Peter, Allan, Caden, Valeriia

Conservation of Momentum

Learning Objectives:

- Observe different types of collision
- Calculate energy loss
- Observe conservation of momentum
- Calculate kinetic energy

Warm-Up

A cart of mass $m1$ is moving on a straight track with a velocity $v0$. It collides with a cart of mass $m2$ that is at rest. The carts stick together and move with a final velocity vf.

1. **Use conservation of momentum to write an expression for the final velocity. Your answer should be written in terms of $v0$, $m1$, and $m2$.**

$$m_1 v_0 + m_2(0) = (m_1 + m_2) v_f$$

$$\frac{m_1 v_0}{m_1 + m_2} = v_f$$

2. **Write an expression for the initial kinetic energy Ki of the system before the collision. Your answer should be in terms of $v0$ and $m1$.**

$$K_i = \frac{1}{2} m_1 v_0^2 + \frac{1}{2} m_2 (0)^2$$

$$K_i = \frac{1}{2} m_1 v_0^2$$

3. **Using your expression for vf, write an expression for the final kinetic energy of the system Kf after the collision. Your answer should be in terms of $v0$, $m1$, and $m2$.**

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$$K = \frac{1}{2} \frac{(m_1 v_0 + m_2 (0))^2}{m_1 + m_2}$$

$$K = \frac{1}{2} \frac{(m_1 v_0)^2}{m_1 + m_2}$$

4. **Write an expression for the fractional change in kinetic energy $\Delta K/Ki$. Your answer should be in terms of only m1 and m2.**

$$\frac{\Delta K}{K_i} = \frac{m_1}{m + m_1} - 1$$

Part 1 - The Losses Due to Friction

Press the power button located on the side of one of the Smart Carts to turn it on. When the Bluetooth indicator light is flashing red, it means the sensor is waiting to be connected to a laptop.

Boot a laptop up into Windows and open the **Conservation of Momentum** Capstone file. Click on the **Hardware Setup** button located on the left side of the screen. This will open a window displaying nearby wireless devices.

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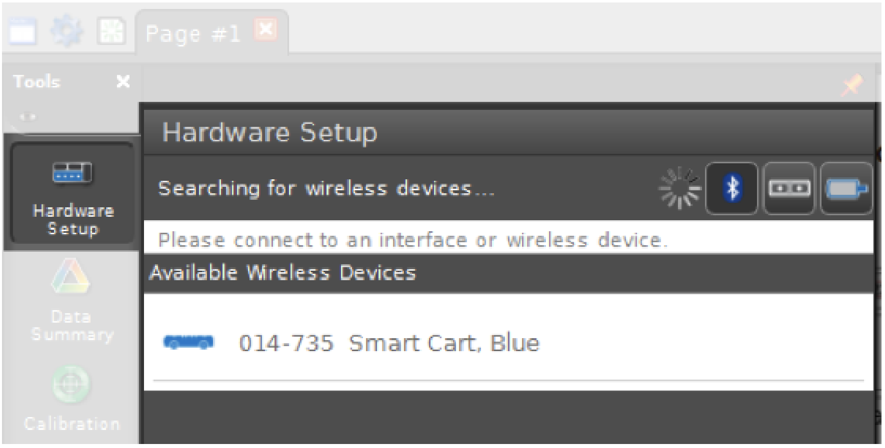
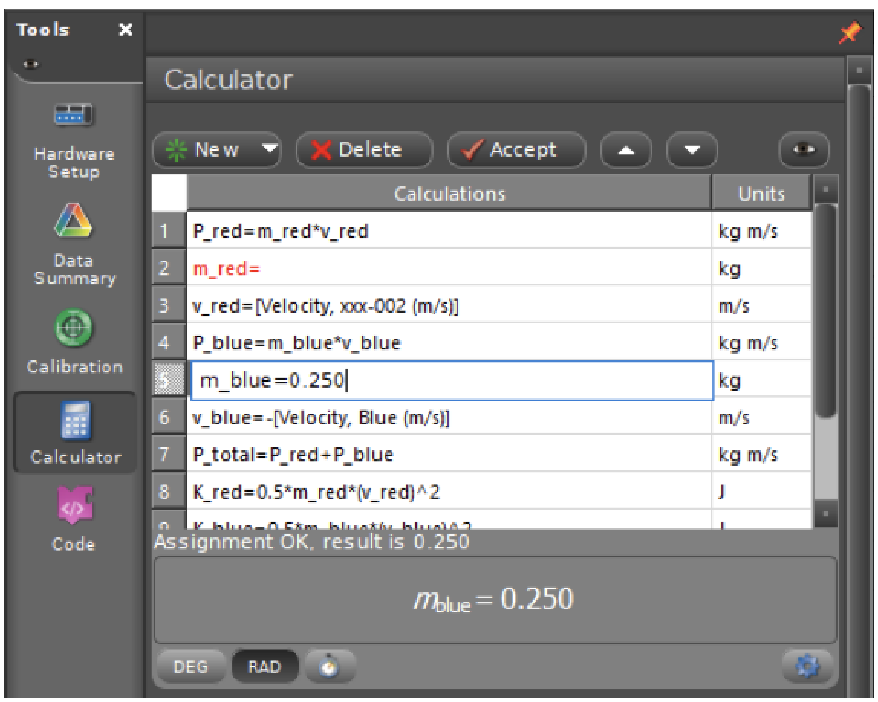


