ECE 101L

LAB 2: Fundamental Circuit Theory Theorems

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Section A

1 Introduction

1.1 Purpose of Experiment

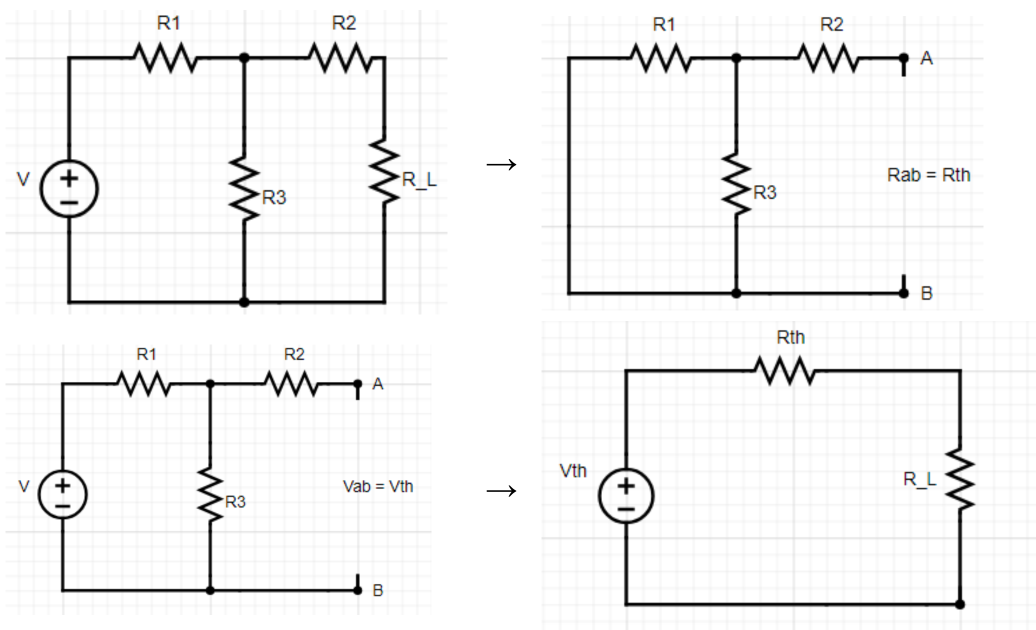
Three theorems fundamental to the study of circuit theory are explored in this laboratory. The first two are classic engineering models designed to represent lumped linear non-ideal voltage and current sources. These are known as the Thevenin equivalent circuit for the non-ideal voltage source, and Norton equivalent circuit for the non-ideal current source. We will then use these engineering models to consider what optimum load resistance is required for maximum power to flow from a non-ideal source into a resistive load that experimentally confirms the maximum power transfer theorem.

1.2 Background Theory

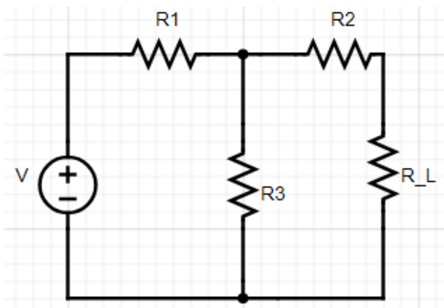
Thévenin's theorem states that *"Any linear electrical network containing only voltage sources, current sources and resistances can be replaced at terminals A–B by an equivalent combination of a voltage source* V_{th} *in a series connection with a resistance* R_{th}*."*

- The equivalent voltage V_{th} is the voltage obtained at terminals *A–B* of the network with terminals *A–B* open circuited.
- The equivalent resistance R_{th} is the resistance that the circuit between terminals *A* and *B* would have if all ideal voltage sources in the circuit were replaced by a short circuit and all ideal current sources were replaced by an open circuit.

- If terminals A and B are connected to one another, the current flowing from A and B will be $\frac{V_{\text{th}}}{R_{\text{th}}}$. This means that R_{th} could alternatively be calculated as V_{th} divided by the short-circuit current between A and B when they are connected together.



Norton's theorem is a simplification that can be applied to networks made of linear time-invariant resistances, voltage sources, and current sources. At a pair of terminals of the network, it can be replaced by a current source and a single resistor in parallel. The Norton equivalent circuit is used to represent any network of linear sources and impedances at a given frequency. To find the equivalent, the Norton current I_{no} is calculated as the current flowing at the terminals into a short circuit (zero resistance between A and B). The Norton resistance R_{no} is found by calculating the output voltage produced with no resistance connected at the terminals; equivalently, this is the resistance between the terminals with all (independent) voltage sources short-circuited and independent current sources open-circuited, which is equivalent to calculating the Thevenin resistance.



