THE RATIO E/M

INTRODUCTION

In this experiment, we will observe the circular path of an electron beam in a partially evacuated glass tube. The path is observable because the electron beam partially ionizes the background helium gas, which in turn gives off a bluish glow. We will see how the charge-to-mass ratio, *q/m,* for these particles closely equals *e/m* for electrons[18], leading us to infer that a beam of electrons indeed is traversing the tube. Although the experiment itself is straightforward, there is a good deal of useful theory and equipment background.

We learned in class that if a charged particle (say an electron or ion), is moving in a region of space where there is a magnetic field but no electric field, then the particle will travel in a circular or a helical path (see Figure 32), and it will neither gain nor lose speed. It turns out that there is a unique relationship between the projected radius of the spiral, the energy of the particle, the magnetic field, and the ratio *q/m* of the particle. This is the principle of the *mass spectrometer*, which is widely used in chemistry and biology to sort ions of distinct mass and/or distinct charge.

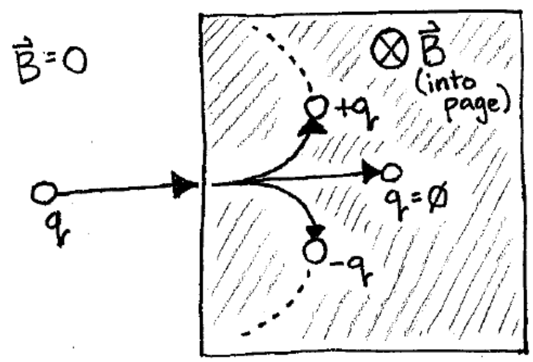


Figure 32. A particle with charge q moving from a region with no magnetic field to one where the field points into the page (shaded region). The particle's path will curve in a direction depending on its charge; an uncharged particle will be unaffected by the field.

The force, $\vec{\mathbf{F}}$, which causes charged particles of charge q to change their direction in a magnetic field, $\vec{\mathbf{B}}$, is called the *Lorentz force:*

$$\vec{\mathbf{F}} = q[\vec{\mathbf{E}} + (\vec{\mathbf{v}} \times \vec{\mathbf{B}})] \tag{25}$$

Here, $\vec{\mathbf{v}}$ is the velocity vector of the charged particle, and $\vec{\mathbf{E}}$ is the electric field. For this experiment, there is no electric field in the region of interest, so the equation becomes:

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \tag{26}$$

[18] e is the charge of the electron, $1.6022 \cdot 10^{-19}$ Coulombs.

In a system where you know the directions of two of the vectors, $\vec{\mathbf{F}}$, $\vec{\mathbf{B}}$, or $\vec{v}$, it is easy to figure out the direction of the third by using the right hand rule (see Figure 33). It is important to note that the direction of $\vec{\mathbf{F}}$ with the right-hand rule is for a *positive* charge. The direction of $\vec{\mathbf{F}}$ for a *negative* charge is opposite that for the positive charge.

