Non-Normal Time Series Estimation

Introduction

Consider a first-order autoregressive time series $X_t = \varphi X_{t-1} + \varepsilon_t$, where the marginal distribution is non-normal and $\{\varepsilon_t\}$ is a sequence of *iid* random variables with characteristic function given by the the Characteristic Function / Laplace-Stieltjes transform:

$$\Phi_\varepsilon(s) = \frac{\Phi_X(s)}{\Phi_X(\varphi s)}$$

Where $\Phi_X(s)$ is the Characteristic Function (CF) of the stationary non-normal series $\{X\}_t$.

Gamma AR(1) Process

In this work, there is an estimation method for the gamma AR(1) process of Sim (1990). It replaces φX_{t-1} of the original model with $\varphi * X_{t-1}$, where $'*'$ is the discrete sum:

$$\varphi * X = \sum_{i=1}^{N(X)} W_i$$

Where $W_i \overset{\text{iid}}{\sim} \text{Exp}(\beta)$ (rate parameter β) and for each fixed value of x, $N(x) \sim \text{Pois}(\alpha\beta x)$.

Then, the AR(1) gamma model places marginal distribution $\text{Gamma}(\nu, \beta(1-\varphi))$ on each X, given by

$$X_t = \varphi * X + \varepsilon_t$$

In the above, $\{\varepsilon_t\}$ are gamma-distributed with rate β and shape ν.

The marginal density of $\{X_t\}$ is given by: