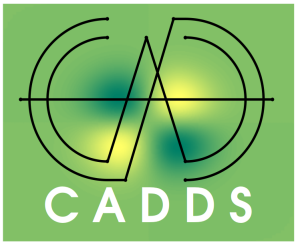
Technical Report 0.1:

EPIDEMIC DYNAMICS INDETIFICACION AND CONTROL VIA UNIVERSAL ALGEBRAIC COTROLLERS



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ÍNDICE

RESUMEN. In this document, an application of universal algebraic controllers (in the sense of [4]) to the computation of predictive models for epidemic propagation in Honduras, is presented. Special attention to COVID-19 propagation, is given.
Some data-driven numerical predictive simulations for the COVID-19 propagation in Honduras, are outlined.

1. INTRODUCTION

The purpose of this document is to present some theoretical and computational techniques for constrained approximation of data-driven predictive models for the propagation of COVID-19 in Honduras during the first quarter of 2020. These models can be interpreted as discrete-time systems that can be *partially* described using the transition block diagram (1.1) as a *black-box device* $\mathfrak{S}$, that needs to be determined in such a way that it can be used

to transform the *present state* x_t into the *next state* x_{t+1}, according to (1.2).

(1.1)

In this study each entry $x_{t,j}$ of the state vector x_t corresponds to the known/predicted number of infected people in Department j, where the index j coincides with the Department's identification number, for instance $x_{t,1}$ is the estimated number of infected people in Atlántida at stage t. We will approach the computation of the *state-transtion* maps corresponding to the device (1.1), applying the algebraic methods developed in [4] and [2] to compute the state-transition matrices that correspond to *matrix solvents* of difference equations of the form

$$\Sigma : \begin{cases} x_{t+1} = T_t x_t,\ t \geq 1 \\ x_1 \in \Sigma \subseteq \mathbb{R}^{18n} \end{cases} \tag{1.2}$$

where $\Sigma \subseteq \mathbb{R}^{18n}$ is the set of *valid* propagation states for the system with $n \in \mathbb{Z}$ fixed, and where the matrices $T_t \in \mathbb{R}^{18n\times 18n}$ are to be determined by the relations (1.2), and in addition need to satisfy the following structural constraints.

$$\begin{cases} T_t = \prod_{j=1}^{18n} \left(I + \hat{e}_j(\tau_{(t,j)} - \hat{e}_j)^\top\right) \\ K_j \circ \tau_{(t,j)}^\top = \tau_{t,j},\ 1 \leq j \leq 18n \end{cases} \tag{1.3}$$

where $\circ$ denotes the Hadamard product, K_j is the jth-row of a connectivity matrix determined by the geographic configuration of Honduras territory under consideration, the matrices $\tau_{(t,j)} \in \mathbb{R}^{18n\times 1}$ are to be determined by (1.2) and 1.3, and where $\hat{e}_{j,n}$ denotes the matrices in $\mathbb{C}^{n\times 1}$ representing the canonical basis of $\mathbb{C}^n$ (the j-column of the $n \times n$ identity matrix I), that are determined by the expression

$$\hat{e}_{j,n} = \begin{bmatrix} \delta_{1,j} & \delta_{2,j} & \cdots & \delta_{n-1,j} & \delta_{n,j} \end{bmatrix}^\top \tag{1.4}$$

for each $1 \leq j \leq n$, where $\delta_{k,j}$ is the Kronecker delta determined by the expression.

$$\delta_{k,j} = \begin{cases} 1,\ k = j \\ 0,\ k \neq j \end{cases} \tag{1.5}$$

2. Universal Algebraic Controllers for the Propagation Model

2.1. Connectivity Matrices. Based on the COVID-19 propagation behavior data available thus far. Let us consider the connectivity matrix $K \in \mathbb{R}^{18\times 18}$ determined by the expression.

$$K = I + adj(G) \tag{2.1}$$

Where $adj(G) = [a_{jk}]$ denotes the adjacency matrix of a graph $G = (V_G, E_G)$ determined by the rules.

$$a_{jk} = \begin{cases} 1,\ \text{if } [v_j, v_k] \in E_G,\ v_j, v_k \in V_G \\ 0,\ \text{otherwise} \end{cases} \tag{2.2}$$

The graph G is determined by the geographical configuration of the Honduras territory, and belongs to the class represented by graphs like the ones in figura 2.1.