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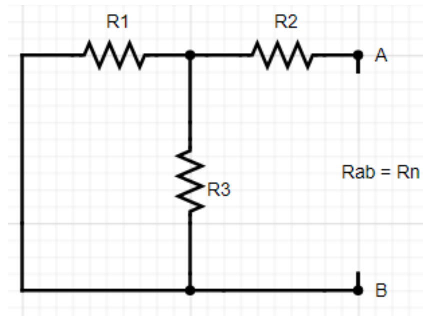

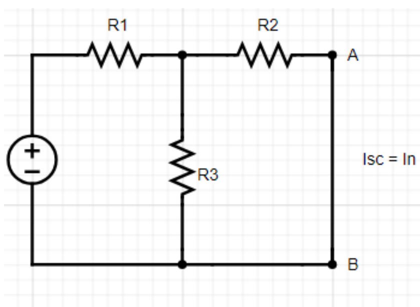

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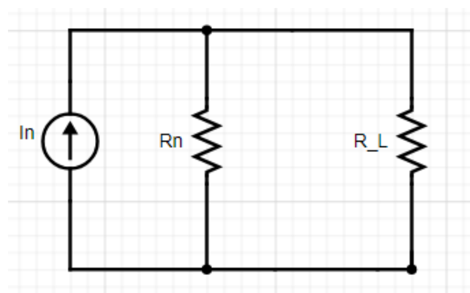


Norton's theorem and its dual, Thévenin's theorem, are widely used for circuit analysis simplification and to study circuit's initial-condition and steady-state response.

The Norton equivalent is related to the Thévenin equivalent by:

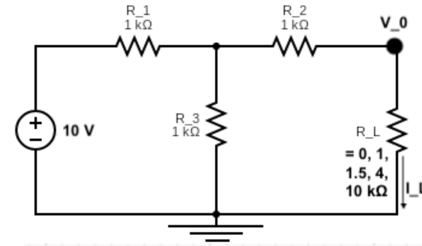
$R_{\text{th}} = R_{\text{no}}$, $V_{\text{th}} = I_{\text{no}} R_{\text{no}}$, and $\dfrac{V_{\text{th}}}{R_{\text{th}}} = I_{\text{no}}$

The maximum power transfer theorem states that, to obtain *maximum* external power from a power source with internal resistance, the resistance of the load must equal the resistance of the source as viewed from its output terminals.

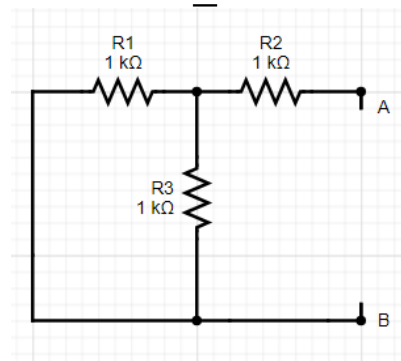
The Maximum Power Transfer Theorem: $\eta = \dfrac{P_{\text{L}}}{P_{\text{TOTAL}}}$ or $P_{max} = \frac{1}{4R_{th}} V_{th}^{2}$

1.3 Expected Results

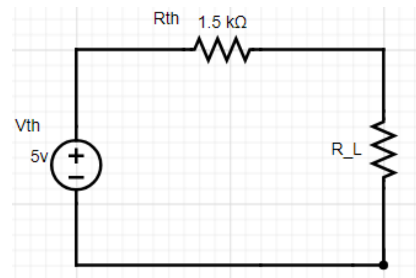
We consider the following circuit, and show calculations for $R_{load} = 0$:



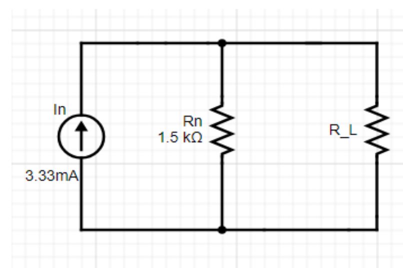
$$R_{\text{th}} = R_2 + R_{\text{load}} + \frac{1}{\frac{1}{R_3} + \frac{1}{R_1}} = 1 + 0 + \frac{1}{\frac{1}{1} + \frac{1}{1}} = 1.5k\Omega \quad \rightarrow$$



$$V_{\text{th}} = \frac{R_1}{R_1 + R_3} * 10V = \frac{1000}{1000 + 1000} * 10 = 5V \quad \rightarrow$$



