Computational analysis of the heat gain of buildings through their roofs using a heat transfer transient nonlinear model solved by numerical methods

Carlos Armando De Castro*

Centro de Investigación en Modelos Naturales para el Ambiente y el Desarrollo, CIMNAD

Bogotá, Colombia

ABSTRACT

In this paper heat transfer through roofs with one layer and with two layers of materials with an air gap between them is modeled. Solar radiation and infrared atmospheric radiation over the exterior surface of the roof are considered, also the convective and radiative heat exchange between the surfaces that limit the air gap in the roof. The transient partial differential heat equation is solved with the nonlinear time variant boundary conditions and the discrete heat equation considered like a system of nonlinear equations for each time step with the Newton-Raphson multidimensional iteration. A code in MATLAB was used to implement the algorithm of solution. With the temperature profile known the heat gain through the roof to the interior of the building is calculated considering a constant internal temperature, with meteorological conditions from the Caribbean Coast of Colombia and varying the roof surfaces'materials and properties to radiation.

Keywords: heat gain, thermal comfort, solar radiation, heat equation, numerical analysis, partial differential equation, finite difference method, nonlinear boundary conditions, atmospheric radiation, natural ventilation, bioclimatic design.

1. INTRODUCTION

In the tropical regions such as Colombia there are several problems with thermal comfort during the day in houses, offices, warehouses and other buildings; this is due to the high ambient temperature (it can reach 40°C in some places) and the high doses of solar radiation that reach the surface during the day. A large part of this thermal discomfort is caused by the heat fluxes that enter the buildings through their roofs because their simple construction or their inadequate design has little to isolate effectively, above all in the houses of people who live in conditions of poverty, whose roof is one single layer of zinc tiles.

The purpose of this research was to model adequately all the parameters that affect the heat flux that enters the building through the roof in order to be able to design better roofs with the focus on thermal comfort, thus reducing the electrical power consumed by buildings that use air conditioning systems or improving the lives of people who cannot afford an HVAC equipment by making them be closer to the conditions of thermal comfort inside their houses.

* Director of Thermal Solar Energy Area
CIMNAD, Bogotá, Colombia
E-mail: cadecastro@cimnad.org
Phone: +57-312-636-9880

Here we consider roofs with one single layer (Figure 1) and two layers with an air gap in the middle (Figure 2). The one-layer roof is the most common design as stated previously.

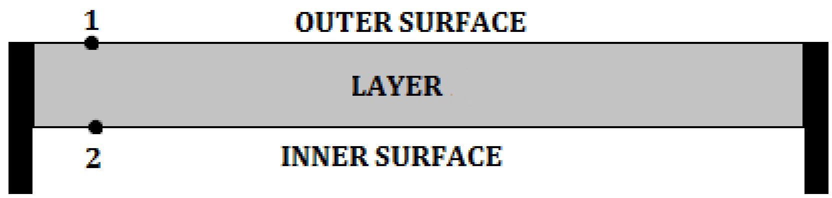

Figure 1. Schematics of the roof with one layer, detailing the surfaces considered in the model.

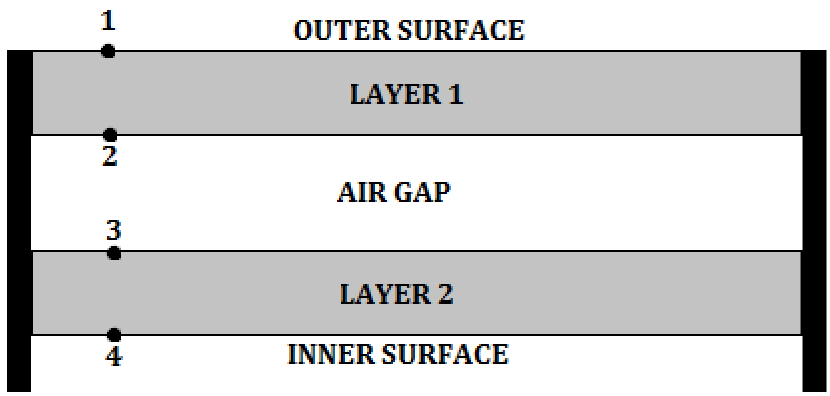

Figure 2. Schematics of the roof with two layers and the air gap in the middle, detailing the surfaces considered in the model.

