ECE 101L
LAB 4: Sinusoidal Steady State Filters
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Section A

Introduction

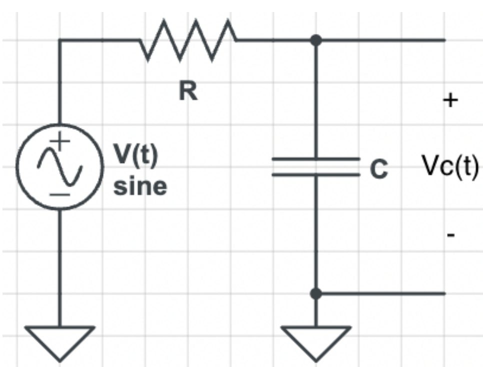
In this laboratory we will investigate the frequency domain characteristics of steady state linear wave filters. We will experimentally measure amplitude and phase responses and correlate them with the underlying theory.

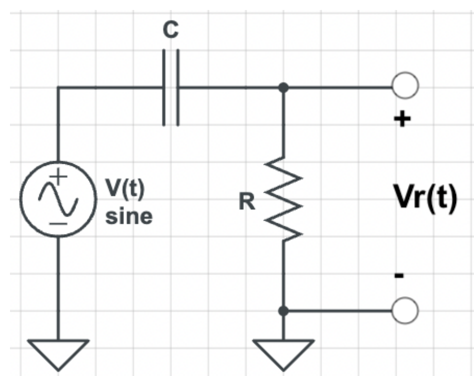
Purpose of Experiment

In this experiment we investigate how steady state wave filters behaved\ when a sinusoidal wave signal is present at different frequencies. In part I, we examine the phase and magnitude responses of two filters: a basic high pass filter made of a wire connecting a capacitor in series with the AC voltage, and a basic RC-low pass filter made of a waveform generator supplying AC voltage in series with a resistor where the output is measured across the capacitor. In part II, we examine the behavior of the phase and magnitude responses of an active low-pass filter configured as an op-amp driven by the waveform generator. We present the data on a Bode plot to show our findings.

Background Theory

Low pass or high pass filters can be built by connecting a resistor and a capacitor in series. Below, we have our simple RC low-pass and high-pass filter setups:





We are interested in two primary responses when we supply steady-state sinusoidal voltage to these circuits. The first is the frequency-dependent AC output voltage. The Bode plot, a log-log scale graph, can be used to explain this response. Our graph will have response and input are expressed as $20log\frac{|V_{out}|}{|V_{in}|}\ dB$ vs. log frequency. We can see the amplitude fluctuation over orders of magnitude in this way.

The phase shift of the response wave with respect to the input is the second primary response in which we are interested. A time delay that is either leading or behind in relation to the input is how we measure this. Using phasors and voltage division we express the low-pass response across the capacitor as

$$V_{out} = V_{in}(\frac{Z_C}{R+Z_c}) => V_{in} = \frac{1}{1+j\omega RC}$$

According to the equation, the response diminishes monotonically with increasing frequency. Accordingly, any frequencies that are allowed to pass through the low-filter to the left of its breakpoint are in the passband, any frequencies that are able to pass through to the output, or are to the right, are in the stop band.

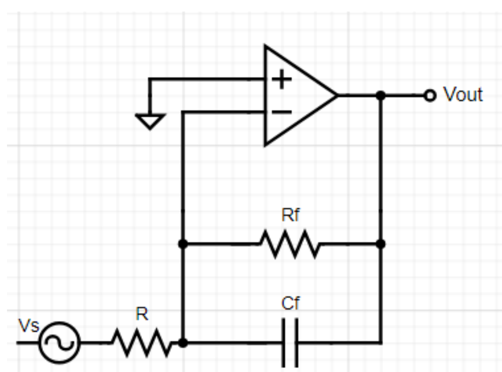
For a high-pass filter we have:

$$V_{out} = V_{in}(\frac{R}{R+Z_C}) => V_{in} = \frac{j\omega RC}{1+j\omega RC}$$

V_{in} is represented as a ratio with a numerator-dependent frequency dependent component. This explains how the numerator term can grow to a suitable size as the frequency increases, guaranteeing that the filter's transfer function approaches a value that allows high frequencies to pass through with less attenuation. As a result, this type of transfer function indicates high-pass filter behavior.

