STATIC MAGNETIC FIELDS

INTRODUCTION

In this set of experiments we investigate, both qualitatively and quantitatively, the magnetic field arising from steady currents in various common configurations. For qualitative observations we will use a gimbal-mounted compass, and for quantitative observations we will use an electronic instrument known as a *Hall probe*.

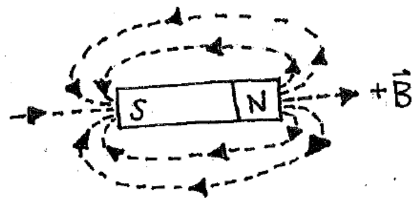

Figure 23. A compass "needle." The dotted lines indicate the external magnetic field lines of the compass. In this illustration, the magnetic field being measured, **B**, points left to right.

The compass: The compass is the earliest known detector of magnetic fields. It comprises a needle of permanently magnetized material that will align itself with magnetic field lines if it is mounted in a frictionless pivot, or *gimbal*. Figure 23 shows the compass "needle" on your compass. The north end is usually painted red. The compass will align itself with the positive direction of the magnetic field, as indicated by the magnetic field vector **B**. An ordinary navigational compass is constrained to rotate about the vertical axis, and so it can only point along the projection of the magnetic field in the horizontal plane.

A word about sign conventions: (see the textbook for further details).

1. The magnetic field vector **B** *external* to a bar magnet tends to point along the field lines *from the north* magnetic pole of a magnet *to the south* magnetic pole.
2. The planet Earth behaves as if a large bar magnet is embedded within it (Figure 24). At the present epoch, the north *geographic* pole of the earth is located near the south *magnetic* pole! However, in a million years or so the magnetic poles will probably reverse, and then the opposite situation will prevail.

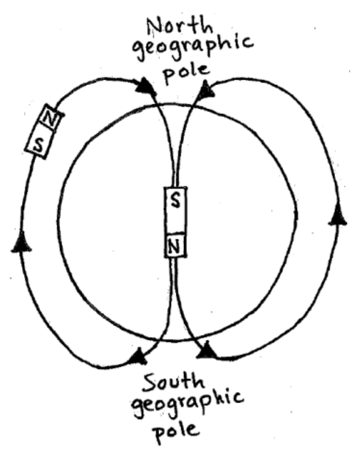

Figure 24. The magnetic field of the Earth.

3. As a consequence of (1) and (2), the Earth's magnetic field presently points from the south *geographic pole* toward the north *geographic pole.*
4. As a consequence of (3), a valid navigational compass needle points in the same direction as the magnetic field.[13]

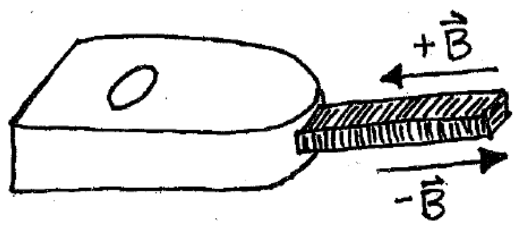

Figure 25. The Hall probe

The Hall probe: The Hall probe that we use measures two components of the magnetic field: one component is along the axis of the probe, and the other component is perpendicular to the probe. (The location of the latter sensor is about 4 mm from the end of the probe, and is indicated by a tiny dot). We concern ourselves entirely with the component along the probe axis; a positive reading indicates a magnetic field pointing into the probe, and vice versa (Figure 25). Unfortunately, the manufacturer has painted the arrows on the probe in the reverse direction of the actual magnetic field!! When in doubt, use the Magnaprobe compass to determine the actual direction of the field. For more information on how a Hall probe works and how to use it, see Appendix 1.

FIELD FROM A LONG, STRAIGHT WIRE

In this section, you will examine the magnetic field direction close to a current-carrying bundle of wires. The magnetic field lines arising from a long, straight bundle of *N* wires are circles concentric with the axis of the wire. We say that the magnetic field points in the 'azimuthal' direction.