Once the model and numerical solution were developed we simulated one solar day considering three types of roofs: the single zinc tile as used commonly ("Techo 1"), roof with two unmodified zinc tiles in the roles of the two layers ("Techo 2"), and roof with two zinc tiles but painted white in the exterior surface and all other surfaces covered by aluminum foil ("Techo 3").

2. MATHEMATICAL MODEL

The mathematical model is based completely on the transient heat diffusion equation in one dimension for each layer of the roof, the boundary conditions on each layer are heat balances considering conduction, convection and radiation. All temperatures are in Kelvin.

2.1. One-layer roof

For the roof layer we have the heat diffusion equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (1)$$

For Surface 1 by heat fluxes balance the boundary condition is:

$$\alpha_s R_s + \epsilon_1 \sigma\left(T_{sky}^4 - T_{s1}^4\right) + h_o(T_a - T_{s1}) - k\frac{\partial T}{\partial x} = 0 \quad (2)$$

For Surface 2 by heat fluxes balance the boundary condition is:

$$\epsilon_2 \sigma\left(T_i^4 - T_{s2}^4\right) + h_i(T_i - T_{s2}) - k\frac{\partial T}{\partial x} = 0 \quad (3)$$

Sky temperature is calculated using Swinbank's equation [1]:

$$T_{sky} = 0.0551 T_a^{1.5} \quad (4)$$

External convection heat transfer is a function of wind speed $u$for horizontal surface [2]:

$$h_o = \begin{cases} 5.6 + 4.0u, & u \leq 5m/s \\ 7.2u^{0.78}, & u > 5m/s \end{cases} \quad (5)$$

With units $\mathrm{W/m^2K}$. Internal convection heat transfer coefficient is considered constant at the value $h_i = 5.6\mathrm{W/m^2K}$ based on the one used by the FEMMASSE, SIMEDIF and HvacLoadExplorer [2].

2.2. Two-layer roof

For layers 1 and 2 we have the heat diffusion equation:

$$\frac{\partial T_1}{\partial t} = \alpha_1 \frac{\partial^2 T_1}{\partial x_1^2} \quad (6)$$

$$\frac{\partial T_2}{\partial t} = \alpha_2 \frac{\partial^2 T_2}{\partial x_2^2} \quad (7)$$

For each surface the boundary conditions of (6) and (7) are the heat fluxes balances, then for Surface 1 the boundary condition is:

$$\alpha_s R_s + \epsilon_1 \sigma\left(T_{sky}^4 - T_{s1}^4\right) + h_o(T_a - T_{s1}) - k_1\frac{\partial T_1}{\partial x_1} = 0 \quad (8)$$

For Surface 2 the boundary condition is:

$$\epsilon_{23}\sigma(T_{s3}^4 - T_{s2}^4) + h_{23}(T_{s3} - T_{s2}) - k_1\frac{\partial T_1}{\partial x_1} = 0 \quad (9)$$

For Surface 3 the boundary condition is:

$$\epsilon_{23}\sigma(T_{s2}^4 - T_{s3}^4) + h_{23}(T_{s2} - T_{s3}) - k_2\frac{\partial T_2}{\partial x_2} = 0 \quad (10)$$

For Surface 4 the boundary condition is:

$$\epsilon_4\sigma\left(T_i^4 - T_{s4}^4\right) + h_i(T_i - T_{s4}) - k_2\frac{\partial T_2}{\partial x_2} = 0 \quad (11)$$

