Practice Question 1

Let $\{X_t\}_{t\in\mathbb{Z}}$ be defined such that $X_t = Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2}$, where $Z_t \overset{\text{iid}}{\sim} \text{WN}(0, \sigma^2)$.

Part A

Given the model equation definitions discussed in this class, what type of process is $\{X_t\}_{t\in\mathbb{Z}}$?

Part B

Given that $\beta_1 = 0.5$ and $\beta_2 = -0.4$, is $\{X_t\}$ invertible? Is it stationary? Justify your answers.

Part C

Compute the normalized spectral density function $f^{\star}(\omega)$ for $\{X_t\}_{t\in\mathbb{Z}}$ for arbitrary coefficients β_1 and β_2. Assume that the process is stationary and invertible. Show your work.

Practice Question 2

Let $\{X_t\}_{t\in\mathbb{Z}}$ be defined as follows, where $Z_t \overset{\text{iid}}{\sim} \text{WN}(0, \sigma^2)$.

$$X_t = 1.2X_{t-1} - 0.4X_{t-2} + Z_t$$

Part A

Is $\{X_t\}$ invertible? Is it stationary? Justify your answers.

Part B

Determine the ACF of $\{X_t\}$ up to order 2.

Part C

Suppose that we have the data points $x_1 = 5$, $x_2 = 3$ and $x_3 = 4$. Given the process defined above, compute the 2-step ahead forecast for x_3.

Practice Question 3

Consider the following bivariate time series $(\{X_t, Y_t\})_{t\in\mathbb{Z}}$, assumed to be stationary for well-defined $\alpha, \beta \in \mathbb{R}$ and $Z_t \overset{\text{iid}}{\sim} \text{WN}(0, \sigma^2)$.

$$X_t = Z_t + \alpha Z_{t-1}, \text{ and } Y_t = Z_t + \beta Z_{t-1}$$

Part A

Compute the cross-covariance function $\gamma_{XY}(h)$ for this bivariate series.

Part B

For a stationary bivariate time series $(\{X_t, Y_t\})_{t\in\mathbb{Z}}$, show that $\gamma_{XY}(-h) = \gamma_{YX}(h)$.