Math 524 Autumn 2023

Homework 2 **Due Friday, January 19**

1. Show that if X is a compact metric space, and $\mathcal{F} \subset C(X)$ is equicontinuous on X, then $\mathcal{F}$ is uniformly equicontinuous: for all $\epsilon > 0$ there is $\delta > 0$ so that, for all $x, y \in X$ and $f \in \mathcal{F}$, $d(x, y) < \delta$ implies $|f(x) - f(y)| < \epsilon$.

2. Show that if X is LCH, then the closure in the topology of uniform convergence on compact sets of an equicontinuous family in $C(X)$ is equicontinuous.

3. Show that a closed subset $\mathcal{F} \subset C_0(X)$, where X is LCH, is compact (in the topology of uniform convergence) iff it is pointwise bounded and equicontinuous on X, and for each $\epsilon > 0$ there is a compact set K such that, for all $f \in \mathcal{F}$, $|f(x)| \leq \epsilon$ on K^c.

4. Folland page 138, Problem 63.

5. Folland page 155, Problem 6.