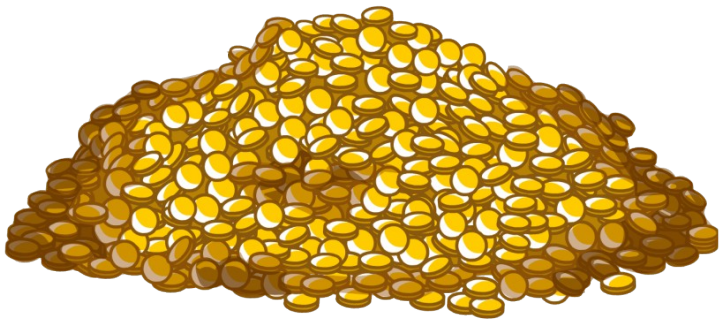
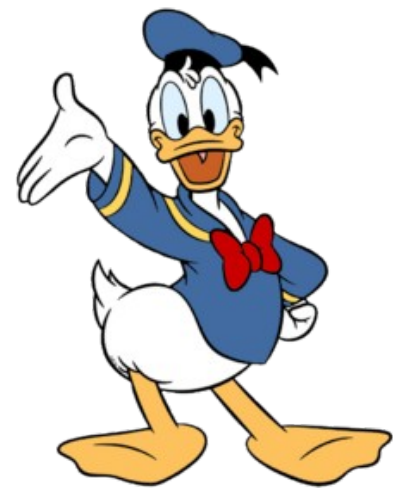
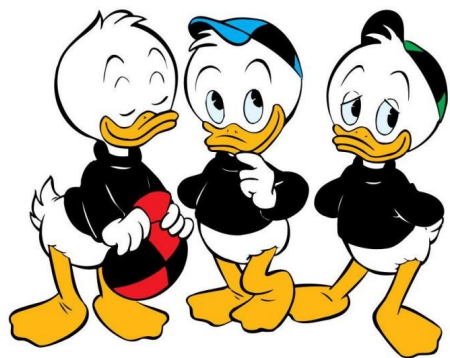
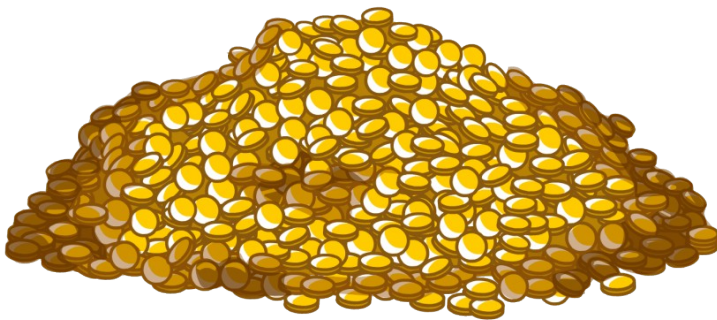
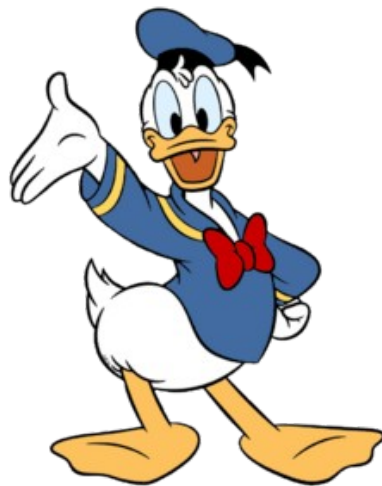
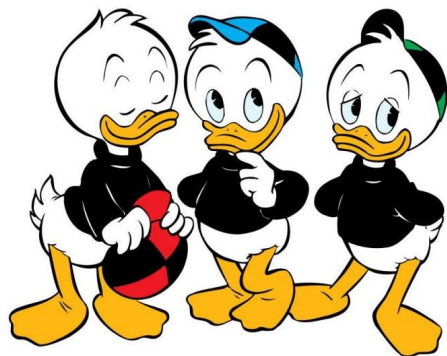


How To Share A Secret

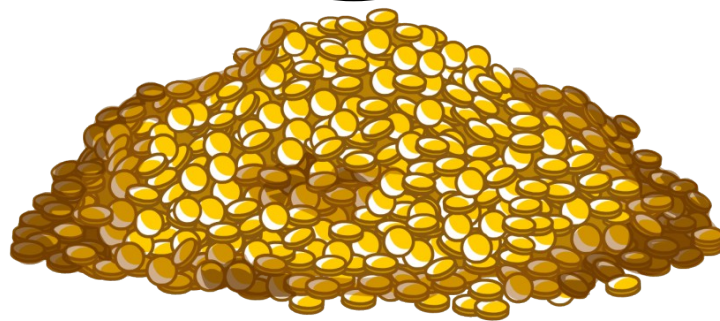
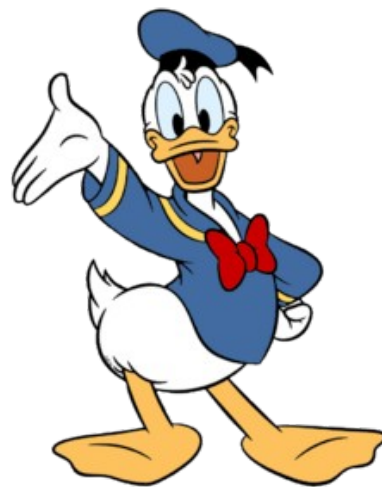
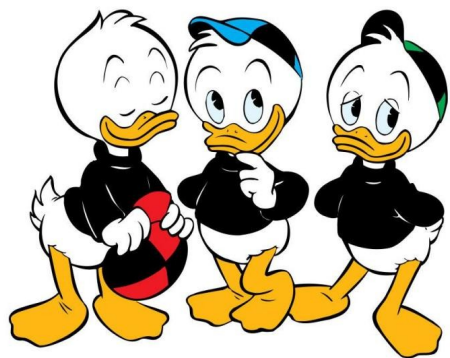
based on an equally named paper by Adi Shamir
(the S in RSA)

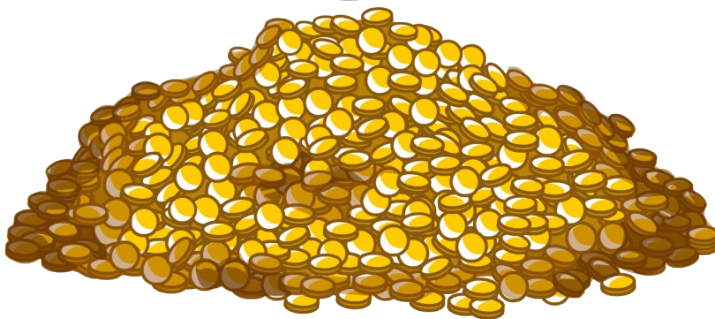
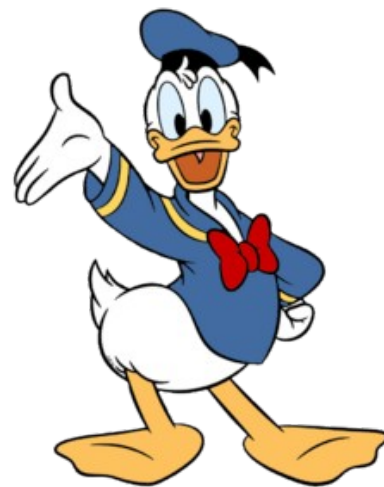
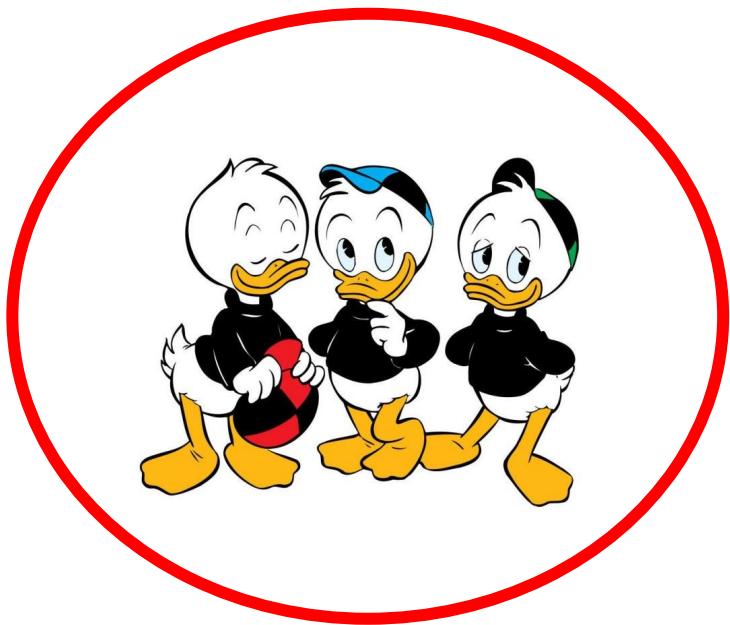


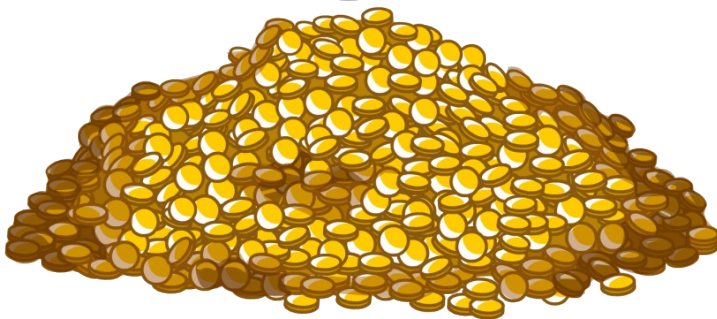
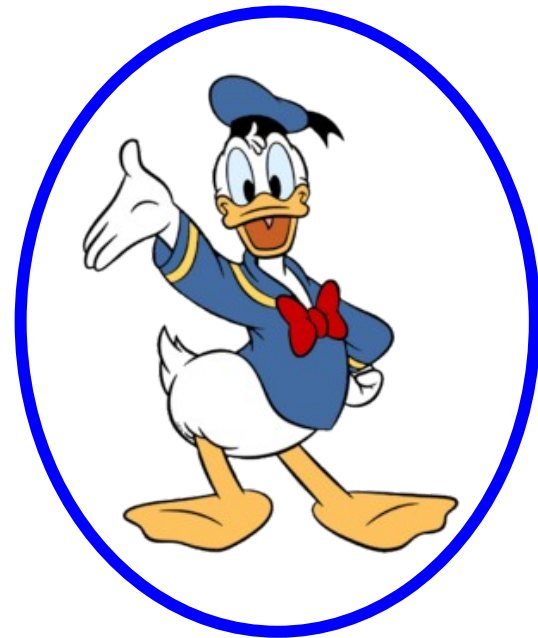
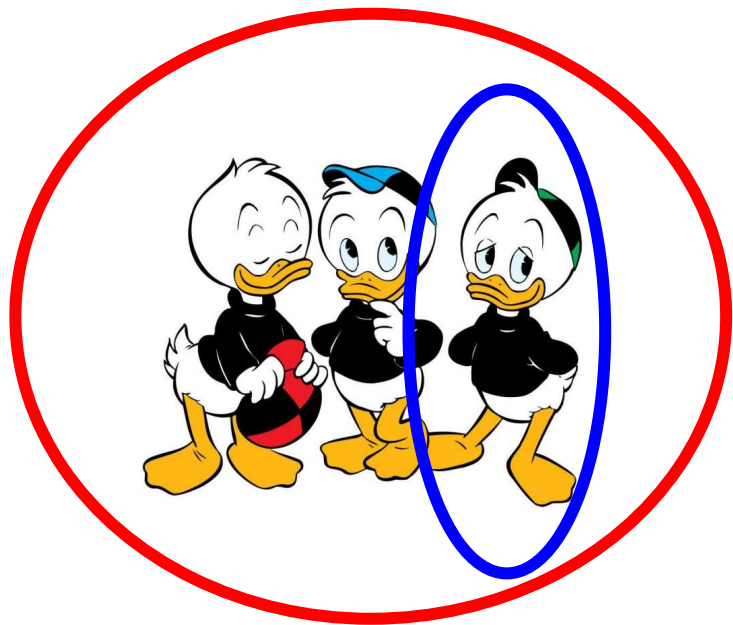