After connecting your wireless Smart Cart, click on the sliders located next to **Smart Cart Acceleration Sensor** and **Smart Cart Gyro Sensor** to disable them. Only the position will be used in this experiment. Click on the **Hardware Setup** button again to close the Hardware Setup window.

Next, click on **Calculator**. A window will open showing the equations for the momentum and kinetic energy of each cart where "*P_red*" is the equation corresponding to the momentum of the red cart, "*m_red*" is its mass, "*v_red*" is its velocity and "*K_red*" is its kinetic energy. Likewise, the equations with the *blue* subscripts correspond to each of these values for the blue cart.



The software now has the information it needs to calculate the momentum and kinetic energy of the cart that you will be using in this first part. Under the **Part 1 – Losses Due to Friction** tab, there should be three graphs that share the same time axis. Click on **<Select Measurement>** along the vertical axis of the top graph.

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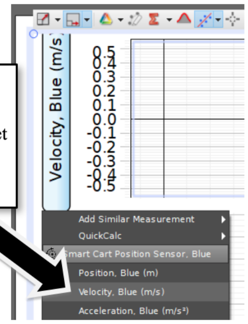
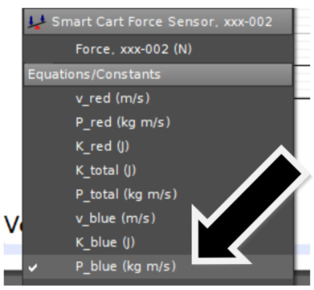
graph.

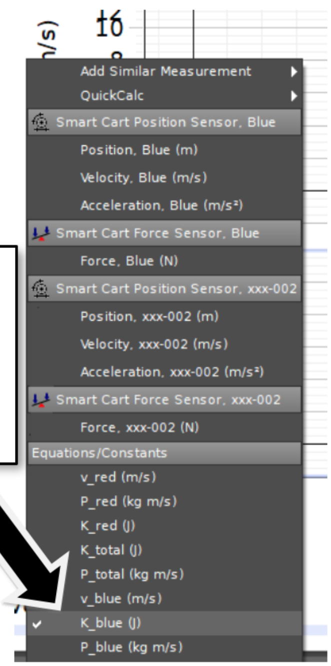
3. From the pop-up menu, click on either **Velocity, Blue (m/s)** or **Velocity, Red (m/s)**, depending on the color of the cart that you have connected, under **Smart Cart Position Sensor**. The top graph is now set to display the velocity of the cart versus time.





4. Next, click on **<Select Measurement>** along the vertical axis of the middle graph. From the pop-up menu, click on either **P_blue (kg m/s)** or **P_red (kg m/s)**, depending on the color of the cart that you have connected, under **Equations/Constants**. The middle graph is now set to display the momentum of the cart versus time.

