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Galileo Ferraris' Contest

Elastostatic analysis

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- * Mechanical behaviour of ductile materials
- * Structural model of the motor
- * Von Mises criterion: an overview
- * Results



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Mechanical behaviour of ductile materials

When subjected to a field of forces, mechanical **stress** arises in the body which is opposed to the deformation of it.

$$\sigma = \frac{F}{A}$$

Strain being the measure of the deformation,

$$\epsilon = \frac{\Delta l}{l}$$

if the deformation is elastic, the mechanical **stress** and the **strain** are proportional and can be described by **Hooke's law**:

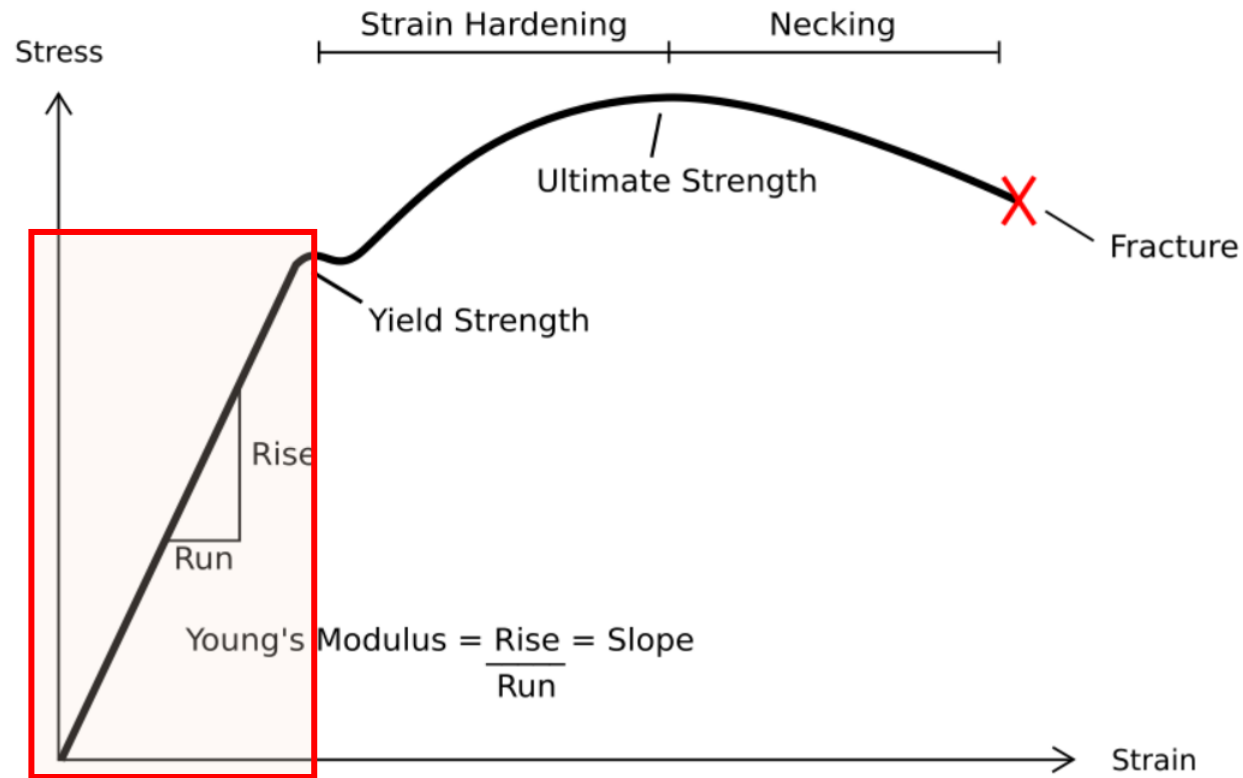
$$\sigma = E\epsilon$$

A deformation is elastic in case the body returns into its initial form after releasing the force. In case it stays deformed after releasing the force, the deformation is called plastic. For our purpose, the materials are modelled for the elastic case since the rotors should be high-fatigue-resistant and no plastic deformation should occur.