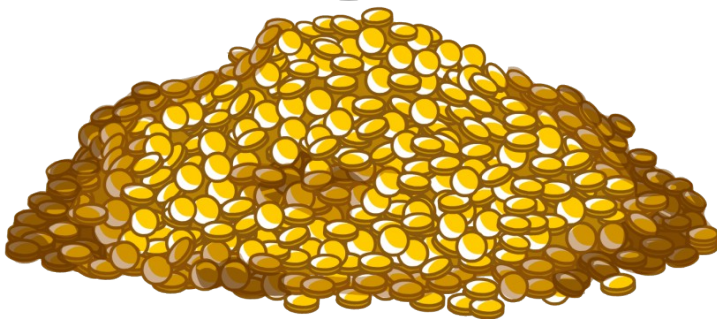
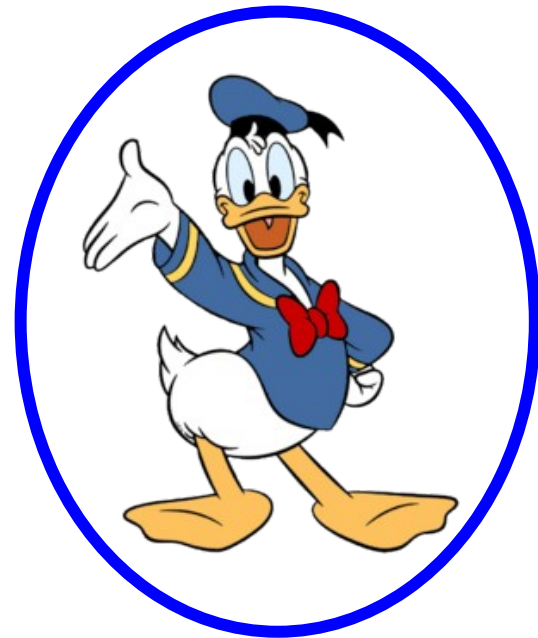
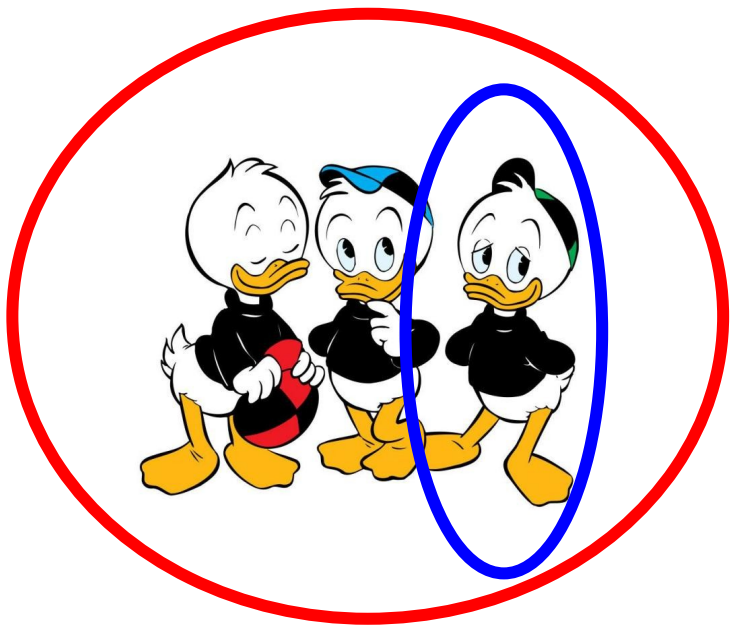


Everyone Gets Access Keys

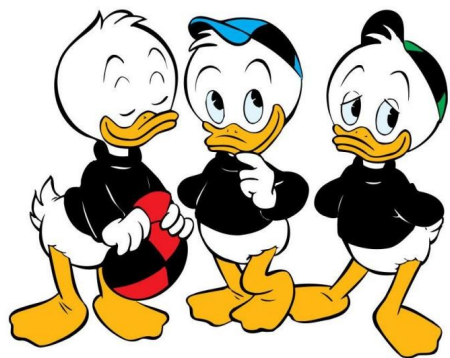


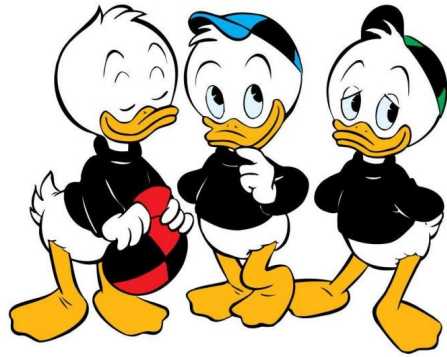






Hierarchical Access





Security Against Theft



How?



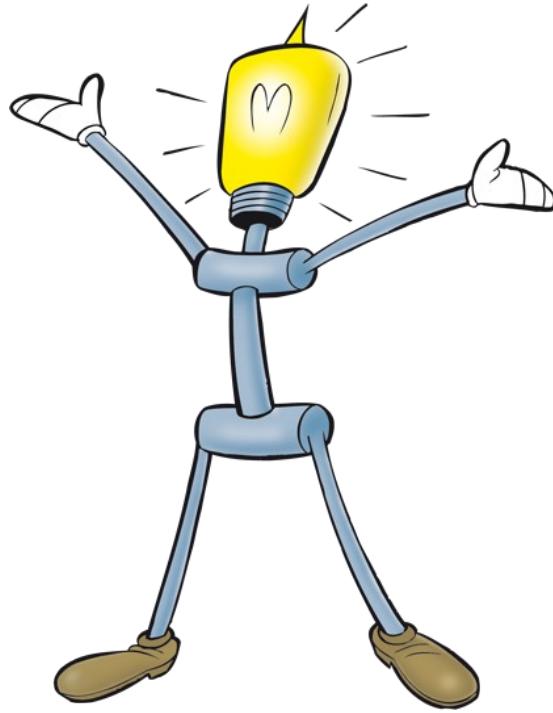
One hidden master key for all?

How?



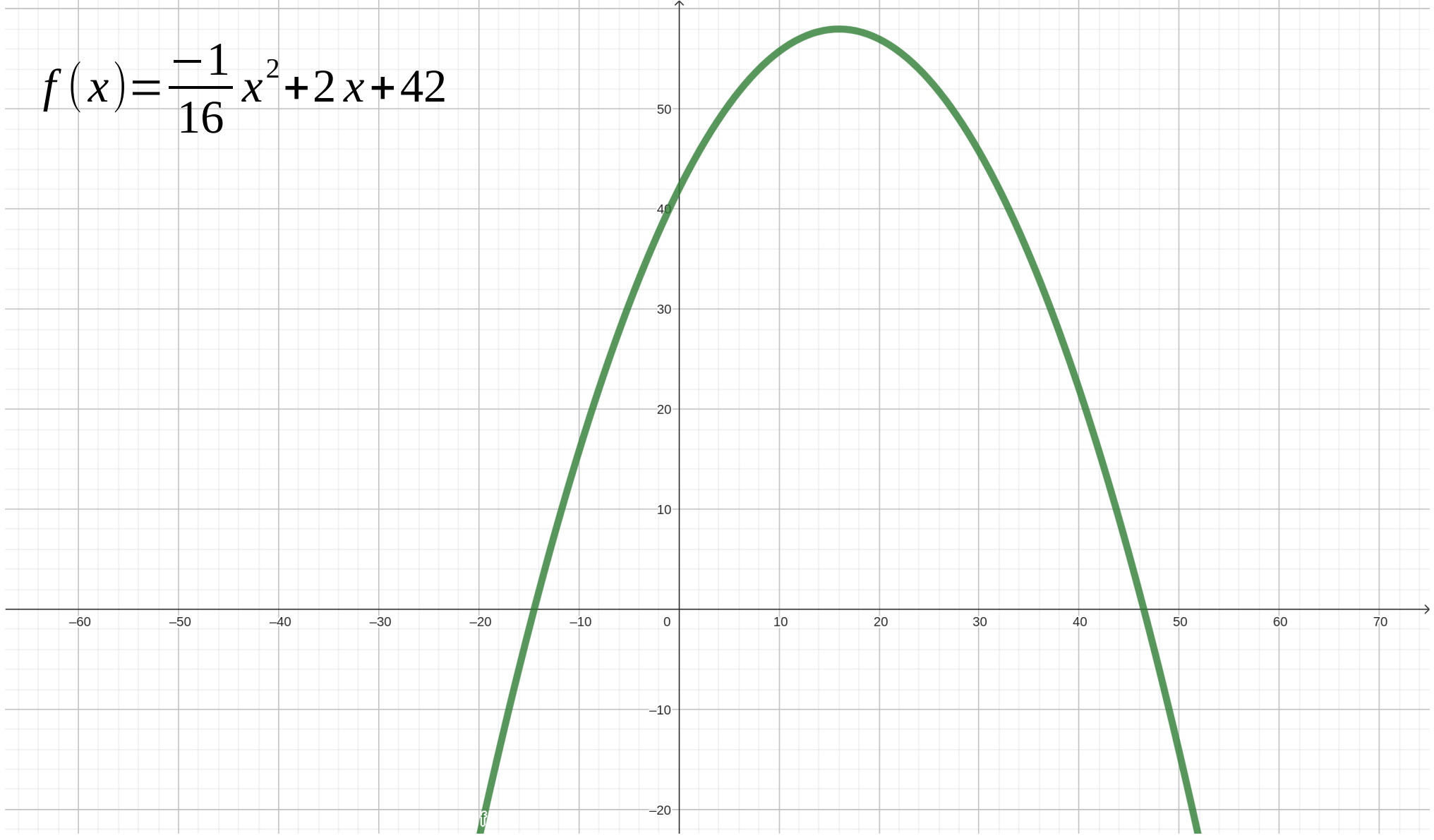
One hidden master key for all?
No hierarchy!
No security against theft or loss!