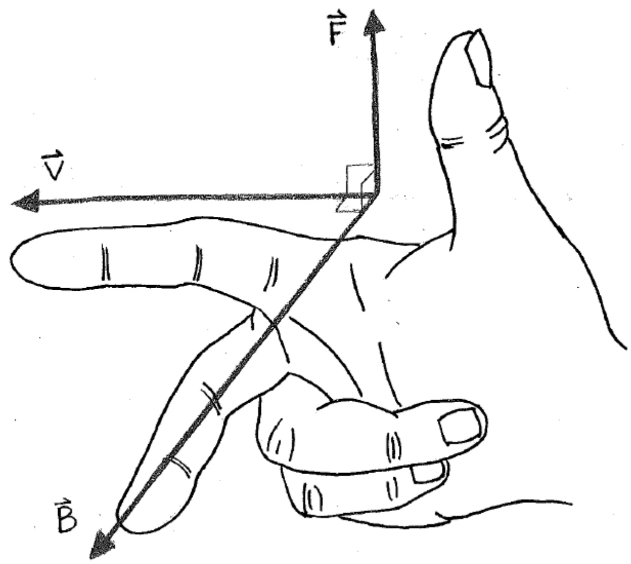


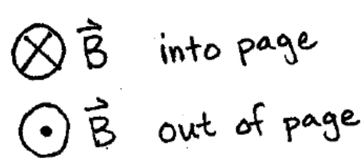
Figure 33. The right-hand rule for the Lorentz force. When aligning your index and middle fingers with $\vec{v}$ and $\vec{\mathbf{B}}$, respectively, your thumb will naturally point along the direction of $\vec{F}$ (for a positive charge q).

If $\vec{\mathbf{F}}$, $\vec{\mathbf{B}}$, and $\vec{v}$ are all perpendicular to one another (as in this experiment), the magnitude of the force, F, can be written as:

$$F = qvB \tag{27}$$

We will return to this equation shortly, as it will be quite useful in determining the charge-to-mass ratio for an electron.

A quick aside on standard notation for magnetic fields: A magnetic field pointing *into* the page is drawn as a circle with an "X" inside, while a field pointing *out of* the page is drawn as a circle with a dot inside. You can imagine this as an arrow pointing towards or away from you – if it is towards you, you can see the tip of the arrowhead, whereas if it is away from you, you can see the cross made by the arrow's feathers.



THE E/M APPARATUS

To study the motion of a charged particle in a magnetic field, we will observe the luminous trail left by an electron beam of known kinetic energy passing through a gas at very low pressure.

A schematic of our two types of *e/m* apparatus tubes are shown in Figure 34. The *heater* is a tiny coil of tungsten that is heated to incandescence, much as it would be in an incandescent light bulb. The tungsten in turn heats a tiny metallic thimble, known as the *cathode*.[19] When the cathode reaches a high enough temperature, the agitated electrons are willingly attracted to the nearby metal disc, called the *anode* or *plate,*[20] which is held at a positive potential, V, of up to 200 Volts. A torrent of electrons collides with the plate, except for a tiny stream that luckily was headed for a tiny aperture in the plate. These lucky electrons emerge in a straight line out of the anode, each with kinetic energy eV.

If we now immerse the tube in a magnetic field that is spatially constant, and which is perpendicular to the emerging stream of electrons, then the electrons will curve along a circular trajectory. We will be able to measure that curvature and accurately determine the ratio *e/m*.

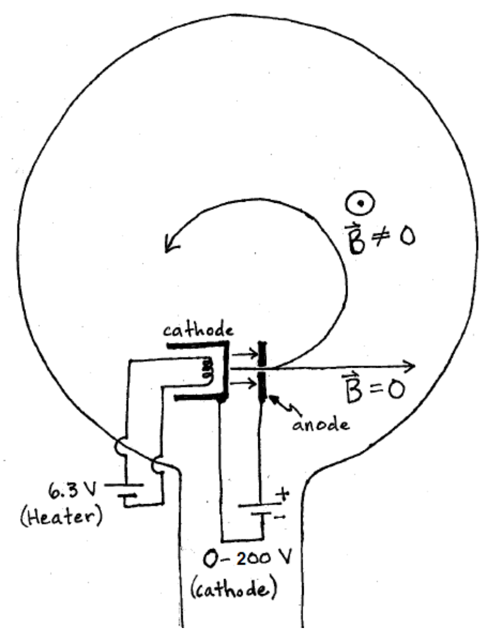


Figure 34. The cathode-ray tube used in the lab setups.

Knowing the plate voltage V, we can calculate the speed of the electrons as they emerge from the tiny hole using energy conservation. If the electrons have been accelerated through a potential difference of V, they will have gained a kinetic energy of eV. Since the kinetic energy is $(1/2)m_e v^2$, we can equate the two:

[19] In chemistry, the *cathode* is the electrode toward which positively charged ions, or cations, migrate.
[20] In chemistry, the *anode* is the electrode toward which negatively charged ions, or anions, migrate. Bare electrons are analogous to anions.

$$\frac{1}{2}m_e v^2 = eV \tag{28}$$

Later, we will need the *square* of the speed of the electron, which is given by

$$v^2 = \frac{2eV}{m_e}. \tag{29}$$

The tube is situated in the center of a pair of coils, similar to the Helmholtz coils we studied in the previous experiment.

