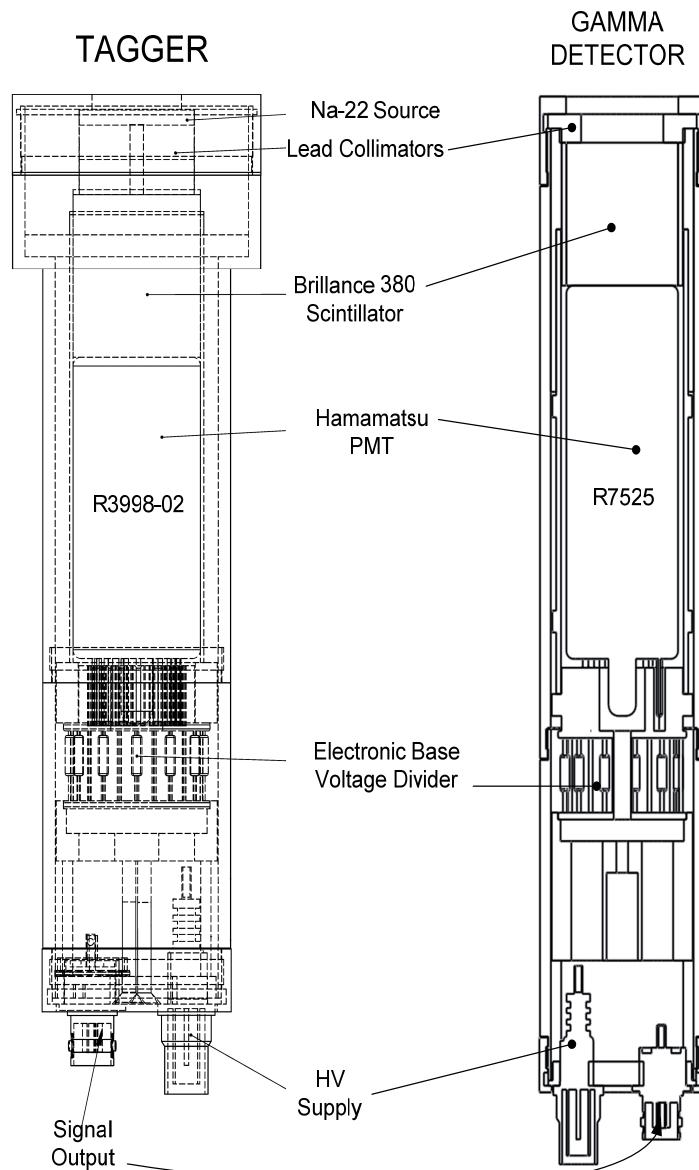


Figure 10: The Tagger and the Gamma Detector. Diagrams are not to scale, so dimensions of Tagger are not comparable to the dimensions of the Gamma Detector.

Sodium 22 is produced in many different ways, but most of them involve proton or alpha particle bombardment of elements [4]. The resulting isotope then might be combined with another chemical to produce a compound. In the case of the *Spectrum Techniques*, they provide a Disc Shaped source, produced by depositing $5 \text{ }[\mu\text{L}]$ of a NaCl Solution in the well of the disc. Then this solution is dried to form an area of around 3 [mm] of Diameter. The amount of Na-22 is then around $1.6 \times 10^{-10} \text{ [g]}$ to get an activity of approximately 1 [\mu Ci]^2 .

² Spectrum Techniques was contacted by email in order to obtain this information.

The Disc Shaped sources and the Collimators are the following ones:

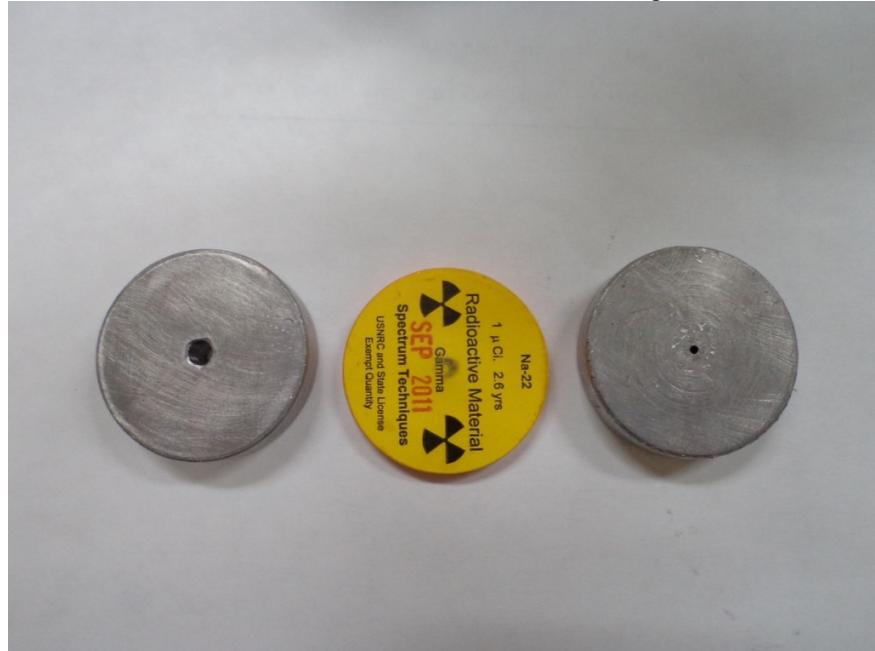


Figure 11: From left to right, the 3 [mm] hole lead collimator, the Na-22 Source and the 1 [mm] hole Collimator. The dimensions of the 1 [mm] hole collimator are the same of figure (8), with the exception of the center hole, which has a diameter of 1 [mm]. This collimator was used during calibration and does not form part of the final prototype.

The assembled tagger is the following one:

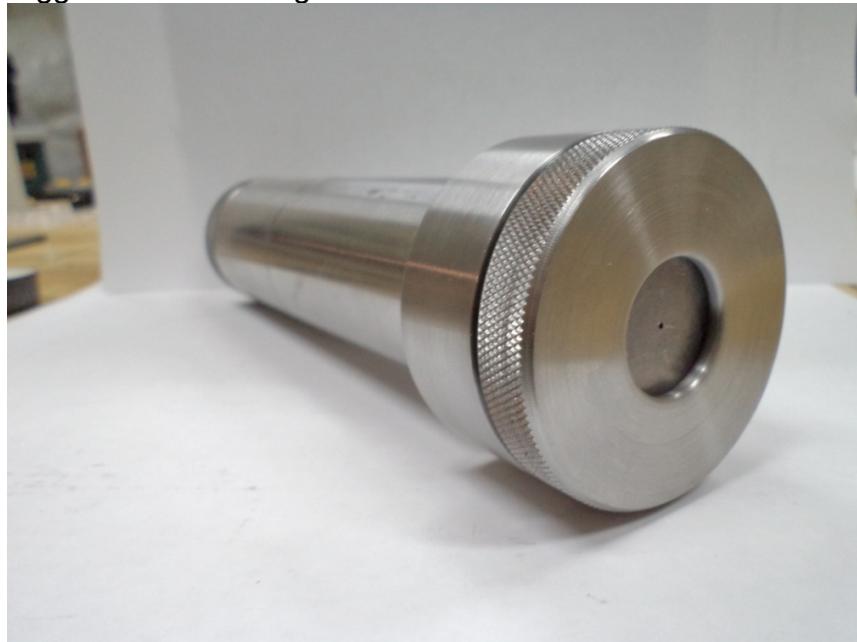


Figure 12: The Tagger. As you may notice, the collimators are misassembled, since the source should be in first position, and the final collimator is the 3 [mm], not the 1 [mm].

The PMT Base Design

The Tagger PMT Base schematic is the following one:

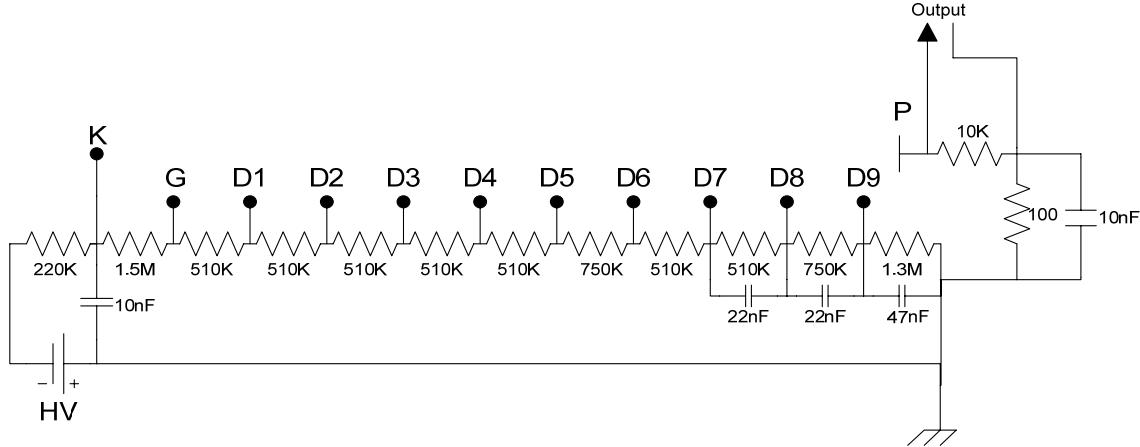


Figure 13: The Tagger PMT Base Schematic. This design was done for a Shashlik Calorimeter that was used in the JLab experiment Two Photon Exchange.

The PMT used for Tagger was the R3998-02, a 9 stages PMT used for High Energy Physics and Scintillation Counting. It has a nominal gain of 1.3×10^6 at 1 [kV].

The network used the following voltage distribution ratio:

K	G	D1	D2	D3	D4	D5	D6	D7	D8	D9	P
3	1	1	1	1	1	1.5	1	1	1.5	2.5	

Table1: R3998-02 tapered distribution of voltages. There was not a tapered network in the Hamamatsu Datasheet, so this distribution was proposed by Prof. Sergey Kuleshov.

As was mentioned before, the design was made for another experiment, with an expected output current of $0.4 \mu\text{A}$. The Shashlik needed to work a gain of 3×10^5 . In order to keep linearity the polarizing current should be at least 100 times bigger than signal current. For the sake of simplicity, a current of $100 \mu\text{A}$ was selected. The resistance was calculated from the graph on figure (14), where for the needed gain a supply voltage of 820 [V] was necessary. Then, the total resistance was calculated using Ohm's Law, giving $8.2 \text{ M}\Omega$. After that, the individual resistance were calculated using the distribution given by table (1), which is a tapered distribution of voltage, which increments the linearity under the cost of less gain.

In the case of the Gamma Detector, the PMT used was the R7525, an 8 stages PMT used for High Energy Physics and Scintillation Counting. It has a nominal gain of 5×10^5 at 1.5 [kV].

The network used the following distribution Ratio:

K	D1	D2	D3	D4	D5	D6	D7	D8	P
4	1	1.5	1	1.2	1.5	2	3.3	3	

Table2: R7525 tapered distribution of voltages. From Hamamatsu PMT Datasheet.

Head-On Types (28mm Dia.)

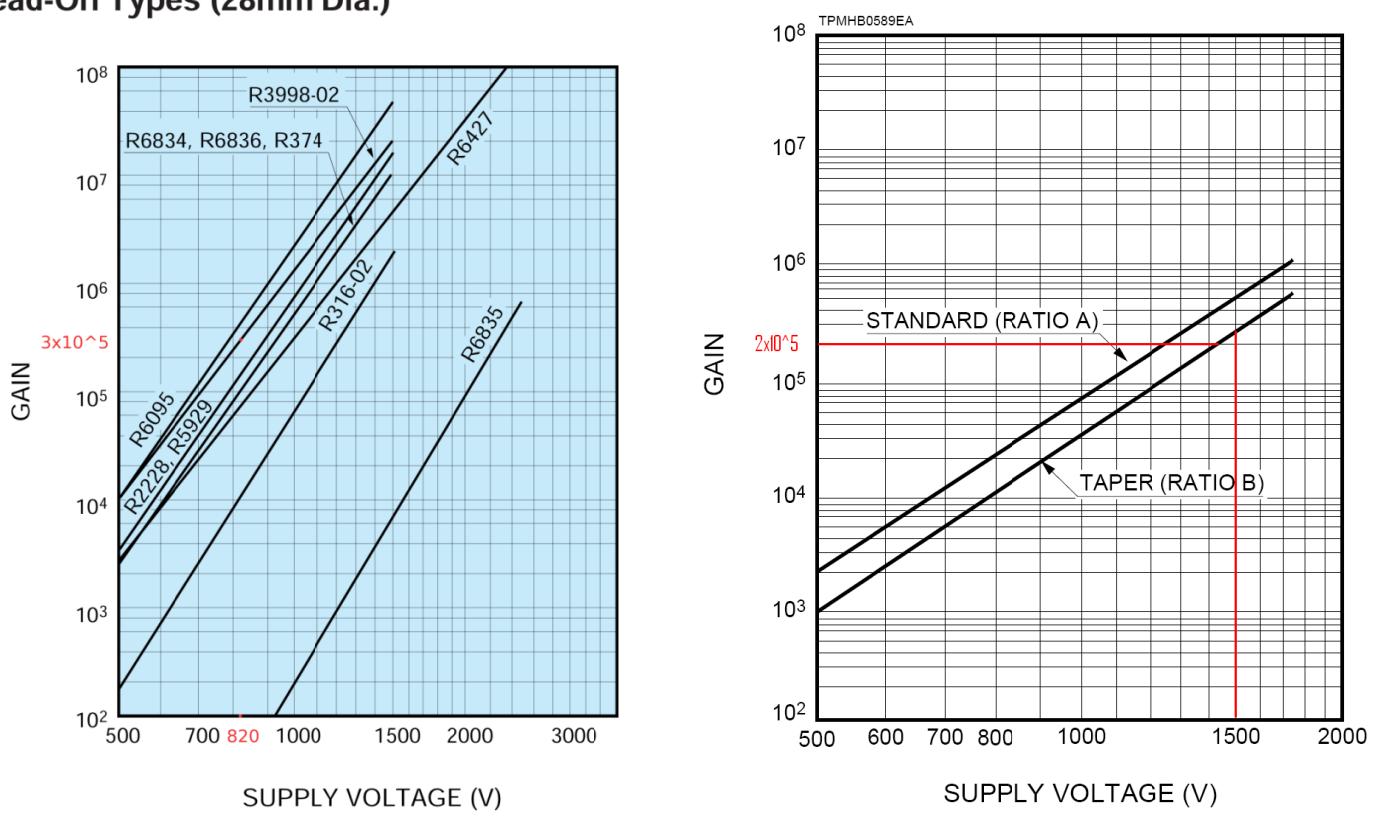


Figure 14: Selected Supply Voltage and Gain for PMTs. R3998-02 on the left, R7525 on the right. From Hamamatsu PMT Catalogue.