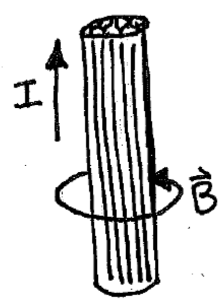

Figure 26. Magnetic field near a straight bundle of wires with the current, *I*, pointing up.

The field diminishes in strength as you move away from the bundle of wires. This field can be expressed mathematically as follows, where *r* is the distance from the wires and where *I* is the current in each wire:[14]

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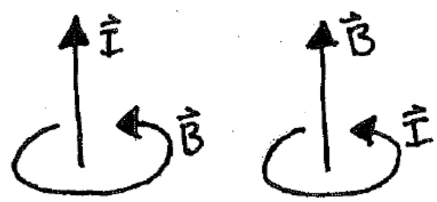
[13] The author has discovered that some really cheap compasses have their needles painted backwards, or in strong magnetic fields can have their magnetization reversed. Beware!

[14] In SI units, $\mu_0 = 4\pi \cdot 10^{-7}$ Tesla-meter per Ampere. Note that $\varepsilon_0\mu_0 = 1/c^2$, where c is the speed of light.

$$\vec{\mathbf{B}} = \frac{\mu_0 NI}{2\pi r}\hat{\boldsymbol{\varphi}} \tag{17}$$

In this equation the symbol $\hat{\boldsymbol{\varphi}}$ signifies a unit vector pointing tangentially to a circle centered on the wire; again, this is called the azimuthal direction.

The right-hand rule is very helpful when trying to determine the direction of **B** if you know the direction of ***I***. In Figure 27(a), when pointing your right thumb along ***I***, your fingers will naturally curl counter-clockwise along **B**. In Figure 27(b), when curling your right fingers counter-clockwise along ***I***, your thumb will point up, in the direction of **B.**



(a) **(b)**

Figure 27. Direction of the magnetic field in the case of a long straight current (a) and in the case of a circular current (b).

Experiment 1

A good approximation to a long, straight bundle of wires is the nearby vicinity of the windings of a coil (Figure 28).

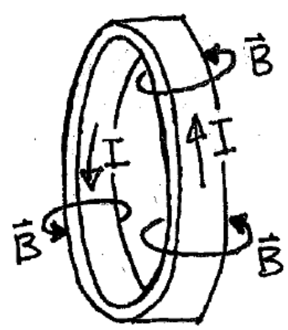


Figure 28. Magnetic field in the vicinity of one coil. The current is moving counter-clockwise.

1. Connect the power supply to one of the pair of large diameter coils, and set the power supply to 1.0 Amperes by adjusting the current control.[15] Is the current moving clockwise or counter-clockwise around the coil?
2. Hold the Magnaprobe near the coil, and record/draw the direction of the magnetic field as you move it in the vicinity of the coil. Verify that the right-hand rule applies here.
3. Turn the current **down to 0 Amps** and turn **off** the power supply. Swap the leads on the coil, and turn the power supply back on and up to 1.0 A. Repeat part (2).

[15] Recall that current will flow from the positive terminal to the negative terminal.

***Note**: In this and other parts of the experiment, it is extremely important to turn the current down to zero before changing any wires or turning off the power supply. A sudden, large change in current induces an enormous voltage in the power supply and coils, which can result in damage to the equipment.*

MAGNETIC FIELD FROM A CIRCULAR COIL

The magnetic field lines arising from a single circular coil are described in detail in the textbook. It is easiest to separately describe the fields in three regions: 1) near the windings of the coil; 2) on the axis of the coil; and 3) at distances far from the center of the coil.

Near the windings of the coil, as we saw in Experiment 1, the field lines become indistinguishable from the field lines near a long straight wire carrying the same *total* current. That is, the field lines are rings, or circles, encircling the coil windings, with a magnitude given by Equation (17).

