Chaotic Initialization

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2024-09-15

Swarm Initialization

We investigate the following swarm initialization procedure using logistic maps with a bifurcation coefficient $\mu \in [\approx 3.569945672, 4]$.

Consider the logistic map, defined as follows:

$$y_{i+1} = \mu \cdot y_i \cdot (1 - y_i), \text{ where } i \in 0, 1, 2, \dots \tag{1}$$

The resulting y_i is distributed in the interval $(0, 1)$, and the initial $y_0 \in (0, 1)$ such that $y_0 \notin \{0, 0.25, 0.50, 0.75, 1\}$.

To match our nomenclature of PSO, we rewrite (1) as follows:

$$y_j^{(i+1)} = \mu \cdot y_j^{(i)} \cdot (1 - y_j^{(i)}), \text{ where } j \in [1, D] \tag{2}$$

Time-Varying Acceleration Coefficients (TVAC)

Time-varying acceleration coefficients (abbreviated as TVAC) is formulated to enhance the global search in the early stages of the optimization and to encourage the particles to converge towards the global optima at the end of the search, which can be achieved by changing the acceleration coefficients c_1 and c_2 with time in such a manner that the cognitive component is reduced while the social component is increased as the search proceeds.

With a large cognitive component and small social component at the beginning, particles are allowed to move around the search space instead of moving toward the population best during early stages. On the other hand, a small cognitive component and a large social component allow the particles to converge to the global optima in the latter stage of the optimization process.

TVAC can be mathematically represented as:

$$c_1(t) = (c_{1i} - c_{1f}) \times \left(\frac{T - t}{T}\right) + c_{1f}$$
$$c_2(t) = (c_{2i} - c_{2f}) \times \left(\frac{T - t}{T}\right) + c_{2f}$$

By default, $c_{1i} = c_{2f} = 0.5$ and $c_{2i} = c_{1f} = 2.5$

Where, from the results (source) it hasbeen observed that best solutions can be determined when changing c_1 from 2.5 to 0.5 and changing c_2 from 0.5 to 2.5, over the full range of search.

We can plot these results below.

Sine-Cosine Acceleration Coefficients (SCAC)

Compared to TVAC, SCAC can better balance the global search ability in the early stage and the global convergence in the later stage.

$$c_1(t) = \varrho \times \sin\left(\left(\frac{T-t}{T}\right) \times \frac{\pi}{2}\right) + \delta$$
$$c_2(t) = \varrho \times \cos\left(\left(\frac{T-t}{T}\right) \times \frac{\pi}{2}\right) + \delta$$
$$\text{By default, } \varrho = 2 \text{ and } \delta = 0.5$$

Nonlinear Dynamic Acceleration Coefficients (NDAC)