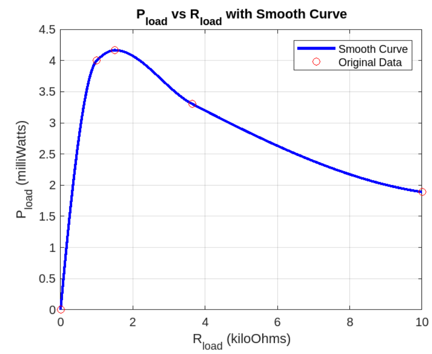
$$I_N = \frac{V_{\text{th}}}{R_{\text{th}}} = 5/1.5 = 3.333mA \quad \rightarrow$$



$$I_L = \frac{10 - \frac{10}{1 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + R_{\text{load}}}}}}{1 + R_{\text{load}}} = \frac{10 - \frac{10}{1 + \frac{1}{\frac{1}{1} + \frac{1}{1+0}}}}{1 + 0} = 3.333mA$$

$$V_0 = 10 - \frac{10}{1 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + R_{\text{load}}}}} - \frac{10 - \frac{10}{1 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + R_{\text{load}}}}}}{1 + R_{\text{load}}} = 10 - \frac{10}{1 + \frac{1}{\frac{1}{1} + \frac{1}{1+0}}} - \frac{10 - \frac{10}{1 + \frac{1}{\frac{1}{1} + \frac{1}{1+0}}}}{1 + 0} = 0V$$

$$P_L = I_L * R_L = \frac{10}{3} * 0 = 0$$



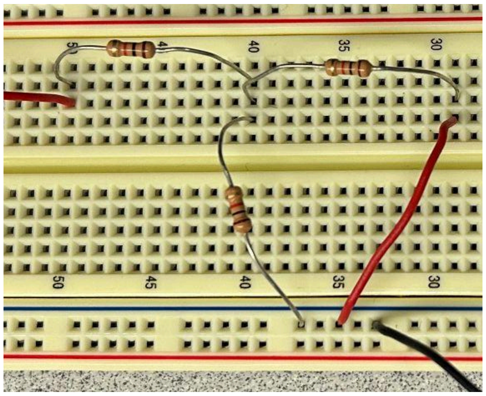
P_L vs R_L graph when the resistance of the load is equivalent to R_{th}:
We calculate max power transfer with $I_L = 1.667mA$ and then $P_{max} = \frac{1}{4R_{th}} V_{th}^2 = P_L = I_L^2 R_L = 4.167mW$
This can be seen in our graph as the maximum power, verifying our Max Power theorem (theoretically).

Theoretical Values, with $R_L = 0, 1, 1.5, 4, 10k\Omega$ are as follows:

R_L kΩ	I_L mA	V_0 V	R_{th} kΩ	V_{th} V	I_N mA	P_L mW
0	3.333	0	1.5	5	3.33	0
1	2	2	1.5	5	3.33	4
1.5	1.667	2.5	1.5	5	3.33	4.167
4	0.9091	3.636	1.5	5	3.33	3.306
10	0.4348	4.348	1.5	5	3.33	1.89

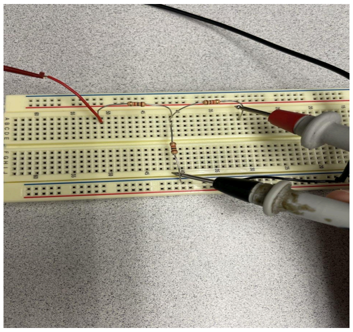
2 Results and Analysis

We build the following circuit for $R_{load} = 0$:



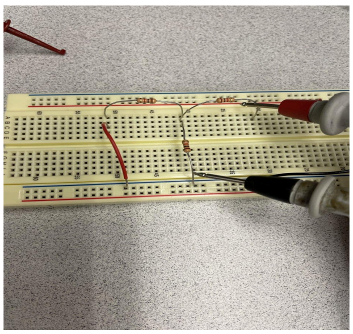
We replace the R_{load} wire with resistors for 1, 1.5, 4, and 10 kΩ and the circuit remains the same, with actual values of $R_1 = 0.9895$, $R_2 = 0.9872$, and $R_3 = 0.9933$.

For the T-circuit above, we first start with taking measurement of the Thevenin resistance by opening the circuit and measuring across the terminals and shorting the source.



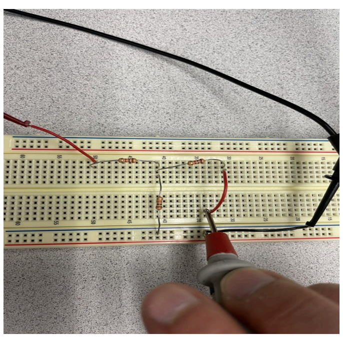
Measurement indicated the Thevenin resistance was $R_{th} = 1.4941k\Omega$.

We measure the Thevenin voltage by measuring the voltage across the open circuit terminals and reconnect the power supply to the input.



Measurement indicated that the Thevenin Voltage was $V_{th} = 4.998v$.

We measure I_{sc} by shorting the terminals with a wire and measuring the current to obtain the measured I_N by placing the ground probe at ground and the input probe at the end of the wire to set the measurement in series with the circuit.



