ECE 101L

LAB 3: Fundamental Circuit Theory Theorems

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Caden Roberts: 2082015

Section A

Introduction

Purpose of Experiment

In this laboratory we will investigate transient voltage and current relationships for single time-constant circuits involving capacitors and inductors. When excited by step voltages, such circuits exhibit exponential responses that we will experimentally observe and analyze to verify relevant circuit theory.

Background Theory

A capacitor reacts to a change in its terminal voltage by either discharging or charging energy into its electrostatic field. Unless the voltage is changing, there is nothing to react to, so there is no current flow and the device looks like an open circuit. Current is governed by $I = i_c(t) - C\frac{dv_c(t)}{dt}$. Resistors connected in series or parallel with capacitors form linear single time-constant (STC) circuits that obey superposition. When excited by a step change in voltage the capacitor either charges or discharges at a rate proportional to $e^{\frac{-t}{\tau}}$, where $\tau = RC$ is generally known as the time-constant . Relevant to this is Lenz's Law:

The current induced in a circuit due to a change in a magnetic field is directed to oppose the change in flux and to exert a mechanical force which opposes the motion.

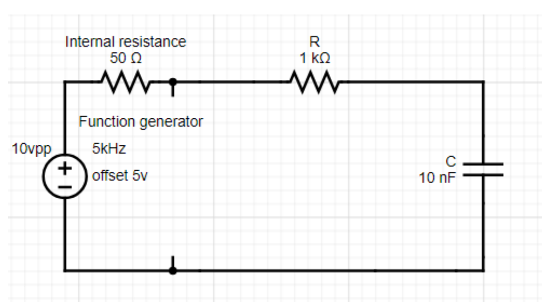
Lenz's Law tells us a capacitor will always respond to a change in voltage by generating a current that opposes it. When driven with a step change in voltage, the response is

$v(t) = v(\infty) + [v(0) - v(\infty)]e^{\frac{-t}{\tau}}$. $v(\infty)$ is the steady-state value after the exponential has gone to zero, and $v(0)$ is the initial voltage at $t = 0^+$. Note $v(0)$ accounts for any energy already stored in the capacitor before $t = 0$ when the step change in voltage occurs.

Inductors are also reactive passive electrical components that store energy. The inductor reacts to a change in its terminal current by either discharging or charging energy in its magnetic field. Unless the current is changing, there is nothing to react to so there is no voltage across the device and it looks like a short circuit. Voltage is governed by $V = v_L(t) - L\frac{di_L(t)}{dt}$. Resistors connected in series with inductors also form linear STC circuits that obey superposition. When excited by a step change in current the inductor either charges or discharges current at a rate proportional to $e^{\frac{-t}{\tau}}$, where $\tau = \frac{L}{R}$ is the time-constant. Lenz's Law tells us an inductor will always respond to a change in current by generating a voltage that opposes it. When driven with a step change in current, the response is $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{\frac{-t}{\tau}}$.

Expected Results

We consider the following circuit, and show calculations for $V_C(t)$ *with* $t = \tau$, $v(\infty) = 10V$, and $v(0) = 0V$:



We consider the voltage supplied from the function generator as a DC voltage of $10v$ to calculate the behavior of the voltage across varying points in time in terms of the time constant

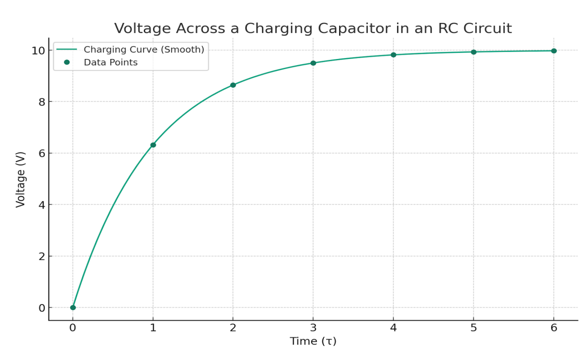
$\tau = (1k\Omega) * (10nF) = 10\mu s$. Through KCL, we see that the current flowing through the resistor is the same as the current flowing through the capacitor. We know that the relationship of current across a capacitor is the product of the capacitance and the change in the voltage with respect to time. Thus, we say $I_s = I_R = I_c \Rightarrow I_c = C\frac{dv_c(t)}{dt}$. By KVL, the sum of the voltage supplied, the voltage across the resistor, and the capacitor must equal 0: $-V_s + V_R + V_c = 0$. We can substitute and rewrite: $-V_s + R(C\frac{dv_c(t)}{dt}) + V_c = 0$.

