Math 525 — Winter 2024

Homework 5 — **Due Friday, February 9**

1. Show that a locally convex vector space contains a bounded open set if and only if the topology is equivalent to one given by a norm. (Here we use the definition of bounded from lecture: $E \subset \mathcal{X}$ is bounded iff each of the seminorms defining the topology on $\mathcal{X}$ is bounded on E.)

2. Show that a vector subspace of a normed vector space is closed in the norm topology if and only if it is weakly closed.

3. Suppose $\mathcal{X}$ is a normed space, and that $f_j \in \mathcal{X}^*$ converges weak* to f. Show that $\|f\|_{\mathcal{X}^*} \leq \liminf_{j\to\infty} \|f_j\|_{\mathcal{X}^*}$, and give an example in which the inequality is strict.

4. Folland page 165, Problem 38.

5. Folland page 170, Problem 48.