Fourier Transform Activity

STAT 443

2024-03-13

Definition 1

Let $h(t)$ be a function of a real varaible t.The Fourier transform (FT) of h is defined as:

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-\mathrm{i}\omega t}dt$$

The inverse Fourier Transform is given by:

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{\mathrm{i}\omega t}d\omega$$

The transform $H(\omega)$ is finite if

$$\int_{-\infty}^{\infty} |h(t)|dt \text{ is finite.}$$

Show that

$$h(t) = h(-t) \implies H(\omega) \propto \int_{-\infty}^{\infty} h(t)\cos(t\omega)\ \mathrm{d}t$$

$$
\begin{aligned}
H(\omega) &= \int_{-\infty}^{\infty} h(t)e^{-\mathrm{i}\omega t}\ \mathrm{d}t \\
H(\omega) &= \int_{-\infty}^{\infty} h(t)[\cos(-\omega t) + \mathrm{i}\sin(-\omega t)]\ \mathrm{d}t \\
H(\omega) &= \int_{-\infty}^{\infty} h(t)[\cos(\omega t) - \mathrm{i}\sin(-\omega t)]\ \mathrm{d}t \\
H(\omega) &= \int_{-\infty}^{\infty} h(t)\cos(\omega t)\mathrm{d}t - \mathrm{i}\int_{-\infty}^{\infty} h(t)\sin(-\omega t)\ \mathrm{d}t \\
H(\omega) &= \int_{-\infty}^{\infty} h(t)\cos(\omega t)\mathrm{d}t - 0 \\
H(\omega) &= \int_{0}^{\infty} h(t)\cos(\omega t)\mathrm{d}t + \int_{-\infty}^{0} h(t)\cos(\omega t)\mathrm{d}t \\
H(\omega) &= 2\int_{0}^{\infty} h(t)\cos(\omega t)\mathrm{d}t \qquad \square
\end{aligned}
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