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Mechanical behaviour of ductile materials



In the first part, the material response is linear meaning that stresses are proportional to strains, with the proportionality constant equal to the elastic modulus of the material (E); upon unloading, full recovery of the strains occurs.



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Strength Criteria - Von Mises stresses

To fully describe the state of stress in a body, the description of a **multiaxial stress state** is needed (stress tensor). For simplifying the analysis of material failure, an equivalent stress theory is adopted.

For ductile materials as steel, the mostly used is the **von Mises equivalent stress**, also called octahedral effective stress:

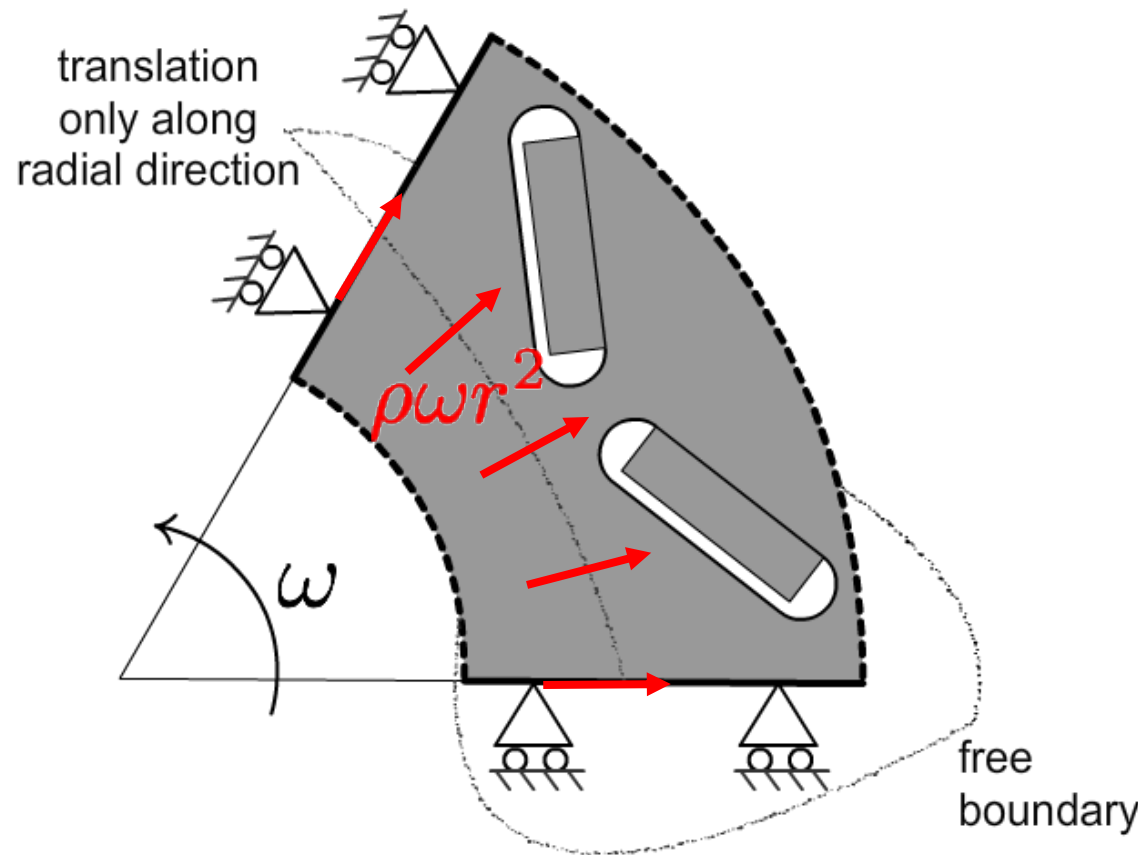
$$\sigma_{vM} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + \sigma_x \sigma_y - \sigma_x \sigma_z - \sigma_y \sigma_z + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)}.$$

The **von Mises equivalent stress** σ_{vM} is a suitable measure to get a global overview of the state of stress in a body and to analyse the mechanical stress in a machine component and is usually compared to the **material yielding strength** σ_y .