2.2. UAC Computation.

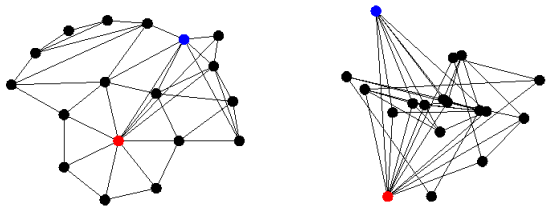

FIGURA 2.1. Homomorphic connectivity graphs corresponding to Honduras departments geographical confuguration. The red dot represents Francisco Morazán, the blue dot represents Cortés.

2.2.1. A sequential UAC Descriptor-Predictor. We start considering a geographically constrained switched UAC model of the form.

$$\begin{cases} x_{t+1} = A_t x_t \\ x_0 \in \mathbb{R}^{18n\times 1} \end{cases}, t \geq 0 \tag{2.3}$$

Where the matrices $A_t \in \mathbb{R}^{18n\times 18n}$ are computed according to the observed propagation's behavior by applying lemma 2.1.

Lemma 2.1. *Let us consider two propagation states $x_t, x_{t+1} \in \Sigma$ and the connectivity matrix $K \in \mathbb{R}^{18n\times 18n}$ determined by* (2.1)*. There is a matrix $T_t \in \mathbb{R}^{18n\times 18n}$ that satisfies* (1.2) *and* (1.3)*, if and only if for each $1 \leq j \leq 18n$, there is $\tau_{(t,j)} \in \mathbb{R}^{18n\times 1}$ such that $\tau_{(t,j)}^{\top} x_t = x_{t+1,j}$ and $K_j \circ \tau_{(t,j)} = \tau_{(t,j)}$, with $x_{t+1} = [x_{t+1,j}]$.*

Demostración. Let us consider the matrix.

$$E_{\tau_{(t,j)}} = I + \hat{e}_j(\tau_{(t,j)} - \hat{e}_j)^{\top} \tag{2.4}$$

Given $x = [x_j] \in \mathbb{R}^{18n\times 1}$, we will have that.

$$\begin{aligned} E_{\tau_{(t,j)}} x &= (I + \hat{e}_j(\tau_{(t,j)} - \hat{e}_j)^{\top} x \\ &= \begin{cases} \tau_{(t,j)}^{\top} x, & k = j \\ x_k, & k \neq j \end{cases} \end{aligned} \tag{2.5}$$

Let us set $T_t = \prod_{j=1}^{18n} E_{\tau_{(t,j)}}$ by (1.3). By (2.4) and (2.5), we will have that the matrix $T_t \in \mathbb{R}^{18n\times 18n}$ that satisfies (1.2) and (1.3), if and only if for each $1 \leq j \leq 18n$, there is $\tau_{(t,j)} \in \mathbb{R}^{18n\times 1}$ such that $\tau_{(t,j)}^{\top} x_t = x_{t+1,j}$ and $K_j \circ \tau_{(t,j)} = \tau_{(t,j)}$. This completes the proof. □

2.2.2. A geographically free Predictor. A geographically free UAC model of the form.

$$\begin{cases} x_{t+1} = T_t x_t \\ x_0 \in \mathbb{R}^{18n\times 1} \end{cases}, t \geq 0 \tag{2.6}$$

Where the matrices $T_t \in \mathbb{R}^{18n\times 18n}$ are computed according to the observed propagation's behavior using the techniques developed in [4, §3.2].

2.2.3. A Discrete Diffusive Predictor. A geographically constrained diffusion model of the form,

$$\begin{cases} x_{t+1} = (I - d(x,t)(I-K))x_t \\ x_0 \in \mathbb{R}^{18n\times 1} \end{cases}, t \geq 0 \tag{2.7}$$

with $K \in \mathbb{R}^{18n\times 18n}$ determined by the expression

$$K_{j,k} = \frac{|\text{sign}(x_{t,j})|\text{adj}(G)_{j,k}}{\sum_{p=1}^{18n}\text{adj}(G)_{p,k}} \tag{2.8}$$

and where the coefficients $d(x,t)$ of the discrete diffusion matrices $I - d(x,t)(I-K) \in \mathbb{R}^{18n\times 18n}$ are computed according to the observed propagation's behavior using lemma 2.2.

