STAT 447 Assignment 3

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Question 1 : Functions on the Unit Interval

For this question, use Simple Monte Carlo. The main twist compared to week one is that you will use a continuous random variable.

Part 1

Write a function called `mc_estimate` that takes a function $f : [0,1] \to \mathbb{R}$ and outputs the Monte Carlo estimato of $\int_0^1 f(x)dx$ using $n = 10,000$ independent samples from unif$(0,1)$.

Part 2

Consider the function $f : [0,1] \to [0,\infty)$ given by:

$$f(x) = \frac{1}{\sqrt[3]{x^2(1-x)}}$$

Note, importantly, that

$$\mathcal{I}_1 = \int_0^1 f(x)\mathrm{d}x = \frac{\pi}{\sin\left(\frac{\pi}{3}\right)}$$

Test your implementation of `mc_estimate` by checking that it produces an answer close to the value above.

```
eq_1_result = pi / (sin(pi/3))
eq_1_result
```

```
## [1] 3.627599
```

Part 3

The following integral, known as the sine integral, does not admit a closed-form expression.

$$\mathcal{I}_2 = \int_0^1 \frac{\sin(t)}{t}\mathrm{d}t$$

It does not admit a closed-form expression. Estimate its value using `mc_estimate(f)`.

Solution

To test our Monte Carlo Approximation, we will evaluate Si(1) using the `pracma` package.

```
Si(1)
```

```
## [1] 0.9460831
```

Question 2 : Posterior and Point Estimates

Part 1

Part 2

Part 3

Part 4

Question 3 : Bayes Action

Part 1

Part 2