How?



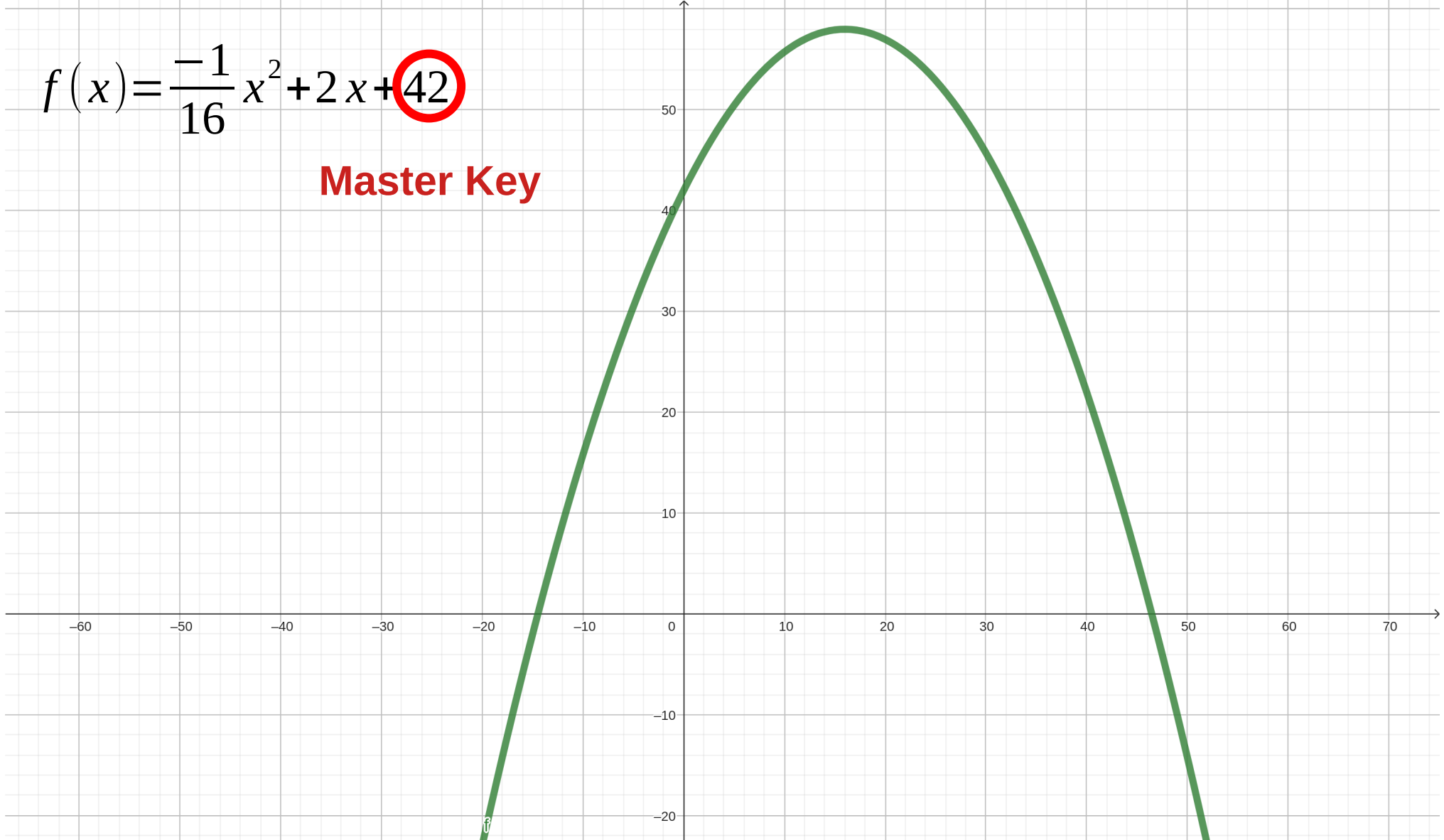
Use a **polynomial** (and a master key)!

$$f(x) = -\frac{1}{16}x^2 + 2x + 42$$



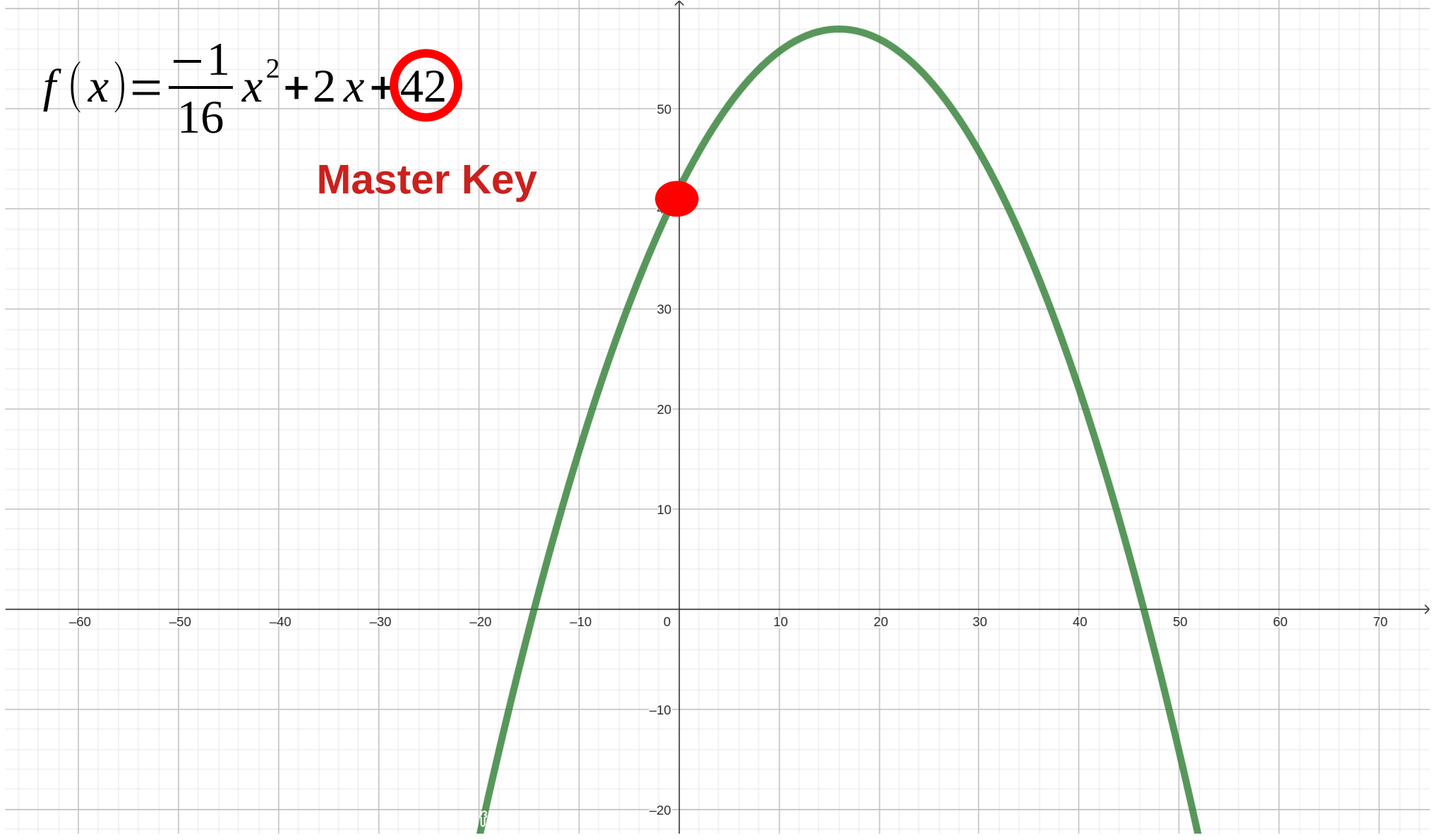
$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