The R7525 resistance network was designed specifically for Gamma Spectrometry. From Brilliance 380 Datasheet we know that the light yield of the crystal is 63 [photons/keV]. Besides that, we know PMT gain and we can estimate the Crystal to PMT transmission efficiency, the Quantum Efficiency (around 20%) and Gamma Ray energy of around 1 [MeV]. With all this information, we can estimate a signal output current of about 2 [μA]:

PMT Gain	Source Rate [Hz]	Crystal Ph/keV [1/keV]	Gamma Energy [keV]	Electron Charge [C]	Transmision Eficiency	Quantum Efficiency	Current [A]
200000	10000	63	1000	1,6E-19	0,5	0,2	2,02E-06

Table 3: Signal Output Calculation. The output current is just the multiplication of all factors.

Finally, if we want a resistor network current of 100 times the signal output, it gives 200 [μA]. We calculated this numbers for gain of 2×10^5 at 1.5 [kV] for a tapered network, as we can see on figure (14). Then, using again Ohm's Law, the equivalent resistance is 7.5 [$M\Omega$]. In order to have resistors easier to find, we finally selected a resistance of 7.31 [$M\Omega$].

Finally, the obtained network is on the following figure:

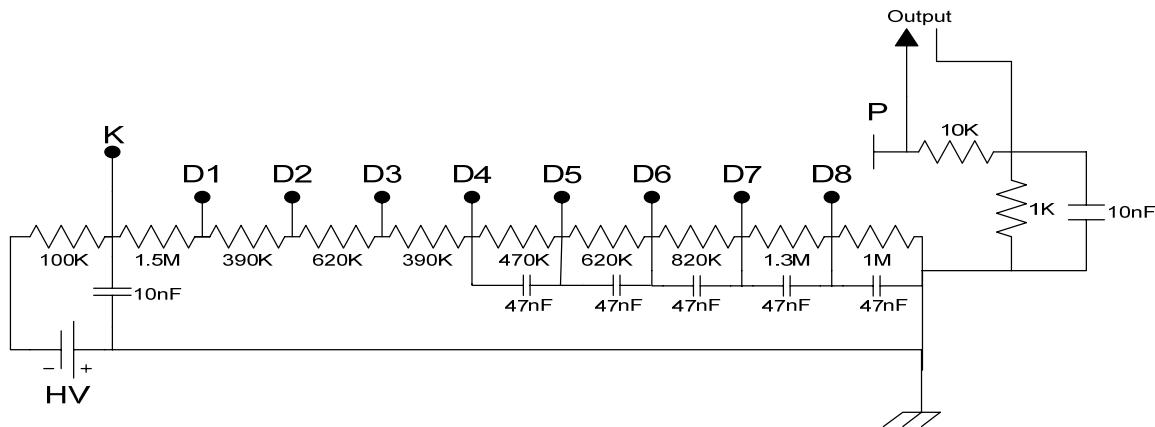


Figure 15: The Gamma Detector PMT Base Schematic.

The Experiment Schematic

The following Diagram Shows the Experiment Schematic:

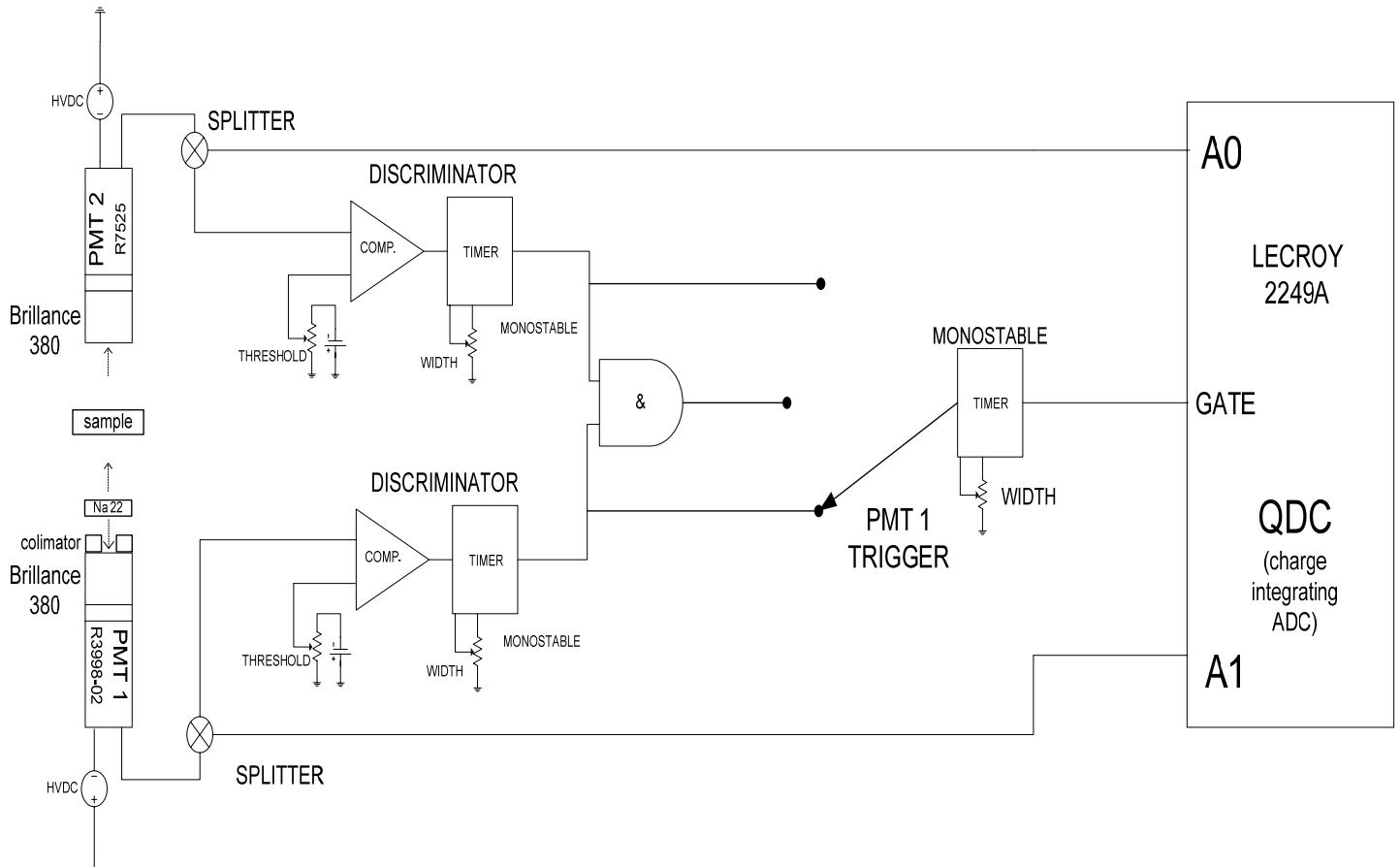


Figure 16: The Experiment Schematic. The Trigger Generation for the CAMAC controller is not included in the diagram.

The Acquisition System consists on a Trigger Generator and a Charge Integration Analog to Digital Converter. The gate generator for integration is a Leading Edge Discriminator, specifically the Lecroy 821 and a Coincidence Unit, the Lecroy 465. As we can see in figure (16), the Coincidence is not being used, instead of that Trigger comes from PMT1, the Tagger PMT.

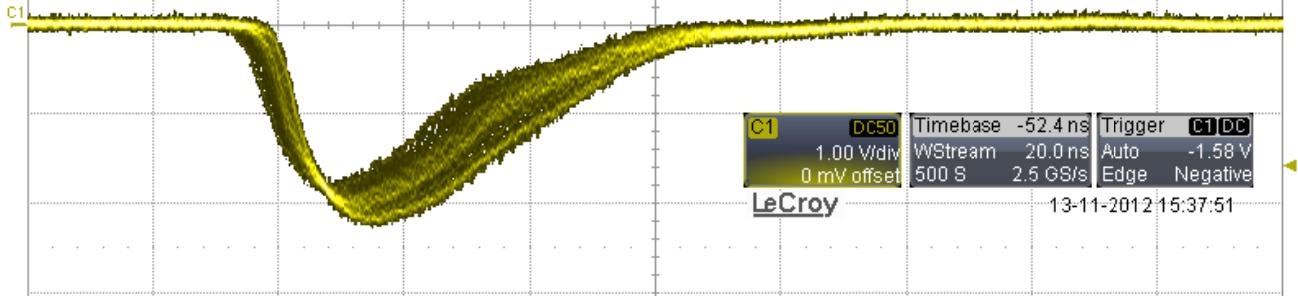


Figure 17: The Tagger PMT Signal Output. As we can see, signal amplitude (at about 1 [kV] PMT polarization) is close to 2 [V]. Signal pulse length is close to 60 [nS].

The information about the deposited energy in the scintillator crystal is hidden in the charge of the signal. In order to obtain this information, the signal must be integrated:

(80)

The integration is done inside the QDC, the Lecroy 2249A CAMAC module.

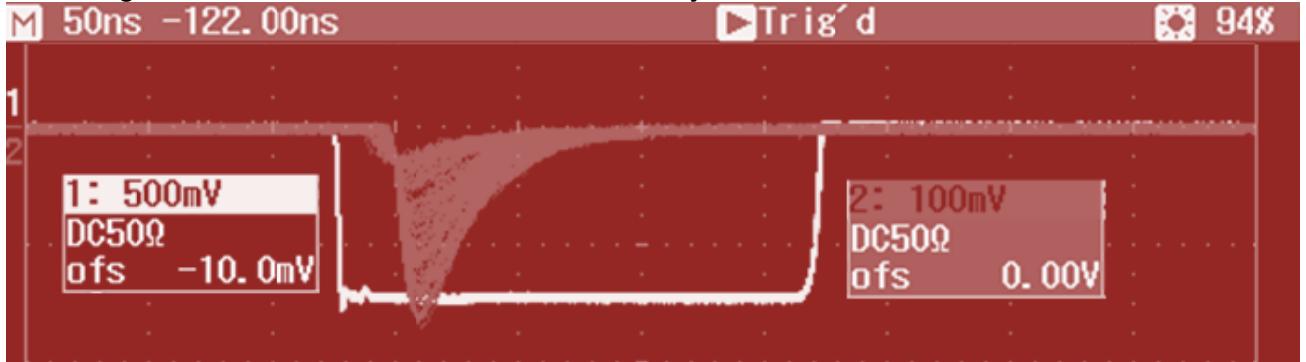


Figure 18: Example of PMT output signal (Ch2) and its integration gate (Ch1). PMT is XP2262, Crystal is Brilliance 380, gamma source is Sodium 22. Supply voltage is unknown.

Lecroy ADC 2249A Characteristics

Channels	12
Resolution (QDC)	10 bit
Full Scale	256 [pC] +-5%
Precision	0.25 [pC]
Gate Impedance	50 [Ohm]
Gate NIM Threshold	600 [mV] or greater enables
Gate Minimum Duration	10 [nS]
Gate Maximum Recommended Duration	200 [nS]
Effective opening and closing times	2 [nS]
Internal Delay	2 [nS]

Table4: QDC Characteristics.

5. The Experiment Calibration

Sodium 22 decays in the following Scheme:

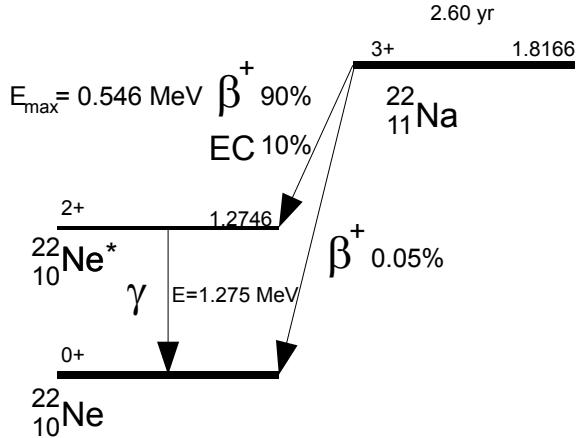


Figure 19: Decay Scheme for Na-22.

This means that most of the time Sodium decays through positron emission. Positron then annihilates with an electron from the material and 2 gamma rays are formed. Besides that, 99.95% of the time Na-22 doesn't go to a ground state of Ne-22, so another gamma of 1.275 [MeV] is formed.

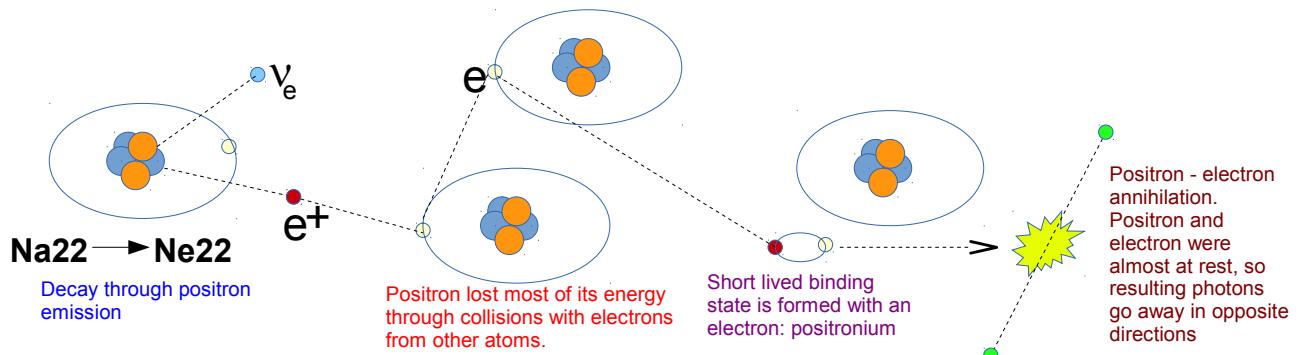


Figure 20: Positron decay of Sodium 22. Figure shows, in a very simplistic way, how gammas are created.

All this means that as we see in figure (7) of the experimental setup, 2 beams of gamma are created: one which goes directly to the Tagger through the collimator and the other one that goes to the Gamma Detector, passing through the material sample. Besides that, as was mentioned in page number 15, source is distributed in the center of the Disc, covering an area with a diameter of 3 [mm]. This means that, using definitions from figure (21):

$$\tan\left(\frac{\alpha}{2}\right) = \frac{3/2}{14/2} = \frac{3}{14} \wedge \tan\left(\frac{\alpha}{2}\right) = \frac{d/2}{x+10} \Rightarrow d = \frac{3}{7}(x + 10) \quad (81)$$

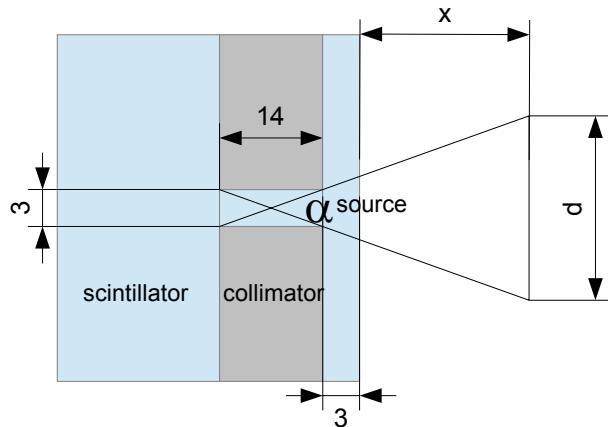


Figure 21: Beam Size. Dimensions are in millimeters, x is distance from source in the front of Tagger, d is diameter of the beam. Drawing is not to scale.