5. Finally, click on **<Select Measurement>** along the vertical axis of the bottom graph. From the pop-up menu, click on either **K_blue (J)** or **K_red (J)**, depending on the color of the cart that you have connected, under **Equations/Constants**. The bottom graph is now set to display the kinetic energy of the cart versus time.



The force of friction acting on the carts is significant enough that we should probably take it into account before beginning the experiment. With only one cart on the track, we will observe and measure the impact that friction has on the cart when it is in motion.

1. When you are ready, position the cart at one end of the track and click **Record** in Capstone. Give the cart a small push. **What causes the cart to slow down once it is in motion? Explain in terms of Newton's laws.**

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Friction causes the cart to slow down when in motion since friction is opposite to the direction of motion. We can apply Newton's second Law Fnet=ma. Fnet=Force of the push - friction, so this decrease in net Force causes a decrease in acceleration.

2. Click **Stop** to end data collection. **Looking at the velocity versus time graph, how much speed per second does the cart lose to friction? Record this value.**

   The cart loses 0. 0606 m/s per second due to friction.

3. **Looking at the momentum versus time graph, how much momentum per second does the cart lose to friction? Record this value.**

   The cart loses 0.0155 kg m/s per second due to friction.

4. **Looking at the kinetic energy versus time graph, how much kinetic energy per second does the cart lose to friction? Record this value.**

   The cart loses 0.0036 J per second due to the friction.

We will be analyzing the momentum and kinetic energy of different types of collisions:

<u>All Collisions</u>

In all collisions, momentum is conserved. If momentum appears to be lost, it is because not every object involved in the collision is being accounted for. For example, when two cars collide they are also colliding with the ground they are driving on and the air they are moving through. If we only account for the momentum of the two cars, the momentum that was transferred to the air and the ground will seem to have disappeared.

<u>Totally Inelastic Collisions</u>

A totally inelastic collision is one where the most possible kinetic energy is lost in a collision (for the given initial conditions). A totally inelastic collision is one in which the objects stick together and move as a single object after the collision. We will be using a Velcro bumper to create a totally inelastic collision.

<u>Elastic Collision</u>

An elastic collision is one in which total kinetic energy is conserved. We will be using a magnetic bumper to create a close approximation to an elastic collision.

Part 2 - Carts of the Same Mass Collide

<u>Trial #1 - Inelastic Collision</u>

Next, click on the **Part 2 – Carts of the Same Mass Collide** tab in Capstone. There should be three graphs set up to show the velocity, momentum, and kinetic energy of each cart versus time.

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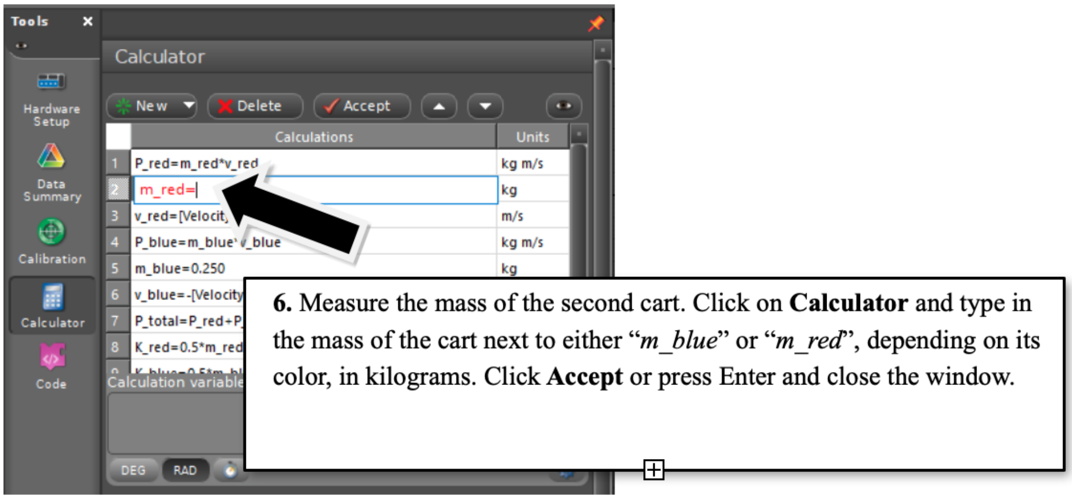
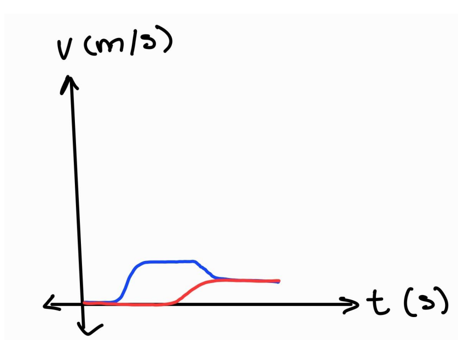