Lemma 2.2. *Given $t \in \mathbb{Z}_0^+$, if $d(x,t) = \text{argmin}\|x_{t+1} - x_t + d(I-K)x_t\|_2^2$, then we will have that.*

$$d(x,t) = -\frac{(x_{t+1}-x_t)^\top (I-K)x_t}{\|(I-K)x_t\|_2^2} \tag{2.9}$$

Demostración. Let us set $f(d) = \|y + dz\|_2^2$ for $y = x_{t+1} - x_t$ and $z = (I-K)x_t$. We will have that $\text{argmin} f(d) = \text{argmin}\|x_{t+1} - x_t + d(I-K)x_t\|_2^2$. Let us consider the equation.

$$0 = f'(d) = 2(y + dz)^\top z$$

This implies that.

$$d = -\frac{y^\top z}{z^\top z} = -\frac{y^\top z}{\|z\|_2^2}$$

This completes the proof. □

2.3. Controllers and Generalized Growth Models. In this section we will build on the ideas for real-time forecasting of epidemic trajectories that were presented in [1]. Let us now consider geographically free Generalized Growth Models (**GGM**) of the form.

$$\frac{dC(t)}{dt} = rC(t)^p \tag{2.10}$$

Where the parameters r and p are to be determined based on the propagation behavior data of each deparment, or based on the global (additive) propagation behavior. In this study we will consider a discrete time approximations of (2.10) of the form,

$$\begin{cases} C_0(t+1) = C_0(t) + r_0\exp(p_0 t) \\ C_1(t+1) = C_1(t) + r_1\exp(p_1 t) \\ C_2(t+1) = C_2(t) + r_2\exp(p_2 t) \end{cases} \tag{2.11}$$

with

$$\begin{aligned} C_0(t_k+1) - C_0(t_k) &\leq C_1(t_k+1) - C_1(t_k) \\ C_1(\tau_j+1) - C_1(\tau_j) &\leq C_2(\tau_j+1) - C_2(\tau_j) \end{aligned} \tag{2.12}$$

for some times t_k, τ_j at which $C(t)$ has been observed/measured.

We then combine the functions in (2.11) to obtain the refined approximation of $C(t)$ determined by the expression.

$$\begin{aligned}\hat{C}(t, w_1, w_2, w_3) &= (C_1(t), C_2(t), C_3(t)) \cdot (w_1, w_2, w_3) \\ &= \sum_{j=1}^{3} w_j C_j(t)\end{aligned} \tag{2.13}$$

Where the coefficients w_j are to be determined and need to satisfy the constraints,

$$(w_1, w_2, w_3) = \operatorname{argmin} \sum_{k=1}^{N} (\hat{C}(t_k, w_1, w_2, w_3) - C(t_k))^2 \tag{2.14}$$

for some times $t_1, \ldots, t_N$ at which $C(t_k)$ has been observed/measured.

3. Algorithms

We can apply lemma 2.1 and lemma 2.2, combined with the techniques developed in [4] and [2], in order to derive three prototypical data-driven approximation algorithms for the propagation model that are described by algoritmo 1, algoritmo 2 and algoritmo 3.

Algorithm 1 First Data-driven (descriptor-corrector) approximation algorithm

Data: Real number $\varepsilon > 0$, State data history: $\{x_t\}_{1 \leq t \leq T}$, $T \in \mathbb{Z}^+$ Connectivity matrix: $K \in \mathbb{R}^{18n \times 18n}$

Result: Approximate matrix realizations: $\{T_t\}_{t=1}^{T-1} \subset \mathbb{R}^{18n \times 18n}$ of $\tilde{\Sigma}$

1. For each $1 \leq t \leq T-1$
   - *a*) Compute $\tau_{(t,j)} \in \mathbb{R}^{18n \times 1}$ such that $K_j \circ \tau_{(t,j)}^{\top} = \tau_{(t,j)}^{\top}$ and $|x_{t+1,j} - \tau_{(t,j)}^{\top} x_t| \leq \varepsilon$ for each $1 \leq j \leq 18n$ and , with $x_t, x_{t+1} \in \Sigma$
   - *b*) Set $T_t = \prod_{j=1}^{18n} E_{\tau_{(t,j)}}$, with $E_{\tau_{(t,j)}}$ defined by (2.4).

return $\{T_t\}_{t=1}^{T-1}$

Algorithm 2 Second Data-driven (predictor) approximation algorithm

Data: Real number $\varepsilon > 0$, State data history: $\{x_t\}_{1 \leq t \leq T}$, $T \in \mathbb{Z}^+$

Result: Approximate state transition matrix: $\mathbf{T} \in \mathbb{R}^{18n \times 18n}$ of $\tilde{\Sigma}$

1. Set $H = [x_{t_1} \ \cdots \ x_{t_1+S}]$ with $t_1 \geq 1$ and $t_1 + S \leq T - 1$
2. Compute the reduced singular value decomposition $H = USV$
3. Compute the perturbation $H_\varepsilon = US_\varepsilon V$ of H according to [4, §3.2: (3.38)].
4. Compute the state-transition matrix $\mathbf{T}$ determined by [4, Corollary 3.8.] according to [4, §3.2: (3.44)].

return $\mathbf{T}$

We can apply the ideas in §2.3, in order to derive three prototypical data-driven corrector schemes for the propagation model that are described by algoritmo 4.