Then, the diameter of the beam has the following behavior:

$$d = \frac{3(x+10)}{7}$$

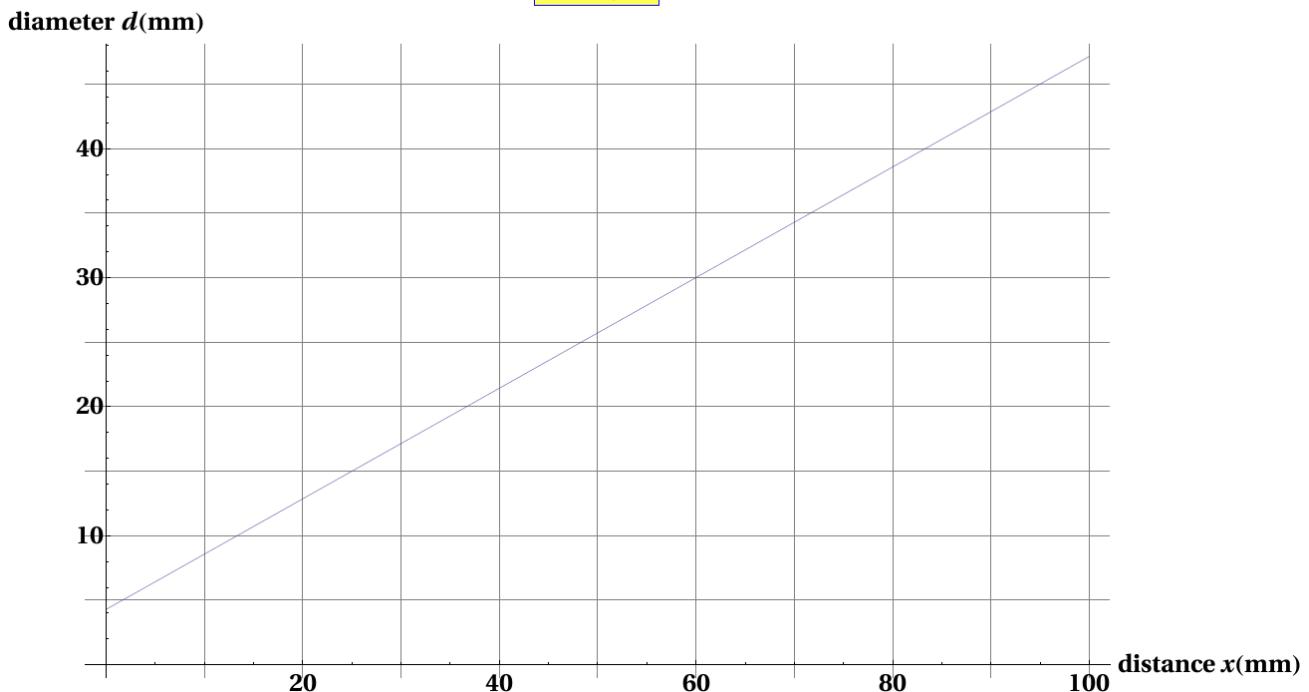


Figure 22: Diameter of Beam as a function of Distance from the Source.

Now, this is the theoretical Diameter, with a lot of assumptions that might not be true. Because of this fact, we need to have a real measured profile of our beam. In order to do this measurement, we have to increase the x distance, step by step, and measure what we will explain in the following section.

Selection of an appropriate distance

The Full Na-22 Spectrum is the following one:

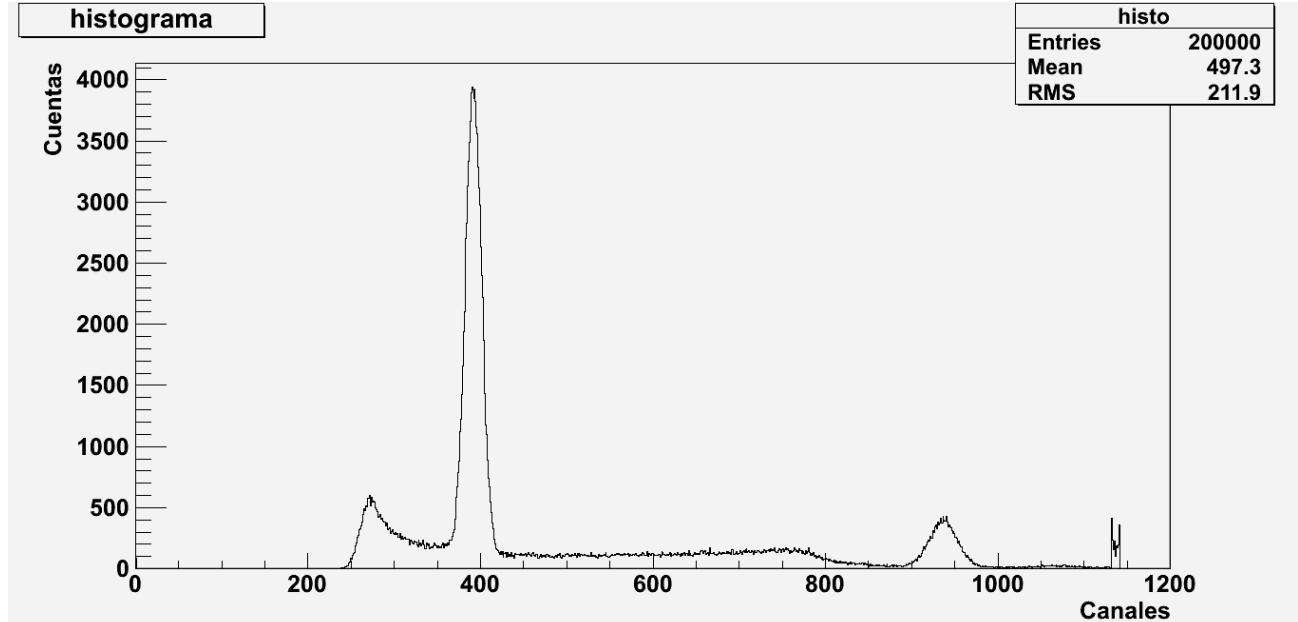


Figure 23: Na 22 Spectrum Histogram. Y axe are events, X axe are QDC Channels. Crystal is Brilliance 380, PMT is XP2262. High Voltage Supply is at 1.43 [kV]. Discriminator threshold at 90 [mV]. Photopeak near channel 400 is the 511 [keV] peak. Photopeak near channel 900 is the 1.275 [MeV] peak. Due to the low gain, in order to see the full spectrum, it is not possible to appreciate all the Compton spectra generated by the 511 [keV] peak.

Using tools from CERN ROOT library it is possible to do a fitting of the 511 [keV] photopeak.

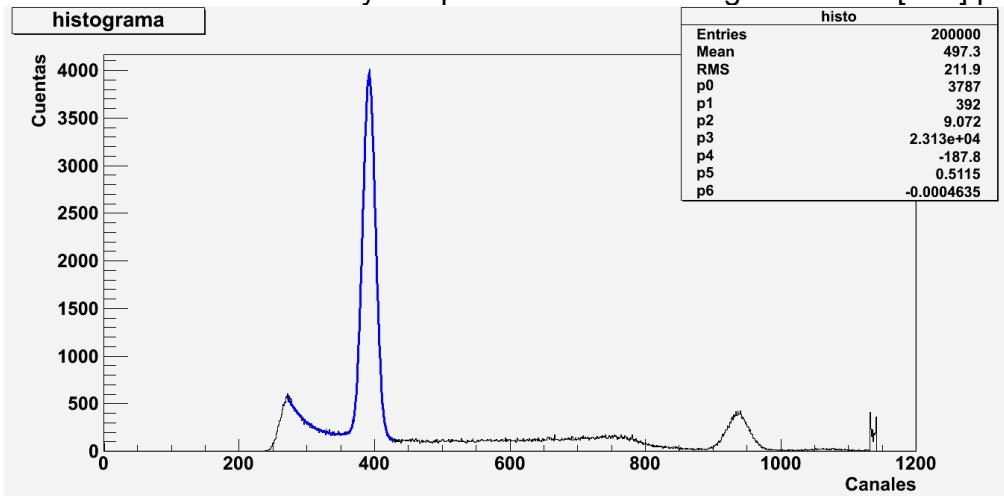


Figure 24: Na 22 Spectrum. Fitting of the 511 [keV] peak. Fitting function is on equation (82), ch means QDC channel. It is clear from the obtained parameters that a Gaussian plus a Parabola, instead a 3rd order polynomial, was a good enough option for that region.

$$\text{Peak Events Fit} = p[0]e^{-\frac{(ch-p[1])^2}{2p[2]^2}} + p[3] + ch \cdot p[4] + ch^2 \cdot p[5] + ch^3 \cdot p[6] \quad (82)$$

Now, for our experiment, we need to concentrate only on the 511 [keV] photopeak, since it comes from photon annihilation and will produce a coincidence for our detector (we have to remember schematic from figure (16)). Also, from now on, Tagger Spectra will be called PMT1 and Gamma Detector Spectra will be called PMT2 (from definition of setup on figure (7)). PMT1 Supply voltage was 830 [V], PMT2 Supply voltage was 1100 [V]. PMT1 was trigger for all following measurements. So, the measured spectrum for PMT1 is the following:

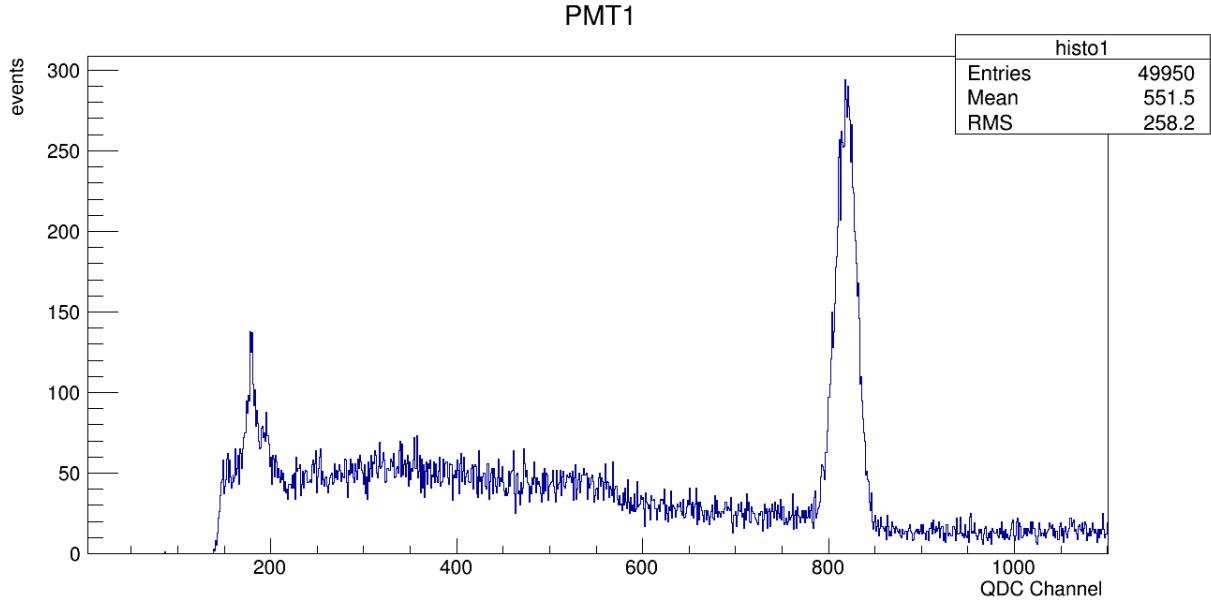


Figure 25: PMT1 Spectrum. As we can see, gain was selected in order to appreciate the long Compton tail. Big photopeak is 511 [keV] peak. Small peak is still a matter of discussion. The overflow was cropped in order to concentrate on relevant data (data over channel 1024 has the overflow bit on).

For PMT2, with PMT1 as trigger, we have the following spectrum:

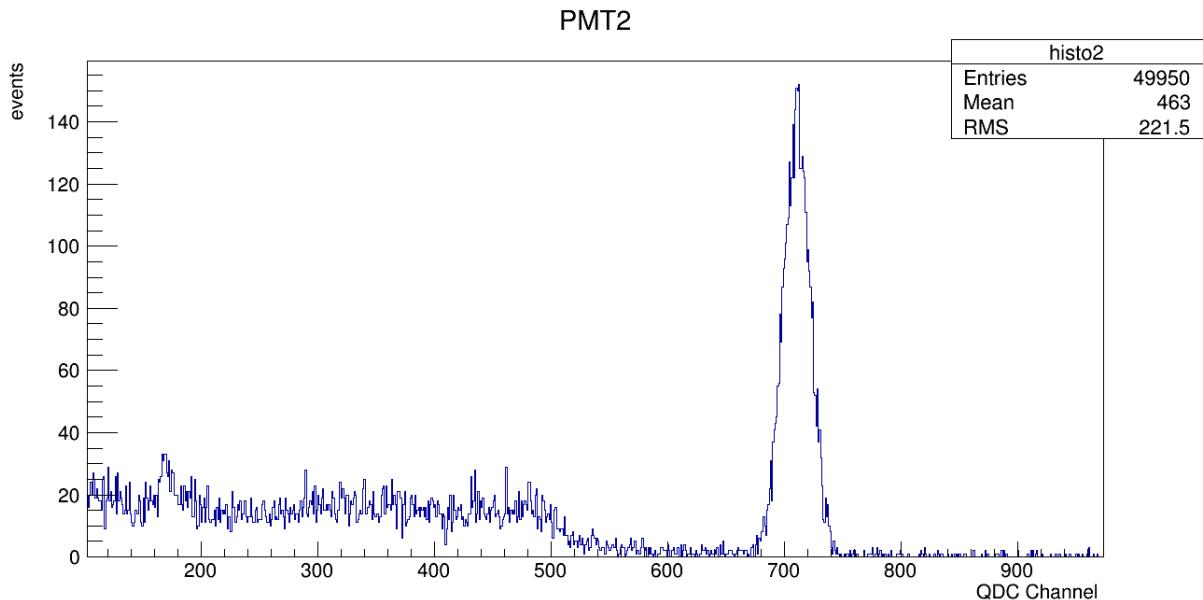


Figure 26: PMT2 Spectrum. In this case, Pedestal was cropped.

Tagger and Gamma Detector were in direct contact, so we will call this distance $0 [mm]$, even though it is not the same $0 [mm]$ from figure (22), because the PMT housing for both detectors prohibit direct contact between the source and the Scintillator on Gamma Detector, as we can see on figure (10) diagrams. However, we will still call this distance the $0 [mm]$ of our measurement.

