SURFACE CHARGE; ELECTROSTATIC POTENTIAL; CAPACITANCE

INTRODUCTION

In the previous experiment we mostly investigated phenomena associated with *insulating* objects. An insulator is an object that can have excess charges attached to it, but the charges are fairly immobile. By contrast, a conductor is a material where the charges are extremely mobile. Within the boundaries of the conducting material the charges will rearrange themselves so as to prevent the existence of an electric field within the material.

Because of the above property, every point within a conducting material (not including points within hollow cavities, which are a special case), and on the surface of a conducting material, is at a constant *electrostatic potential* Φ, which we will refer to as V in this lab. In the non-conducting volumes (like air or vacuum), V is not necessarily constant, and can be a function of the observation point (x,y,z). Finally, we are free to choose an additive constant that fixes V to a convenient value at a convenient point. We usually (but not always) choose this additive constant so that V is equal to zero at infinite distances from our object of interest.

PROPERTIES OF A CHARGED CONDUCTING SPHERE

In the lecture course, we have discovered some remarkable properties of a charged, conducting sphere. If charges are attached to a solid or hollow conducting sphere that is isolated from other conducting surfaces, the following phenomena will occur:

- Because the charges are free and strongly repel one another, the charges will very quickly migrate to the outer surface of the sphere so as to be as far away from one another as possible.
- Since the curvature of the spherical surface is uniform, the charges will distribute themselves uniformly on the surface of the sphere. If the total charge on the sphere is Q, the sphere has surface area A, and the radius of the sphere is R, then the charge density will be

$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi R^2} \tag{1}$$

- We also recall that, for a uniformly charged sphere, the electric field at the surface of the sphere is the same as the field that would have resulted from a point charge at the origin. Thus,[5]

$$E(R) = \frac{\sigma}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 R^2} \tag{2}$$

[5] In SI units, $\varepsilon_0 = 8.854\cdot 10^{-12}$ Farad/meter and is called the *permittivity of free space*.

- Finally, we recall that the electrostatic *potential*, V, of a charged sphere is the same as the electrostatic potential that would have resulted from a point charge at the origin.[6] Thus,

$$V(R) = \frac{Q}{4\pi\varepsilon_0 R} \tag{3}$$

We can now put these properties to good use. If we connect an external voltage source to the sphere, with voltage V, then by combining the preceding equations we can determine the charge density σ:

$$Q = 4\pi\varepsilon_0 RV \quad \text{so} \; \sigma = \frac{Q}{4\pi R^2} = \frac{\varepsilon_0 V}{R} \tag{4}$$

This simple result tells us the value of the charge density on the surface of a sphere if it is charged to a potential V.

It turns out there is a very convenient way to indirectly observe the surface charge σ. If you take a small conducting surface, say a flat metal disc with radius r, and touch it tangentially to the surface of the sphere, charges on the part of the sphere that is "covered" by the disc will migrate outward to the disc itself, as if it were actually part of the sphere (see Figure 6). Then, if we then remove the disc, it will hold an amount of charge given by its area times the surface charge density:

$$Q_{disc} = \pi r^2 \sigma \tag{5}$$

A disc used in this way is called a "proof plane."

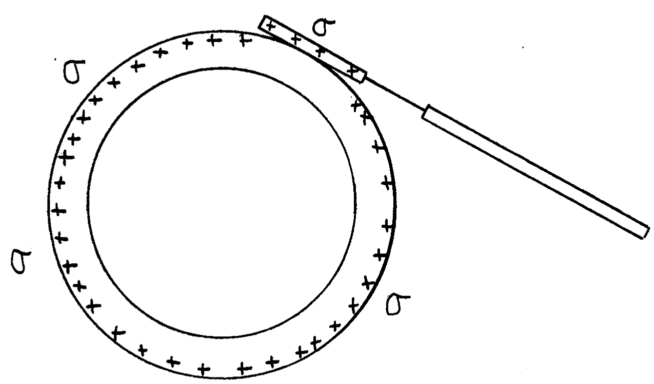

Figure 6 The distribution of charges between a sphere and a proof plane.

We can now utilize a nice feature of the Faraday cage introduced in the previous laboratory. If the conducting disc of the proof plane is held inside the Faraday cage, the electrometer will indicate the charge on the proof plane, *without* discharging the plane. However, if the proof plane touches the inner cylinder of the Faraday cage, all of the charge

[6] Remember that we are free to add a constant to the potential. Here, we choose the constant so that the potential is zero at infinite distance.

on the proof plane will flow to the inner cylinder. The electrometer will still read the correct charge, but the proof plane will itself have been neutralized.

In the following experiment we will observe the charge density on a conducting sphere that has been charged to a potential of 1,000 Volts.