Master Key

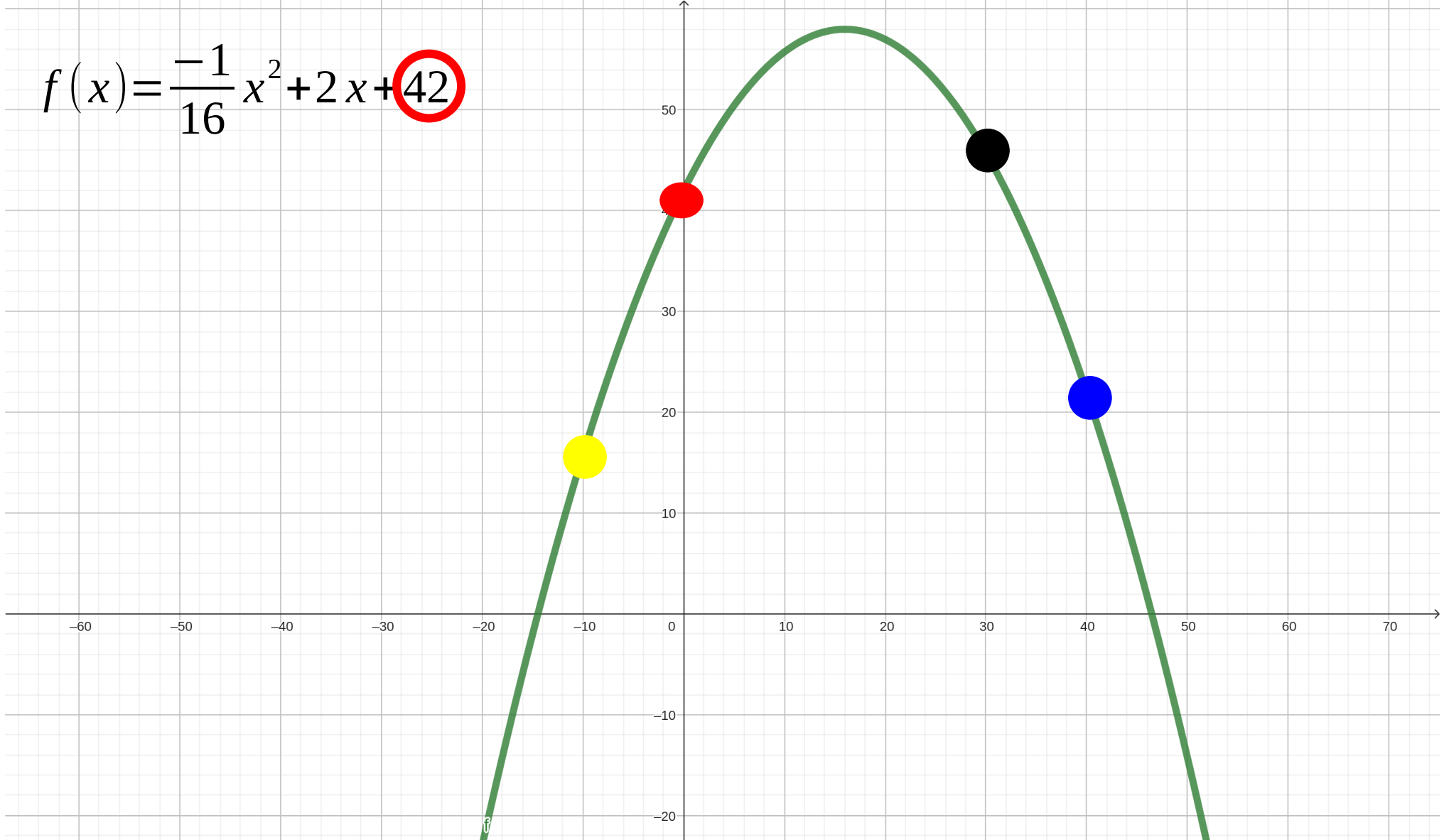


$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

Master Key



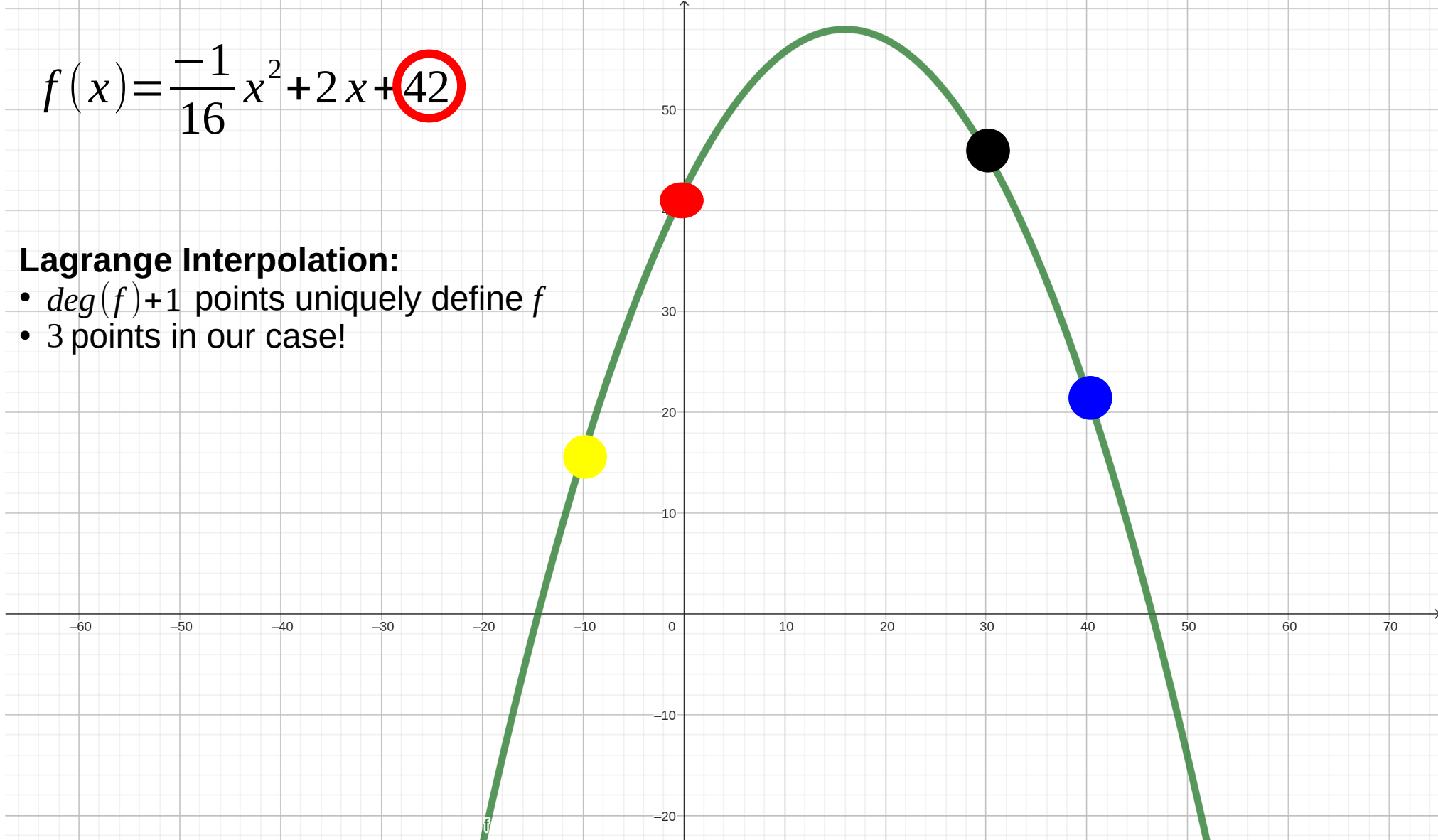
$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$



$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

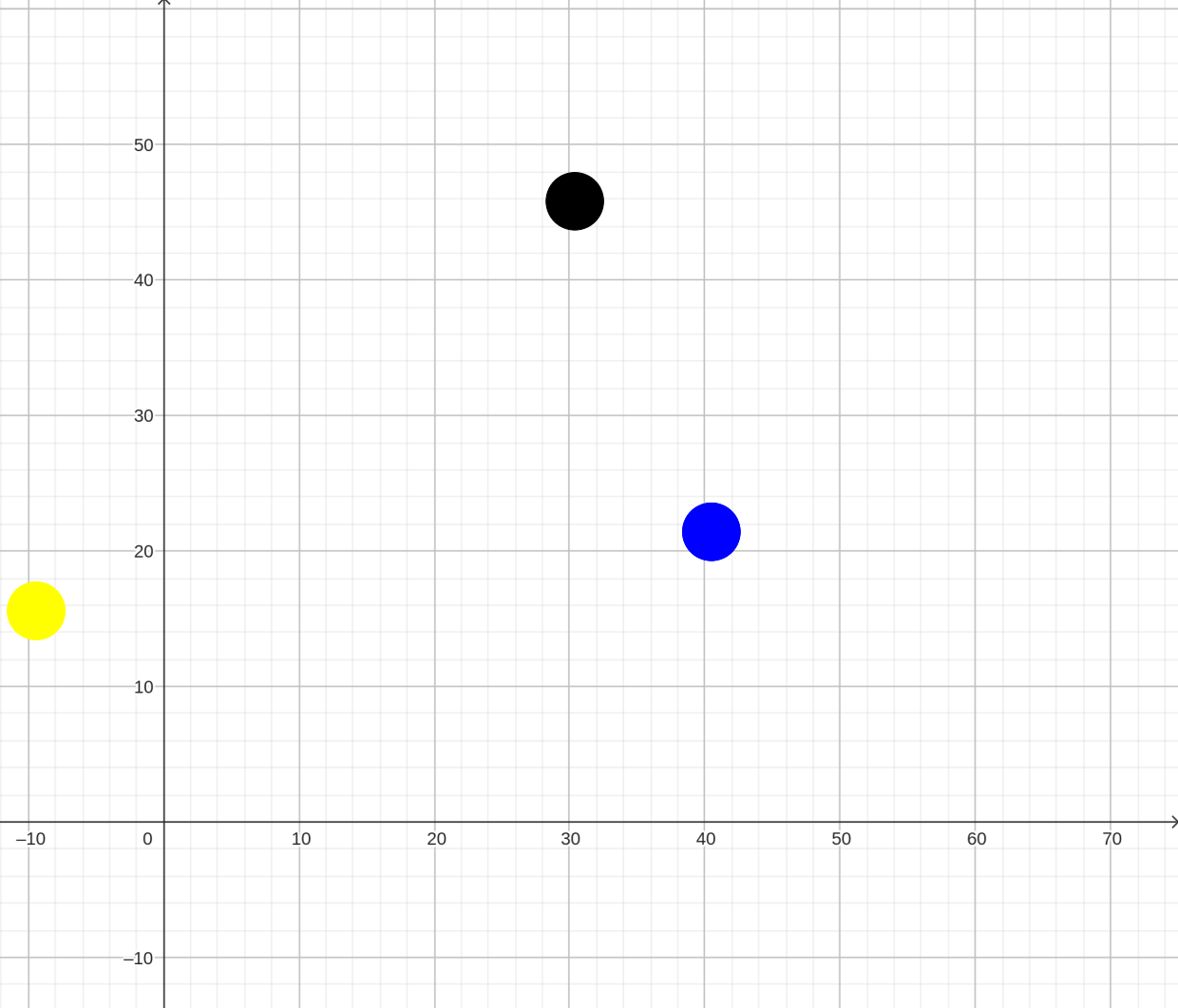
Lagrange Interpolation:

- $\deg(f)+1$ points uniquely define f
- 3 points in our case!



Lagrange Interpolation:

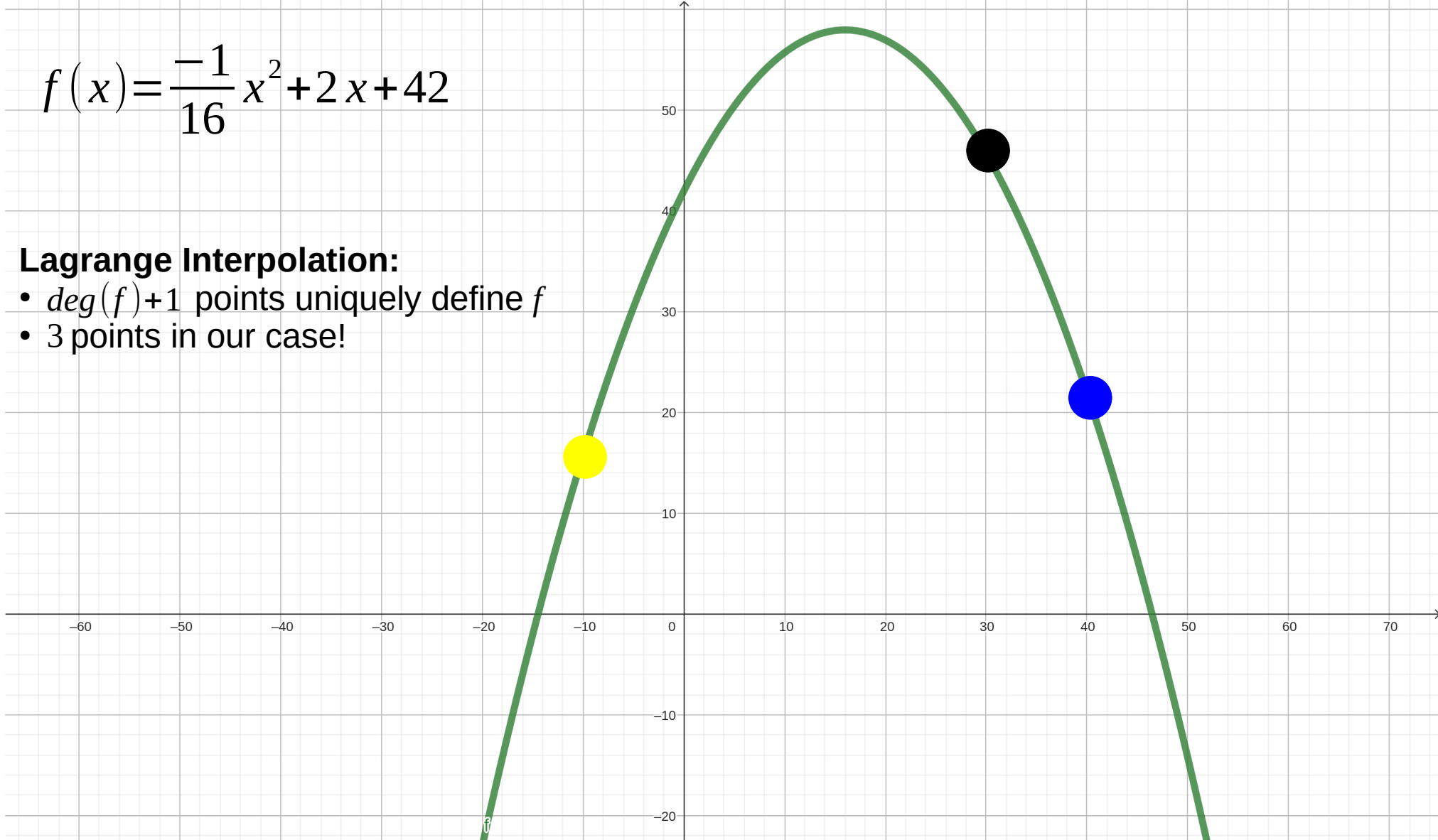
- $\deg(f)+1$ points uniquely define f
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$$f(x) = -\frac{1}{16}x^2 + 2x + 42$$

Lagrange Interpolation:

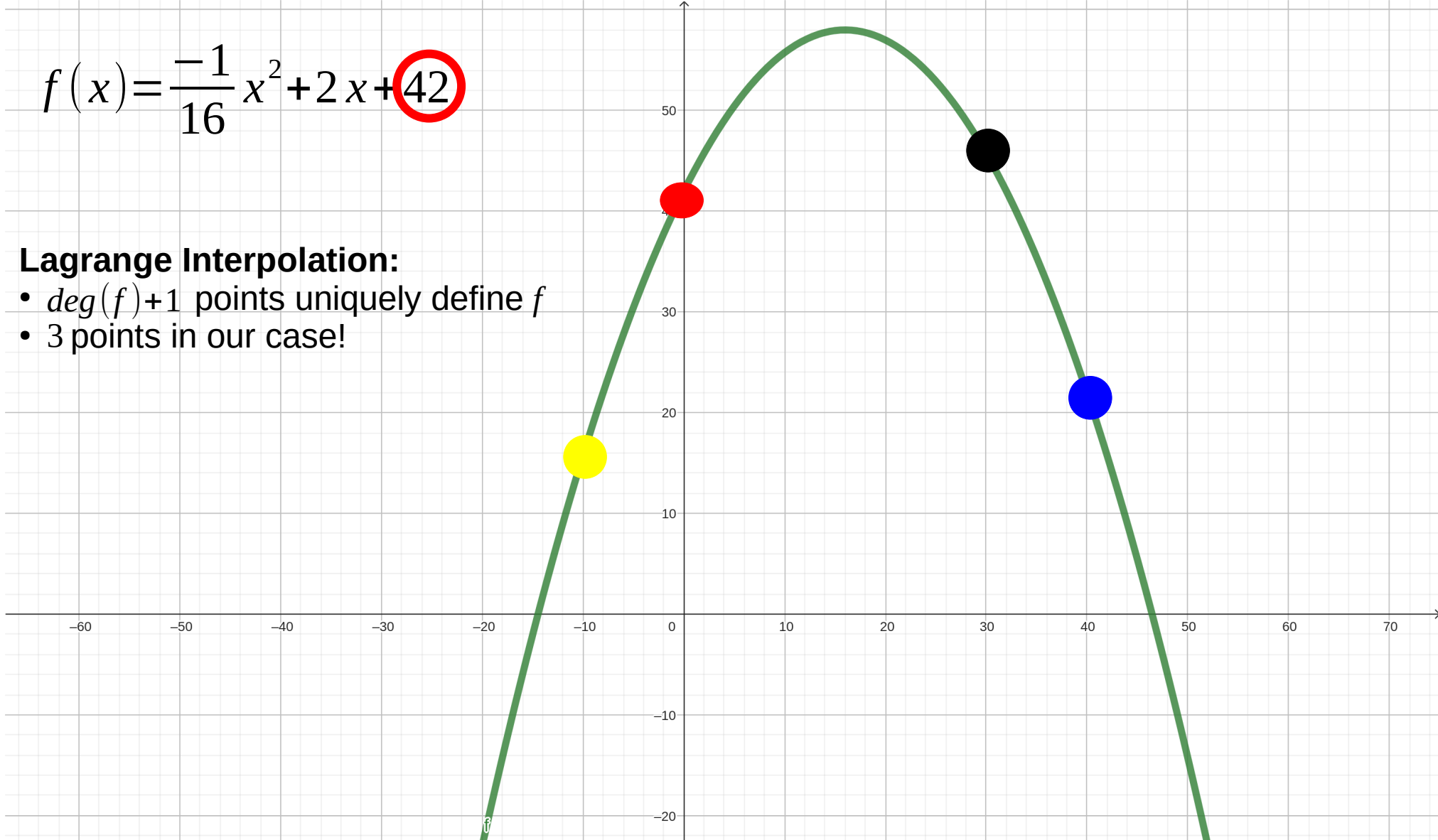
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Lagrange Interpolation:

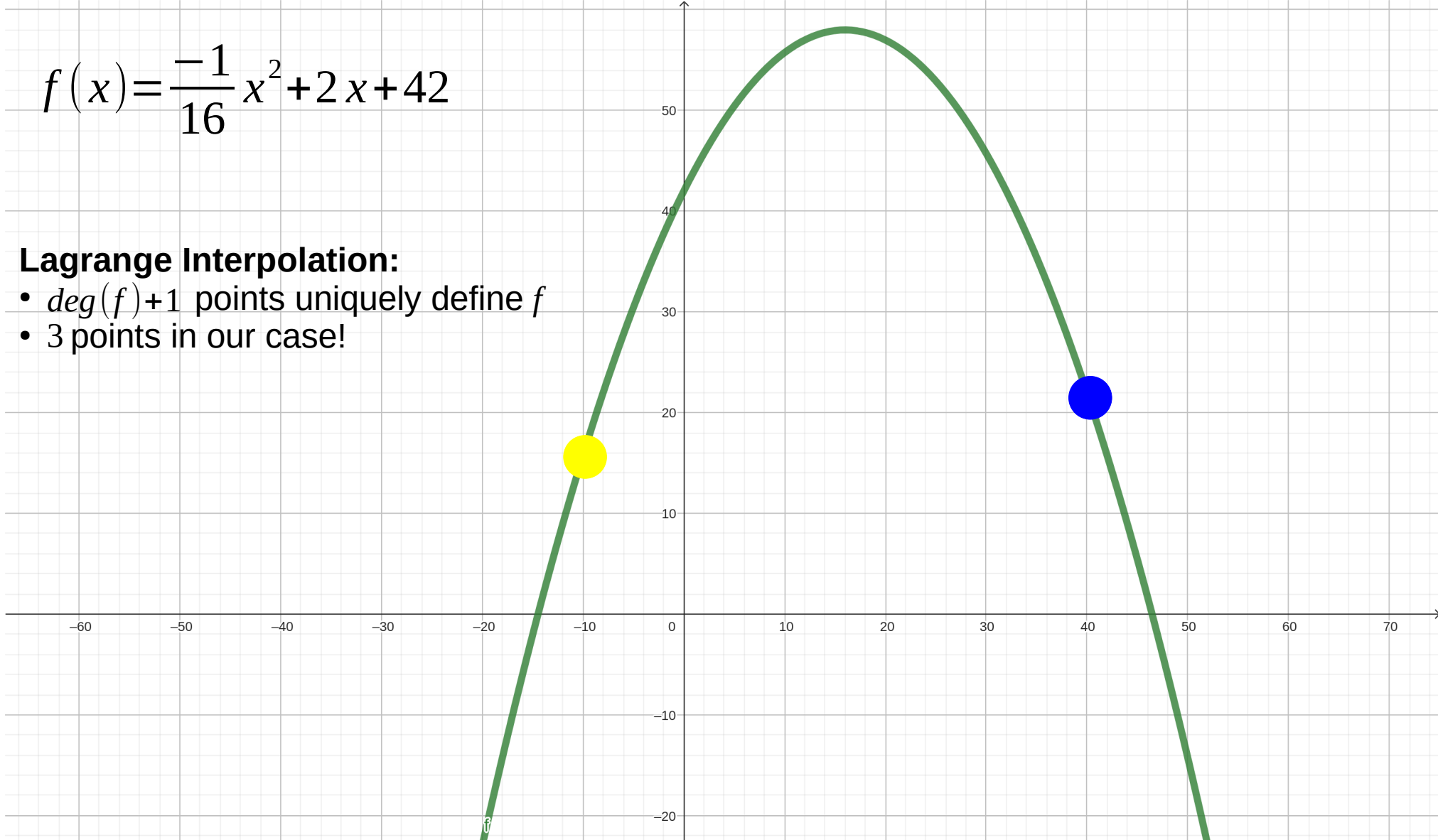
- $\deg(f)+1$ points uniquely define f
- 3 points in our case!



$$f(x) = -\frac{1}{16}x^2 + 2x + 42$$

Lagrange Interpolation:

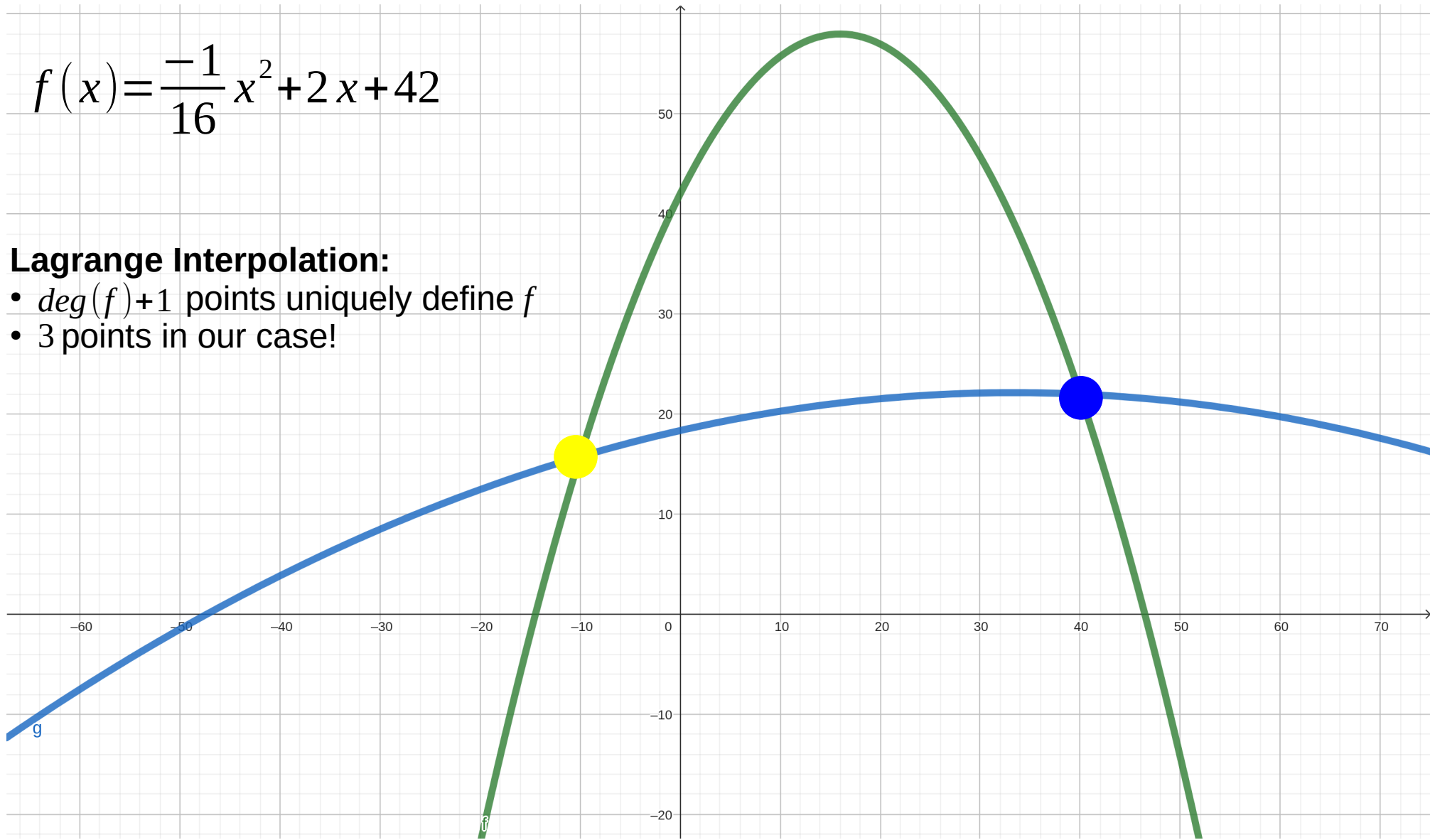
- $\deg(f)+1$ points uniquely define f
- 3 points in our case!



$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

Lagrange Interpolation:

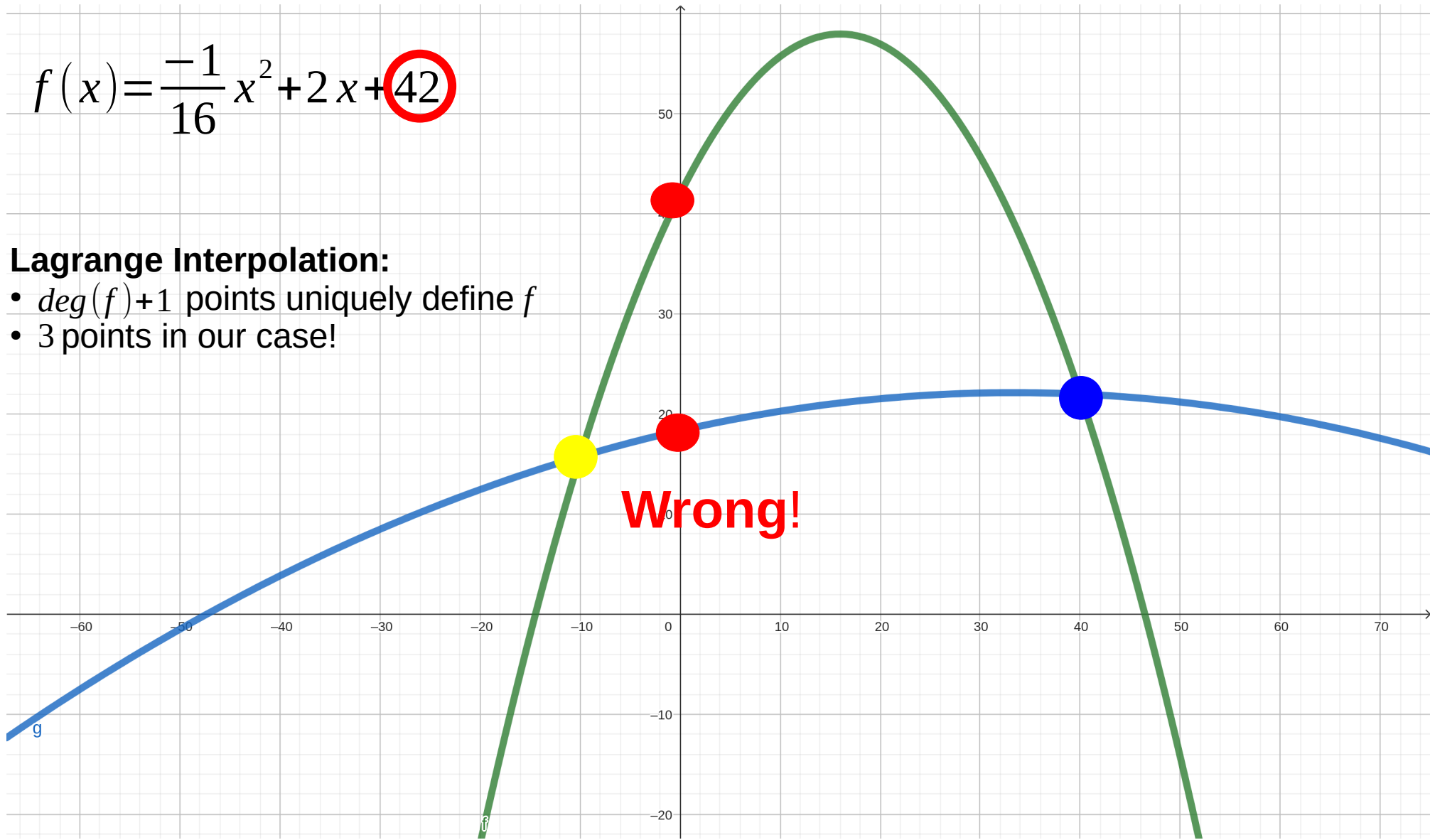
- $\deg(f)+1$ points uniquely define f
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$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

Lagrange Interpolation:

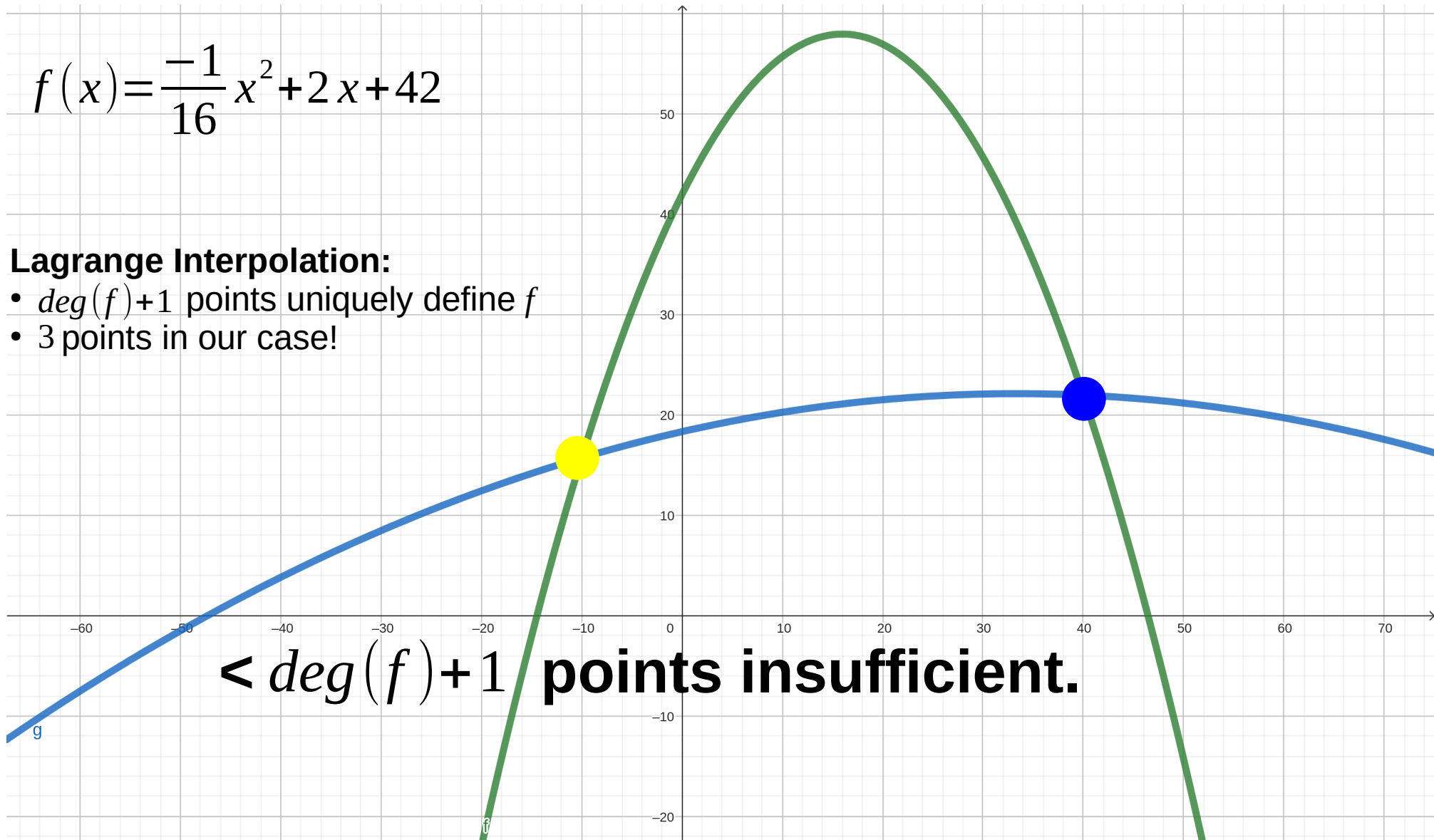
- $\deg(f)+1$ points uniquely define f
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$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

Lagrange Interpolation:

- $\deg(f)+1$ points uniquely define f
- 3 points in our case!

