SVFs and GVFs in PSO

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Slowly-Varying Functions

A function $\ell(x)$ is said to be a Slowly Varying Function (SVF) as $x \to \infty$ if for any constant $c > 0$

$$\lim_{x\to\infty}\left(\frac{\ell(cx)}{\ell(x)}\right) = 1$$

The authors (link) use $\ell(x) = (10\ln(x))^\alpha$. We can verify that ℓ is an SVF as follows:

$$\begin{aligned}
\lim_{x\to\infty}\left(\frac{\ell(cx)}{\ell(x)}\right) &= \lim_{x\to\infty}\left(\frac{10^\alpha \ln(cx)^\alpha}{10^\alpha \ln(x)^\alpha}\right) \\
\lim_{x\to\infty}\left(\frac{\ell(cx)}{\ell(x)}\right) &= \lim_{x\to\infty}\left(\frac{\ln(x)^\alpha + \ln(c)^\alpha}{\ln(x)^\alpha}\right) \\
\lim_{x\to\infty}\left(\frac{\ell(cx)}{\ell(x)}\right) &\overset{x \text{ dominates}}{=} \lim_{x\to\infty}\left(\frac{\ln(x)^\alpha}{\ln(x)^\alpha}\right) = \boxed{1}
\end{aligned}$$

Regular-Varying Functions (RVF)

A Regular Varying Function (RVF) is one that behaves like a power of x, meaning that it grows at a regular rate as x increases. Mathematically, a function $f(x)$ is regularly varying if it satisfies:

$$\lim_{x\to\infty}\left(\frac{f(cx)}{f(x)}\right) = c^\delta$$

For some constant δ.The authors let $L(x) = x^\delta \ell(x)$ as an RVF. This makes sense as an RVF because multiplying an SVF by x^δ transforms the slowly varying function into one that grows or shrinks at a regular rate. The control parameter δ lets you fine-tune this behavior to promote faster or slower convergence depending on the stage of the optimization process.

In Update Equations

The SVF $\ell(x)$ and RVF $L(x)$ are used in the PSO Update Equations based on t. For small t, the RVF is used to perturb the $\mathbf{X}$-values as follows:

$$\mathbf{X}_{t+1}^{(i)} = \begin{cases} \mathbf{X}_t^{(i)} + \mathbf{V}_t^{(i)} + \text{RVF}(\mathbf{X}_t^{(i)}), & \text{if } t \leq \lceil \rho T \rfloor \\ \mathbf{X}_t^{(i)} + \mathbf{V}_t^{(i)} + \text{SVF}(\mathbf{X}_t^{(i)}), & \text{otherwise} \end{cases}$$

Where t is the current iteration, T is the max iteration and ρ is a control parameter dictating when the update perturbations switch from RVF to SVF.

Implementation:

```
SVF <- function(x, alpha = 0.25){
  return((10 * log(x))^(alpha))
}
x = seq(from = 1, to = 10, length.out = 1000)
plot( SVF(x), type = 'l', ylim = c(0, 7) )

RVF <- function(x, delta = 0.5){
  return( x^(delta)*SVF(x))
}

lines( RVF(x) , col = 'red')
```