Experiment 1

1. Set up the Faraday cage as in the previous lab.
2. Connect the positive terminal of the high voltage power supply to the spherical conductor, and the negative terminal to the aluminum pie tin on the table. Set the high voltage to 1,000 volts.[7]
3. You should have at your station a plastic wand with a metal disc. Touch this proof plane tangentially to the charged sphere, and then observe the charge on the proof plane by touching the proof plane to the inner cylinder of the Faraday cage. Note down the voltage reading on the electrometer. Is the charge transferred positive or negative?
4. Repeat step 3 a few times *without* re-zeroing the electrometer. What happens to the electrometer reading? (Remember to set the electrometer to a higher scale if the needle goes off-scale).
5. Try transferring charge from different parts of the sphere (e.g. top, bottom, left, right, etc). Is the charge transferred roughly the same each time? Would you expect this to still be the case if we had an irregularly shaped object instead of a sphere? Explain.
6. If there is time, test your prediction from step 5 with the irregularly shaped charged object in the back of the lab room.

PARALLEL PLATE CAPACITOR

If two conducting objects are not in electrical contact with one another, and if they have equal and opposite charges, then the objects will be at different electrostatic potentials.[8] In other words, positive free charges will be attracted to the negative object and repelled by the positive object. The potential difference, or voltage difference V, will depend only upon the charge and the geometry, according to the following formula:

$$Q = CV \tag{6}$$

The coefficient C is known as the *capacitance*, and depends upon the geometry for simple cases as follows:

[7] The power supply is not dangerous because it is in series with a $120 \cdot 10^6$ ohm resistor, which limits the current to about 10^{-5} ampere, which is harmless.

[8] Recall from the introduction that the electrostatic potential is constant everywhere within, and on the surface of, a conductor.

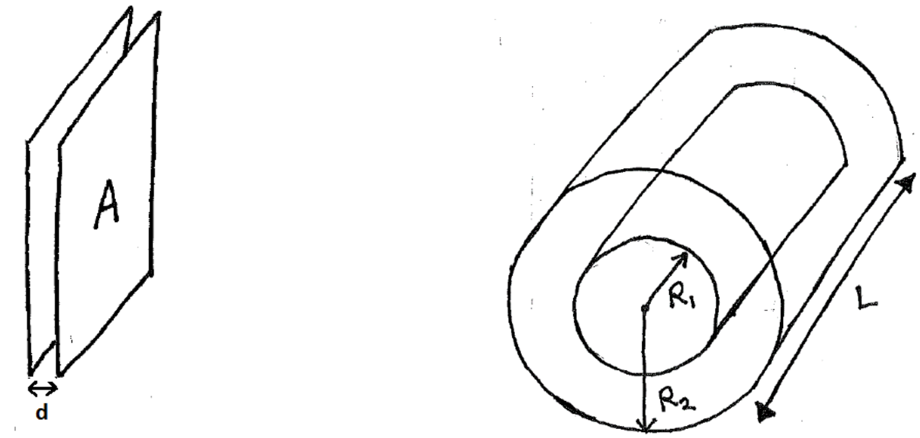

Figure 7. (a) Parallel plate capacitor. (b) Concentric cylinders.

For a parallel plate capacitor (Figure 7a) with two plates of area A and separation d, with $d \ll \sqrt{A}$:

$$C = \frac{\varepsilon_0 A}{d} \tag{7}$$

Note that for the above equation to hold, d must be much smaller than the length scale of A. If this were not the case, we would have to take into account the changes in capacitance near the edges of the plates. This would complicate the equation significantly!

For two concentric cylinders (Figure 7b) with length L, smaller radius R_1 and larger radius R_2:

$$C = \frac{2\pi\varepsilon_0 L}{\ln(R_2 / R_1)} \tag{8}$$

For two identical parallel wires (Figure 8a) with length L, radius R, and separation d (where $d >> R$):

$$C = \frac{\pi\varepsilon_0 L}{\ln(d / R)} \tag{9}$$

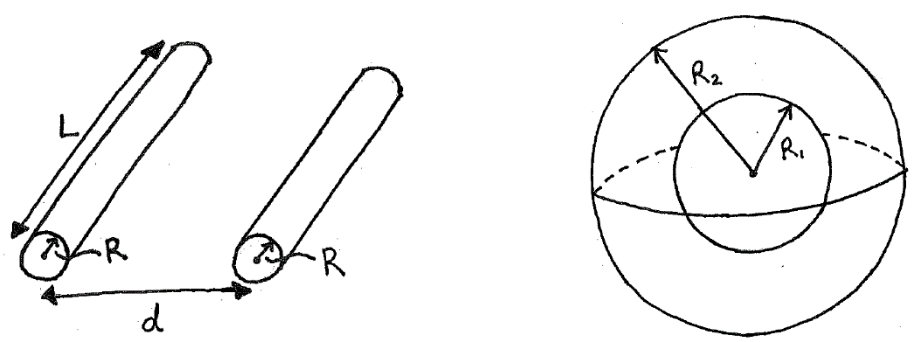

Figure 8. (a) Two parallel wires. (b) Two concentric spheres.