Sky temperature and external convection heat transfer coefficient are given by equations (4) and (5). Emissivity between surfaces 2 and 3 is calculated by [3]:

$$\epsilon_{23} = \frac{1}{1/\epsilon_2 + 1/\epsilon_3 - 1} \quad (12)$$

Convection heat transfer coefficient between surfaces 2 and 3 is given by:

$$h_{23} = \mathrm{Nu}\frac{k_{air}}{D} \quad (13)$$

Where D is the distance between the surfaces (thus the height of the air gap) and the Nusselt number is $\mathrm{Nu} = 1$ if surface 2 is at a higher temperature than Surface 3 because in that case there are no convection cells and heat transfer is due only by conduction in the air; otherwise the correlation by Hollands [3] for horizontal enclosures is used [3]:

$$\mathrm{Nu} = 1 + 1.44\left[1 - \frac{1708}{\mathrm{Ra_D}}\right]^+ + \left[\frac{\mathrm{Ra_D^{1/3}}}{18} - 1\right]^+ \quad (14)$$

Where notation $[\quad]^+$ indicates that if the expression inside the brackets is less than zero the whole term is made equal to zero. Rayleigh number in this case is:

$$\mathrm{Ra_D} = \frac{g\beta|T_{s2} - T_{s3}|D^3}{\nu^2}Pr \quad (15)$$

Where air properties are calculated at the average temperature $T_{av} = (T_{s2} + T_{s3})/2$. Air density is calculated as an ideal gas, conductivity and viscosity of air are considered temperature-dependent according to Sutherland's approximation:

$$\rho_{air} = \frac{P_{atm}}{R_{air}T_{av}} \quad (16)$$

$$k_{air} = 0.0241\left(\frac{T_{av}}{273.15}\right)^{1.5}\left(\frac{469}{T_{av} + 196}\right) \quad (17)$$

$$\mu_{air} = 1.716 \times 10^{-5}\left(\frac{T_{av}}{273.15}\right)^{1.5}\left(\frac{384}{T_{av} + 111}\right) \quad (18)$$

$$\nu = \mu_{air}/\rho_{air} \quad (19)$$

3. NUMERICAL SOLUTION

Equations (1), (6) y (7) are discretized by the finite difference method in the domain corresponding to each one, more precisely, we partition each roof layer in M internal nodes where the heat equation applies and 2 more nodes corresponding to each surface of the layer where the boundary conditions apply.

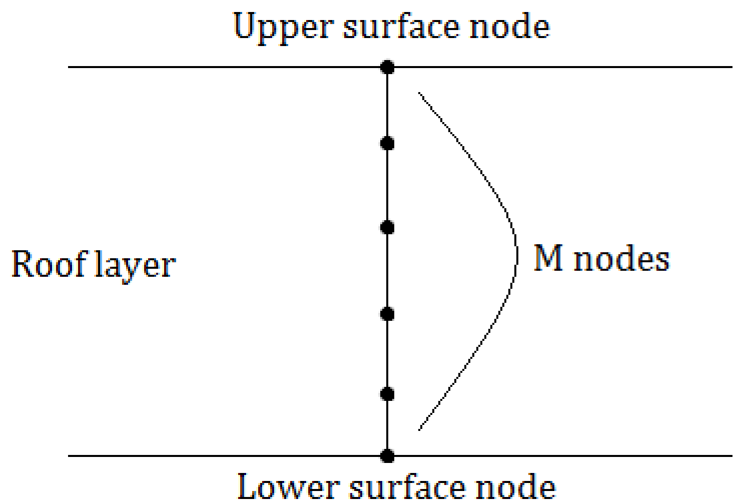

Figure 3. Domain discretization to solve the heat equation.

The heat transfer equation discretization is for node *i* at time interval *n* [4]:

$$\frac{T_{i,n+1} - T_{i,n}}{\Delta t} = \alpha \frac{T_{i+1,n} - 2T_{i,n} + T_{i-1,n}}{\Delta x^2} \quad (20)$$

Equation (20) actually describes M algebraic equations for each roof layer, these coupled with the equations given by the boundary conditions comprise a system of equations that is nonlinear. For the one-layer roof there are M+2 equations in the system and for the two-layer roof there are 2M+4 equations. These systems of equations are solved by the Newton-Raphson multidimensional method [4], this method is used for a system of equations given by

$$\begin{aligned} &f_1(x_1, x_2, \dots, x_n) = 0 \\ &f_2(x_1, x_2, \dots, x_n) = 0 \\ &\vdots \\ &f_n(x_1, x_2, \dots, x_n) = 0 \end{aligned}$$

There are defined two vectors $\mathbf{x}$ and $\mathbf{F}(\mathbf{x})$ such that:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{F}(\mathbf{x}) = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

The jacobian matrix of the system is then the matrix whose components are:

$$J_{ij} = \frac{\partial f_i}{\partial x_j} \qquad (21)$$

With an initial guess for the solution $\mathbf{x}_0$, the iteration to solve is:

$$\mathbf{x}_{i+1} = \mathbf{x}_i - J(\mathbf{x}_i)^{-1}\mathbf{F}(\mathbf{x}_i) \qquad (22)$$

This solution is calculated for each time step *n* that is considered in the simulations.

4. SIMULATIONS

The simulations were done with the inputs shown in Figures 4 to 6 for 12 hours of simulation time (one solar day) and an initial temperature for all nodes of 20°C:

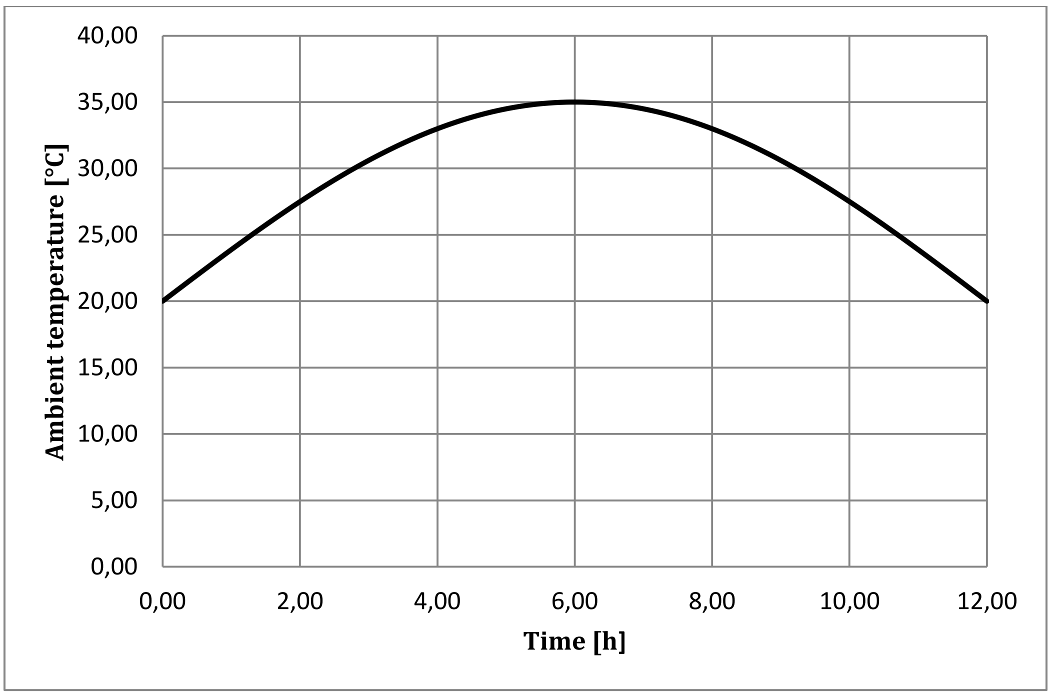

Figure 4. Ambient temperature for the simulation.

These meteorological data values are based on those encountered typically in the Caribbean Coast of Colombia and other hot regions of the country such as Middle Magdalena or the Eastern Plains.

