Math 524 Autumn 2023

Homework 4 **Due Friday, October 27**

1. For a bounded real-valued function $f(x)$ defined on a metric space, define the upper and lower envelope functions by the rule:

$$\overline{f}(x) = \lim_{r>0}\Big(\sup_{d(y,x)<r} f(y)\Big)\,, \qquad \underline{f}(x) = \lim_{r>0}\Big(\inf_{d(y,x)<r} f(y)\Big)\,,$$

and define the *oscillation function* for f as

$$\mathrm{osc} f(x) = \overline{f}(x) - \underline{f}(x)\,.$$

Show that

(a.) The function f is continuous at x if and only if $\omega(x) = 0$.

(b.) For each $\epsilon > 0$, the set $\{\, x : \omega(x) < \epsilon \,\}$ is an open set.

(c.) Show that if f is a bounded function on a metric space, then the set of x where $f(x)$ is continuous is a Borel set.

2. Folland Page 48, Problem 9, parts (a) and (b).

3. Folland Page 52, Problem 14.

4. Folland Page 52, Problem 15.

5. Folland Page 52, Problem 16.