The software now has the information it needs to calculate the momentum and kinetic energy of each cart. Position both carts on the track with their Velcro bumpers facing each other. Next, set one cart near the middle of the track and the other at one end.

Click on **Record** and push the cart at the end of the track towards the cart at the center. Since the Velcro bumpers on the cart are facing each other, after they collide, they will stick together, as shown in the Momentum and Impulse video at 0:50. Click **Stop** to end data collection.

Look at the **velocity vs. time** graph for the carts. **Sketch the curves that you see. Make it clear which curve goes with which cart. Explain how these curves demonstrate the motions you observed.**



The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

The curves explain how the blue cart in motion hits the red cart. The moments where the red slope rises is the impulse and then the velcro keeps the carts together when the lines connect.

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From the graph, read and record the velocity of the initially moving cart right before the collision. Also, read and record the velocity of the two carts moving after the collision.

The velocity of the blue cart (initially moving) before the collision was 0.58 m/s. The velocity of the carts after the collision is 0.28 m/s.

Use the result from the warm-up, and the velocity of the initially moving cart before the collision, to predict the velocity of the two carts after the collision.

0.29 m/s

1

$$\frac{(0.270 \cdot 0.58)}{0.270 + 0.270}$$

$$= 0.29$$

How does this predicted final velocity compare to the measured final velocity? If the measured value is less than the predicted value, is it consistent considering the frictional losses?

4% difference

2

$$\frac{0.29 - 0.28}{\frac{0.29 + 0.28}{2}} \cdot 100$$

$$= 3.50877192982$$

Look at the **momentum vs. time** graph and s**ketch the curves. Make it clear what each curve corresponds to.**

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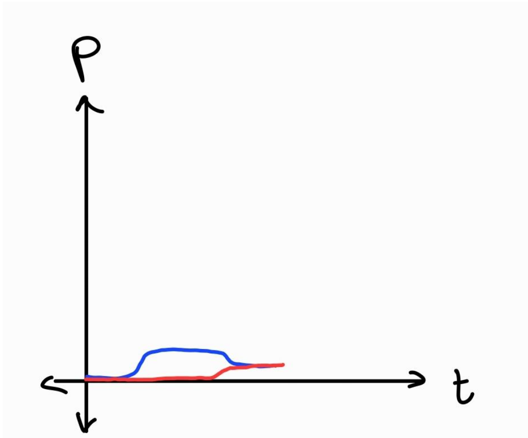


*Goes noticeably shorter than the velocity graph

The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

Explain how these curves demonstrate the momentums involved.

The blue cart in motion collides with the sill red cart and then carries the momentum. Their momentums are the same after the velcro which is where the lines are connected.

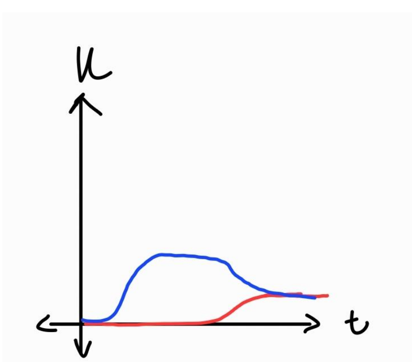
Does momentum appear to be conserved within the errors due to frictional losses?