After manipulating the equation we find $V_c(t) = V_s + (V_c - V_s)e^{-\frac{t-t_o}{RC}}$ where $\tau = RC = 10$. V_S, or $V(\infty)$, is the steady state voltage value, which is when the capacitor is fully charged and no longer in its transient state. In this case, $V(\infty) = 10v$. V_C is the initial voltage value across the capacitor $V_o(t_o)$. We see there is no voltage across the capacitor initially, therefore, $V_o(t_o) = 0$. Shown is calculation of $V_C(t)$ for $t = \tau$:

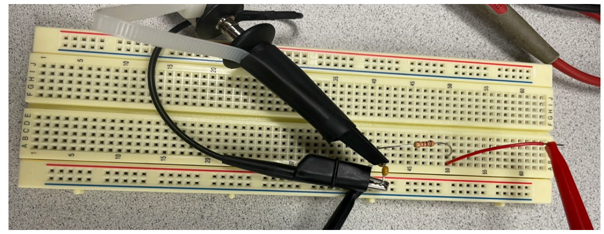
$$V_C(t) = v(\infty) + [v(0) - v(\infty)]e^{\frac{-t}{\tau}} = 10 + (0-10)e^{-1} = 6.321$$

τ	0	1τ	2τ	3τ	4τ	5τ	6τ
Voltage v	0	6.321	8.647	9.502	9.817	9.933	9.975

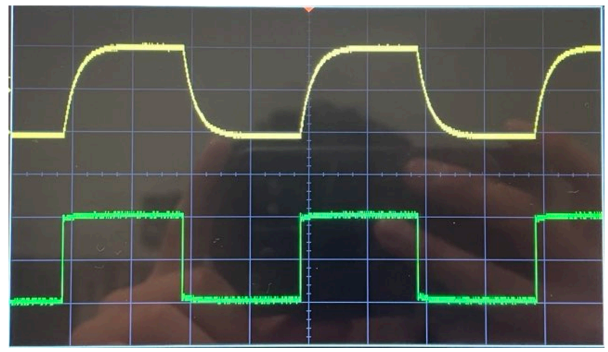
We plot the data on a voltage vs. time graph, finding an asymptotic exponential growth towards the maximum voltage (voltage supplied):



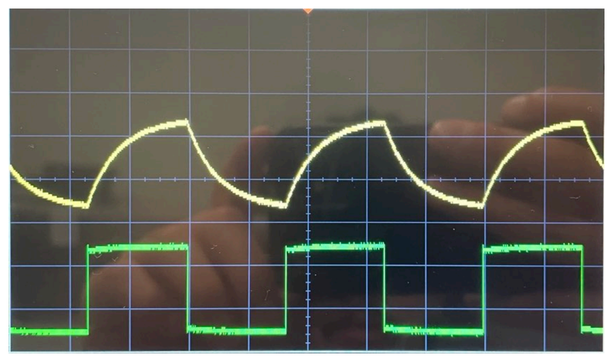
Results and Analysis



For the circuit above, we construct it and measure across the capacitor with the oscilloscope probes. We observe the following waveforms of input voltage and the voltage across the capacitor at 5kHz. The channel two probes were connected such that input signal waveform is displayed on the oscilloscope.



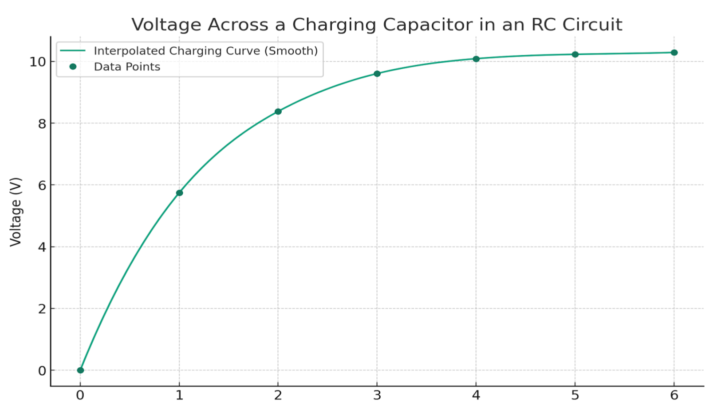
Then, we observe the waveform of the input voltage and the voltage across the capacitor at 15kHz. We see the waveform expectedly models a faster growth towards the maximum voltage across the capacitor due to the signal frequency increase.



