Math 525 Winter 2024

Homework 7 **Due Friday, February 23**

1. Show that if $1 \leq p \leq q < \infty$, and (X, μ), (Y, ν) are σ-finite, then

$$\left(\left(\int_X |f(x,y)|^p \, d\mu(x)\right)^{\frac{q}{p}} d\nu(y)\right)^{\frac{1}{q}} \leq \left(\left(\int_X |f(x,y)|^q \, d\nu(y)\right)^{\frac{p}{q}} d\mu(x)\right)^{\frac{1}{p}}.$$

Give an example to show that this inequality can fail if $q < p$. Hint: consider a function g that is periodic with period 1, and $f(x,y) = g(x-y)$ on the unit square.

2. Folland page 192, Problem 21.

3. Folland page 196, Problem 31.

4. Folland page 197, Problem 32.

5. Folland page 199, Problem 36.