The results of the simulations are shown in Figures 7 and 8, where it can be seen that the roof with double layer is better to isolate the interior of the building from the harsh external conditions. The relative difference the double layer roof Techo 2 with respect to the commonly used Techo 1 for the same conditions, material and radiative properties is 83.8% less energy per square meter needed to maintain a desired comfortable interior temperature of 20°C the whole 12-hour day. With our design Techo 3 this difference is a 96.1% saving of energy compared to Techo 1.

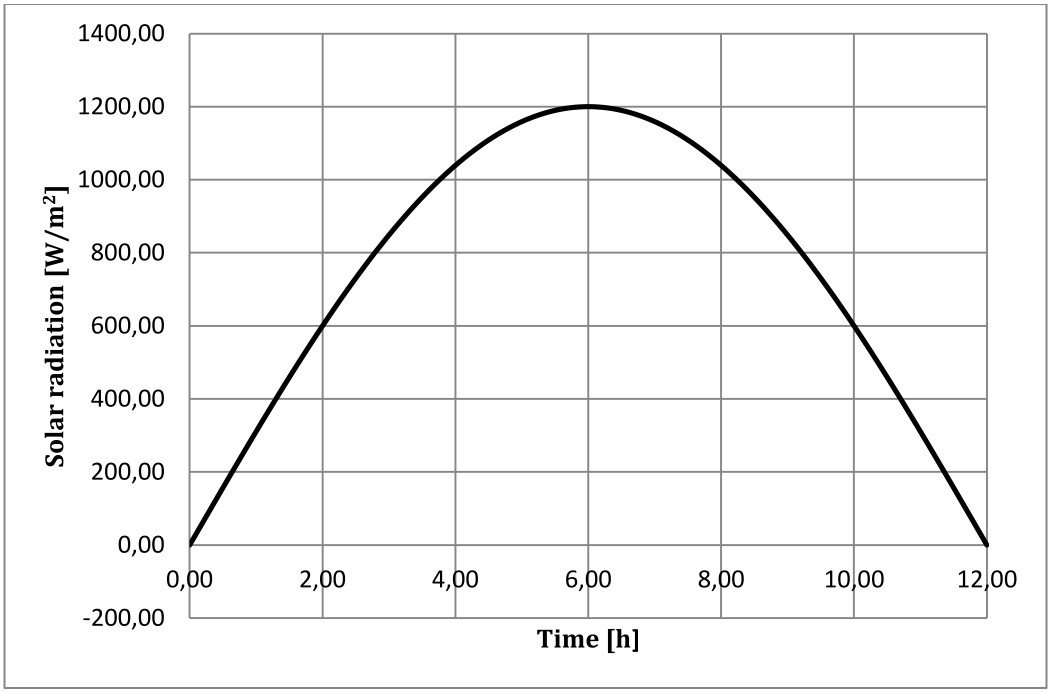

Figure 5. Solar radiation for the simulation.