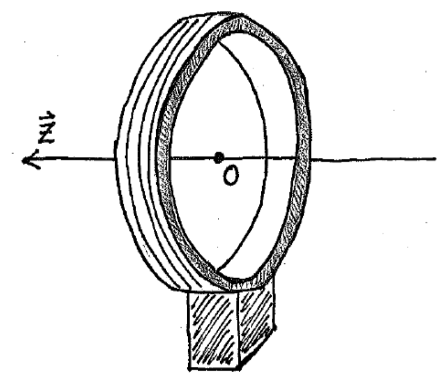


Figure 29. Single coil. The z-axis is shown along the axis of the coil, with the origin at the coil center.

On the *axis* of the coil, the field line is a straight line coincident with the axis of the coil (call it the z-axis). The direction of the field on the axis is also given by the right-hand rule. To determine the direction, simulate the circular direction of the current with the fingers of your right hand, recalling that the direction of the current is from the positive terminal of the power supply to the negative terminal. Your right thumb will then point along the direction of the field. The magnitude of the field is given by the formula below, where R is the radius of the coil, I is the current in a *single* wire, and N is the number of turns:

$$\vec{\mathbf{B}} = \frac{\mu_0 NIR^2}{2(z^2 + R^2)^{3/2}} \hat{\mathbf{z}} \tag{18}$$

Note that the magnitude of the magnetic field will decrease as the distance, z, from the origin increases along the z-axis. Where will the field be at a maximum?

Experiment 2

We will be measuring the properties of coils that are mounted on a metal base. Each coil has the following properties:

Number of turns: $N = 200$ turns
Average radius: $R = 103$ mm

1. Set up the Hall probe as described in Appendix 1. Before taking any measurements, be sure to "zero" the probe by turning down the power supply current to zero and pressing the **tare** button on the probe.
2. Connect the power supply to just one coil and set the current to 1.0 Ampere, taking note of the direction of the current as indicated on the coil's label.
3. Measure the magnitude and direction of the magnetic field along the axis of a single coil with the Hall probe. Start measurements at $z = 0$, and take at least two other measurements at distances on either side of the origin (at +5 and -5 cm, for example). Compare your results with Equation (18).
4. According to Equation (18), how far away do you need to be before the probe reads a magnetic field of 0? Test this with the Hall probe – how are your measurements limited by the instruments?

MAGNETIC FIELD FROM A PAIR OF COILS

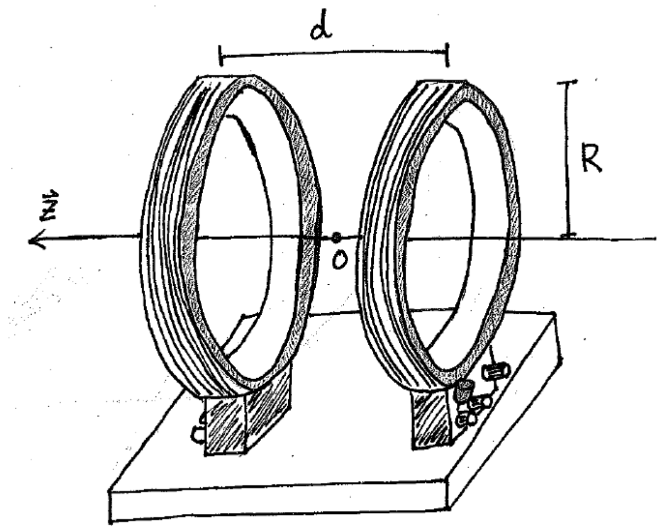


Figure 30. Two coaxial coils, each with radius R and separated by a distance d. The origin of the z-axis is midway between the coils.

The magnetic field along the axis of a *single* coil peaks at the center of the coil, and then falls off with distance according to Equation (18). If two identical coils are mounted on a common axis with their centers at distances $\pm d/2$ from the origin, and if each has a current I flowing through each turn of each coil, by the principle of superposition, the field *along the z-axis* will be just the sum of the two contributions:

$$\vec{\mathbf{B}}(0,0,z) = \frac{\mu_0 N I R^2}{2}\left[\frac{1}{\left((z-d/2)^2+R^2\right)^{3/2}} + \frac{1}{\left((z+d/2)^2+R^2\right)^{3/2}}\right]\hat{\mathbf{z}} \tag{19}$$