Wave-filters are frequently implemented using operational amplifiers. An op-amp low-pass filter configured in first-order RC is displayed below.

Consider the active low-pass filter circuit:



We find that the response is expressed as:

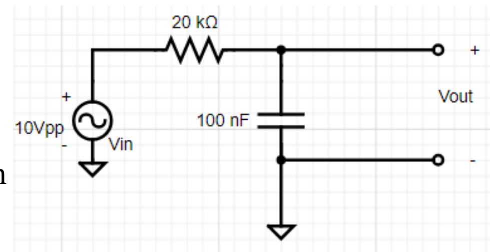
$$V_{out} = V_{in}(\frac{Z_f}{R}) \text{ where } Z_f = \frac{R_f Z_C}{R_f + Z_C}$$

$$=> V_{out} = V_{in}(-\frac{R_1}{R}\frac{1}{(1+j\omega R_1 C)})$$

$$=> f_c = \frac{1}{2\pi R_1 C_1}$$

Now, we analyze the behavior of two simple RC filter circuits by measuring the delay, period, and frequencies to determine the response with Bode and phase plots.

Results and Analysis



We have the circuit as shown:
A waveform generator supplies a sinusoidal voltage with 10Vpp amplitude and we vary the frequencies from $10Hz$ up to $10MHz$. We analyzed the behavior by taking measurements of output voltage, time delay and phase within the frequency range.