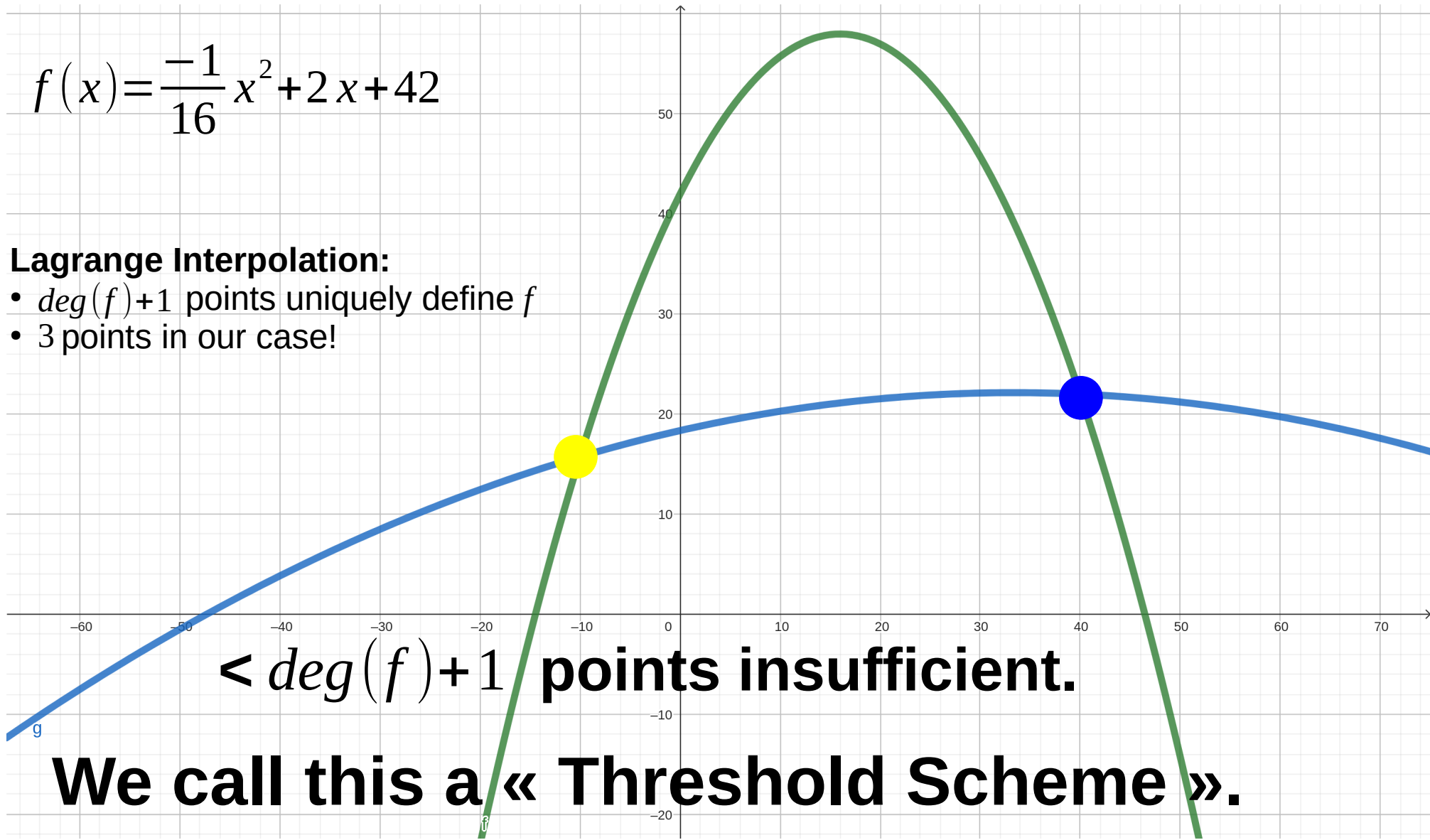


$2 < \deg(f) + 1$ points insufficient.

$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

Lagrange Interpolation:

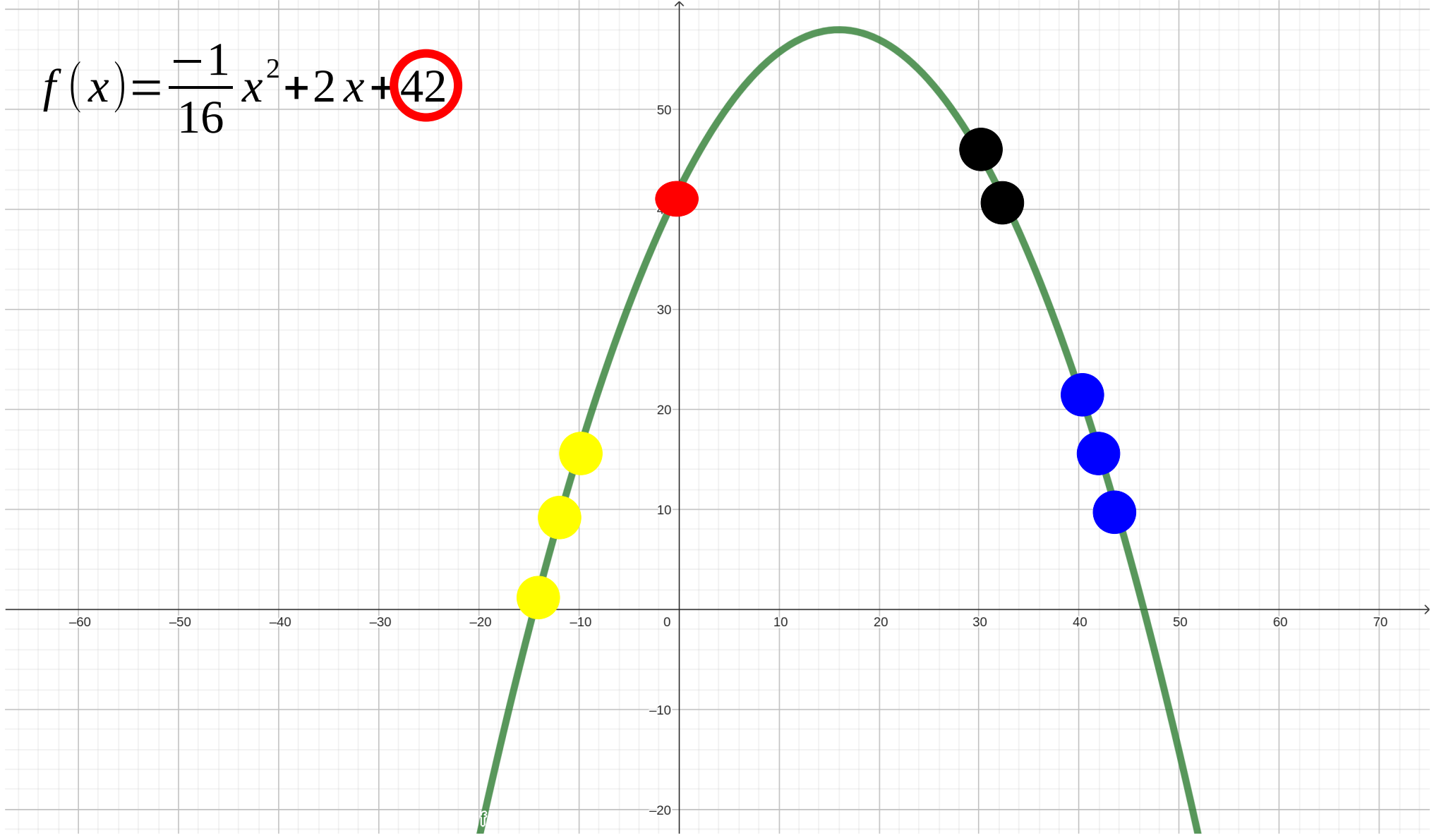
- $\deg(f)+1$ points uniquely define f
- 3 points in our case!



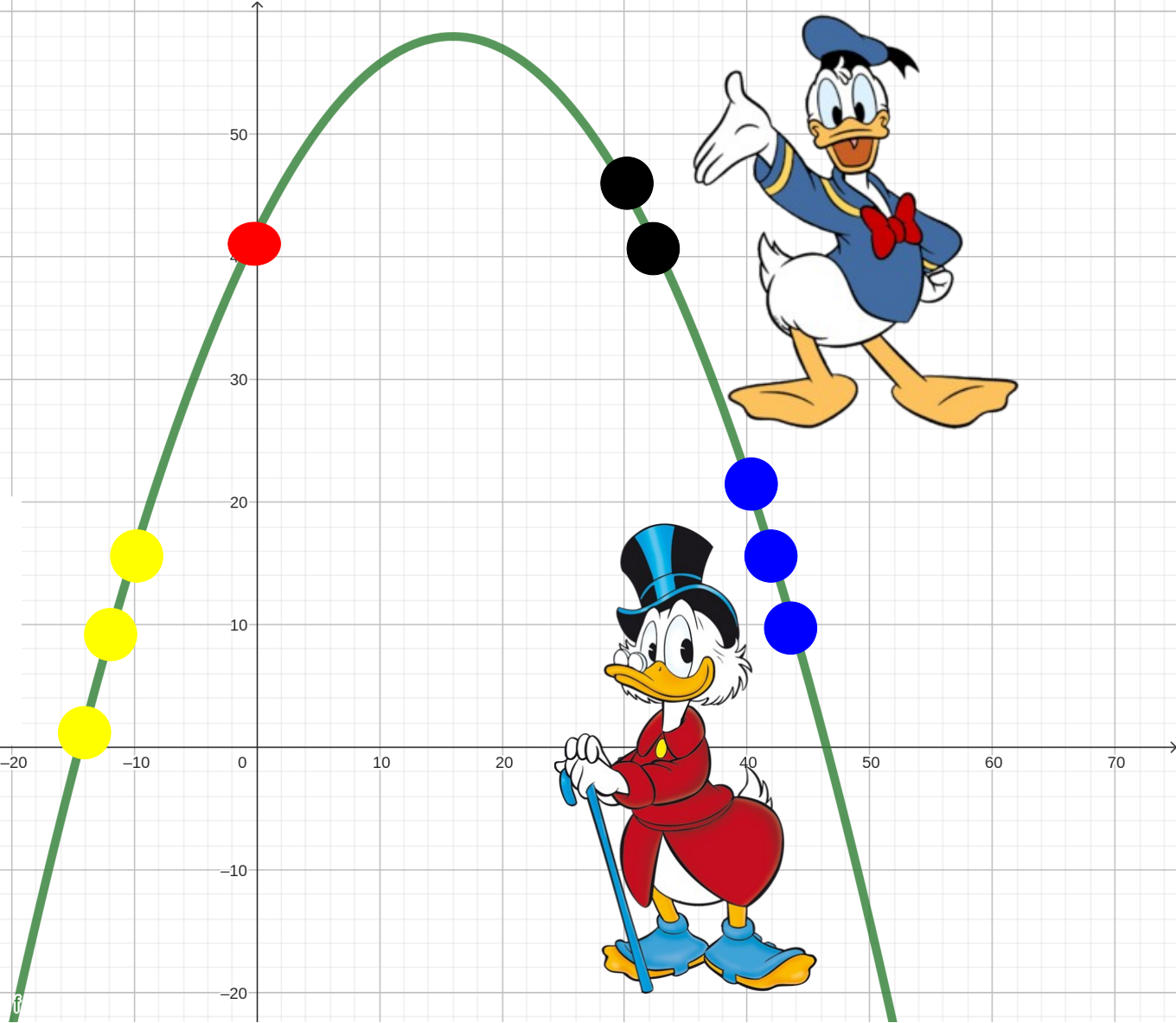
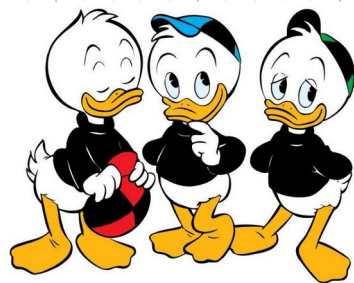
$\leq \deg(f)+1$ points insufficient.