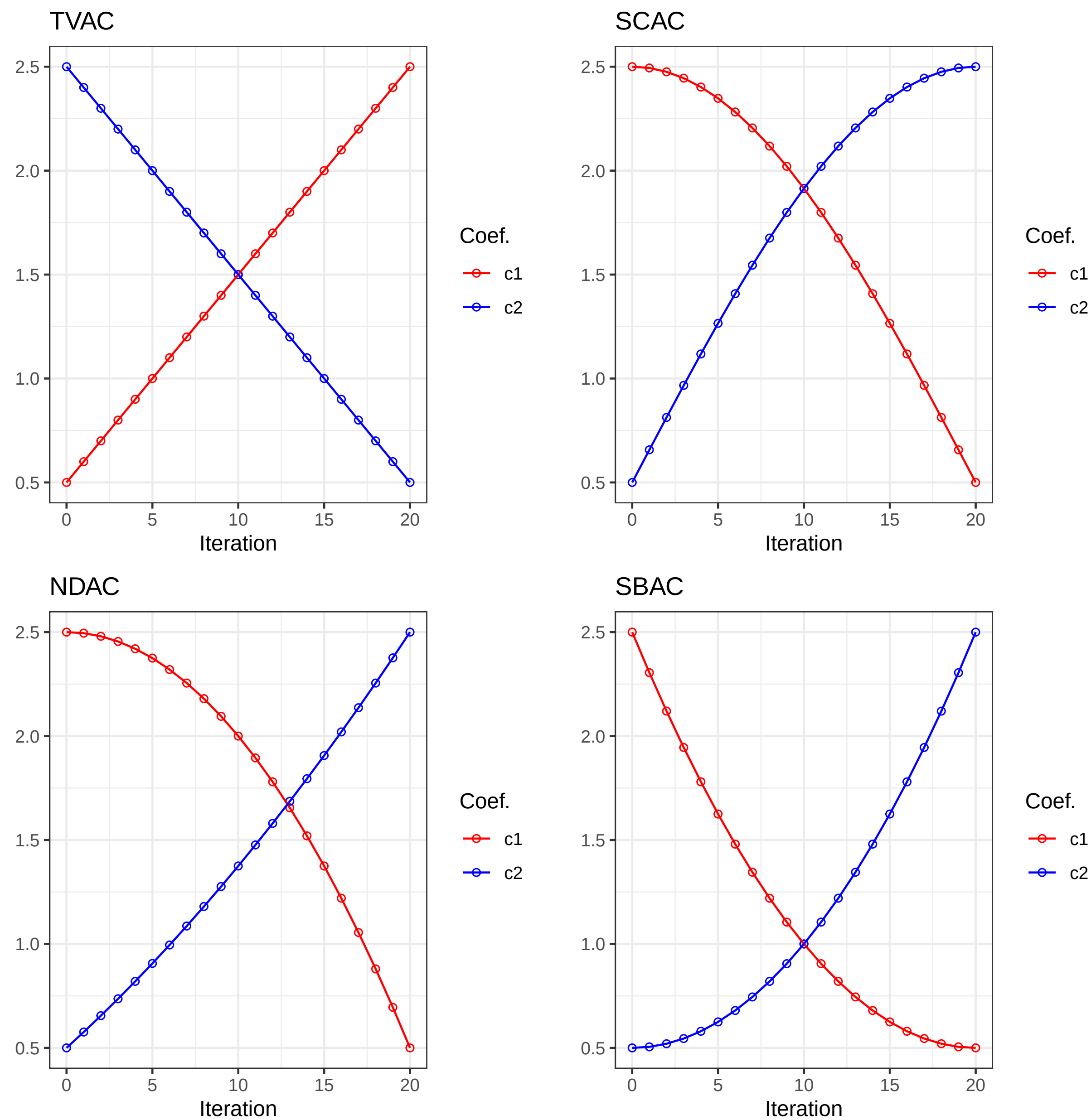
In literature (source 7), the nonlinear dynamic acceleration coefficients (NDAC) is added in the PSO method as a parameter update mechanism that has powerful capability of tuning cognitive component c1 and social component c2 .

$$c_1(t) = -(c_{1f} - c_{1i}) \times \left(\frac{t}{T}\right)^2 + c_{1f}$$
$$c_2(t) = c_{1i} \times \left(\frac{T-t}{T}\right)^2 + c_{1f} \times \left(\frac{t}{T}\right)$$
$$\text{By default, } c_{1i} = 0.5 \text{ and } c_{1f} = 2.5$$

Sigmoid-Based Acceleration Coefficients (NDAC)

Proposed in the CPSOS work!

$$c_1(t) = \frac{1}{1+\exp(-\lambda t/T)} + (c_{1f} - c_{1i})\left(\frac{t-T}{T}\right)^2$$
$$c_1(t) = \frac{1}{1+\exp(-\lambda t/T)} + (c_{1f} - c_{1i})\left(\frac{t}{T}\right)^2$$
$$\text{By default, } c_{1i} = 0.5, c_{1f} = 2.5 \text{ and } \lambda = 0.0001$$


TVAC
SCAC
NDAC
SBAC
2.5
2.0
1.5
1.0
0.5
0
5
10
15
20
Iteration
Coef.
c1
c2