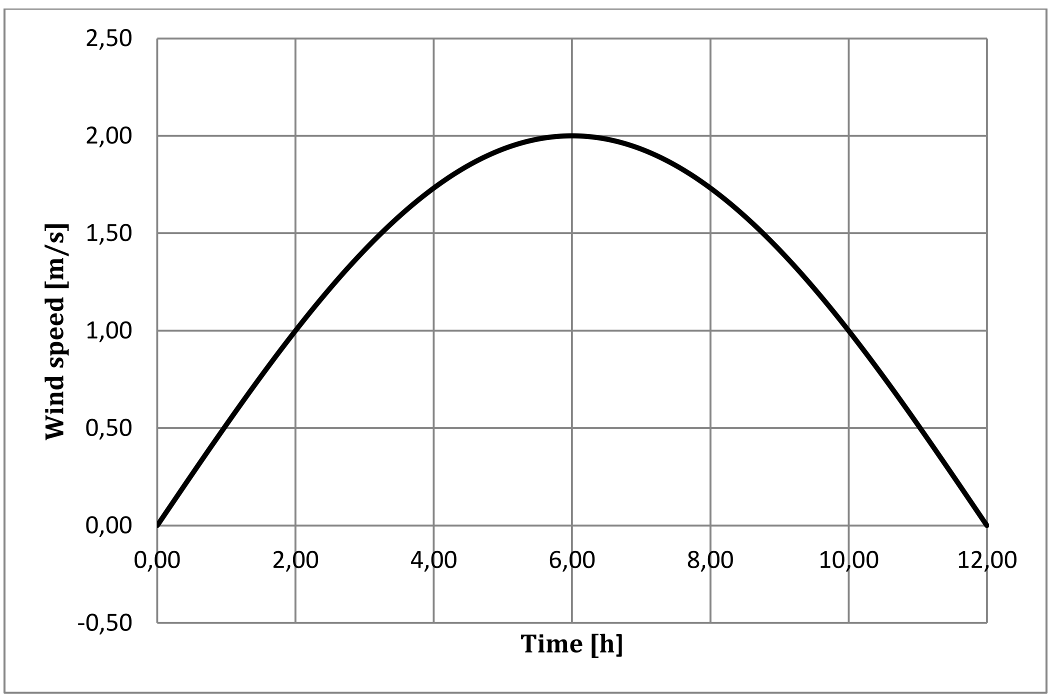

Figure 6. Wind speed for the simulation.

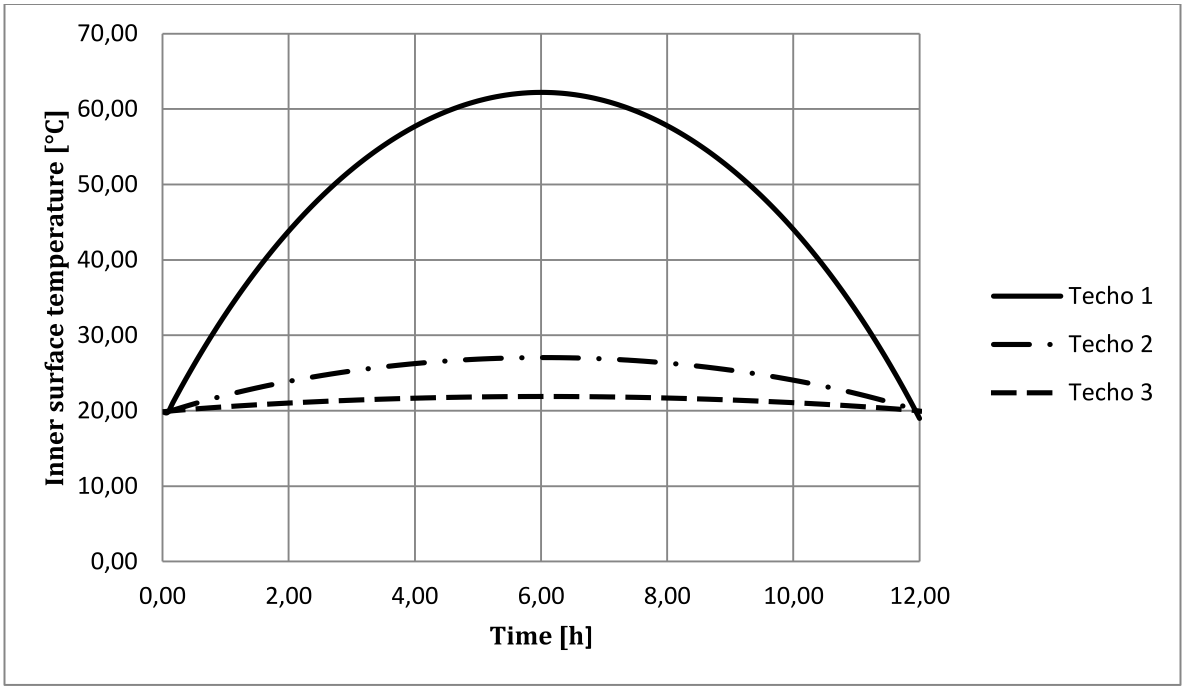

Figure 7. Temperature of the inner roof surface.