Now, going back to measurements, we can see that statistics drop down a lot. We can understand this better if we see both pictures as they are in the reality:

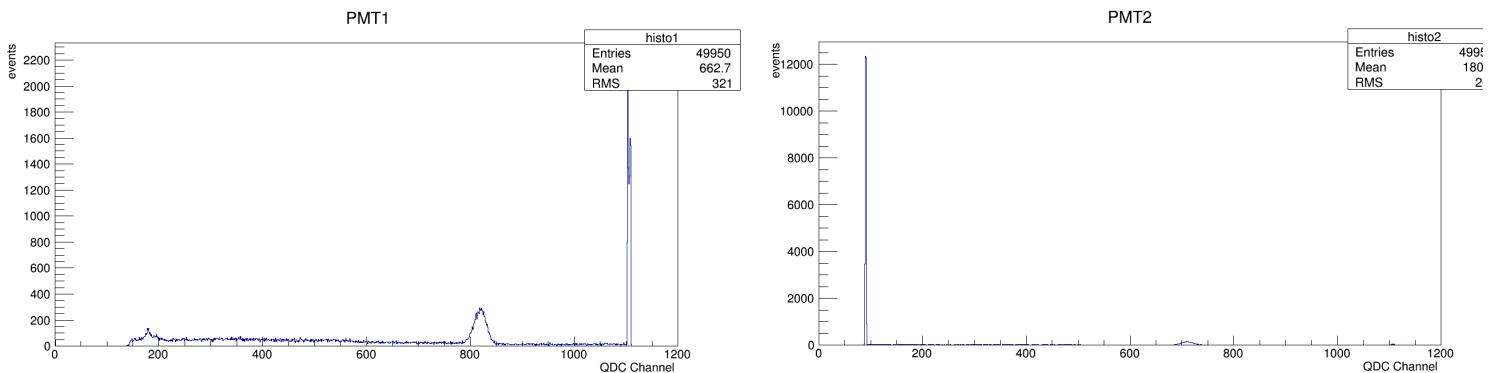


Figure 27: PMT1 and PMT2 Spectra. On PMT1, overflow wasn't cropped. On PMT2, pedestal wasn't cropped. In this way, we can appreciate how big is the proportion of pedestal events on PMT2. PMT1 tagger was in direct contact with PMT2 gamma detector.

Let's remember that if a Gamma Ray doesn't come directly to the Scintillator, the probability to produce a Photopeak (or produce a fully absorption of its energy) drops down.

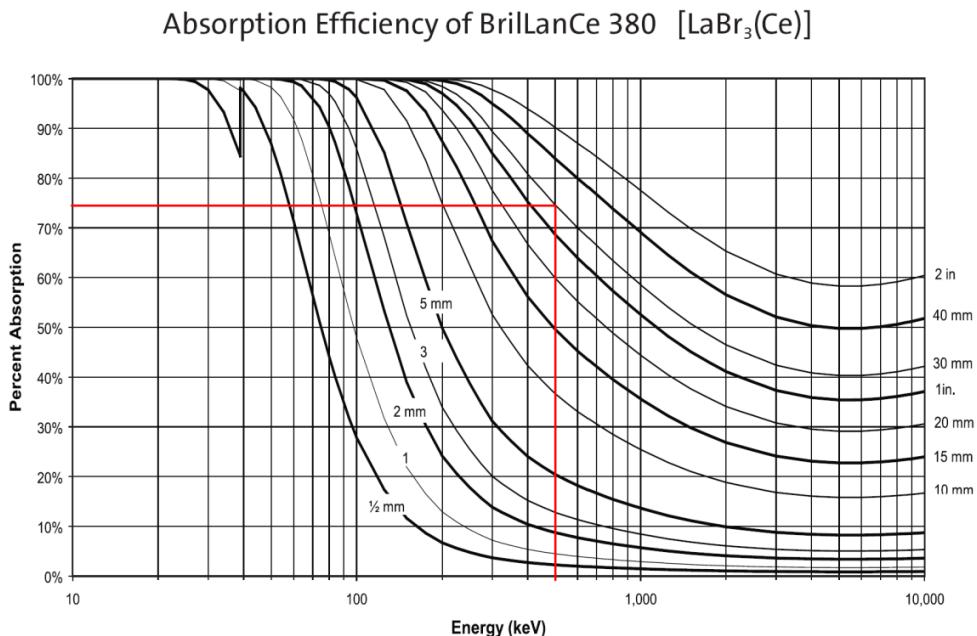


Figure 28: Absorption Efficiency. Lines designate, more or less, gamma absorption efficiency for our Scintillator when a 511 [keV] gamma hits its center. From Brilliance's datasheet.

Since we are more interested in gamma events which go directly to the Scintillator, we selected for PMT1 the events that formed part of the 511 [keV] photopeak.

We want to calculate density of materials due to Compton Scattering of gamma particles. So this means that the drop down in events, after placing a material between the Tagger and Detector, will be due to Compton Scattering. So that's the reason why we concentrate on photopeaks fitting, in order to see a change on this statistic.

So, in order to see this, first a Search for Peaks was performed, using a ROOT algorithm. After that, a fitting of PMT1 photopeak was done. From this peak, events on PMT2 were selected and plotted on histogram of PMT2 only if PMT1 events were on photopeak in that measurement. The fitting function was the following one:

$$\text{PMT1 fitting function} = p[0]e^{-\frac{(ch-p[1])^2}{2p[2]^2}} + p[3] + ch \cdot p[4] \quad (83)$$

Polynomial was added in order to reach a good fitting of the Gaussian function. The plotted random variable might be defined in the following way:

$$\text{Plotted Events PMT2} = \text{Events on PMT when PMT1 was on PMT1 peak} \quad (84)$$

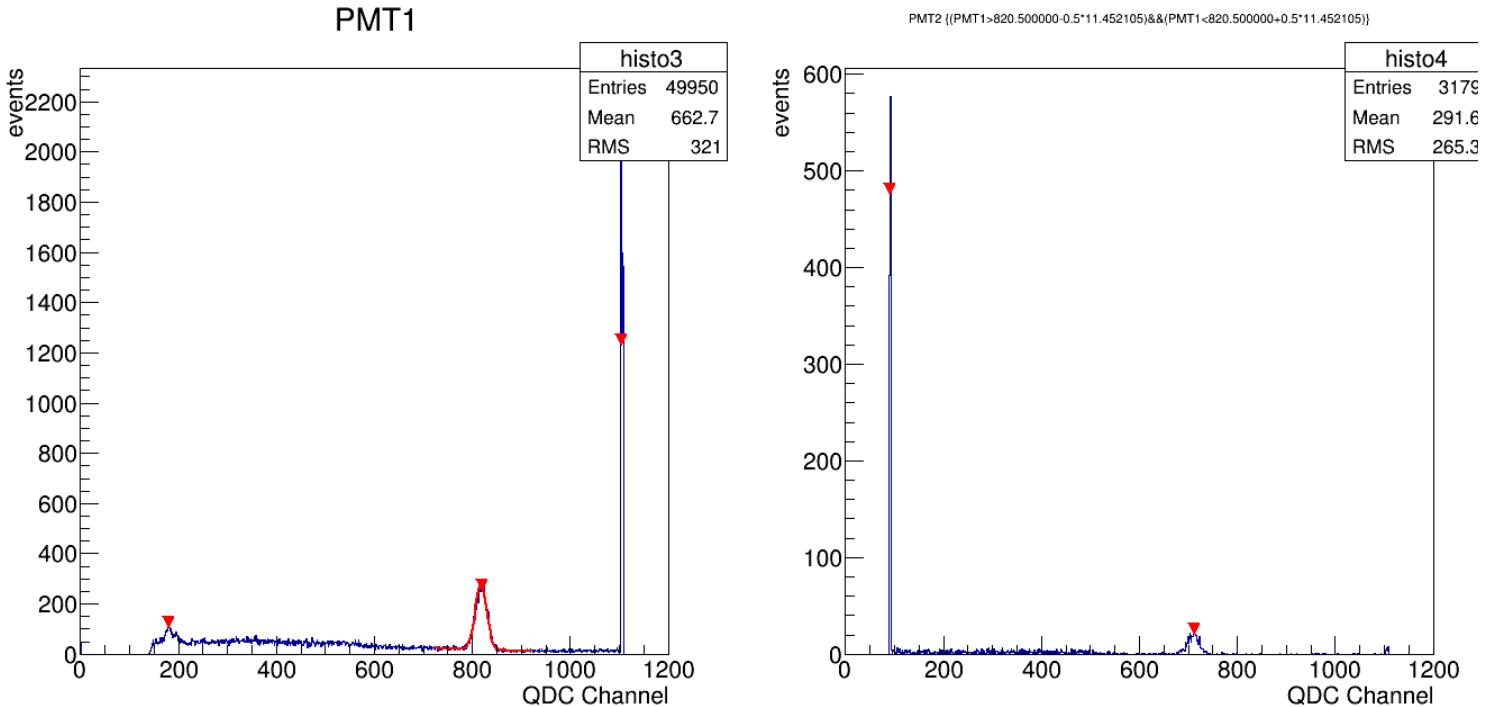


Figure 29: PMT1 photopeak fitting and PMT2 conditional histogram. Histogram of PMT2 was filled with PMT2 event data only if this data corresponded to an event when PMT1 was on the photopeak, given by the Gaussian Fit.

Since, from histogram of PMT2 on picture (26), we know more or less where the PMT2 photopeak is located, we can then select even when we have a peak on both PMT.

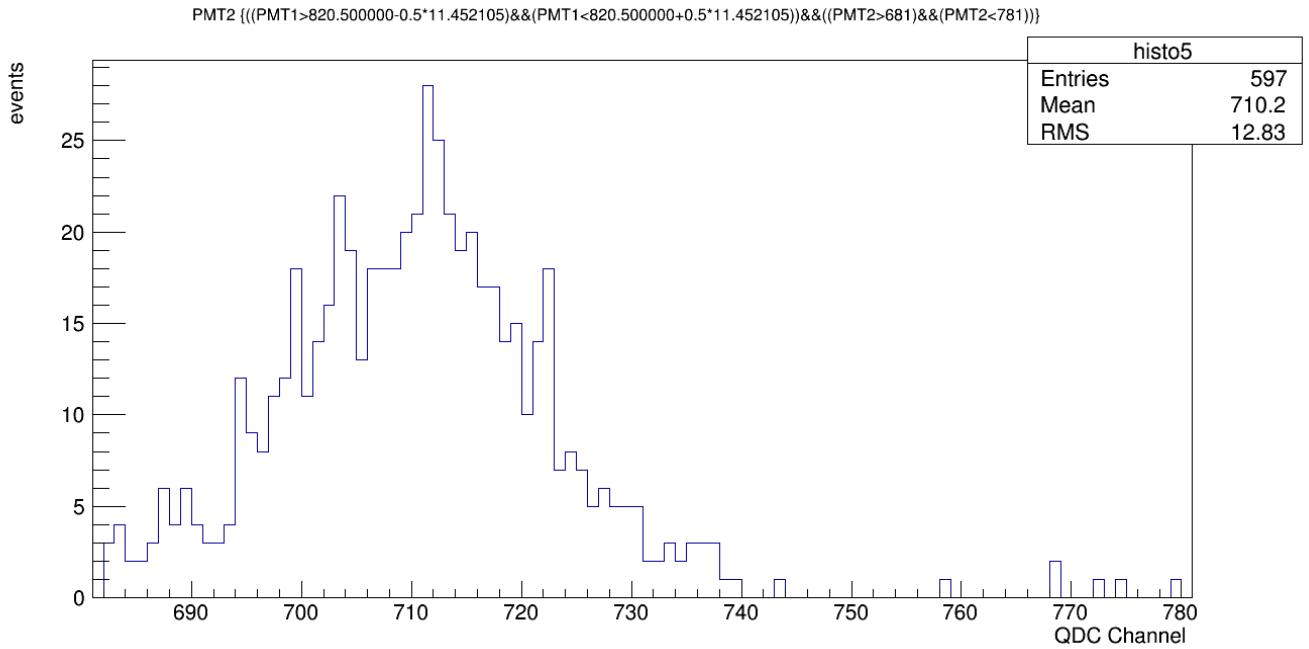


Figure 30: PMT2 photopeak when PMT1 events are also in photopeak. This is the joint distribution, as we can see on the title of the histogram. The statistic is very poor.

This is the same than doing a coincidence. Now we proceed to define the following amount:

$$\text{event probability} = \frac{(\text{Events of PMT2 when PMT1 was on peak}) - (\text{Events of PMT2 on Pedestal when PMT1 was on peak})}{\text{Events on PMT1 peak}} \quad (85)$$

Now, we proceed to plot this amount as we increase the distance between PMT1 and PMT2:

Conditional Probability of Event Occurrence on PMT2 since there were events on 511 keV Photopeak of PMT1

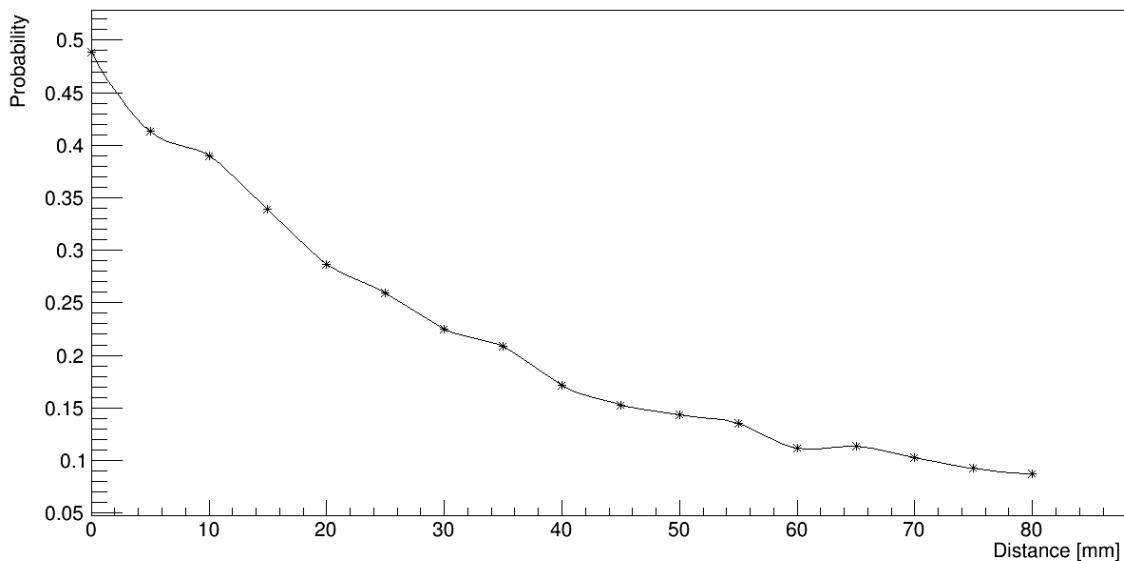


Figure 31: Probability to measure a gamma on PMT2 since we measured one on PMT1 peak.

What is this amount?

This is simply the probability to measure a gamma on PMT2 since we measured one on PMT1 photopeak.

What is the utility of this?