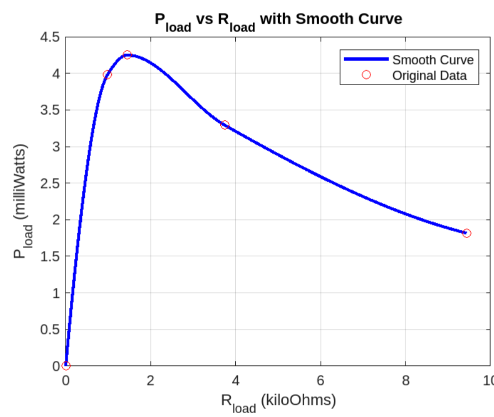
Measurement indicated that the Norton Current was $I_N = 3.3467mA$

After collecting the data for V_{th}, R_{th}, and , I_N we reconstruct the circuit seen above with the appropriate load resistances, starting with the 0th case, and collect data for values V_o, I_L, P_L and repeat measurement procedures for the respective values of each R_L case.

Experimental Values are as follows:

R_L kΩ	I_L mA	V_0 V	R_{th} kΩ	V_{th} V	I_N mA	P_L mW
0.000119	3.3467	0.007	1.4941	4.998	3.3467	0.001333
0.98448	2.0113	1.9945	1.4941	4.998	3.3467	3.982544
1.44744	1.7183	2.4731	1.4941	4.998	3.3467	4.249527
3.7355	0.9392	3.5971	1.4941	4.998	3.3467	3.295072
9.4389	0.43871	4.3411	1.4941	4.998	3.3467	1.816672

Plotting the measured data into a P_L vs R_L graph shows the general behavior of the data, theoretical or experimental, is the same:



This verifies the Max Power Theorem.

3 Error Analysis

With % error calculated as follows, $\% \text{ Error } = 100 * \frac{\text{Calculated} - \text{Measured}}{\text{Calculated}}$ *, we find:*

Equivalent circuit theorem values:

Value	Theoretical	Measured	% Error
V_{th}	$5v$	$4.998v$	0.04
R_{th}	$1.5k\Omega$	$1.4941k\Omega$	0.393
I_N	$3.33mA$	$3.3467mA$	0.502

The 0th R_{load} kΩ case:

Value	Theoretical	Measured	% Error
R_L	$0k\Omega$	$0.000119k\Omega$	100
V_o	$0v$	$0.007v$	100
I_L	$3.33mA$	$3.3467mA$	0.393
P_L	$0mW$	$0.001333mW$	100

The 1kΩ R_{load} case:

Value	Theoretical	Measured	% Error
R_L	$1k\Omega$	$0.98448k\Omega$	1.5
V_o	$2v$	$1.9945v$	0.275
I_L	$2mA$	$2.0113mA$	0.565
P_L	$4mW$	$3.982544mW$	0.4364

The 1.5kΩ R_{load} case:

Value	Theoretical	Measured	% Error
R_L	$1.5k\Omega$	$1.44744k\Omega$	3.504
V_o	$2.5v$	$2.4731v$	1.076
I_L	$1.667mA$	$1.7183mA$	3.077
P_L	$4.167mW$	$4.249527mW$	1.98

The 4kΩ R_{load} case:

Value	Theoretical	Measured	% Error
R_L	$4k\Omega$	$3.7355k\Omega$	6.61

V_o	$3.363v$	$3.5971v$	6.96
I_L	$0.9091mA$	$0.9392mA$	3.31
P_L	$3.306mW$	$3.295072mW$	0.33

The 10kΩ R_{load} case:

Value	Theoretical	Measured	% Error
R_L	$10k\Omega$	$9.4389k\Omega$	5.611
V_o	$4.348v$	$4.3411v$	0.159
I_L	$0.4348mA$	$0.43871mA$	0.899
P_L	$1.89mW$	$1.816672mW$	3.880

Conclusion

Through conducting this lab, we thoroughly understand and have proven both the theory and the application of the Thevenin and Norton circuit theorems. They provide two viable techniques to determine the maximum power transfer across a load resistance. We also can recognize the two techniques' relationship. We experimentally verified the maximum power transfer theorem as demonstrated by the behavior of the graphs of both the expected and measured data. Our % errors were all within a reasonable margin (except the expected 100% error when we find % error for a calculated value of 0).

Sources

Wikipedia Contributors. "Thévenin's Theorem." *Wikipedia*, Wikimedia Foundation, 17 Oct. 2019, en.wikipedia.org/wiki/Th%C3%A9venin%27s_theorem.

Wikipedia Contributors. "Norton's Theorem." *Wikipedia*, Wikimedia Foundation, 24 Sept. 2022, en.wikipedia.org/wiki/Norton%27s_theorem.

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