We call this a « Threshold Scheme ».

$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

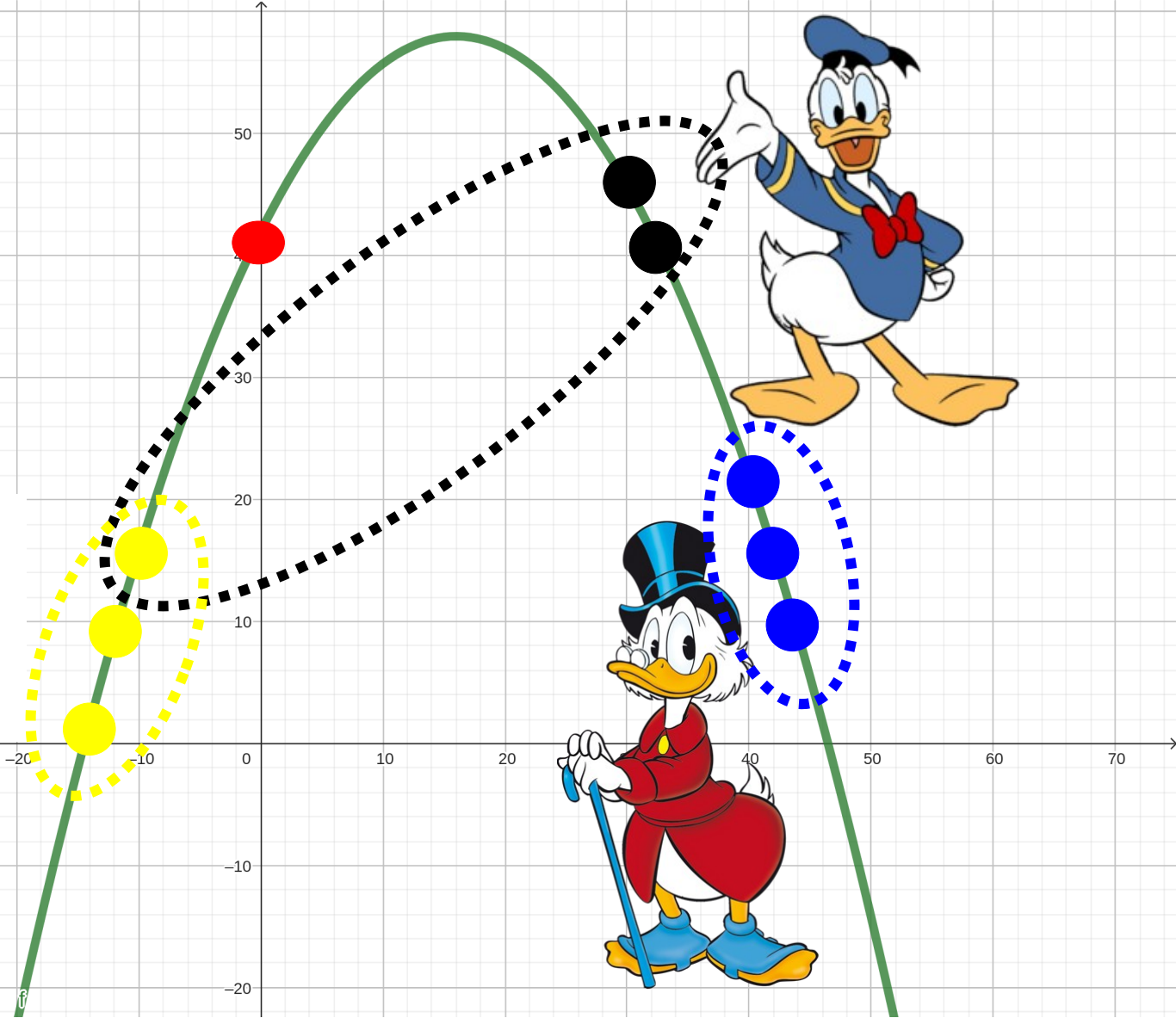
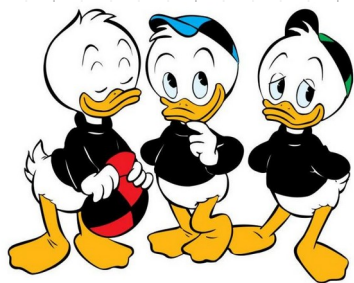


$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$



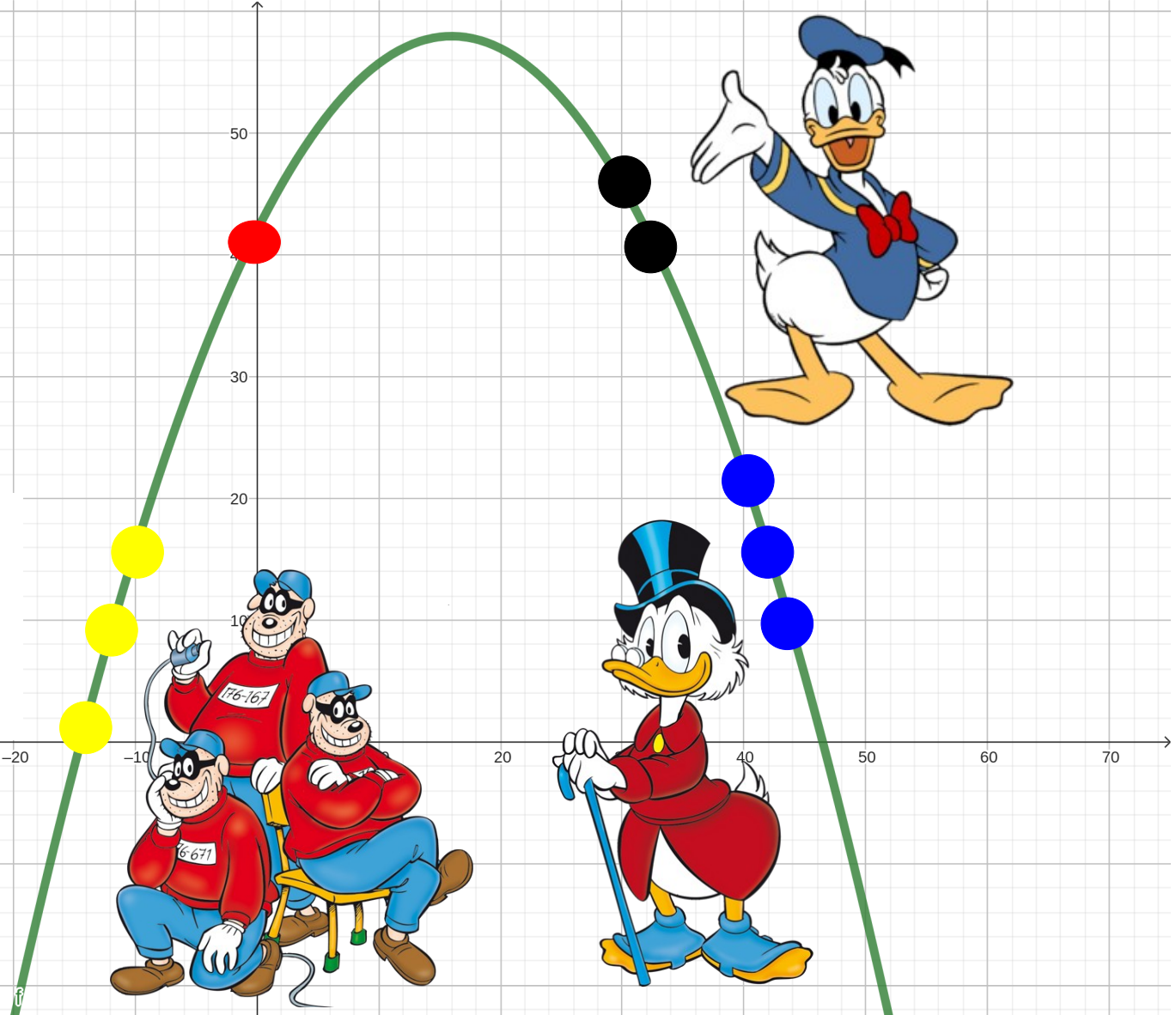
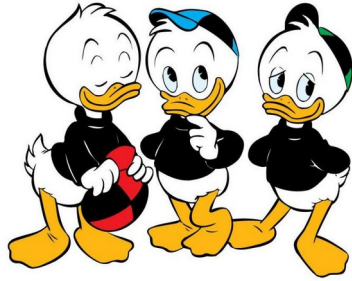
$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

1. Hierarchical Access



$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

1. Hierarchical Access
2. Security Against Theft



How to Share a Secret: **Threshold Schemes!**

How to Share a Secret: **Threshold Schemes!**

Used in Secure Multi-Party Computation

Notable Use Case: Cryptocurrencies



Sources

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$$f(x) = \frac{-1}{16}x^2 + 2x + 42$$