Although this is a very complicated expression, it boils down at $z = 0$ (now the mid-point between the coils) to the following:

$$\boxed{\vec{\mathbf{B}}(0,0,0) = \frac{\mu_0 N I R^2}{((d/2)^2+R^2)^{3/2}}\hat{\mathbf{z}}} \tag{20}$$

Also, it turns out that when the distance d between the coils is equal to R, the radius of the coils, the magnetic field at the mid-point between the coils is *extremely* uniform. This fact, which was discovered by Helmholtz, arises from the fact that the field is diminishing with distance from one coil at nearly the exact rate that it is increasing from the other coil. Coils exactly satisfying this condition are called *Helmholtz coils*. Substituting $d = R$ in Equation (20), we get

$$\boxed{\vec{\mathbf{B}}(0,0,0) = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 N I}{R}\hat{\mathbf{z}} \quad (\text{when } d = R)} \tag{21}$$

Experiment 3

1. Connect the two coils *in series* and *in the same direction*. The direction of the current is indicated on the coils near the jacks. You should only need three banana cables to do this. Remember that the current will flow from the positive terminal on the power supply to the negative terminal.
2. Set the power supply to provide a current of 1.0 Ampere, so that each turn of each coil carries the same current I.
3. Measure the direction and magnitude of the magnetic field along the axis of the pair of coils (separated by R), near the midpoint between the coils. Take measurements at $z = 0$ and at the center of each coil.
    a. Does your result at the midpoint agree with your results from the prelab?
    b. How far from the origin (along the axis) do you have to go before the field drops to 90% of its maximum value?
    c. How far do you have to go before the field drops to 0?
4. Turn down the power supply and switch the input and output cables on one of the coils. The currents in the coils will thus remain equal, but will now flow in opposing directions. Set the current back to 1.0 A.
5. Observe the magnitude of the magnetic field at the origin. Take measurements at $z = 0$, at the center of each coil, and outside the coils. Are your results consistent with what you would expect from an application of the principle of superposition? That is, if you added up the magnetic field from each single coil, what magnitude of field would you expect at the origin? At the center of each coil?

MAGNETIC FIELD FROM A SOLENOID

Perhaps the simplest geometry of all is the geometry of the *solenoid*. A solenoid is a long[16] coil of wire of radius R, length L, and N turns. Let $n=N/L$ be the number of turns per meter. It happens that the field is even more uniform at the center of a solenoid than it is for the Helmholtz coil; furthermore, the field is independent of the diameter D, and is given by

$$\boxed{\vec{\mathbf{B}}(0,0,0)=\mu_0 nI\,\hat{\mathbf{z}}} \tag{22}$$

The above formula is correct for a very long coil; *i.e.* $L >> D$. If this inequality is not satisfied, the field at the center of the coil will be diminished by the factor

$$F=\frac{L}{\sqrt{L^2+D^2}} \tag{23}$$

The actual field at the center will thus be given by

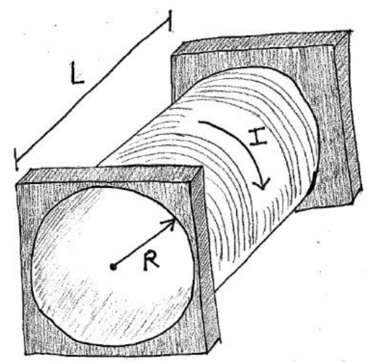
$$\vec{\mathbf{B}}_{actual}=F\,\vec{\mathbf{B}}(0,0,0)=F\,\mu_0 nI\,\hat{\mathbf{z}} \tag{24}$$

Experiment 4

Our coil has 565 turns and it is 146 mm long.

1. Connect your solenoid to the power supply. With a current of 1.0 Ampere, measure the direction and magnitude of the magnetic field along the axis of the solenoid by inserting the Hall probe into the solenoid.

2. Calculate the expected magnetic field for an extremely long coil (Equation (22)) and the correction factor (Equation (23)) for this solenoid.