Using the cursor function on the oscilloscope, we find the measured values at varying counts of $t = \tau$ at the $5kHz$ frequency since the waveform allows for an ideal measurement:

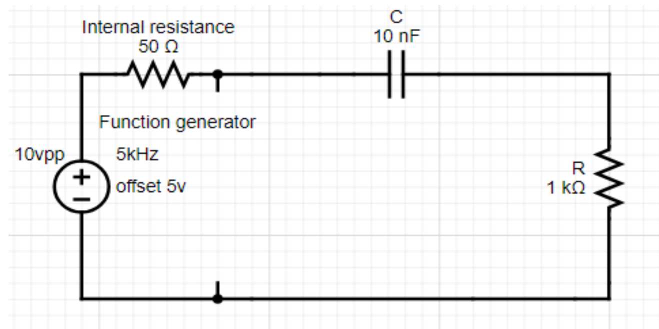
τ	0	1τ	2τ	3τ	4τ	5τ	6τ
Voltage v	0	5.74625	8.382	9.608	10.090	10.231	10.2915

Plotting this data verifies our expected results:



Part II

Expected Results

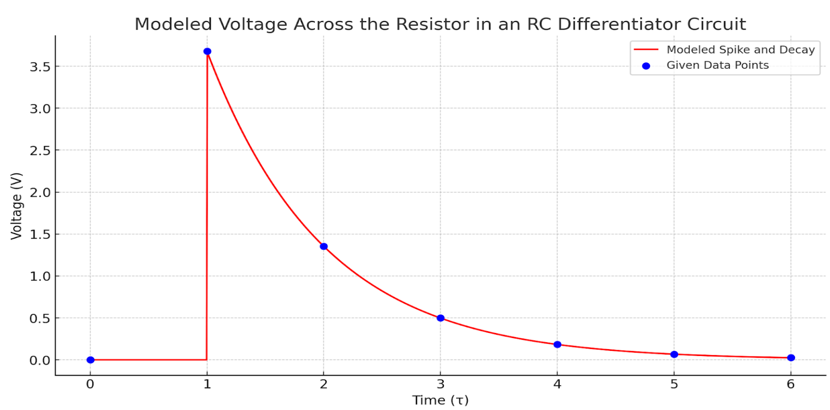


We reconstruct the circuit into the RC differentiator configuration and calculate the resistance as a function of time.

We use the values found by the calculations of $V_c(t)$ in part I and take the difference from the supplied voltage, yielding the dissipated voltage across the resistor $V_R(t)$ relative to the voltage across the capacitor as a function of time $V_R(t) = 10 - V_c(t)$:

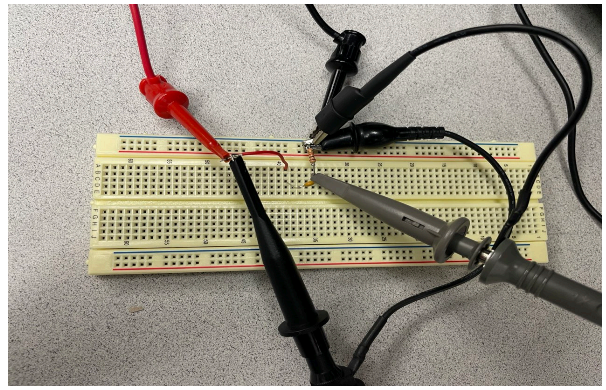
$Time$ τ	0	1τ	2τ	3τ	4τ	5τ	6τ
Voltage v	0	3.679	1.353	0.498	0.183	0.067	0.025

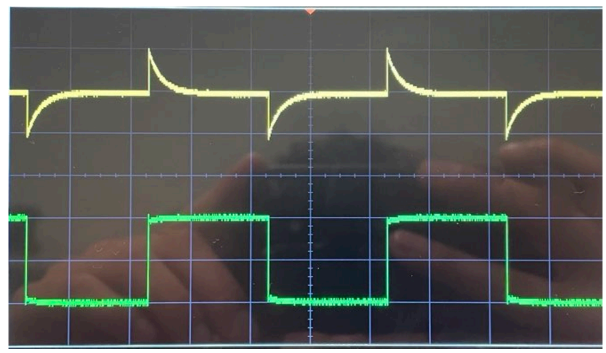
Plotting this data, we find that the behavior of the graph resembles the waveform later observed in measurement. Initially there is no voltage, followed by a sudden spike in voltage and exponential decay.