For two concentric spheres (Figure 8b) with smaller radius R_1 and larger radius R_2:

$$C = \frac{4\pi\varepsilon_0}{(1/R_1 - 1/R_2)} \tag{10}$$

Capacitance is expressed in units of *Coulombs per Volt*, otherwise known as *Farads*.

In this experiment we will qualitatively verify the formula for the parallel plate capacitor given above, using charge scooped off of the charged sphere. Please perform, record, and interpret the following steps:

Experiment 2

1. Set up the charged sphere at 1000 V as in the previous experiment.
2. Disconnect the electrometer from the Faraday cage, and connect it to the parallel plate capacitor. Set the plate separation to 1.0 mm (this is the thickness of the three tiny Teflon spacers glued to the plates).
3. Using the proof plane, scoop charge from the 1000 V sphere onto the plate of the capacitor that is connected to the (+) terminal of the electrometer. A capacitor voltage of 5 – 10 V is ideal.
4. Make a prediction: If $Q = CV$, and C is inversely proportional to the distance d between the plates (Equation (7)), what *should* happen to V as d increases?
5. Now test your prediction. With the capacitor charged up, increase the separation of the plates in 1 mm steps, taking care not to touch the plates or otherwise discharge them. For each plate separation (d) up to about 15 mm, record the potential (V) - after the first few separations, you can increase the step size to 2 mm. Plot your values, with d on the horizontal axis and V on the vertical axis (you should have at least 10 points).

Does the potential on the plates increase or decrease? Is that what you expected from your prediction in step 4?

Now, take a closer look at your results. Is your plot a straight line, or is it curved? Can you explain why the potential stops changing very much when the plates are very far apart? Take a look at Appendix 1 if you're stuck on this one.

MEASUREMENT OF CAPACITANCE

Fortunately, relatively inexpensive instruments are available that can quickly and easily measure capacitance. First, disconnect the electrometer, then remove any excess charge from the parallel plate capacitor and the Faraday cage by touching them (as in the Electrostatics lab). Using the capacitance meter provided (see Appendix 2 for a description of the instrument), perform and record the following steps.

Experiment 3

1. Connect the capacitance meter to the parallel plate capacitor by way of two banana leads and alligator clips.
2. Measure the capacitance of the **parallel plate capacitor** at 3 different plate separations from 1.0 mm up to 10.0 mm. For each measurement, compare your experimental *C* with the value predicted by the capacitance formula (Equation (7)). If your observed value is much larger than the predicted value at 10.0 mm, it means that the banana leads are contributing an additive capacitance to each measurement (see Appendix 1). Try subtracting off this excess capacitance and see if your predictions better fit your data.
3. Now, connect the capacitance meter to the **Faraday cage** and measure its capacitance. After correcting for the capacitance of the test leads, compare with the value predicted by the capacitance formula for two concentric cylinders (Equation (8)). What should you use for *L* - the height of the inner, shorter cylinder, or that of the outer, taller cylinder?

CAPACITANCE IN CIRCUITS

You will now take a look at some commercial capacitors - the kind found in electronics such as computers, cell phones, calculators, etc. These capacitors have a *C* on the order of nanofarads. Will the capacitance of the cables be significant in these measurements?

Figure 9. Symbol for a capacitor and capacitance meter.

In circuits, capacitors are drawn as two parallel lines and capacitance meters are drawn as a "C" within a circle (Figure 9). Capacitors can be combined either in *series* or in *parallel.* In Figure 10, two capacitors with capacitances C_1 and C_2 are shown in series with each other (a), and two capacitors are shown in parallel with each other (b). In each case, the capacitance meter measures the combined capacitance.

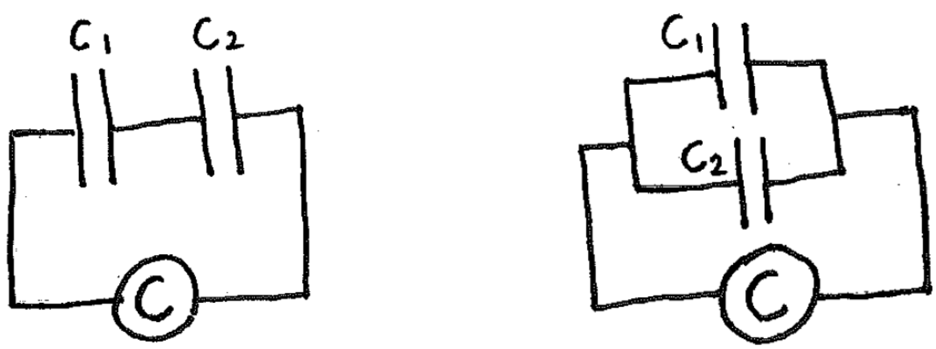

Figure 10. (a) Two capacitors in series. (b) Two capacitors in parallel.