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Structural analysis and boundary conditions



The structural analysis assesses the mechanical stresses on the rotor caused by the centrifugal load when rotating at overspeed.

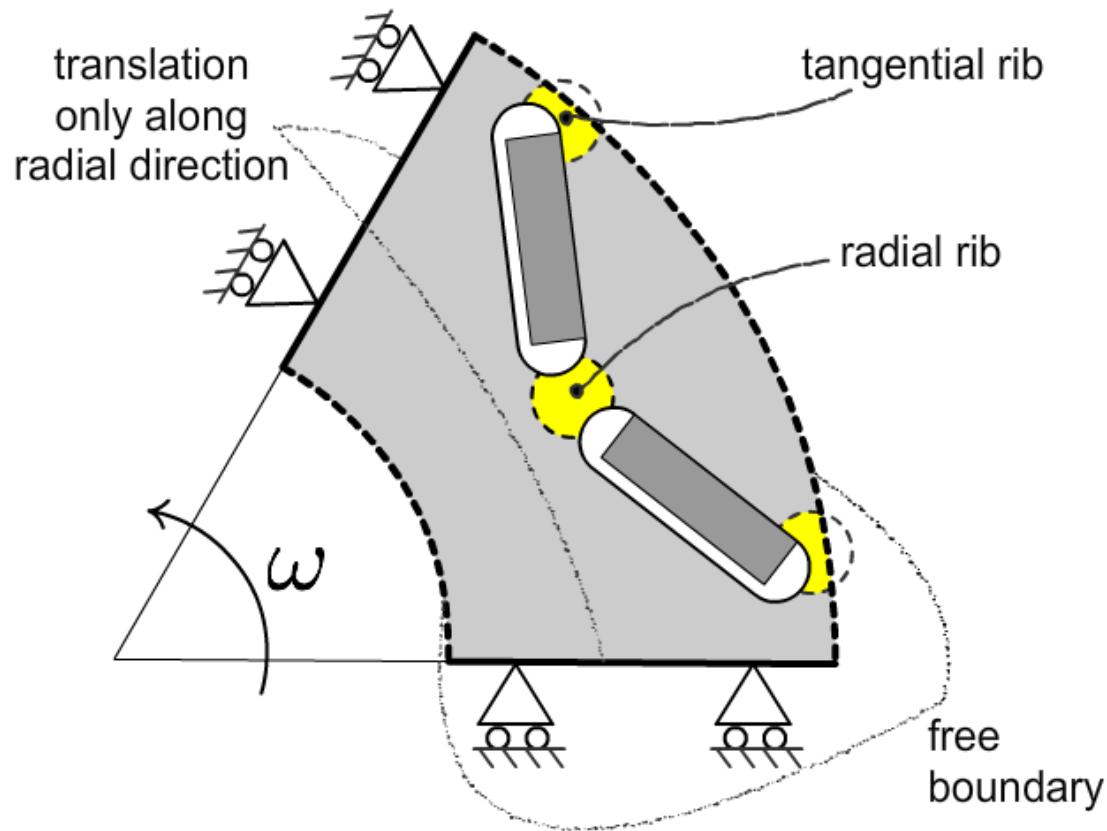
These are computed performing a 2D finite element analysis (FEA) under the plain strain hypothesis.

Due to symmetry conditions, it is sufficient to simulate only a part (one pole) of the rotor using appropriate boundary conditions



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Structural analysis and boundary conditions



Highlighted are the most stressed regions and in need of an advanced and accurate design.

High-accuracy meshing is crucial for ribs and shared across magnetic, thermal, and structural domains to maintain consistency.

From a structural point of view they should be increased in thickness...