Algorithm 3 Third Data-driven diffusive (predictor) approximation algorithm

Data: REAL NUMBER $\varepsilon > 0$, STATE DATA HISTORY: $\{x_t\}_{1\leq t\leq T}, T \in \mathbb{Z}^+$ CONNECTIVITY MATRIX: $K \in \mathbb{R}^{18n\times 18n}$

Result: APPROXIMATE DIFFUSIVE STATE TRANSITION MATRIX: $\mathbf{D} \in \mathbb{R}^{18n\times 18n}$ of $\tilde{\Sigma}$

1. Set $x = x_{t_1}, y = x_{t_2}$ with $t_1 \geq 1 \leq t_2 \leq T-1$
2. Compute coefficient d according (2.9)
3. Compute the (diffusive) state-transition matrix $\mathbf{D} = I - d(I - K)$ according to (2.8).

return $\mathbf{D}$

Algorithm 4 Data-driven Generalized Growth Model

Data: INTEGER INDEX: k,STATE DATA HISTORY: $\{x_t\}_{1\leq t\leq T}, T \in \mathbb{Z}^+$

Result: APPROXIMATE GGM: $\mathbf{C}(t, w_1, w_2, w_3)$ for $\tilde{\Sigma}$

1. Set $x_k = (x_{1,k}, \ldots, x_{T,k})$
2. Compute functions $C_j(t)$ that satisfy (2.12)
3. Compute the coefficients w_j that satisfy (2.14).
4. Compute the GGM $\mathbf{C}(t, w_1, w_2, w_3)$ according to (2.13).

return $\mathbf{C}(t, w_1, w_2, w_3)$

4. NUMERICAL EXPERIMENTS

We have created two spreadsheets named `COVID19History.xlsx` and `HNConnect.xlsx`, where we have collected the data corresponding to observed COVID-19 propagation history in Honduras thus far and to the geographical configuration of Honduran Departments, respectively.
We have written a GNU Octave program named `COVID19.m` that implements algoritmo 1 based on the data in `COVID19History.xlsx` and `HNConnect.xlsx`. The GNU Octave code of `COVID19.m` is presented below.

```
## Copyright (C) 2020 Fredy Vides
##
## This program is free software: you can
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## and/or modify it under the terms of the
## GNU General
## Public License as published by the Free
## Software
## Foundation, either version 3 of the
## License, or
## (at your option) any later version.
##
## This program is distributed in the hope
## that it will be
## useful, but WITHOUT ANY WARRANTY; without
## even the
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```

```
## FITNESS FOR A
## PARTICULAR PURPOSE.  See the
## GNU General Public License for more
## details.
##
## You should have received a copy of the
## GNU General
## Public License
## along with this program.  If not, see
## <https://www.gnu.org/licenses/>.
##
##
##
## function [K,T,x0,x]=COVID19(m,n,tol,graph)
##
## Example:
## [K,T01,x0,x1]=COVID19(0,1,eps);
## [K,T12,x1,x2]=COVID19(1,2,eps);
## [K,T23,x2,x3]=COVID19(2,3,eps);
## [K,T03,x0,x3]=COVID19(0,3,eps);
## norm(x3-T03*x0,1)+norm((T23*T12*T01-T03)*x0,1)

## Author: fredy <fredy@HPCLAB>
## Created: 2020-03-17

function [K,T,x0,x]=COVID19(m,n,tol,graph)
m=m+1;
n=n+1;
pkg load io;
COVIDHist=xlsread ('COVID19History.xlsx');
HNConnect=xlsread ('HNConnect.xlsx');
A=HNConnect (1:18,1:18);
[M,N]=size(A);
E=eye(M,N);
K=A+E;
if nargin<=3
graph=1;
end
if graph==1
r=.5;
z1=(r*exp(2*pi*i*(0:6)/7)).';
z2=(2.0*r*exp(20*pi*i*(0:8)/(9*21))).';
z3=2.4*r*exp((pi+.1)*i/4);
xy=zeros(M,2);
xy([15 18 4 12 17 2 7],:)=[real(z1),imag(z1)];
xy([9 3 1 6 16 5 14 13 10],:)=[real(z2),imag(z2)];
xy(11,:)=[real(z3),imag(z3)];
```