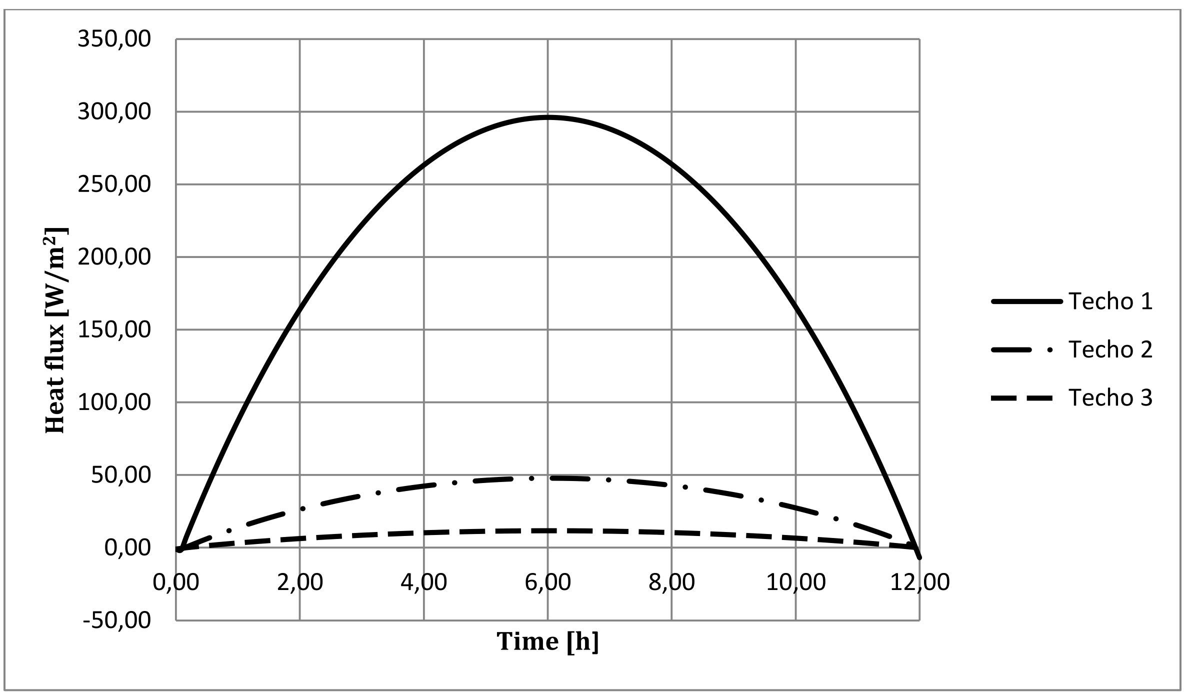

Figure 8. Heat flux effectively entering the building through the roof.

5. CONCLUSIONS

We have developed a computational model for estimating the heat gain through two types of roofs, the MATLAB code developed is very useful for our research on natural ventilation and energy efficient construction of buildings. This code is being used in consultancies to bioclimatic design of warehouses for grain storages; also, as shown in this papers, it is useful to be used in the design of economic houses for people in conditions of poverty who also deserve to live in thermal comfort.

REFERENCES

[1] Bilbao, J. and De Miguel, A.. 2007. Estimation of Daylight Downward Longwave Atmospheric Irradiance under Clear-Sky and All-Sky Conditions. Journal of Applied Meteorology and Climatology. 46: 878-889.

[2] Drakos, Nikos. A Computer Model to Predict the Surface Temperature and Time-of-Wetness of Concrete Pavements and Bridge Decks. http://ciks.cbt.nist.gov/bentz/nistir6551/node5.html

[3] Incropera, F. and DeWitt, D. Introduction to Heat Transfer. 5th Edition.Wiley. 2006.

[4] De Castro, C. A. Métodos Numéricos Básicos para Ingeniería. 1st Edition. Asesorías en Matemáticas, Física e Ingeniería. 2009.