The total capacitance, C_{total}, read by the meter will depend on the configuration of C_1 and C_2. For a series configuration:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{11}$$

For a parallel configuration:

$$C_{total} = C_1 + C_2 \tag{12}$$

These equations are known as the *series capacitance law* and the *parallel capacitance law*.

Experiment 4

1. Measure the capacitance of each of the three commercial capacitors provided (three measurements) using the capacitance meter. Compare with the values marked on the board.
2. Connect two of the three commercial capacitors in *series* and measure the resulting capacitance.[9] Compare with the value predicted by the series capacitance law (Equation (11)). Repeat, connecting all three of the capacitors in series.
3. Connect two of the three commercial capacitors in *parallel* and measure the resulting capacitance. Compare with the value predicted by the parallel capacitance law (Equation (12)). Repeat, connecting all three of the capacitors in parallel.

APPENDIX 1. CAPACITANCE IN CABLES AND LEADS

Capacitance shows up in a few unexpected places here – it is not just limited to the parallel plates, Faraday cage, and commercial capacitors! The BNC cables and banana leads that we use to connect the various components together *also* have an inherent capacitance.

A BNC cable is essentially two coaxial cylinders (Figure 7b) with a potential difference between them. The result is a small inherent capacitance on the order of a hundred or so picofarads. The two banana leads are essentially two parallel wires (Figure 8), which also have a capacitance. This capacitance increases if you push the wires closer together, decreases if you spread them apart, and is on the order of a few tens of picofarads. Try this out!

These effects will only become significant if the capacitance of the object you are working with is also on the order of picofarads (e.g. with the parallel plate capacitor). The extra capacitance from the cables or leads (C_{cables}) will be in *parallel* with what you are trying to measure (C_0), so your total capacitance C_{total} will be the sum of the two:

$$C_{total} = C_0 + C_{cables} \tag{13}$$

If C_0 becomes significantly smaller than C_{cables}, then C_{total} will become approximately equal to C_{cables}.

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[9] If you are unsure how to connect your capacitors together, ask your TA or refer to the DC Circuits lab in this manual.

APPENDIX 2. THE CAPACITANCE METER

In this lab, you will be using a capacitance meter (shown in Figure 11). With this meter, you can measure capacitance over a range of 200 picofarads (200pF) to 20 millifarads (20mF).

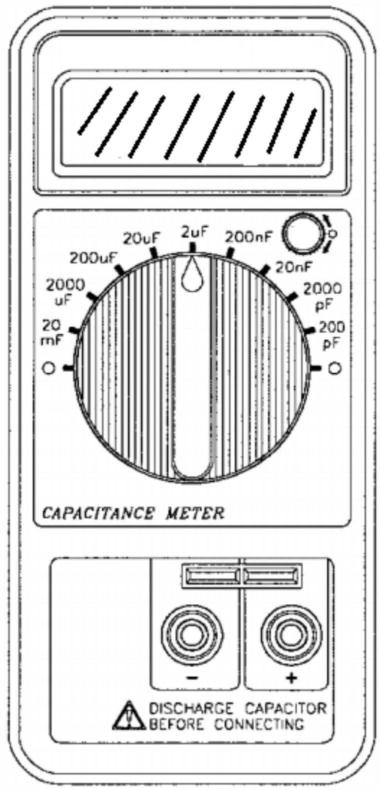

Figure 11. The BK 810C capacitance meter.

Most simple capacitance meters work by charging up the capacitor with a known current, then measuring how quickly the voltage across the capacitor increases. The voltage of a large capacitor (on the order of μF) will increase slowly, while that of a small capacitor (on the order of pF) will increase quickly.

PRE-LABORATORY

1. Assume a conducting sphere of radius 6.5 cm is connected to a 1000 V power supply. What is the charge density on the surface of the sphere in C/m^2?
2. Assume that a proof plane is a conducting disc with a radius of 1.27 cm and is used to transfer some charge from the sphere described above. How much charge will be transferred in one "scoop?"
3. What is the capacitance of two parallel circular plates, each of radius 10 cm, and separated by 1.0 mm?
4. Two capacitors, of capacitance 50 nF and 100 nF, are connected in parallel. This combination is then connected in series to a 200 nF capacitor. What is the total capacitance of the network?[10]

[10] 1 μF = 10^{-6} F; 1 nF = 10^{-9} F; 1 pF = 10^{-12} F.