```
subplot(211);
gplot (A,xy,'k-');
hold on;
plot(xy(:,1),xy(:,2),'k.','markersize',20,...
xy(8,1),xy(8,2),'r.','markersize',20,xy(6,1)...
,xy(6,2),'b.','markersize',20);
hold off;
axis off;
axis square;
subplot(212);
XY=randn(M,2);
gplot (A,XY,'k-');
hold on;
plot(XY(:,1),XY(:,2),'k.','markersize',20,XY(8,1),...
XY(8,2),'r.','markersize',20,XY(6,1),XY(6,2),...
'b.','markersize',20);
hold off;
axis off;
axis square;
end
x0=COVIDHist (1:18,m);
f0=find(abs(x0)<=tol);
x=COVIDHist (1:18,n);
f1=find(abs(x)<=tol);
f2=find(abs(x)>tol);
x0(f0)=0;
x(f1)=0;
T=E;
for k=f2
T0=E;
T0(k,:)=K(k,:).*(x(k)/x0);
T=T0*T;
end
T0=ones(M,1);
y0=T*x0;
T0(f2)=x(f2)./y0(f2);
T=diag(T0)*T;
K=A+E;
end
```

One can run program `COVID19.m` using the following command lines in GNU Octave.

```
>> [K,T01,x0,x1]=COVID19(0,1,eps);
>> [K,T12,x1,x2]=COVID19(1,2,eps);
>> [K,T23,x2,x3]=COVID19(2,3,eps);
>> [K,T03,x0,x3]=COVID19(0,3,eps);
>> norm(x3-T03*x0,1)+norm((T23*T12*T01-T03)*x0,1)
ans =    3.0531e-15
```

We have written a GNU Octave program named `UACPredictor.m` that implements algoritmo 3 based on the data in `COVID19History.xlsx`. The GNU Octave code of `UACPredictor.m` is presented below.

```
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##
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## and/or modify it under the terms of the
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## Public License as published by the Free
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##
## This program is distributed in the hope
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## details.
##
## You should have received a copy of the
## GNU General
## Public License
## along with this program.  If not, see
## <https://www.gnu.org/licenses/>.
##
##
## function [Xh,T,EIHUB,EIHLB,EGHUB,EGHLB]=
# UACPredictor(n,r,tol)
##
## Example:
## [Xh,T,EIHUB,EIHLB,EGHUB,EGHLB]=...
## UACPredictor(9,12,1e-12);

## Author: fredy <fredy@HPCLAB>
## Created: 2020-03-28

function [Xh,T,EIHUB,EIHLB,EGHUB,EGHLB]=...
UACPredictor(n,r,tol)
pkg load io;
Xh=xlsread ('COVID19History.xlsx');
[p,m]=size(Xh);
```

```
Xh=Xh(1:(p-1),(n+1):(r+1));
[p,m]=size(Xh);
Xh0=Xh(:,1:(m-1));
[uh,sh,vh]=svd(Xh0,0);
sh0=diag(sh);
f=find(sh0<=tol);
sh0(f)=tol;
sh0=diag(sh0);
T=Xh0\Xh(:,m);
T=[[zeros(1,m-2);eye(m-2)] T];
T=uh*sh0*vh'*T*(vh/sh0)*uh';
EIHUB=Xh(:,1);
EGHUB=Xh(:,1);
EIHLB=EIHUB;
EGHLB=EGHUB;
for k=1:(m-1)
EIHUB = [EIHUB (Xh(:,k+1)>0).*ceil(T*Xh(:,k))];
EIHLB = [EIHLB (Xh(:,k+1)>0).*floor(T*Xh(:,k))];
EGHUB = [EGHUB (Xh(:,k+1)>0).*ceil(T*EGHUB(:,k))];
EGHLB = [EGHLB (Xh(:,k+1)>0).*floor(T*EGHLB(:,k))];
end
end
```

One can run program `UACPredictor.m` using the following command lines in GNU Octave.