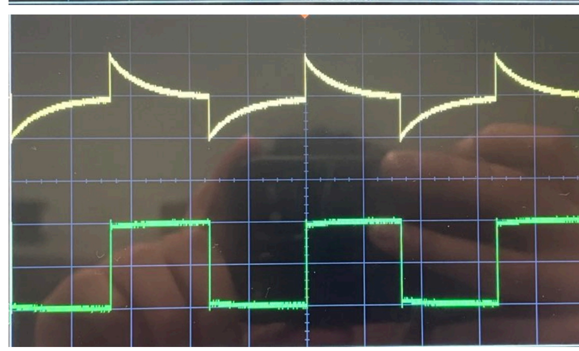


This models how the capacitor starts to behave as an open circuit as it nears max voltage; letting very little power left to be dissipated across the resistor.

Results and Analysis





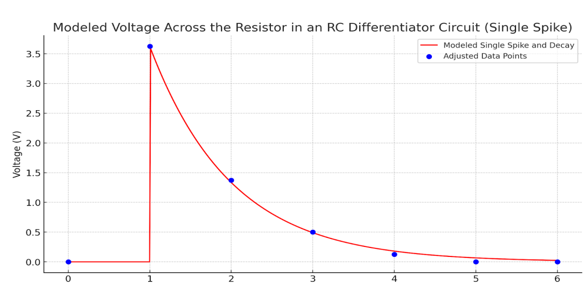


For the RC differentiator configuration, we switch the resistor and capacitor positions and measure across the resistor using the oscilloscope probes. The channel two oscilloscope probes are connected to observe both the waveforms generated across the capacitor and the input waveform of the differentiator circuit. We observe the $5kHz$ waveform[middle]. Notice that the waveform matches what was modeled from our theoretical data. Over time, the resistor dissipates less power due to the capacitor nearing max voltage as early discussed. Observing the waveform at $15kHz$:

Now, we measure the behavior of the voltage across the resistor using the $5kHz$ waveform with the cursor function on the oscilloscope:

τ	0	1τ	2τ	3τ	4τ	5τ	6τ
Voltage v	0	3.625	1.375	0.5	0.125	0	0

Plotting this data shows the oscilloscope is proving our theoretical behavior.. Therefore, our expected results are experimentally verified.



Error Analysis

With % error calculated as follows, $\% \text{ Error } = 100 * \frac{\text{Calculated} - \text{Measured}}{\text{Calculated}}$*, we find:*

Part I **(**integrator configuration case):

τ	0	1τ	2τ	3τ	4τ	5τ	6τ
Measured Voltage	$0v$	$5.74625v$	$8.382v$	$9.608v$	$10.090v$	$10.231v$	$10.2915v$
Expected Voltage	$0v$	$6.321v$	$8.647v$	$9.502v$	$9.817v$	$9.933v$	$9.975v$
Error	0%	9.09%	3.06%	1.11%	2.78%	3%	3.173%

Part II (differentiator configuration case):

τ	0	1τ	2τ	3τ	4τ	5τ	6τ
Measured Voltage v	$0v$	$3.625v$	$1.375v$	$0.5v$	$0.125v$	$0v$	$0v$
Expected Voltage v	$0v$	$3.679v$	$1.353v$	$0.498v$	$0.183v$	$0.067v$	$0.025v$
Error %	0%	1.46%	1.63%	0.4%	31.6%	100%	100%

Conclusion

Through this experiment, we learned about RC circuits, and RC integrator and differentiator configurations. We found that the relationship between the capacitor and the power dissipated across the resistor in the differentiator configuration verifies that a capacitor acts as an open terminal in the circuit as it nears max voltage. The drastic error margins in the RC differentiator table are due to the oscilloscope being unable to obtain highly precise values of negligible voltages. This verifies that the capacitor hardly lets more current flow through once it reaches its maximum voltage. Beyond instrument measurement precision limitations, the error margins were within considerably close range, hence verifying our theoretically calculated results.

Sources

Wikipedia Contributors. "Lenz's Law." *Wikipedia*, Wikimedia Foundation, 26 Aug. 2019, en.wikipedia.org/wiki/Lenz%27s_law.