The momentum before the collision was 0.18 kg m/s and the momentum after the collision was 0.08 kg m/s. There is a change in momentum of 0.1 kg m/s, therefore the momentum was not conserved within the errors.

How much momentum does one cart lose and how much does the other cart gain?

The blue cart (moving) loses 0.1 kg m/s of momentum and the red car (stationary) gains 0.1 kg m/s.

Look at the **kinetic energy vs. time** graph**; sketch the curves. Make it clear what each curve corresponds to.**



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The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

Explain how these curves demonstrate the kinetic energies involved.

The blue cart gains energy from the push and then the energy gets absorbed by the red cart from the impact which then share the energy together when velcroed together.

Does kinetic energy appear to be conserved?

The average Kinetic energy before the collision is 0.06 J and after is 0.01 J. Therefore, during the collision some Kinetic energy is lost.

How much kinetic energy does one cart gain and the other lose?

The blue cart (moving) gains 0.06 J and then later loses 0.05 J upon the impact. The red card gains 0.01 J upon the impact.

Is your experimental result consistent with your prediction from the warm-up given the frictional losses of the experiment?

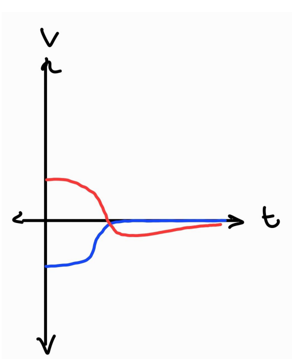
Our data and our calculations are somewhat consistent with each other. Through our calculations we determined that Ki = 0.05 J and Kf = 0.02 J. But, our data showed that the average Ki = 0.06 J and the average Kf= 0.01 J. The difference in our Kf is most likely due to friction slowing down the carts and due to the difference between the recorded and calculated final velocity. The difference between our recorded and the calculated Ki is due to margin of error and possibly not accurate enough mass measurement.

Trial #2 - Elastic Collision

Next, position both carts on the track, this time with their magnetic bumpers facing each other. Set one cart near the middle of the track and the other at one end as before.

Click on **Record** and push the cart at the end of the track towards the cart in the middle. Since the magnetic bumpers on the cart are facing each other, the moving cart will transfer its momentum to the stationary cart. Click **Stop** to end data collection.

Look at the **velocity vs. time** graph and **sketch the curves that you see. Make it clear which curve goes with which cart. Explain how these curves demonstrate the motions you observed.**



The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.
The graph shows the instance of collision at the point where the red and the blue line intersect. After the collision, the red cart began moving the left (negative) as shown by the negative velocity, while the blue cart came to a stop as shown by the blue line approaching zero.

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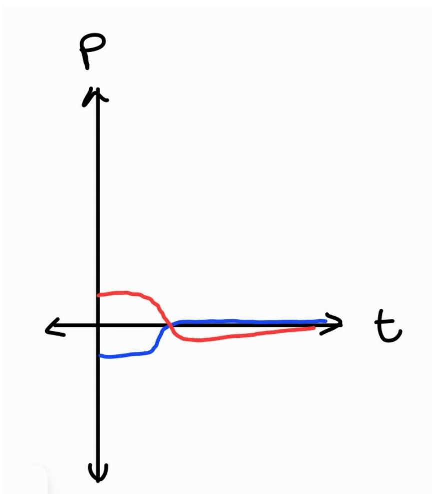
From the graph, read and record the velocity of the initially moving cart right before and after the collision. Read and record the velocity of the cart that is initially stationary right before and after the collision.

How does the initial speed of the initially moving cart compare with the final speed of the cart that is initially stationary?
The initial speed of the blue (moving) cart and the final speed of the red (stationary) cart was 0.08 m/s .

Look at the graph of **momentum vs. time** and s**ketch the curves. Make it clear what each of the curves corresponds to.**



The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

Explain how these curves demonstrate the momentums involved.

The point at which the curves intersect is the collision point. After the collision, the momentum of the red cart becomes negative since it's moving to the left or negative direction. The momentum of the blue cart, after the collision, is zero which represents how the blue cart stops.