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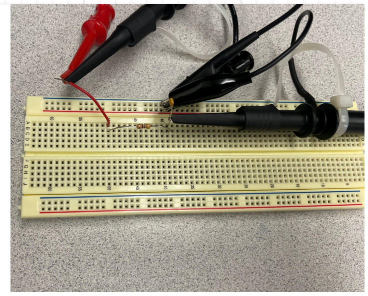
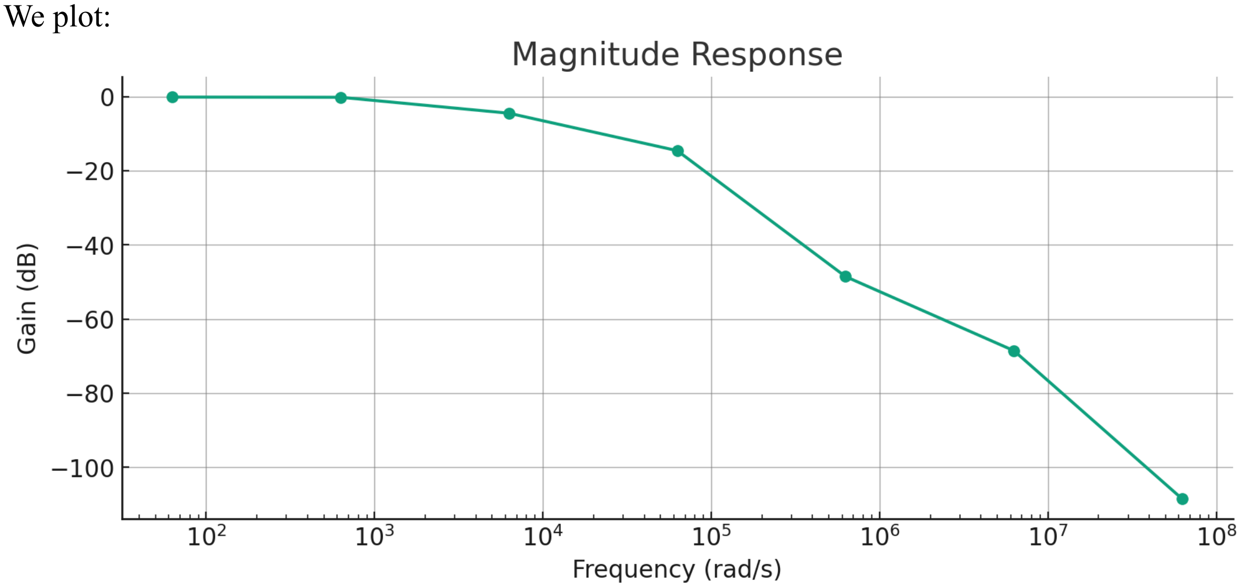
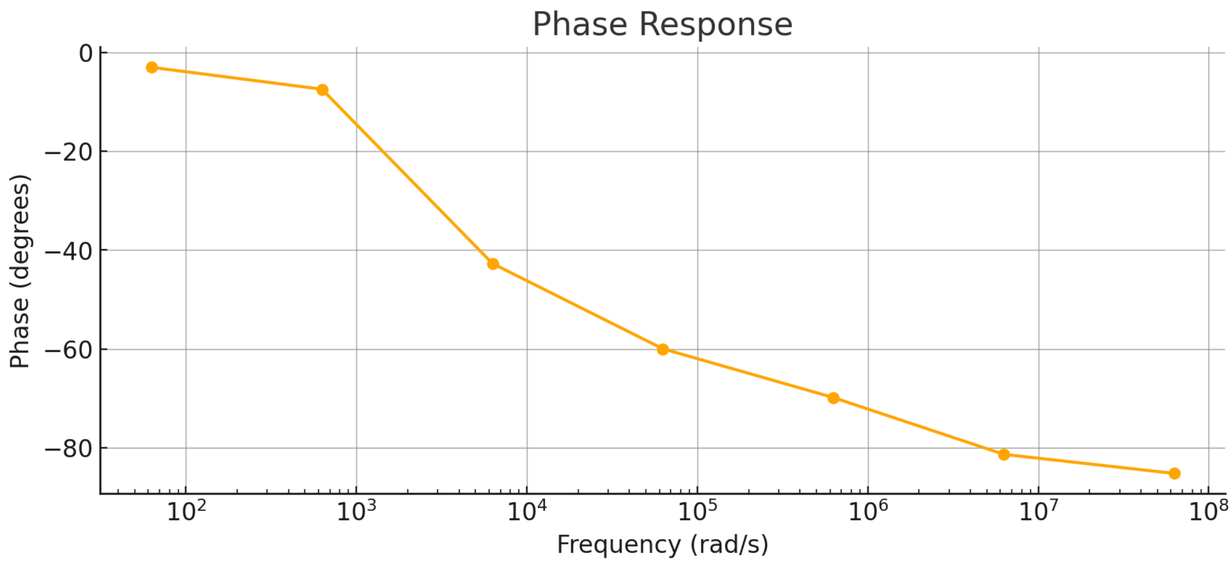


The intermediary values analysis was conducted are shown (using negative phase angle to indicate a lagging shift):

Frequency	Input Voltage (Vpp)	Output Voltage (Vpp)	Time Delay	Phase (Degrees)
10 Hz	10.7 V	10.56 V	840 µs	-3.02
100 Hz	10.7 V	10.5 V	208 µs	-7.45
1kHz	10.7 V	6.4 V	147 µs	-42.73
10KHz	10.7 V	2 V	50 µs	-60
100KHz	10.7 V	40 mV	20 µs	-69.87
1MHz	10.7V	4 mV	4.76 µs	-81.37
10MHz	10.7V	0.04mV	500ns	-85.23

We plot:

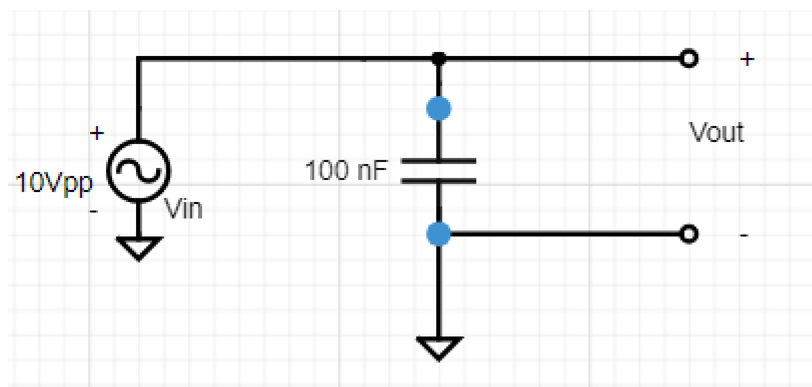




We see the expected behavior of a low-pass filter. The frequency increases in the magnitude response and the slope continues to monotonically decrease. The phase plot's behavior shows that as the frequency increases, the lag in the phase angle shift decreases. Greater time delays and the phase shifts reflect a larger attenuation to higher frequency signals.

Now, we consider how a simple RC high-pass filter circuit behaves:

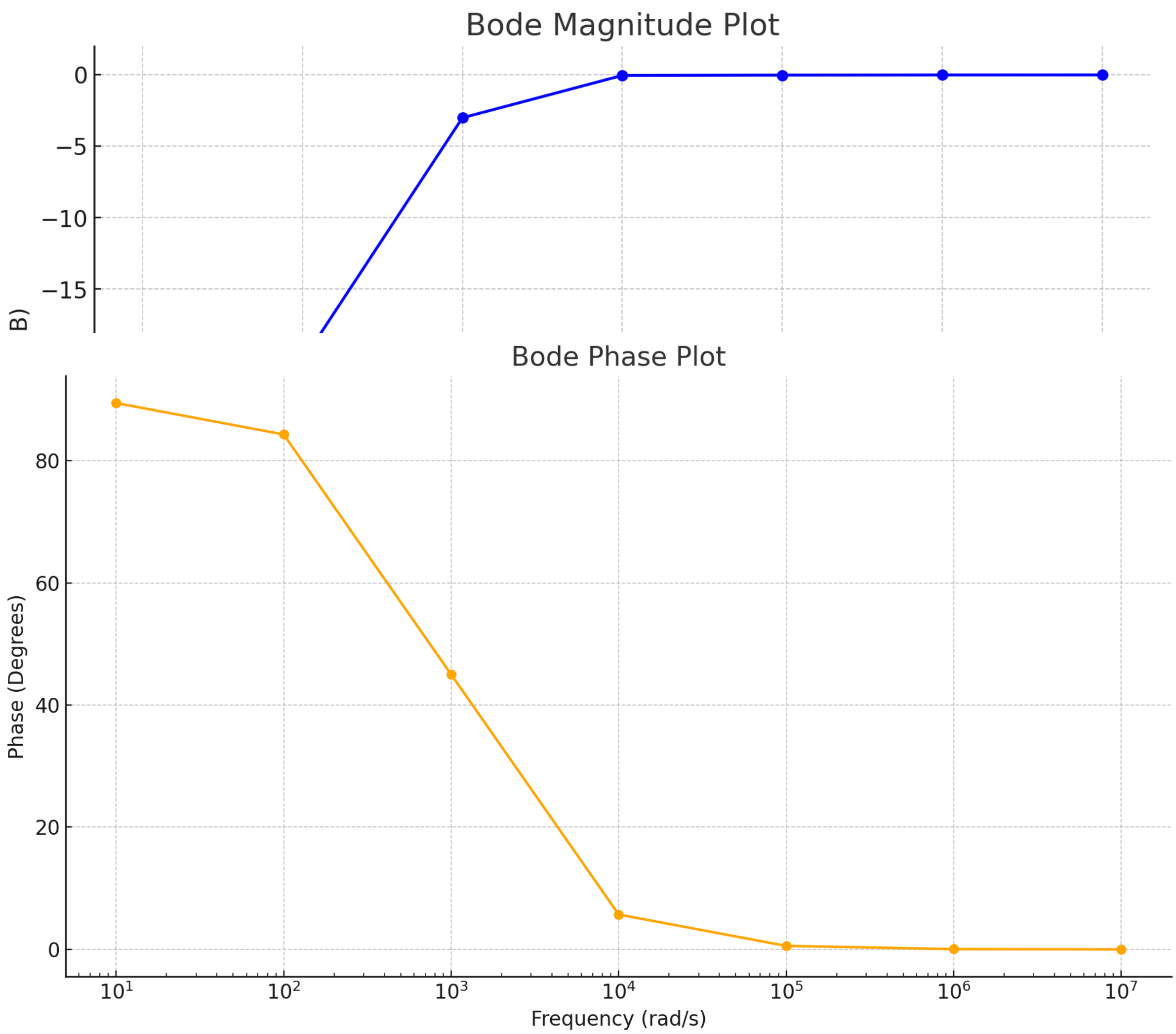
We construct this circuit by replacing the resistor used in the low-pass RC filter circuit with a wire. We find that the internal resistance of the wave generator of 50Ω and the addition of the wire is negligible such that the circuit may act as a high-pass filter.