Apparatus	N (number of turns)	R (radius of coil)	Coil separation
Large spherical tube	130	158 mm	158 mm

Table 1. Properties of the Helmholtz coils.

When the coils are energized, the tube will be immersed in a very uniform magnetic field, given by the expression in the previous lab:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R} \tag{30}$$

With the field coils energized, the electron beam should now be moving in a beautiful circular arc,[21] of radius r. We can easily calculate r by looking at the net force on the electrons, given by:

$$F = \frac{m_e v^2}{r} \tag{31}$$

Here, the net force is given by the electron mass multiplied by the centripetal acceleration (since the electrons are moving in a circular path). Since the only other significant force acting on the electrons is the Lorentz force, we can equate it to the net force:

$$\begin{aligned} evB &= \frac{m_e v^2}{r} \\ \Rightarrow v &= \frac{eBr}{m_e} \end{aligned} \tag{32}$$

Now, square this formula to get v^2, and set it equal to Equation (29):

[21] The tube is evacuated, except for a trace of helium. The visible trajectory is the trail of ions left behind caused by collisions of the electrons with the background gas.

$$\frac{e^2B^2r^2}{m_e^2} = \frac{2eV}{m_e} \tag{33}$$

After some rearranging and cancellations, we can solve for *e/m* in terms of the measured quantities V, B, and r:

$$\frac{e}{m_e} = \frac{2V}{B^2r^2} \tag{34}$$

The so-called *mass spectrometer* (or "*mass spec*") works on the principles given above. If the beam of charged particles consists of charged atoms or molecules, the beam will be dispersed according to the charge-to-mass ratio of particles. This is a very common way of analyzing the atomic or molecular composition of a sample, and it can be used even with very massive molecules, such as proteins.

SETUP

The setup consists of the following elements:

1. The **heater/filament power supply**[22] sends current to the tungsten filament, which heats up the cathode. This has been set up to provide a constant 6.3 V across the filament and does not need adjusting. An orange glow near the filament portion of the cathode ray tube indicates that the heater is working properly.
2. The **plate/anode voltage power supply** provides a potential difference between the cathode and the anode, accelerating the electrons. This can be varied between 0 – 200 V
3. The **e/m apparatus** consists of two Helmholtz coils, a cathode ray tube, and connections to various power supplies. The electron beam can be turned on and off with the red push-button on the apparatus near the tube.
4. The **current power supply** provides current to the Helmholtz coils in order to generate a magnetic field. The voltage knob should be turned all the way clockwise to allow for the full range of current values. The current knob should only be adjusted on this power supply. ALWAYS TURN THE CURRENT TO ZERO before turning off or disconnecting this power supply. <u>Only run the current long enough to take measurements, otherwise the coils will overheat</u>!

CAUTION: NEVER SET THE CATHODE TO PLATE (ANODE) VOLTAGE TO GREATER THAN 200 **VOLTS. TO DO SO MAY PERMANENTLY DAMAGE THE TUBE.**

[22] The make and model of the power supply may vary. Specific instructions for your power supply will be at your lab station.

EXPERIMENTS

Preliminary experiments:

Turn on the main power switch of the heater/filament power supply (make sure the plate/anode voltage is set to zero), and let the tube warm up for about two minutes.

Now turn up the plate voltage and observe the path of the electrons. What happens as you increase the voltage? Specifically, what is happening to the velocity of the electrons, v?

Turn on the current power supply and increase the current. What happens to the path of the electrons? Does a larger magnetic field make the electrons curve more tightly or more loosely? Is this what you would expect from Equation (34)?

Now that there is a magnetic field present, try changing the plate voltage. Do the electrons curve more tightly for a larger or a smaller value of the voltage V? Is this what you would expect?