```
>> s=9;R=12;
>> [Xh,T,EIHUB,EIHLB,EGHUB,EGHLB]=...
UACPredictor(s,R,1e-12);
>> t=1:(R-s+1);
>> subplot(221),plot(t,Xh(8,:),'k.-',...
'linewidth',6,t,EIHLB(8,:),'r.-',...
'linewidth',2,t,EIHUB(8,:),'b.-',...
'linewidth',2)
>> subplot(222),plot(t,Xh(8,:),'k.-',...
'linewidth',6,t,EGHLB(8,:),'r.-',...
'linewidth',2,t,EGHUB(8,:),'b.-',...
'linewidth',2)
>> subplot(223),plot(t,Xh(6,:),'k.-',...
'linewidth',6,t,EIHLB(6,:),'r.-',...
'linewidth',2,t,EIHUB(6,:),'b.-',...
'linewidth',2)
>> subplot(224),plot(t,Xh(6,:),'k.-',...
'linewidth',6,t,EGHLB(6,:),'r.-',...
'linewidth',2,t,EGHUB(6,:),'b.-',...
'linewidth',2)
```

The previous lines produce the graphical outputs illustrated in figura 4.1.

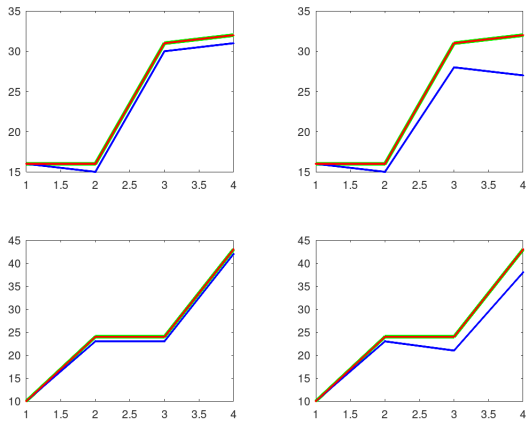

FIGURA 4.1. Four stages forcast for Francisco Morazán (top): Local time estimates (left) and Global time estimates (right). Four stages forcast for Cortés (bottom): Local time estimates (left) and Global time estimates (right). Green dotted lines represent observed values, blue dotted lines represent lower bounds for expected-predicted values, and red dotted lines represent upper bounds for expected-predicted values

We have written a GNU Octave program named `DiffCOVID19.m` that implements algoritmo 2 based on the data in `COVID19History.xlsx`, `HNConnect0.xlsx` and `HNConnect1.xlsx`. The GNU Octave code of `DiffCOVID19.m` is presented below.

```
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## details.
##
```

```
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## GNU General
## Public License
## along with this program.  If not, see
## <https://www.gnu.org/licenses/>.
##
##
## [K1718,A,x17,x18,xx,yy,zz,f]=...
## DiffCOVID19(17,18,1,eps,1);
##
## Example:
## [K1718,A,x0,x,xx,yy,zz,f]=...
## DiffCOVID19(17,18,1,eps,1);

## Author: fredy <fredy@HPCLAB>
## Created: 2020-04-01

function [K,A,x0,x,xx,yy,zz,f]=...
DiffCOVID19(m,n,p,tol,graph)
m=m+1;
n=n+1;
pkg load io;
COVIDHist=xlsread ('COVID19History.xlsx');
%HNConnect=xlsread ('HNConnect0.xlsx');
HNConnect=xlsread ('HNConnect1.xlsx');
A=HNConnect (1:18,1:18);
[M,N]=size(A);
E=eye(M,N);
if nargin<=4
graph=1;
end
x0=COVIDHist (1:18,m);
x=COVIDHist (1:18,n);
K=diag(x0>0)*A*diag(1./sum(A));
K=E-K;
d=K*x0;
d=-(x-x0)'*d/(d'*d);
K=E-d*K;
[xx,yy]=meshgrid(-1.5:3/30:1.5);
f=[];
r=.5;
z1=(r*exp(2*pi*i*(0:6)/7)).';
z2=(2.0*r*exp(20*pi*i*(0:8)/(9*21))).';
z3=2.4*r*exp((pi+.1)*i/4);
xy=zeros(M,2);
xy([15 18 4 12 17 2 7],:)=[real(z1)...
,imag(z1)];
```

```
xy([9 3 1 6 16 5 14 13 10],:)=...
[real(z2),imag(z2)];
xy(11,:)=[real(z3),imag(z3)];
for k=1:18
z=sqrt((xx-xy(k,1)).^2+(yy-xy(k,2)).^2);
m=min(min(z));
f=[f;find(z==m)];
end
  [X,Y]=meshgrid (-1.5:3/150:1.5);
  zz=zeros(size(xx));
  zz(f)=Mxvec(K,x0,p);
  Z=interp2(xx,yy,zz,X,Y,'spline');
if graph==1
  figure;
  contour(X,Y,Z,64);
  colormap summer;
  hold on;
  gplot (A,[xx(f) yy(f)],'k-');
  plot(xx(f),yy(f),'k.','markersize',...
  20,xx(f(8)),...
  yy(f(8)),'r.','markersize',20,...
  xx(f(6)),yy(f(6)),...
  'b.','markersize',20);
  hold off;
  axis off;
  axis square;
end
end
function y=Mxvec(A,b,n)
y=b;
for k=1:n
y=A*y;
end
end
```

One can run program `DiffCOVID19.m` using the following command lines in GNU Octave.