Does momentum appear to be conserved within the errors due to frictional losses?
The momentum seems to not be conserved within the errors due to friction because the initial momentum was 0.1 kg m/s and after was 0.04 kg m/s.

How much momentum does one cart lose and how much does the other cart gain?

The blue (moving) cart lost 0.1 kg m/s in momentum and the red (stationary) cart gained 0.04 kg m/s and the rest was lost due to friction and human error.

Look at the graph of **kinetic energy vs. time** and **sketch the curves. Make it clear what each of the curves corresponds to.**

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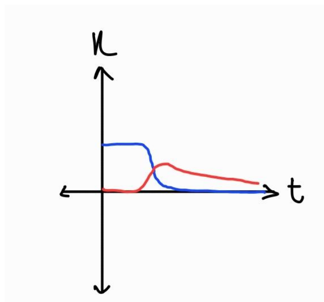


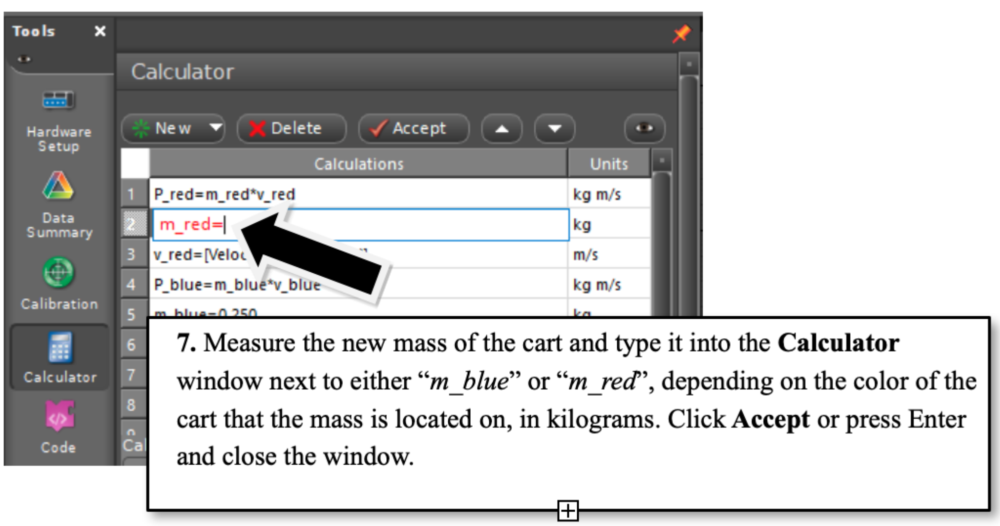
Does kinetic energy appear to be conserved before and after the collision?

No because the energy goes from 0.02 J to 0.01 J.

Part 3 – Carts of Different Mass Collide

Trial #3 - Inelastic Collision

Next, click on the **Part 3 – Carts of Different Mass Collide** tab. Again, there should be three graphs set up that will show the velocity, momentum, and kinetic energy of each cart versus time. Add a 250-gram cart mass to the top of one of the carts. Since its mass has increased, we will need to update our equations in the software so that it can correctly calculate the momentum and kinetic energy of the cart.



Next, position both carts on the track with their Velcro bumpers facing each other. Again, set one cart near the middle of the track and the other at one end.

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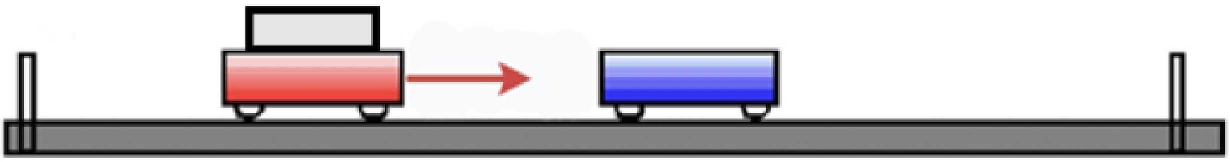
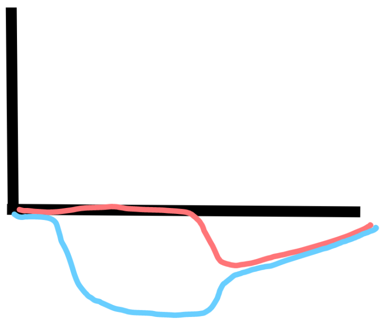


Click on **Record** and push the cart at the end of the track towards the cart at the center. Click **Stop** to end data collection.