In the following experiment, you will determine the *e/m* ratio graphically, by plotting the voltage V against the square of the current, I^2. By combining Equations (30) and (34),

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R} \qquad \text{and} \qquad \frac{e}{m_e} = \frac{2V}{B^2 r^2},$$

we can rearrange to obtain V in terms of I^2:

$$V = \left(\frac{4}{5}\right)^3 \frac{e}{m_e} \frac{r^2 \mu_0^2 N^2}{2R^2} I^2 \tag{35}$$

For the two available e/m apparatus, this reduces to

$$V = \frac{e}{m_e} r^2 (2.737 \cdot 10^{-7}) I^2 \tag{36}$$

and using the slope of your best fit line will allow you to obtain *e/m* as

$$\frac{e}{m_e} = \frac{slope}{r^2 (2.737 \cdot 10^{-7})} \tag{37}$$

How to measure the radius *r*:

The radius of the circular path can be determined using the light-up ruler behind the tube. The ruler's position can be adjusted using the silver knobs at the ends of the ruler. As shown in Figure 35, you should position your eye such that the electron beam exactly overlays its reflection in the ruler, and read off the position in cm. Note that you will get the most accurate values of r if you place the ruler along the diameter of the circular path.

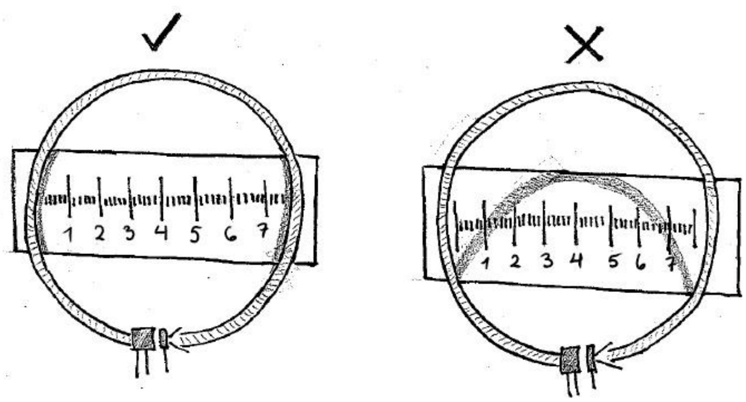


Figure 35. The correct way to align the beam with its reflection is shown on the left. An incorrect alignment is shown on the right.

In order to get an accurate measurement of r, take readings on both the right and left side of the circular beam (note the ruler is not centered at zero!). Subtract the left side reading from the right side reading and divide by two to get the radius of the beam:

$$r = \frac{r_{right} - r_{left}}{2}$$

Experiment

1. Set your beam to a radius between 3 and 6 cm. Record the coil current and plate voltage.
2. Change I and V such that you again get the exact same radius (it may be easiest to read the radius along the outside edge of the beam). Record these new values, and repeat until you have at least 4 different values of I and V for the same radius r.
3. In your lab notebook, make a graph and carefully plot I^2 on the x-axis and V on the y-axis.
4. Draw a best-fit line through the points and determine the slope of the line. The best-fit line should go through the point (0,0).[23] Plug this into Equation (37) along with your radius r in meters to get the ratio e/m. Compare your value with the currently accepted value of $1.7588 \cdot 10^{11}$ Coulomb/kg.

[23] If your line of best-fit does *not* go through the origin, it may be due to random fluctuations or a systematic error in the data. Compare the slope from your line to a straight-line fit that *does* go through the origin: which one corresponds better to the accepted value of e/m?

PRE-LABORATORY

1. Take *e/m* to be the accepted value of $1.76 \cdot 10^{11}$ C/kg. For a plate voltage of 100 volts and a circular beam of radius 3.0 cm, what would you expect the required magnetic field B to be?

2. Assume that the above magnetic field is produced by two Helmholtz coils, and assume that each coil has 119 turns, a radius of 106 mm, and a separation of 106 mm. What current I is required to achieve the expected magnetic field?