```
>> [K1718,A,x17,x18,xx,yy,zz,f]=...
DiffCOVID19(17,18,1,eps,1);
```

The previous lines produce the graphical outputs illustrated in figura 4.2.
One can now combine all predictive algorithms using the following command lines in GNU Octave.

```
>> [K,T1718,x17,x18]=COVID19(17,18,eps,0);
>> [K,T1819,x18,x19]=COVID19(18,19,eps,0);
>> [Xh,T,EIHUB,EIHLB,EGHUB,EGHLB]=...
UACPredictor(17,19,1e-18);
>> [K1718,A,x17,x18,xx,yy,zz,f]=...
```

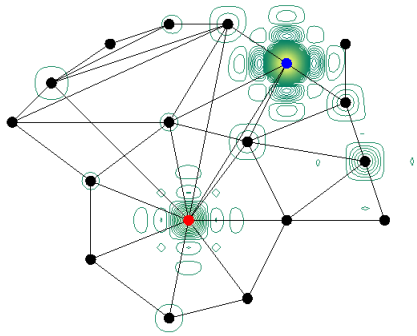

FIGURA 4.2. Diffusion map generated by the diffusive model.

```
DiffCOVID19(17,18,1,eps,0);
>> [x19 T1718^2*x17 T^2*x17 K1718^2*x17]
ans =

    16.00000    11.00000    11.01820    11.04176
     2.00000     2.25486     2.00331     2.00328
    20.00000    20.00000    20.03310    19.98064
     2.00000     2.01402     2.15116     1.05549
     1.00000     1.00000     1.00165     1.00138
   195.00000   196.36313   195.32268   194.77527
     0.00000     0.00000     0.00000     0.00000
    54.00000    53.37141    53.97481    46.99580
     0.00000     0.00000     0.00000     0.00000
     0.00000     0.00000     0.00000     0.00000
     0.00000     0.00000     0.00000     0.00000
     1.00000     1.12800     1.00165     1.00471
     4.00000     4.49258     4.00662     4.00288
     0.00000     0.00000     0.00000     0.00000
     0.00000     0.00000     0.00000     0.00000
     8.00000     8.05616     8.01324     8.04501
     0.00000     0.00000     0.00000     0.00000
     9.00000     8.05546     8.01324     8.05306
```

We have written a GNU Octave program named `WeightedGGM.m` that implements algoritmo 4 based on the data in `COVID19HistoryJoint.xlsx`. The GNU Octave code of `DiffCOVID19.m` is presented below.

```
## Copyright (C) 2020 Fredy Vides
##
## This program is free software:
## you can redistribute it and/or modify
## it under the terms of the GNU General
## Public License as published by
```

```
## the Free Software Foundation, either
## version 3 of the License, or
## (at your option) any later version.
##
## This program is distributed in the hope
## that it will be useful, but
## WITHOUT ANY WARRANTY; without even the
## implied warranty of
## MERCHANTABILITY or FITNESS FOR A
## PARTICULAR PURPOSE.  See the
## GNU General Public License for more
## details.
##
## You should have received a copy of the
## GNU General Public License
## along with this program.  If not, see
## <https://www.gnu.org/licenses/>.

##
## function [Y,t]=...
WeightedGGM(department,graph)
##
## Example: [Y,t]=WeightedGGM(19);
##

## Author: fredy <fredy@HPCLAB>
## Created: 2020-04-10

function [Y,t]=WeightedGGM(department,graph)

if nargin==0, department=19;end
if nargin<=1, graph=1;end

pkg load io;
Xh = xlsread ('COVID19HistoryJoint.xlsx');
X=Xh(department,:);

if sum(X)==0
Y1=Y2=Y3=Y=X;
t=1:length(X);
else
[Y1,t,D,ts]=SDiffLB(X,1);
[Y2,t]=EMod(X);
[Y3,t,D,ts]=SDiffUB(X,1);
Y=[Y1;Y2;Y3];
w=Y'\X';
Y=w'*Y;
```