We measure and plot:

Frequency	Input Voltage (Vpp)	Output Voltage	Time Delay	Phase (Degrees)
10 Hz	10.7 V	107mV	49.84 ms	89.43
100 Hz	10.7 V	1.065 V	4.79 ms	84.29

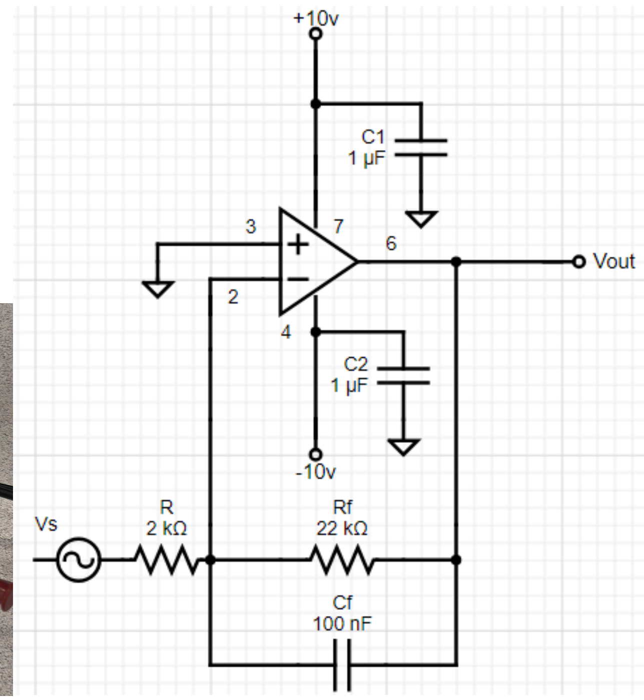
1kHz	10.7 V	7.566 V	380 µs	45
10 KHz	10.7 V	10.647 V	23.66 µs	5.71
100 KHz	10.7 V	10.673 V	3 µs	0.573
1MHz	10.7 V	10.6832 V	0.0573 µs	0.0487
10MHz	10.7 V	10.687	0.006 µs	0.005



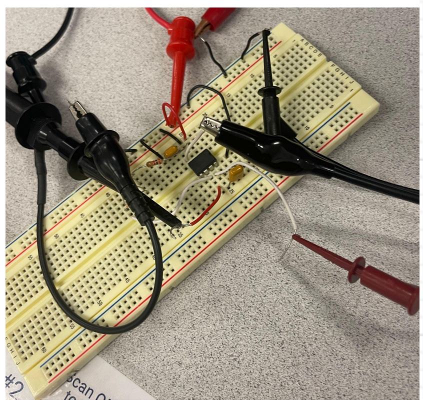
The magnitude graph behavior reflects the change in magnitude as the frequencies grow. The graph stabilizes due to a constant gain for frequencies higher than the cut off and the reactive impedance is negligible. The phase behaves such that the leading shift of the output starts to minimize towards the time of the input signal. This indicates that there is minimal output alteration as the filter passes higher frequencies.