3. Now compare your measurement at the center with that of the corrected theoretical formula (Equation (24)). How far from the origin (along the axis) do you have to go before the field drops to 90% of its maximum value?

APPENDIX 1 – THE HALL PROBE

A Hall probe consists of a small wafer of silicon that has been doped with a small concentration of impurity atoms, which ionize at room temperature and thus provide the wafer with charge carriers. A small, steady current, ***I***, is forced through the chip, and the voltage between the sides of the chip is measured. The probe makes use of the *Hall effect,* which describes the effect of a magnetic field on a current-carrying conductor.

[16] In this context, "long" implies that the length L is much greater than the diameter D.

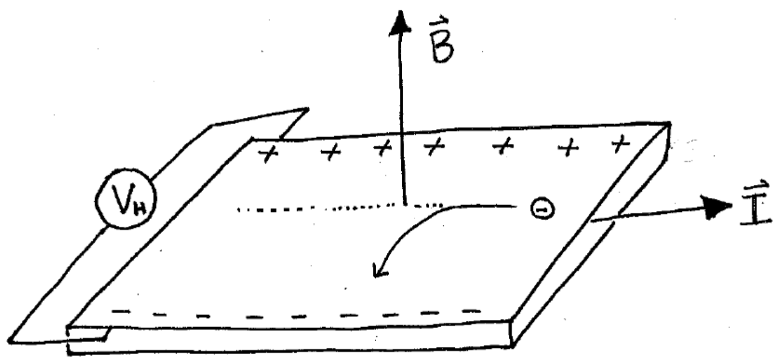


Figure 31. The Hall effect on a current-carrying conducting chip. The magnetic field B is perpendicular to the current *I*, so negative charges experience a resulting force tugging them downwards. V_H is the Hall voltage measured by the probe.

If the chip is suspended in a magnetic field **B**, the negative charges will be tugged toward one edge of the chip, causing a side-to-side *Hall voltage* (V_H) to appear that is proportional to the applied magnetic field. The effect will be largest when the normal to the wafer is parallel to the direction of the magnetic field. Thus, to measure the direction and magnitude of the field, orient the probe for a maximum signal, and read off the voltage induced. For a commercial Hall probe, the voltage is calibrated so that the field can be read out directly in Gauss.[17]

The Hall probe we use simultaneously records two numbers: the component of the magnetic field along the axis of the probe, and a component of the field perpendicular to the probe axis. We are concerned only with the former for this lab.

How to use the Hall Probe: You will find at your station a small, pistol-shaped object labeled “2-axis Magnetic Field.” It is attached by a short cable to a handheld readout device labeled “XPlorer GLX.” Here are the steps to activate the probe:

1. Depress the power button on the lower right hand corner of the readout device to turn on the power.
2. Now press the button with the “home” icon, which will bring up on the display twelve panels.
3. With the directional buttons, highlight the second panel at the top, labeled “digits.”
4. Now press the check mark button.
5. Finally, press the big right arrow button. The panel will now display the two components of magnetic field, in units called Gauss. To convert to Tesla, which is the SI unit used in this course, multiply the value in Gauss by 10^{-4}.

The probe now needs to be properly zeroed. To do this, turn off the current to any nearby magnets, and press the “Tare” button on the side of the probe. The probe should now read

[17] The Tesla is the SI unit of magnetic field. The Gauss is an old-fashioned, but still common unit of magnetic field. 1 Tesla equals 10^4 Gauss.

approximately 0 Gauss (with some fluctuations). You should repeat this operation whenever you do a new experiment, as the electrical zero of the probe tends to drift with time.

PRE-LABORATORY

Suppose I have a current source of 1.0 Amperes flowing through each of the following coils. What is the magnetic field in Tesla and in Gauss at the stipulated points?

1. At the center of a single 200-turn coil with a radius of 103 mm.
2. At the midpoint between two 200-turn coaxial coils of radii 103 mm and separated by a distance of 103 mm. (Assume the currents are arranged so that the fields add constructively).
3. At the center of a 565-turn, 146 mm-long solenoid of radius 34.1 mm.