With this we can select an appropriate working distance. Placing a material between Tagger and Gamma Detector will decrease probability of figure (31) even more. For that reason we selected a working distance of 30 [mm].

```
shape_80_3mm.lst
Warning in <TCanvas::Constructor>: Deleting canvas with same name: c1
Warning in <TCanvas::Constructor>: Deleting canvas with same name: c2
Warning in <TCanvas::Constructor>: Deleting canvas with same name: canvas1
Se encontraron los peaks del PMT1, donde peaks[1] es 831.5y peaks[2] es 182.5
FCN=262.173 FROM MIGRAD    STATUS=CONVERGED    372 CALLS    373 TOTAL
                           EDM=3.87605e-09   STRATEGY= 1    ERROR MATRIX ACCURATE
EXT PARAMETER                      STEP          FIRST
NO.   NAME      VALUE       ERROR        SIZE      DERIVATIVE
 1 p0        2.52392e+02  3.93915e+00  2.97017e-05 -1.30981e-02
 2 p1        8.32680e+02  1.60694e-01  1.26729e-05 -1.53705e-02
 3 p2        1.17712e+01  1.41026e-01  1.80202e-05 -2.60277e-02
 4 p3        7.50169e+01  4.49706e+00  1.49535e-05 -1.45385e-03
 5 p4       -6.83633e-02  5.27897e-03  1.24488e-05  1.00770e-03
Warning in <TCanvas::Constructor>: Deleting canvas with same name: c3
*****events on Histograms*****
Counts on Fitted Peak of PMT1 3159
Counts of Histogram of PMT2 data when PMT1 was on peak 3135
Pedestal Events on PMT2 when PMT1 was on peak: 2859
*****AIR PMT1 & PMT2*****
Probability of events on peak PMT1: 0.06324
probability of events in photopeak pmt2: 0.02711324
Probability of having peak in PMT1 AND PMT2: 0.00171473
Probability of having events on PMT2 if PMT1 was on peak: 0.08736942
Counts PMT2 680-780 85
*****
(int)0
root [1] ■
```

Figure 32: ROOT Data Analysis Algorithm output.

6. Measuring Density

We measured density for the following materials: Lead, Duralumin and Polystyrene.

The information of these materials is in the following tables:

	Duralumin				Lead		Polystyrene	
Density [g/cm ³]	2,79	-	-	-	11,34	Density [g/cm ³]	1,05	-
Composition	Cu	Mg	Mn	Al	Pb	Composition	C	H
Weight Fraction wi	0,044	0,015	0,006	0,935	1	Molecular Amount m	8	8
Z	29	12	25	13	82	Z	6	1
A	63,55	24,33	54,94	26,98	207,2	A	12,011	1
Z/A	0,4563	0,493	0,455	0,48	0,396	m*Z	48	8
wi*Z/A	0,0201	0,007	0,0027	0,451	0,396	m*A	96,088	8,1
Z/A equivalent	0,4807	-	-	-	0,396	Z/A equivalent	0,5377	-
Thickness [cm]	1,6	-	-	-	1,0	Thickness [cm]	1,0	-

Table 5: Materials Characteristics. For Duralumin and Lead, Z/A equivalent ratio was calculated as on equation (67). For Polystyrene Z/A equivalent ratio was calculated as on equation (44).

Now, for a distance of 30 [mm] between Tagger and Gamma Detector, with the different materials placed as shown on figure (7), the same fitting of photopeak of PMT1 was done. ~50000 events per measurement. Fitting function was the same from equation (83).

For Air:

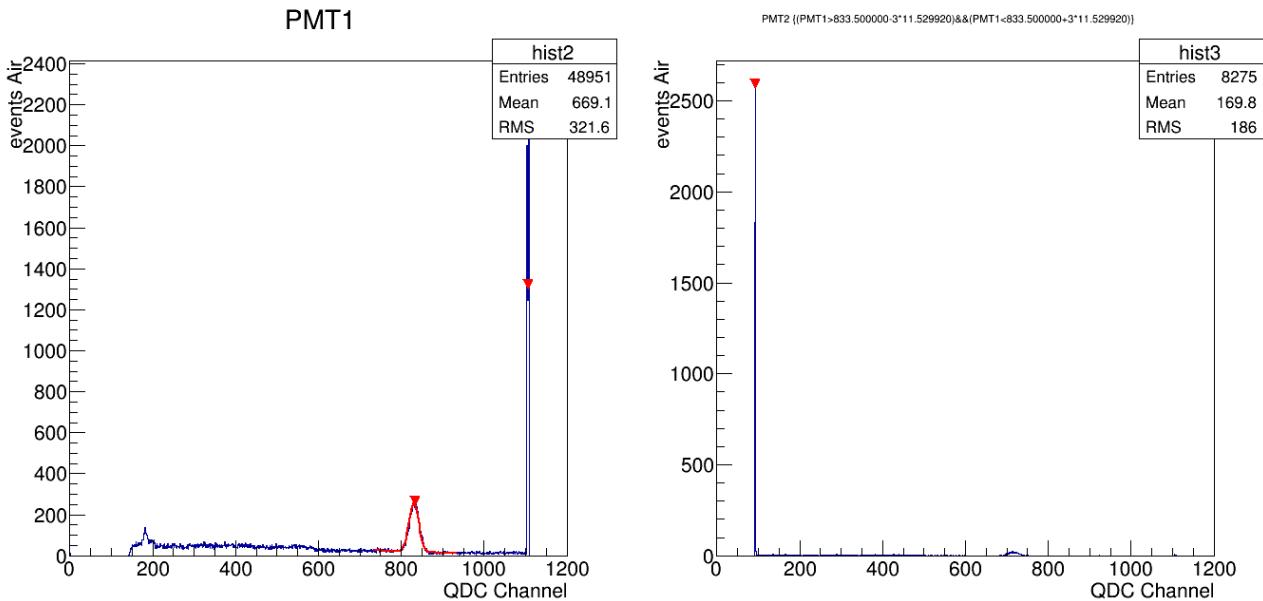


Figure 33: PMT1 photopeak fitting and PMT2 conditional histogram. Air between Tagger and Gamma Detector.

Photopeak of PMT2 with Air between PMT:

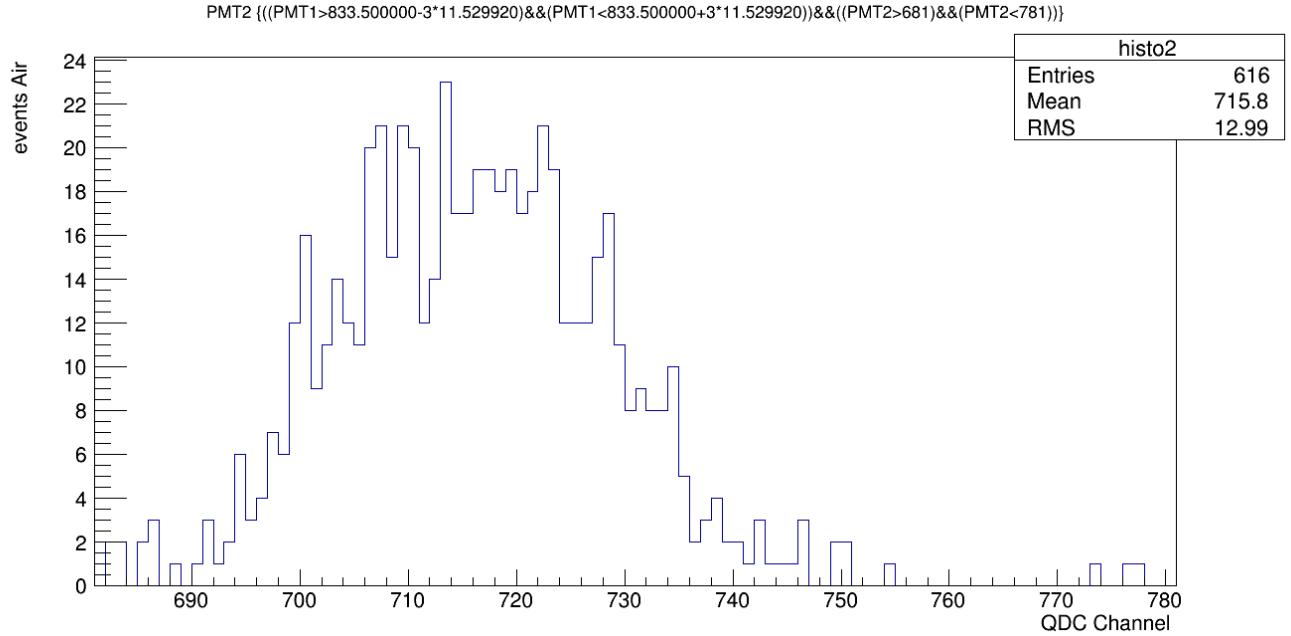


Figure 34: PMT2 conditioned photopeak histogram. Air between PMT1 and PMT2.

So, as we can see, even though on figure (32) we appreciate the low statistics of PMT2, when we do a conditioned plot for PMT2 photopeak we can still get good enough statistics, as we can see on figure (33).

Now, let's change material to Polystyrene:

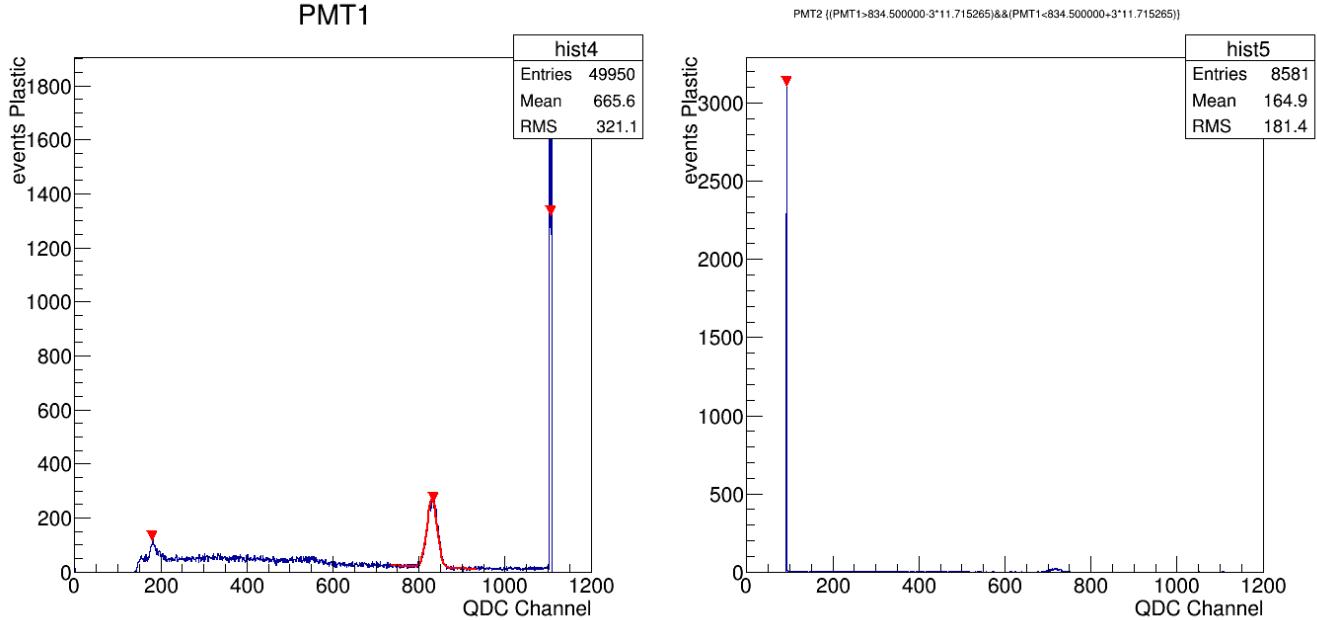


Figure 35: PMT1 photopeak fitting and PMT2 conditional histogram. Polystyrene between Tagger and Gamma Detector.

Photopeak of PMT2 with Polystyrene between PMT:

PMT2 {((PMT1>834.500000-3*11.715265)&&(PMT1<834.500000+3*11.715265))&&((PMT2>681)&&(PMT2<781))}

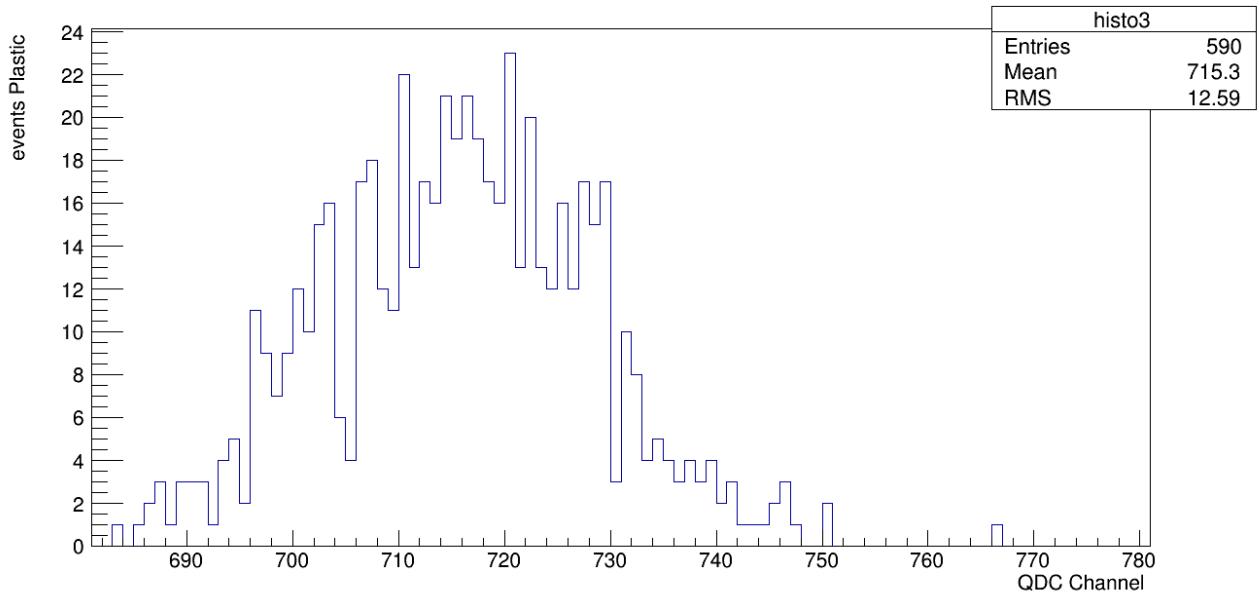


Figure 36: PMT2 conditioned photopeak histogram. Polystyrene between PMT1 and PMT2.

As we can see, there is a slight difference between statistics of histogram on figure (34) and the one from above, on figure (36). This is because density of plastic is very low.