```
end
if graph==1
plot(t,X,'k.-','markersize',...
12,t,Y1,...
'b.-','markersize',12,t,Y2,...
'c.-','markersize',12,t,Y3,...
'r.-','markersize',12,t,Y,...
'g.-','markersize',12);
legend('C(t)','C_0(t)',...
'C_1(t)','C_2(t)','C_{w}(t)');
grid on;
end
end

function [Yt,t,D,ts]=SDiffLB(S,s)
Ls=length(S);
t=0:(Ls-1);
ds=diff(S);
ff=find(ds>0);
D=ds(ff(1));
ts=t(ff(1));
Lff=length(ff);
for k=2:Lff-1
if s==0,if sum(ds(ff(k))<=...
ds(ff((k+1):Lff)))>=Lff-k-1 ...
&& ds(ff(k))<=ds(ff(k+1)), ...
D=[D ds(ff(k))];ts=[ts ...
t(ff(k))];end;end
if s==1,if sum(ds(ff(k))<...
ds(ff((k+1):Lff)))>=Lff-k-1 ...
&& ds(ff(k))<ds(ff(k+1)), D=[D ...
ds(ff(k))];ts=[ts t(ff(k))];end;
end
end
D=[D ds(ff(Lff))];
ts=[ts t(ff(Lff))];
LD=log(D);
p=polyfit(ts,LD,1);
Yt=S(1);
for k=1:(Ls-1)
Yt=[Yt Yt(k)+...
exp(polyval(p,k-1))];
end
end

function [Yt,t,D,ts]=SDiffUB(S,s)
Ls=length(S);
```

```
t=0:(Ls-1);
ds=diff(S);
ff=find(ds>0);
D=ds(ff(1));
ts=t(ff(1));
for k=2:length(ff)
if s==0,if sum(ds(ff(k))>=...
ds(ff(1:(k-1))))>=k-1, D=...
[D ds(ff(k))];ts=[ts ...
t(ff(k))];end;end
if s==1,if sum(ds(ff(k))>...
ds(ff(1:(k-1))))>=k-1, D=...
[D ds(ff(k))];ts=[ts ...
t(ff(k))];end;end
end
LD=log(D);
p=polyfit(ts,LD,1);
Yt=S(1);
for k=1:(Ls-1)
Yt=[Yt Yt(k)+...
exp(polyval(p,k-1))];
end
end

function [Yt,t]=EMod(S)
Ls=length(S);
t=0:(Ls-1);
D=diff(S);
ff=find(D>0);
ts=t(ff);
LD=log(D(ff));
p=polyfit(ts,LD,1);
Yt=S(1);
for k=1:(Ls-1)
Yt=[Yt Yt(k)+...
exp(polyval(p,k-1))];
end
end
```

One can implement `WeightedGGM.m` using the following command lines.

```
>> [Y,t]=CombinedExpModel(19);
```

This produces the graphical output shown in figura 4.3.

The spreadsheet data files together with a copy of the program `COVID19.m` are available at [3].

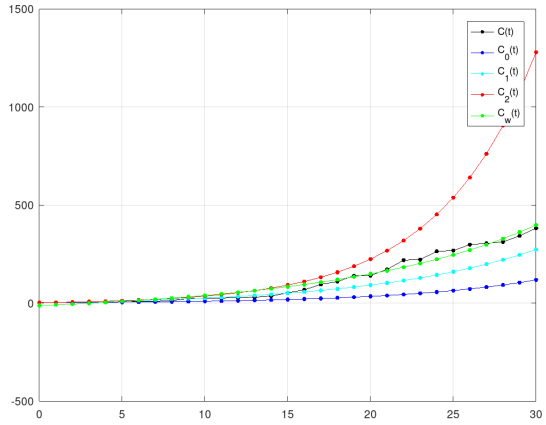

FIGURA 4.3. Graphical output.

5. CONCLUSION AND FUTURE DIRECTIONS

The results in §2 can be used to derive predictive numerical simulation algorithms like algoritmo 1, algoritmo 2 and algoritmo 3.
Thus far, given the present state and conditions of available data, by applying algoritmo 1, algoritmo 2, algoritmo 3 and algoritmo 4, one can get very good predictions in a time span that ranges from two to five days in the future.
Once more accurate COVID-19 behavior data become available, we plan to extend algoritmo 1, algoritmo 2, algoritmo 3 and algoritmo 4, to describe other aspects of the COVID-19 propagation in Honduras, for longer time periods. An extension of the ideas presented in this document to more complex geographical configuration graphs will be the subject of future communications.

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