Sketch the curves that you see in the velocity vs. time graph for the inelastic collision. Make it clear which curve goes with which cart. Explain how these curves demonstrate the motions you observed.



The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

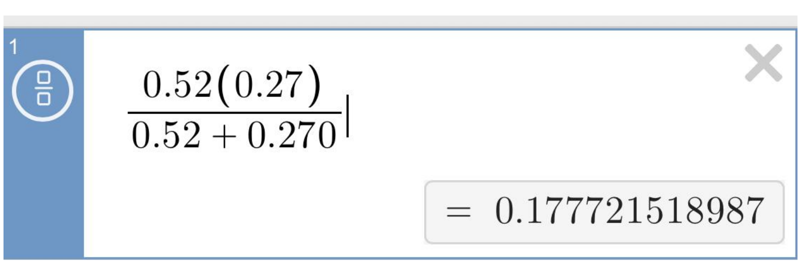
The point where they collide is shown when the curves come together. Then their velocities after the collision are the same which shows how the two cart stuck together and moved together.

From the graph, read and record the velocity of the initially moving cart right before the collision and record the velocity of the two carts moving after the collision.

From the graph, the Initial velocity of the blue (moving) cart was 0.27 m/s and the final velocity of the carts together was 0.16 m/s.

Use the result from the warm-up, and the velocity of the initially moving cart before the collision, to predict the velocity of the two carts after the collision.

Vf=0.18m/s



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How does the predicted final velocity compare to the measured final velocity? If the measured value is less than the predicted value, is it consistent considering the losses due to friction?

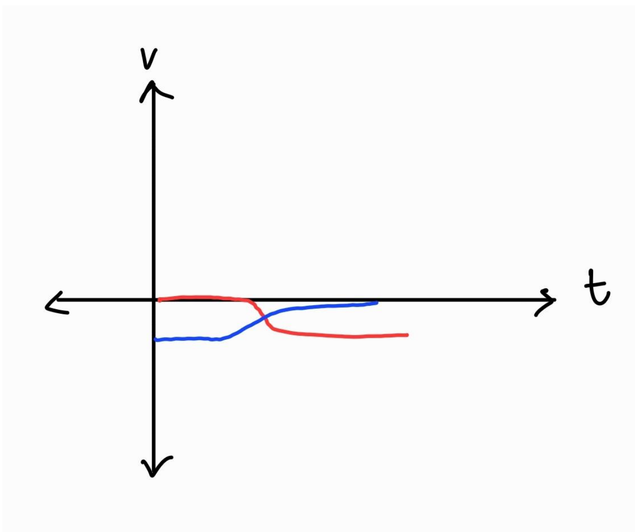
The calculated and the recorded final velocities are pretty close. The two measurements have about 10% difference. However, this is still a big margin of error due to friction.

Trial #4 - Elastic Collision

With the 250-gram mass still on top of one of the carts, position them on the track so that their magnetic bumpers are facing each other. Again, set one cart near the middle of the track and the other at the end.

Click on the red **Record** button and push the cart at the end of the track towards the stationary cart at the center. Click the **Stop** button to end data collection after the carts have stopped moving.

Sketch the curves that you see in the velocity vs. time graph. Make it clear which curve goes with which cart.



The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

Explain how these curves demonstrate the motions you observed.

The blue cart is pushed and then bounced back magnetically the opposite direction while the previously still red card now moves in the initial direction of the blue cart.

How does the velocity of the light cart after the collision compare to the velocity of the heavy cart before the collision? (Is it moving faster or slower? Is it moving in the same or opposite direction?)

