

Stat 443: Time Series and Forecasting

Assignment 2: Time Series Models

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Questions

1. Stationary Second-Order AR Process

Consider the stationary second-order AR process:

$$X_t = \frac{7}{10}X_{t-1} - \frac{1}{10}X_{t-2} + Z_t$$

where $Z_{t \in \mathbb{Z}}$ denotes a white noise process with mean zero and variance σ^2 .

a) Yule-Walker Equations

Derive the Yule-Walker equations and find the autocorrelation functions of $X_{t \in \mathbb{Z}}$.

b) Simulation and Plot

Assume $\sigma^2 = 1$. Using `set.seed(443)` to set the simulation seed and `arima.sim()` function simulate 1000 observations from the AR(2) process defined above. Plot its sample ACF for the first 15 lags along with the theoretical ACF obtained in part (a). Compare the sample and theoretical ACFs.

```
# R code block for simulation and plotting  
set.seed(443)  
# Further simulation and plotting code goes here
```

2. ARMA(2,1) Process

Consider the ARMA(2,1) process:

$$X_t = -0.5X_{t-2} + Z_t + 0.5Z_{t-1}$$

a) Stationarity and Invertibility

Check whether the process is stationary and invertible. Justify your answers.

We will begin by converting $\{X_t\}$ into a function of the characteristic polynomials $\varphi(B)$ and $\theta(B)$. We let B denote the backshift operator.

By rearrangement,

$$\begin{aligned}X_t &= -0.5X_{t-2} + Z_t + 0.5Z_{t-1} \\X_t + 0.5X_{t-2} &= Z_t + 0.5Z_{t-1} \\X_t(B^0 + 0.5B^2) &= Z_t(B^0 + 0.5B^1) \\X_t(1 + 0.5B^2) &= Z_t(1 + 0.5B)\end{aligned}$$

Therefore, we can write our ARMA(2,1) process as:

$$\text{ARMA}(2,1) : \varphi(B)X_t = \theta(B)Z_t \text{ Where } \begin{cases} \varphi(B) = 1 + 0.5B^2 \\ \theta(B) = 1 + 0.5B \end{cases}$$

b) Pure AR Process

Write the above ARMA(2,1) process as a pure AR process.

c) Pure MA Process

Write the above ARMA(2,1) process as a pure MA process.

d) Autocorrelation Function

Find the ACF of $X_{tt \in \mathbb{Z}}$.

3. AR(2) Process

Consider the AR(2) process given by:

$$2aX_t + \frac{3}{6a}X_{t-1} - \frac{1}{6a}X_{t-2} = Z_t, \quad a \in \mathbb{R}$$

a) Stationarity Conditions

Under which conditions on constant a is the process $X_{tt \in \mathbb{Z}}$ stationary?

b) Autocorrelation Function

Assuming that a satisfies conditions found in part (a), find the ACF of $X_{tt \in \mathbb{Z}}$.

4. SARIMA Model

Show that SARIMA(2,1,1) \times (0,1,1)₁₂ can be written as an ARMA(p,q) process and specify values of p and q .