Math 524 Autumn 2023

Homework 6 **Due Monday, November 27**

1. Suppose that $f, g \in L^1(\mathbb{R}, \mathcal{B}_\mathbb{R}, m)$. Show that, for almost all $x \in \mathbb{R}$, the function $f(x-y)g(y)$ is integrable in y. For such x define

$$(f * g)(x) = \int f(x-y)g(y)\,dx$$

Show that $f * g = g * f$ almost everywhere, and that

$$\|f * g\|_1 \leq \|f\|_1 \|g\|_1.$$

2. Folland Page 92, Problem 11.

3. Folland Page 93, Problem 17.

4. Folland Page 94, Problem 21.

5. Recall the algebra $\mathcal{A}_n \subset \mathcal{B}_{(0,1]}$ on $(0,1]$ from Lecture 1, generated by sets of the form $\left(\frac{k}{2^n}, \frac{k+1}{2^n}\right]$ with $0 \leq k < 2^n - 1$. (This is finite so it is a σ-algebra as well.) Given $f \in L^1\big((0,1], \mathcal{B}_{(0,1]}, m\big)$, find the conditional expectation of f on $\mathcal{A}_n$ as defined in problem 17 above.