Math 524 | Autumn 2023

Homework 5 | **Due Friday, November 3**

1. Recall that a Borel measure μ on $\mathbb{R}^n$ is called *regular* if $\mu(K) < \infty$ for all compact sets K, and if for all Borel sets E we have

$$\begin{aligned}\mu(E) &= \sup\{\mu(K) \,:\, K \subseteq E\,, \text{ K compact}\}\\ &= \inf\{\mu(U) \,:\, U \supseteq E\,, \text{ U open}\}\end{aligned}$$

Show that if μ and ν are regular Borel measures, and

$$\int \phi \, d\mu = \int \phi \, d\nu$$

for all $\phi \in C_c(\mathbb{R}^n)$, then $\mu = \nu$.

2. Folland Page 59, Problem 20.

3. Folland Page 59, Problem 21.

4. Folland Page 63, Problem 33 (you can use results from that section of the text).

5. Folland Page 68, Problem 46.