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Structural-magnetic interaction

Radial and tangential ribs design does not depend only on structural needs but interacts with magnetic needs

Magnetic and Structural Trade-offs

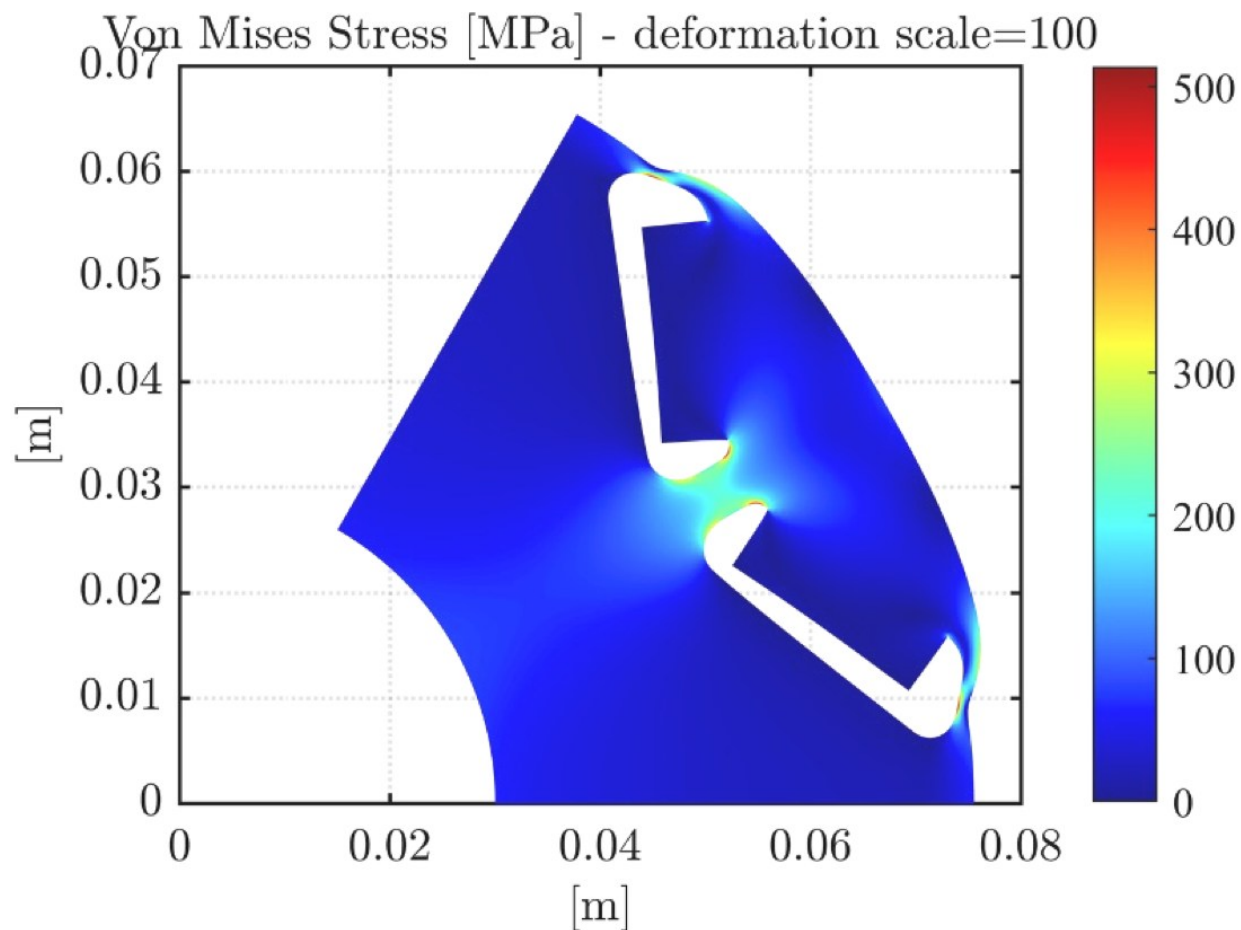
OPEN PROBLEMS:

- Ribs divert magnetic flux from the PMs, reducing magnetic anisotropy but are essential for structural stability.
- Rib thickness is minimized to ensure integrity during overspeed while not affecting magnetic performance.



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Von Mises Stresses - results



- To avoid inaccuracies due to local mesh variability, a reliable stress metric (VM99) is calculated as the average stress **above the 99th percentile** in rib regions.
- The deformed rotor shape and stress distributions highlight the ribs as critical stress regions, with VM99 used as the structural performance metric.