Now we can start with more dense materials, such as Duralumin:

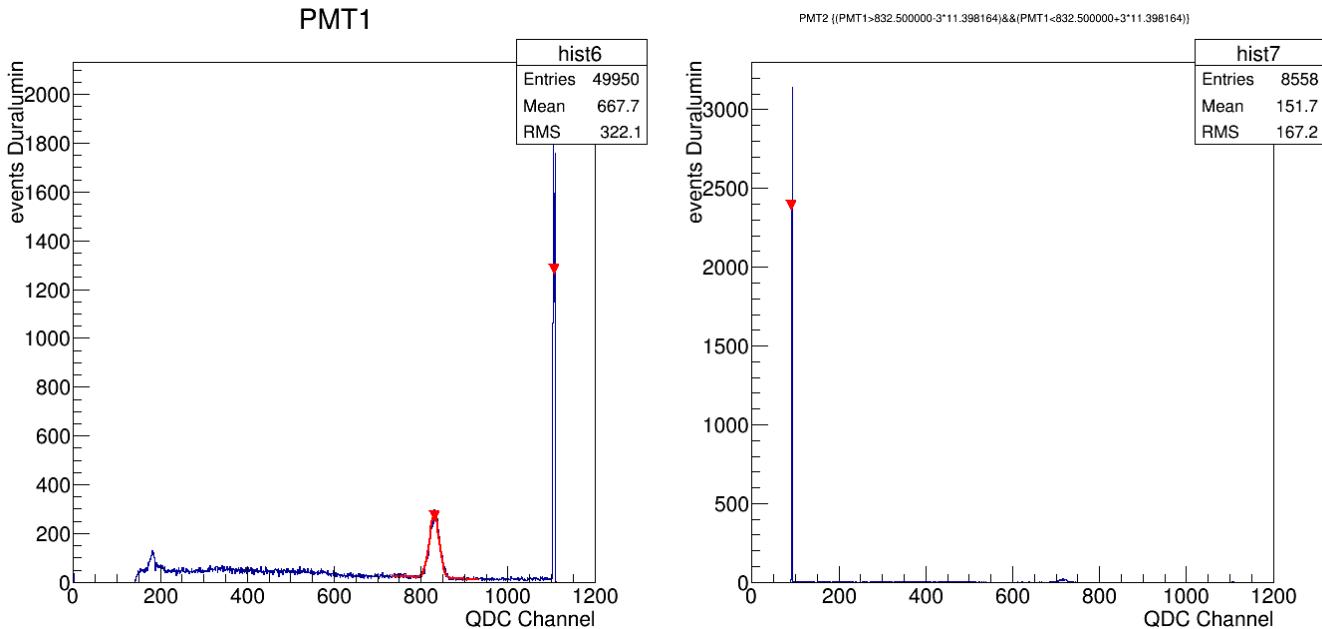


Figure 37: PMT1 photopeak fitting and PMT2 conditional histogram. Duralumin between Tagger and Gamma Detector.

As we can see on figure (37), even though on PMT2 pedestal statistics hasn't grown up significantly, the overflow peak on PMT1 has grown up. This might be due to backscattering of the 1.27 [MeV] photons on the Duralumin, but all of this is just matter of speculation because no calculation on that has been done by us. Let's see the PMT2 peak:

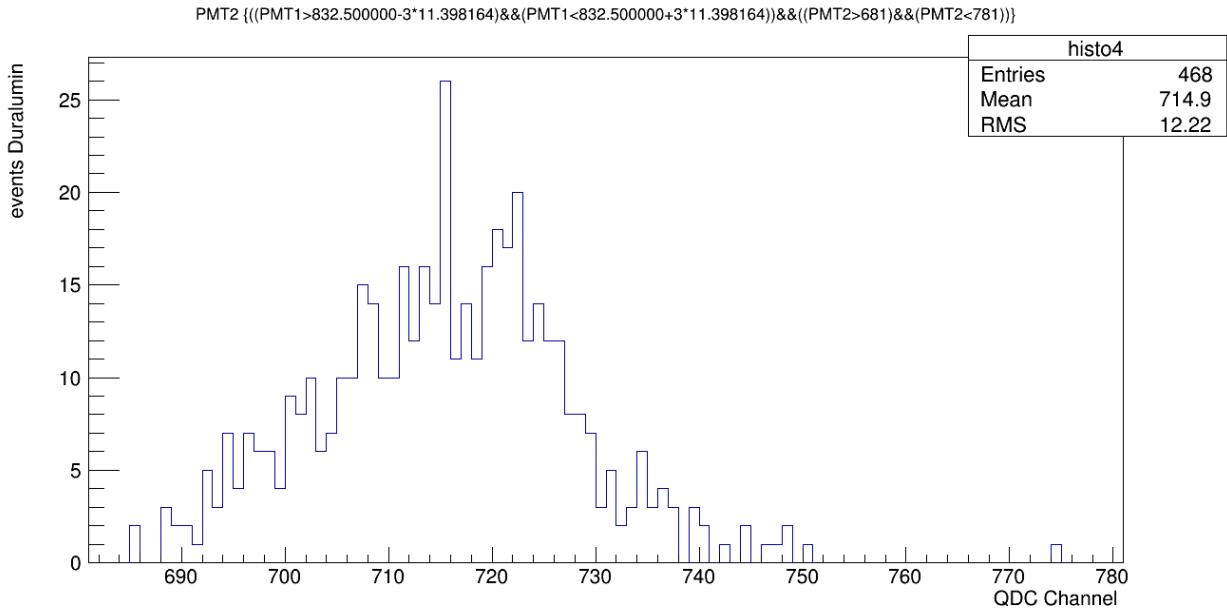


Figure 38: PMT2 conditioned photopeak histogram. Duralumin between PMT1 and PMT2.

We are able to see that photopeak statistics fell a lot on figure (38). This means that there were more scattered particles than with plastic or air. Let's check what happens with Lead.

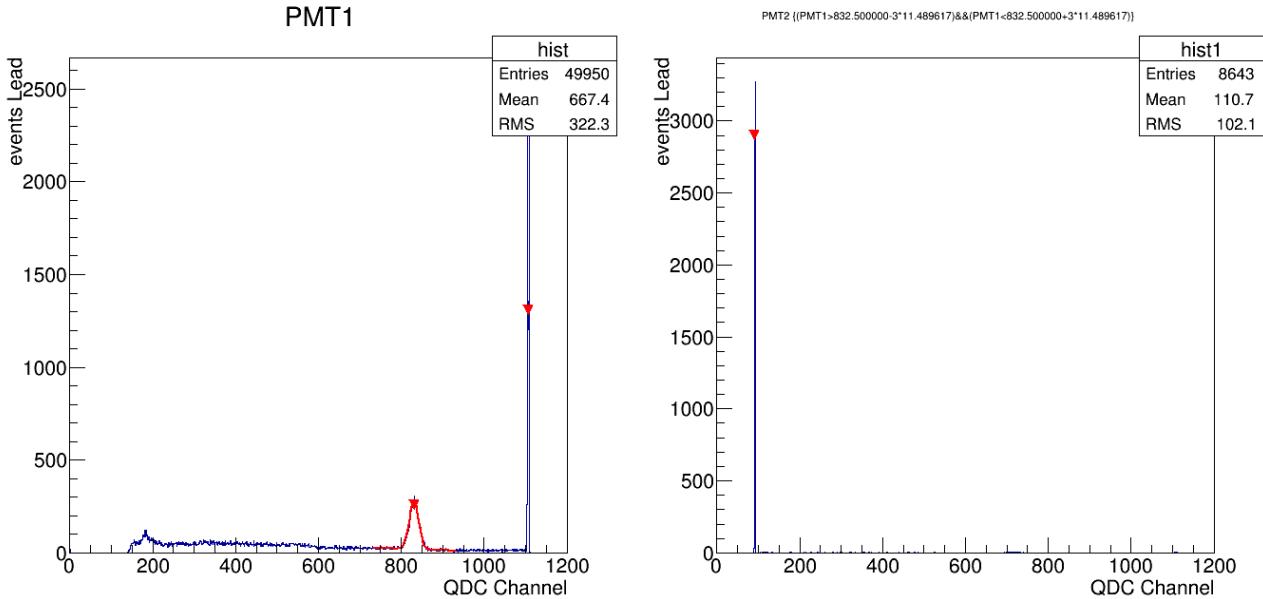


Figure 39: PMT1 photopeak fitting and PMT2 conditional histogram. Lead between Tagger and Gamma Detector.

Now the photopeak of PMT2 for Lead:

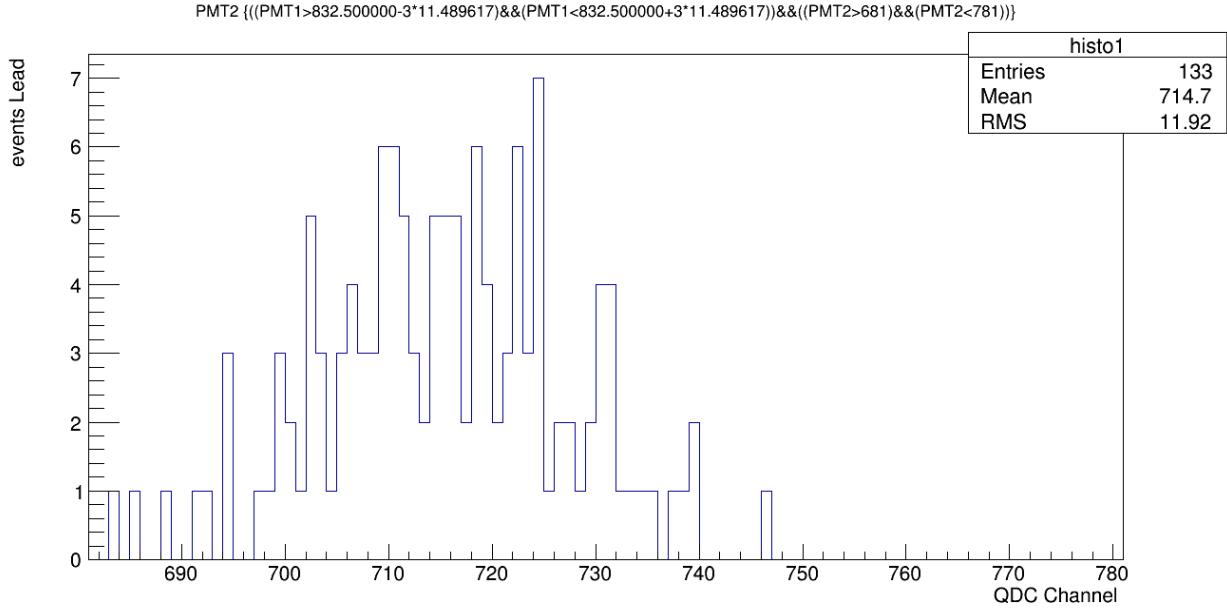


Figure 40: PMT2 conditioned photopeak histogram. Lead between PMT1 and PMT2.

As we can see on figure (40), statistics are very low. Since errors are given by the Poisson distribution, they are the square root of the bin events. Plotting these errors leads to:

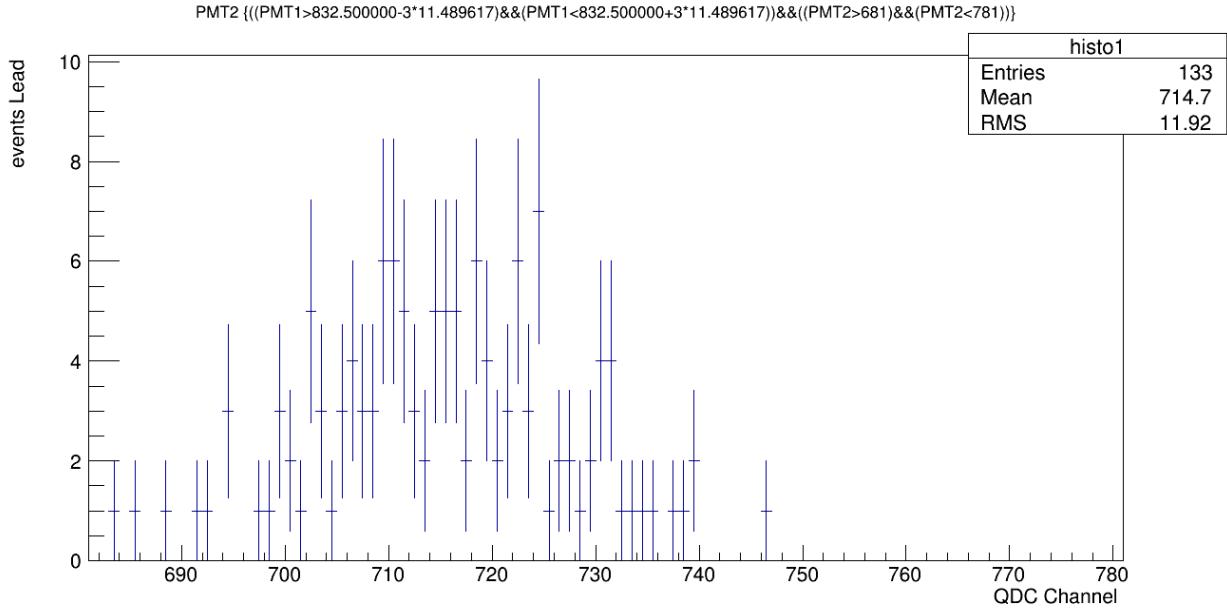


Figure 41: PMT2 conditioned photopeak histogram. Lead between PMT1 and PMT2. Poisson distribution dispersion for each bin is plotted on the histogram.

Those big errors might have consequences on density calculations. We will check this on the next section of the document.

Computing Density

First of all, we need to do some definitions.

To calculate the attenuation coefficient, we need to use equation number (18). Just in case it is not remembered, we will write down it again:

$$\frac{1}{\lambda_1} = \frac{\ln\left(\frac{I_0}{I_1}\right)}{x_1} \quad (18)$$

Therefore, we need to define the incident intensity of gamma rays, I_0 . It will be done in the following way:

$$I_0 = \frac{\text{events on peak of PMT1 with air between PMT1 and PMT2}}{\text{events on peak of PMT2 with air between PMT1 and PMT2}} \quad (86)$$

Hence, this is just the normalized probability of having events on peak of PMT2 when we have an event on peak of PMT1.

What will be I_1 ? Again, we use the same definition from equation (86):

$$I_1 = \frac{\text{events on peak of PMT1 with a material between PMT1 and PMT2}}{\text{events on peak of PMT2 with a material between PMT1 and PMT2}} \quad (87)$$

Again, this number is normalized, since not necessarily the amount of measured events is the same on the measurement with a material and on the measurement without a material.

Finally, the thickness of the material x_1 is given on table (5).

So, the measured attenuation coefficients are the following ones:

		Thickness [cm]	Attenuation [cm ⁻¹]
Normalized incoming photons	0,069	-	-
Normalized out coming photons Polystyrene	0,064	1	0,075
Normalized out coming photons Duralumin	0,0509	1,6	0,19
Normalized out coming photons Lead	0,0144	1	1,567

Table 6: Measured Attenuation Coefficients.

Then, we proceed to compute the densities. First, as on equation (25), without considering Z/A ratio, and second as on equation (22), considering Z/A ratio.

$$\rho_2 = \frac{\frac{Z_{eq1}}{A_{eq1}} \lambda_2 \rho_1}{\frac{Z_{eq2}}{A_{eq2}} \lambda_1} \quad (22) \qquad \rho_2 = \frac{\frac{1}{\lambda_2} \rho_1}{\frac{1}{\lambda_1}} \quad (25)$$

Lead measurement was used as a parameter. The results are resumed in the following table:

without Z/A correction	Density	Density in terms of Duralumin Density	Relative error	Density in terms of Lead Density	Relative error	Density in terms of Polystyrene Density	Relative error
Polystyrene	1,05	1,1106	5,77%	0,5459	48%	-	-
Duralumin	2,79	-	-	1,3714	51%	2,6379	5,5%
with Z/A correction	Density	Density in terms of Duralumin Density	Relative error	Density in terms of Lead Density	Relative error	Density in terms of Polystyrene Density	Relative error
Polystyrene	1,05	0,9928	5,45%	0,4018	62%	-	-
Duralumin	2,79	-	-	1,129	60%	2,9507	5,8%

Table 7: Measured Densities and their relative errors. Dimensions are in [g/cm³].