Part II

We observe the response of an op-amp implementation of a low-pass filter circuit and measure as we did before. Consider the circuit:

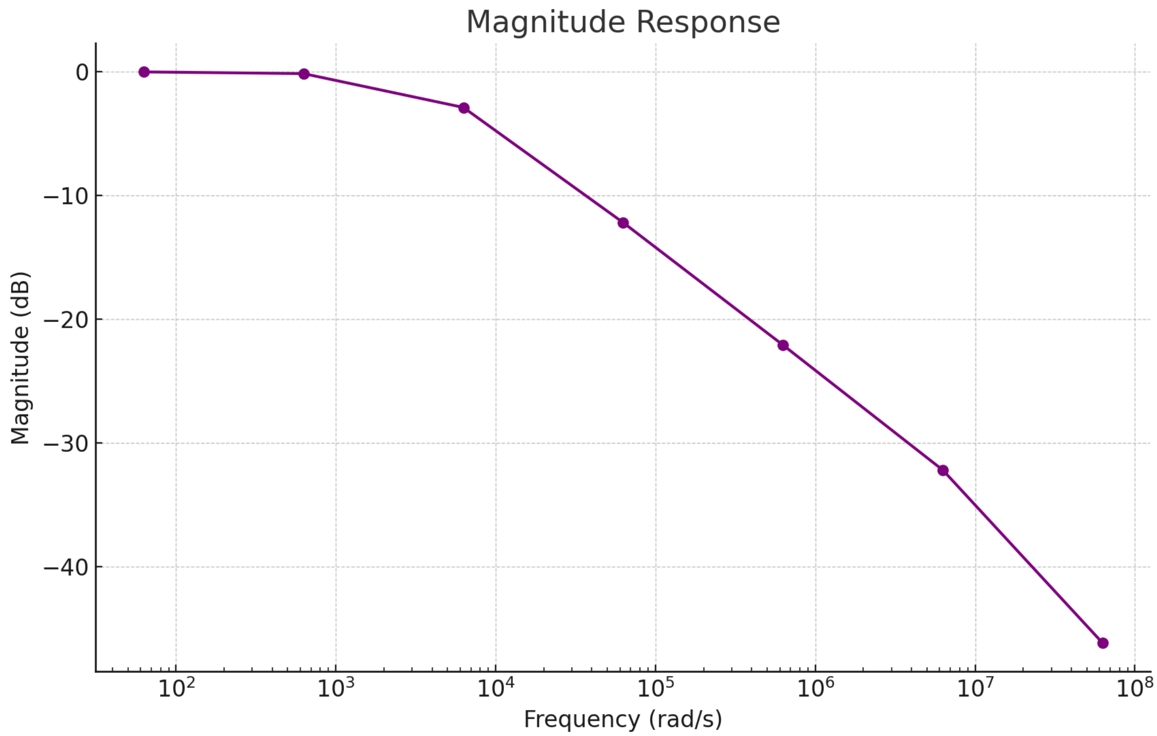


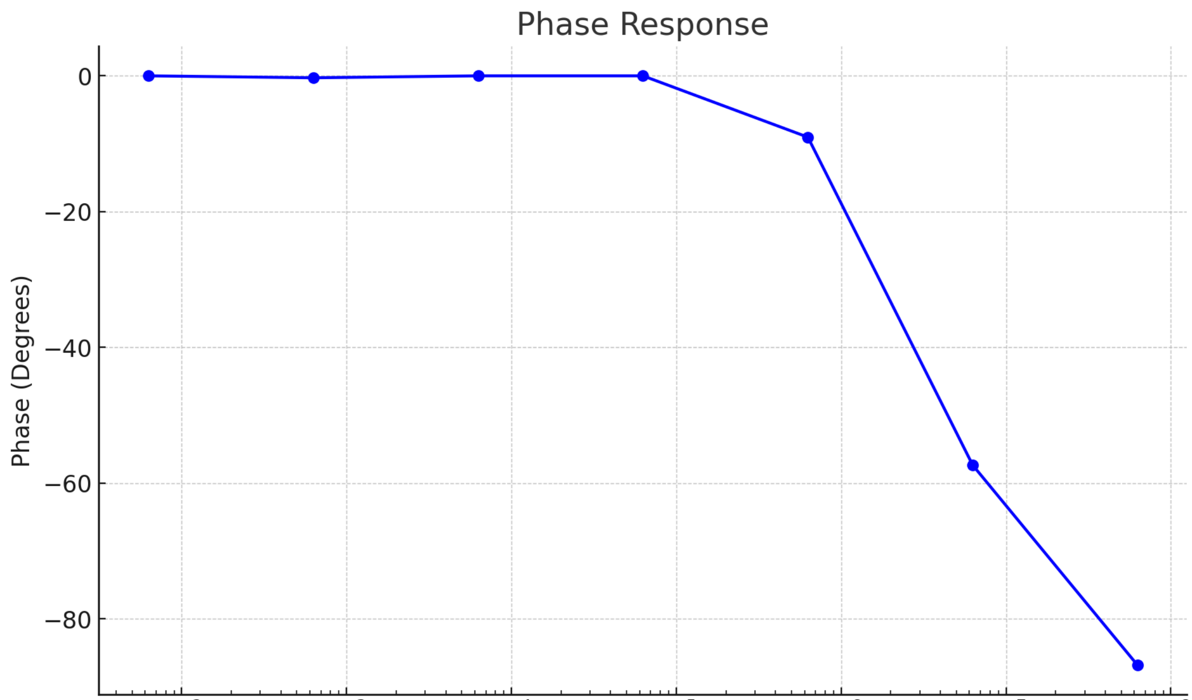
We have an operational-amplifier implementation of an active low-pass filter. We construct this circuit using the LM741 op-amp provided in the lab kit materials.



We measure and plot:

Frequency	Input Voltage (Vpp)	Output Voltage	Time Delay	Phase (Degrees)
10 Hz	10.2 V	10.16 V	100 ns	-0.00036
100 Hz	10.2 V	10 V	7.39 µs	-0.29
1kHz	10.2 V	7.3 V	0 µs	-42.76
10 KHz	10.2 V	1.02 V	0 µs	-76
100 KHz	10.2 V	106.35 mV	250.91 ns	-9.03
1MHz	10.2 V	10.43 mV	159.33 ns	-57.36
10MHz	10.2 V	1.02 mV	24.12 ns	-86.84





The magnitude response indicates what we'd expect from an active low-pass filter as the frequency starts to increase. The gain starts to become small due to the output signal being attenuated as measurements are taken at higher frequencies past the cut off. Analyzing the phase response, the shift is quite stable until it approaches the point where the frequencies are past the cut-off. The higher the frequency of the signal is, the more out of phase the circuit becomes.

4 Conclusion

We now understand the behavior of simple low and high pass circuits using capacitors. We learned about the implementation of an active low-pass filter and how its behavior does not deviate from its simpler counterparts. Investigating how signals are attenuated at high frequencies, the implications of a wave filter's value in application became apparent, especially when stabilizing systems. Despite challenges in the construction of the op-amp circuit and taking measurements with the oscilloscope, we were able to obtain enough data points to analyze the behavior of filters and identify them graphically instead of computationally.