The velocity of the light cart is more than the large cart's initial speed. They both move in the same direction. The Blue cart starts traveling backwards after the collision. This is because the force to move the heavy cart is part shared to the lighter cart. Since the lighter cart needs less force to move, It actually travels faster despite having less force than the heavier cart's top speed.

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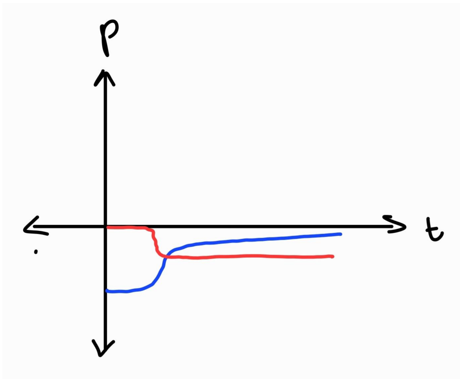
How does the velocity of the heavy cart after the collision compare to the velocity it had before the collision? (Is it moving faster or slower? Is it moving in the same or opposite direction?)

The velocity of the heavy cart before the collision is greater than its velocity after the collision. However the direction is still the same.

Sketch the curves in the momentum vs. time graph. Make it clear what each of the curves corresponds to. Explain how these curves demonstrate the momentums involved.



The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

Does momentum appear to be conserved within the errors due to losses from friction?

The initial total momentum is 0.24 kg m/s and the total final momentum 0.19 kg m/s. Therefore, the momentum was not conserved and does not fall within the errors due to losses from friction.

How much momentum does one cart lose and how much does the other cart gain?

The blue cart loses 0.17kg m/s while the red cart gains 0.13kg m/s after the collision.

Sketch the curves of the kinetic energy vs. time graph. Make it clear what each curve corresponds to. Explain how these curves demonstrate the kinetic energies involved.

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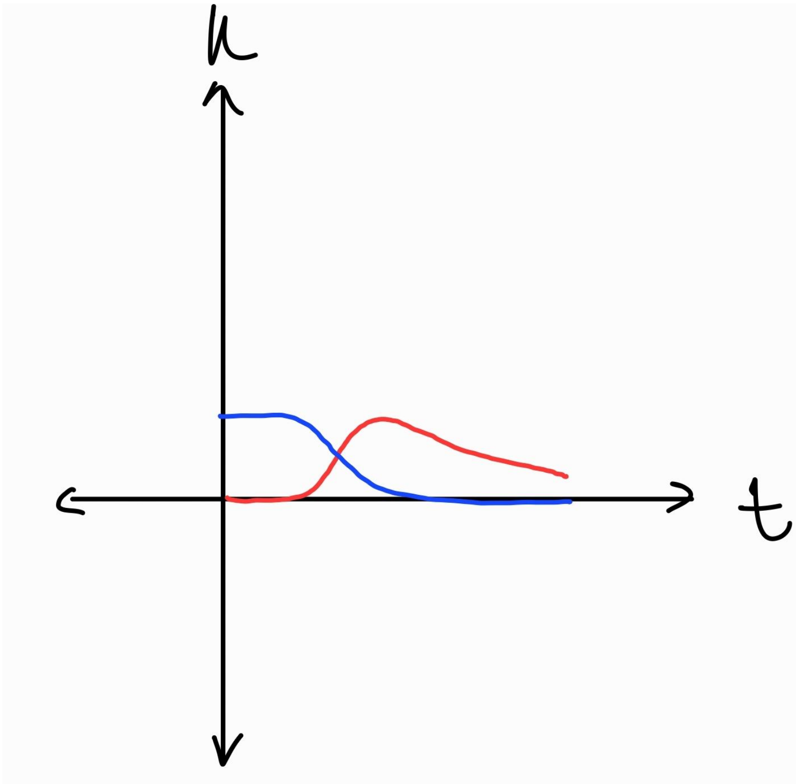

The curves are color coded. The red line is for the red cart and the blue line is for the blue cart.

Does kinetic energy appear to be conserved before and after the collision?

The kinetic energy of the system does not appear to be conserved before and after the collision. Our initial kinetic energy was 0.06 J and final was 0.03 J. Therefore about half of the kinetic energy was lost due to friction.