So, as we can see on table (7) the relative errors are when Polystyrene and Duralumin are related between them. But, when it comes to Lead, the errors grow a lot. I'm not sure if this is due to the apparition of other implied physics or if it is because we have a very low statistic. This topic should be analyzed more deeply.

In order to understand this better, we will zoom in figures (33) ,(35), (37), (39), letting out either pedestal or overflow bins, as is required for the specific plot.

For Air:

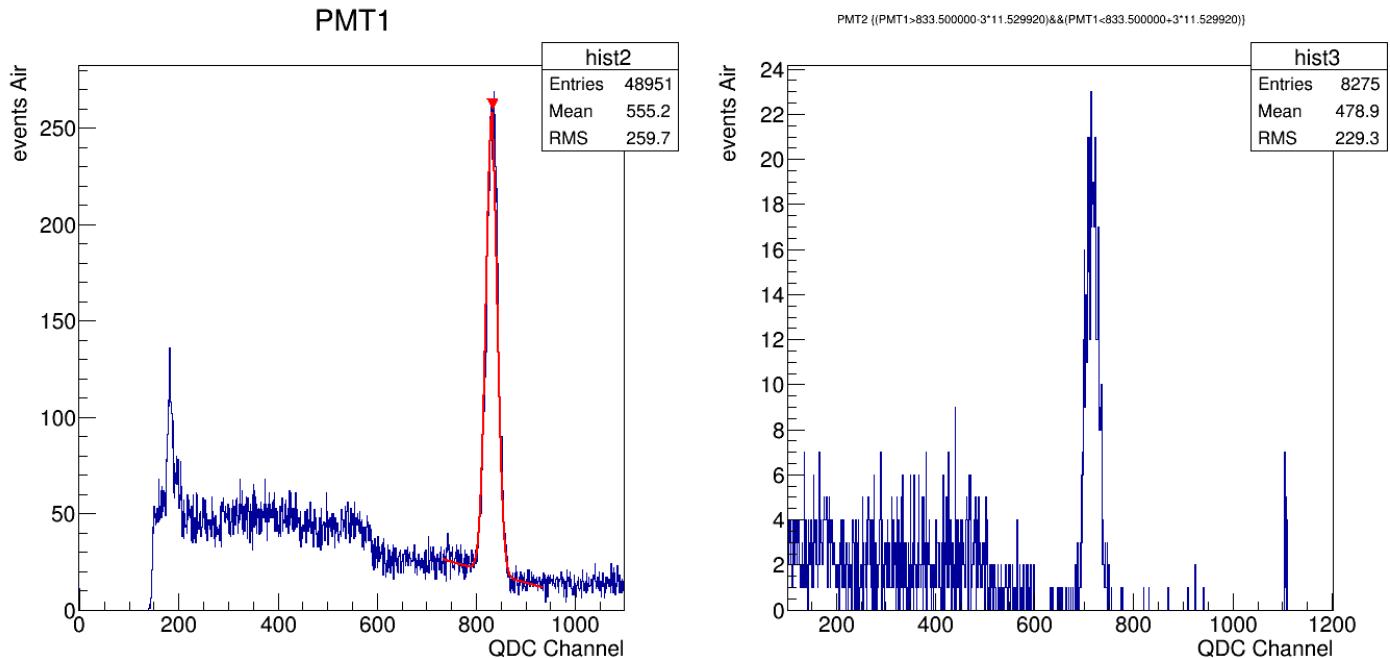


Figure 42: PMT1 photopeak fitting and PMT2 conditional histogram. Air between Tagger and Gamma Detector. Overflow on PMT1 and pedestal on PMT2 were cropped.

For Polystyrene:

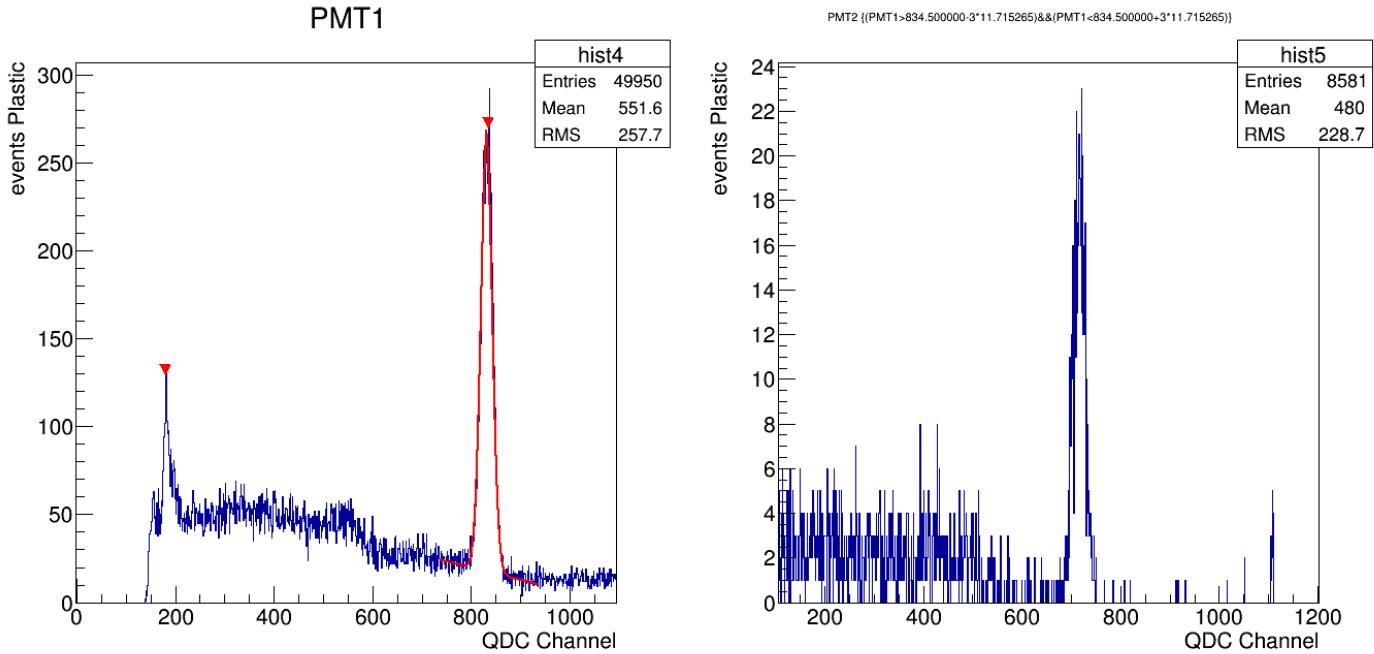


Figure 43: PMT1 photopeak fitting and PMT2 conditional histogram. Polystyrene between Tagger and Gamma Detector. Overflow on PMT1 and pedestal on PMT2 were cropped.

For Duralumin:

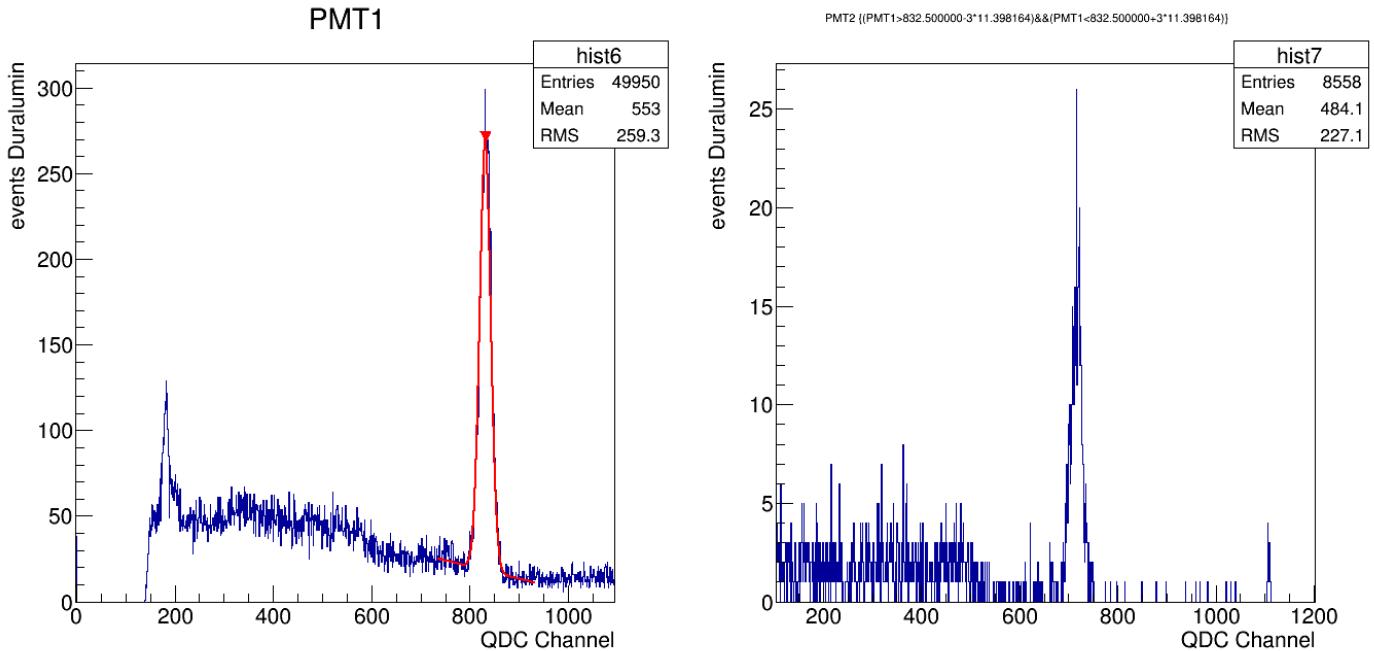


Figure 44: PMT1 photopeak fitting and PMT2 conditional histogram. Duralumin between Tagger and Gamma Detector. Overflow on PMT1 and pedestal on PMT2 were cropped.

Finally, for Lead:

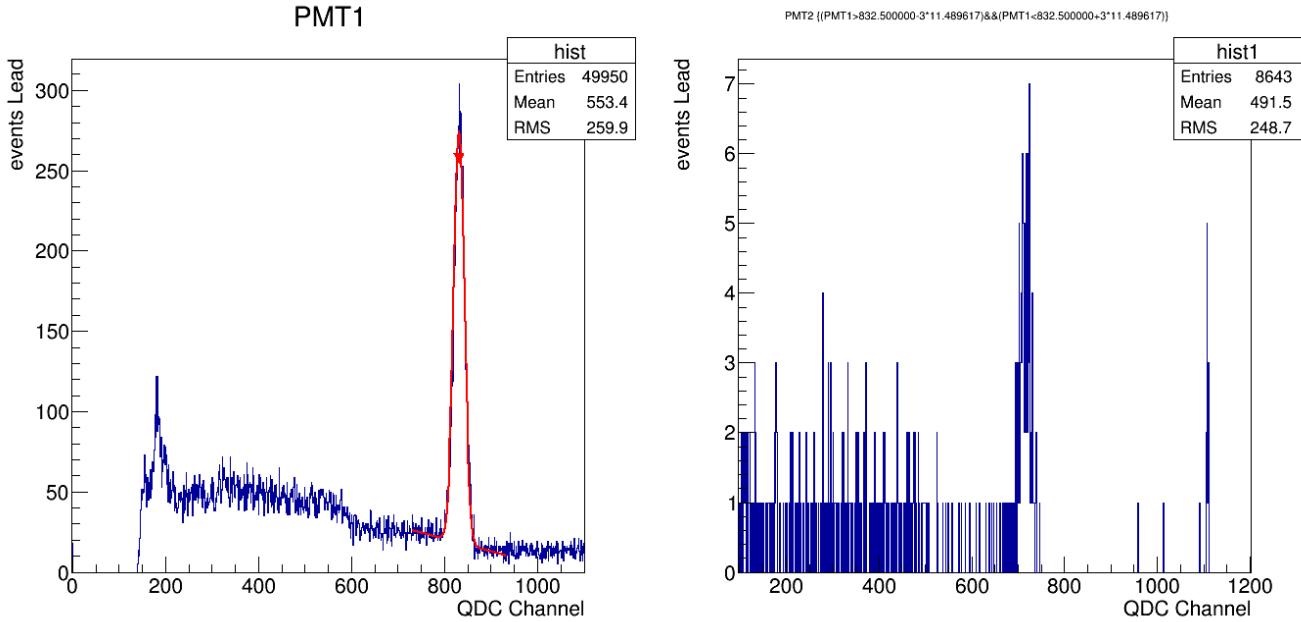


Figure 45: PMT1 photopeak fitting and PMT2 conditional histogram. Lead between Tagger and Gamma Detector. Overflow on PMT1 and pedestal on PMT2 were cropped.

So, just by looking at picture (45) we can notice that statistics for PMT2 are very low and this is the most probable source of errors. The output from code is the following:

```
*****air PMT1 & PMT2*****
Counts PMT1 780-880 8931
Counts PMT2 680-780 616
Normalized Incoming Photons I0=0.0690
*****LEAD PMT1 & PMT2*****
Counts PMT1 780-880 9313
Counts PMT2 680-780 134
10 mm of lead, ln(0.0690/0.0144)/1 cm=1.567
Density of Lead : 11.34 [gr/cm3]
*****Plastic PMT1 & PMT2*****
Counts PMT1 780-880 9240
Counts PMT2 680-780 591
10 mm of Plastic, ln(0.0690/0.0640)/1 cm=0.075
Density of Plastic, in terms of Duralum = 1.1106
Density of Plastic, in terms of Duralum, with ratio correction= 0.9928
Density of Plastic, in terms of Lead = 0.5459
Density of Plastic, in terms of Lead, with ratio correction = 0.4018
Nominal Density of Plastic = 1.05
*****Aluminium PMT1 & PMT2*****
Counts PMT1 780-880 9189
Counts PMT2 680-780 468
16 mm of Duraluminium, ln(0.0690/0.0509)/1.6 cm=0.190
Density of Duralum, in terms of Plastic = 2.6379
Density of Duralum, in terms of Plastic, with ratio correction= 2.9507
Density of Duralum, in terms of Lead = 1.3714
Density of Duralum, in terms of Lead, with ratio correction= 1.1290
Nominal Density of Duralum = 2.79
*****
root [2] ■
```

Figure 46: The ROOT Data Analysis Algorithm Output

7. Application Example: Quantification of Gold in Gold Ore

Typically, Gold appears in ore as an intrusion in rock, without forming a chemical bond, or sometimes forming some alloy with Silver or Mercury.



Figure 47: High-grade gold ore from a quartz vein near Alma, Colorado, USA. The appearance is typical of a very good gold-quartz ore. From Wikipedia.

Many times gold is associated to quartz, which has a much lower density.

	Gold		Quartz	
Density [g/cm ³]	19,3	Density [g/cm ³]	2,65	-
Composition	Au	Composition	Si	O
Weight Fraction w _i	1	Molecular Amount m	1	2
Z	79	Z	14	8
A	196,97	A	28,085	15,999
Z/A	0,4011	m*Z	14	16
w _i *Z/A	0,4011	m*A	28,085	31,998
Z/A equivalent	0,4011	Z/A equivalent	0,499	-

Table8: Gold and Quartz Characteristics

Now, let's think that we want to calculate the density of a Gold Ore rock, made of quartz.

First, let's check again what we know about densities.

Calculation of Density for a Sample made of 2 Materials

Let's remember a few simple facts:

$$M = M_a + M_b \wedge M = \rho V \Rightarrow \rho V = \rho_a V_a + \rho_b V_b \quad (88)$$

Now we define the Volume Fraction $f_{a,b}$:

$$f_{a,b} = \frac{V_{a,b}}{V_a + V_b} = \frac{V_{a,b}}{V} \Rightarrow f_a + f_b = 1 \Rightarrow f_b = 1 - f_a \quad (89)$$

Hence, we can rewrite (88) in the following way:

$$\rho V = \rho_a V f_a + \rho_b V (1 - f_a) \Rightarrow \rho = \rho_a f_a + \rho_b (1 - f_a) \quad (90)$$

We know that we need to work with the Weight Fraction $w_{a,b}$ instead of the Volume Fraction, so let's write more conveniently this equation:

$$f_a = \frac{V_a}{V_a + V_b} = \frac{\frac{M_a}{\rho_a}}{\frac{M_a}{\rho_a} + \frac{M_b}{\rho_b}} = \frac{\frac{w_a M}{\rho_a}}{\frac{w_a M}{\rho_a} + \frac{w_b M}{\rho_b}} = \frac{\frac{w_a M}{\rho_a}}{\frac{w_a M}{\rho_a} + \frac{(1-w_a)M}{\rho_b}} = \frac{\frac{w_a M}{\rho_a}}{w_a + \frac{(1-w_a)}{\rho_b} \rho_a} \quad (91)$$

So now we have defined the Volume Fraction in terms of the Weight Fraction.

Thus now what's left is to replace (91) on (90):

$$\rho = \rho_a \frac{w_a}{w_a + \frac{(1-w_a)}{\rho_b} \rho_a} + \rho_b \left(1 - \frac{w_a}{w_a + \frac{(1-w_a)}{\rho_b} \rho_a} \right) = \frac{w_a \rho_a \rho_b}{w_a \rho_b + (1-w_a) \rho_a} + \rho_b - \frac{w_a \rho_b^2}{w_a \rho_b + (1-w_a) \rho_a} \quad (92)$$

Then finally the Density becomes:

$$\rho = \frac{w_a \rho_b (\rho_a - \rho_b)}{w_a \rho_b + (1-w_a) \rho_a} + \rho_b \quad (93)$$

Now, let's do this with Gold and Quartz.

Calculus of Attenuation Constant of Quartz Gold Ore

From equation (67) we know that:

$$\frac{1}{\lambda_{mix}} = N_a \rho_{mix} \sigma_C \sum_{i=1}^{n_{el}} \frac{w_i Z_i}{A_i} \quad (67)$$

We can rewrite for Gold Ore (Au sub index for Gold, Q sub index for Quartz):

$$\frac{1}{\lambda_{ore}} = N_a \rho_{ore} \sigma_C \left(w_{Au} \frac{Z_{Au}}{A_{Au}} + w_Q \frac{Z_Q}{A_Q} \right) = N_a \rho_{ore} \sigma_C \left\{ w_{Au} \left(\frac{Z_{Au}}{A_{Au}} - \frac{Z_Q}{A_Q} \right) + \frac{Z_Q}{A_Q} \right\} \quad (94)$$

And if we use definition from equation (92):

$$\frac{1}{\lambda_{ore}} = N_a \left(\frac{w_{Au} \rho_Q (\rho_{Au} - \rho_Q)}{w_{Au} \rho_Q + (1-w_{Au}) \rho_{Au}} + \rho_Q \right) \sigma_C \left\{ w_{Au} \left(\frac{Z_{Au}}{A_{Au}} - \frac{Z_Q}{A_Q} \right) + \frac{Z_Q}{A_Q} \right\} \quad (95)$$

So, with this, let's suppose that in order to have a reference material, as we did before, we use a pure Quartz Rock.

$$\frac{1}{\lambda_Q} = N_a \rho_Q \sigma_C \frac{Z_Q}{A_Q} \quad (96)$$

Therefore, using the ratio between (95) and (96):

$$\frac{\frac{1}{\lambda_{ore}}}{\frac{1}{\lambda_Q}} = \frac{\left(\frac{w_{Au}(\rho_{Au} - \rho_Q)}{w_{Au}\rho_Q + (1-w_{Au})\rho_{Au}} + 1 \right) \left\{ w_{Au} \left(\frac{Z_{Au}}{A_{Au}} - \frac{Z_Q}{A_Q} \right) + \frac{Z_Q}{A_Q} \right\}}{\frac{Z_Q}{A_Q}} = \frac{\left(\frac{16.65 w_{Au}}{2.65 w_{Au} + 19.3(1-w_{Au})} + 1 \right) (0.499 - 0.0982 w_{Au})}{0.499} \quad (97)$$

So in this way we can see how differently the Attenuation Constant Behave:

$$2.00401 \left(\frac{16.65 w}{19.3 (1-w) + 2.65 w} + 1 \right) (0.499 - 0.0982 w)$$

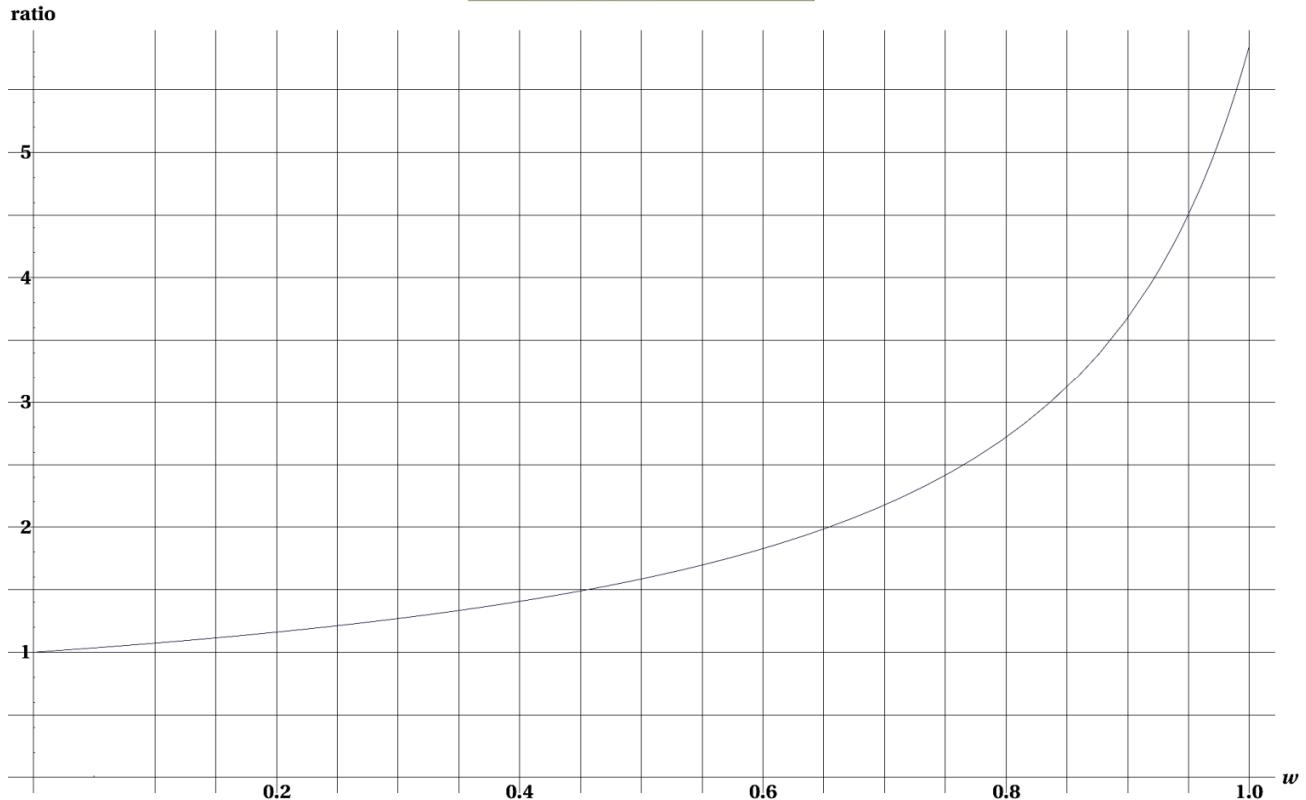


Figure 48: Ratio of Attenuation Constants as Defined on Equation (97). X Axe is increasing weight Fraction of Gold, Y Axe is the Ratio as a function of Gold Weight Fraction.

Now, if we do the following calculation:

$$\rho_{ore^*} = \frac{\frac{1}{\lambda_{ore}}}{\frac{1}{\lambda_Q}} \rho_Q \quad (98)$$

We know that ore density in reality behaves as:

$$\rho_{ore} = \frac{w_{Au}\rho_Q(\rho_{Au}-\rho_Q)}{w_{Au}\rho_Q+(1-w_{Au})\rho_{Au}} + \rho_Q = 2.65 + \frac{44.123*w_{Au}}{2.65*w_{Au}+19.3*(1-w_{Au})} \quad (99)$$

Then let's again define a ratio:

$$\frac{\rho_{ore^*}}{\rho_{ore}} = \frac{\frac{\left(\frac{16.65w_{Au}}{2.65w_{Au}+19.3(1-w_{Au})}+1\right)(0.499-0.0982w_{Au})*2.65}{0.499}}{2.65+\frac{44.123*w_{Au}}{2.65*w_{Au}+19.3*(1-w_{Au})}} \quad (100)$$

And plotting this function we get:

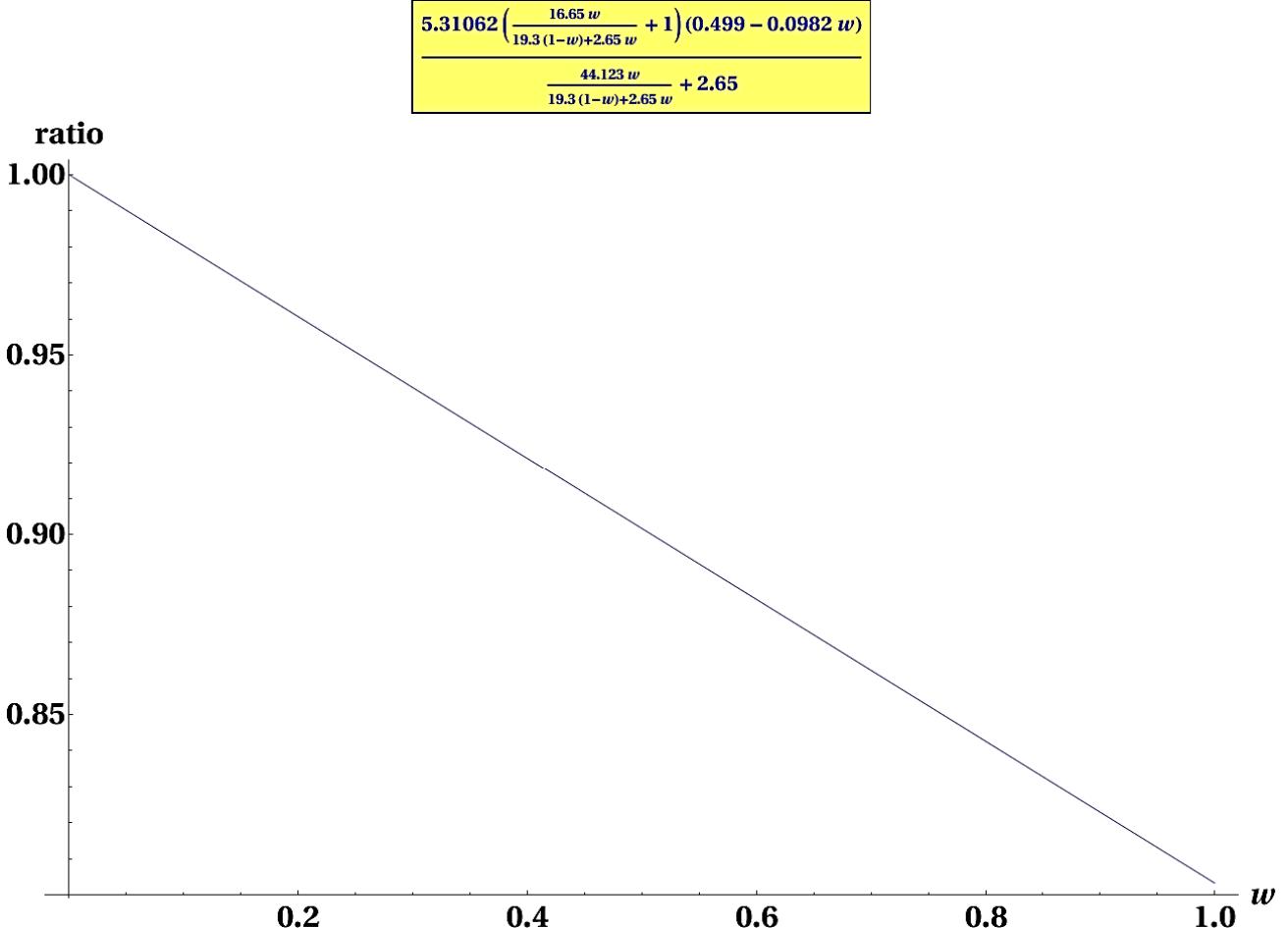


Figure 49: Ratio between measured density of ore and real ore density as a Function of Gold weight fraction on ore. It means that as the amount of Gold grows, the measured density starts to be less than the real density.

8. Suggestion and Conclusions

The project was successful. In order to obtain better results we need a faster acquisition system and a source with bigger activity. For instance, I had to expend a whole day measuring only to obtain 5 points of the plot on figure (31).

Besides that, we need to calculate the statistical errors in order to understand better the behavior of the system.

9. References

- [1] W. R. Leo, Techniques for Nuclear and Particle Physics Experiments: A How-to Approach, Chapter 2.
- [2] Knoll, Glenn F., Radiation Detection and Measurement, 2nd Edition, Chapter 2.
- [3] G. Nelson, D. Reilly "Gamma-Ray Interactions with Matter", in *Passive Nondestructive Analysis of Nuclear Materials*, Los Alamos National Laboratory, NUREG/CR-5550, LA-UR-90-732 ,1991, pp. 27-42.
- [4] T.D. Steiger et al., Development of intense, long-lived positron sources, Nuclear Instruments and Methods in Physics Research A299 (1990), pp. 255-260.
- [5] R. D. Evan, Atomic Nucleus (McGraw-Hill New York, 1955), Chaps 23 – 25.