CptS 451- Introduction to Database Systems

Relational Design Theory (DMS Ch-19)

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Relational Design Theory (topics)



- Motivation & Overview
- Functional Dependencies
- Boyce-Codd Normal Form
- 3NF

Database Design: A 6-Step Program



- Requirements Analysis: data requirements, critical operations on the data
- Conceptual DB Design: high-level description of data and constraints - typically using ER model
- 3. Logical DB Design: conversion into a schema
 - pick a type of DBMS, relational DBMS is most popular and is our focus
- 4. Schema Refinement: normalization (eliminating redundancy)
- Physical DB Design: consider workloads, indexes and clustering of data
- 6. Application/Security Design

Designing/Refining a Database Schema



- Usually many designs possible
 - Often use higher-level design tools.
 - Some designers go straight to relations
 - Useful to understand why tools produce certain schemas
- Some are (much) better than others!
- How do we choose?
 - Theory for relational database design



Example1: College application info.

- SSN and name
- Colleges applying to
- High schools attended (with city)
- Hobbies

Apply(SSN, sName, cName, HS, HScity, hobby)





Example1: College application info.

Apply(SSN, sName, cName, HS, HScity, hobby)

Kyle (with SSN 111) from PHS (Pullman) swims and plays trumpet and he applied to WSU, UW, and OSU.

SSN	sName	cName	HS	HScity	hobby
1111	Kyle	WSU	PHS	Pullman	Swim
1111	Kyle	WSU	PHS	Pullman	Trumpet
1111	Kyle	UW	PHS	Pullman	Swim
1111	Kyle	UW	PHS	Pullman	Trumpet
1111	Kyle	OSU	PHS	Pullman	Swim
1111	Kyle	OSU	PHS	Pullman	Trumpet



Example1: College application info.

Apply(SSN, sName, cName, HS, HScity, hobby)

Design "anomalies"

1) Redundancy

- Captures information multiple times
 - For example:
 - SSN, sName, HS, Hscity are repeated per (cName, hobby) pair.

• There is functional dependency
between SSN and sName, HS and
HScity.

SSN	sName	cName	HS	HScity	hobby
1111	Kyle	WSU	PHS	Pullman	Swim
1111	Kyle	WSU	PHS	Pullman	Trumpet
1111	Kyle	UW	PHS	Pullman	Swim
1111	Kyle	UW	PHS	Pullman	Trumpet
1111	Kyle	OSU	PHS	Pullman	Swim
1111	Kyle	OSU	PHS	Pullman	Trumpet



Example1: College application info.

Apply(SSN, sName, cName, HS, HScity, hobby)

Design "anomalies"

- 1) Redundancy
- 2) Update anomaly
- if we decide to call the instrument cornet (instead of trumpet) we need to modify it in each of the tuples in which it is stored (one per cName). Else, database will be inconsistent.

SSN	sName	cName	HS	HScity	hobby
1111	Kyle	WSU	PHS	Pullman	Swim
1111	Kyle	WSU	PHS	Pullman	Trumpet Cornet
1111	Kyle	UW	PHS	Pullman	Swim
1111	Kyle	UW	PHS	Pullman	Trumpet
1111	Kyle	OSU	PHS	Pullman	Swim
1111	Kyle	OSU	PHS	Pullman	Trumpet



Example1: College application info.

Apply(SSN, sName, cName, HS, HScity, hobby)

Design "anomalies"

- 1)Redundancy
- 2) Update anomaly
- 3) Deletion anomaly
- How to delete "Trumpet" hoby without deleting applicant information
 - possible solution: use null values in the hobby field

SSN	sName	cName	HS	HScity	hobby
1111	Kyle	WSU	PHS	Pullman	Swim
1111	Kyle	WSU	PHS	Pullman	Trumpet -NULL
1111	Kyle	UW	PHS	Pullman	Swim
1111	Kyle	UW	PHS	Pullman	Trumpet NULL
1111	Kyle	OSU	PHS	Pullman	Trumpet NULL



Example: College application info.

- SSN and name
- Colleges applying to
- High schools attended (with city)
- Hobbies

Student(SSN, sName)
Apply(SSN, cName)
HighSchool(SSN, HS)
Located(HS, HScity)
Hobbies(SSN, hobby)

- Decompose the relation into multiple relations
 - No anomalies
 - Can reconstruct the original relations (no loss of information)



Example: College application info.

- SSN and name
- Colleges applying to
- High schools attended (with city)
- Hobbies

```
Student(SSN, sName)
Apply(SSN, cName)
HighSchool(SSN, HS)
Located(HS, HScity)
Hobbies(SSN, hobby)
```

HobbyList(hobby, desc)
CollegeList(cName)

 The best design, for an application for relational databases depend not only on constructing the relations well, but also in what the data is representing in the real world.

Redundancy and Anomalies in Relational Schema – Example 2



CptS451 Projects	Student	Proj title	Date	Room#
	Kyle S.	Yelp	04/28/22	EME102A
	Aaron B.	Yelp	04/28/22	EME102A
	Jeromy J.	Yelp	04/28/22	EME102A
	Kelly K.	OODB	04/30/22	ETRL101

Redundancy:

 date of presentation and room# are repeated per member of project group

Redundancy and Anomalies in Relational Schema – Example 2 (cont.)



_	CptS451 Projects	Student	Proj title	Date	Room#	Error in updating.
		Kyle S.	Yelp	04/30/14	EME102A	Forgot to update all entries.
		Aaron B.	Yelp	04/28/14	EME102A	
		Jeromy J.	Yelp	04/28/14	EME102A	
		Kelly K.	OODB	04/30/14	ETRL101	

Update Anomaly:

 if we modify presentation date for the "yelp" project, we need to modify the date in each of the tuples in which it is stored (one per member). Else, database will be inconsistent.

Redundancy and Anomalies in Relational Schema – Example 2 (cont.)



CptS451 Projects	Student	Proj title	Date	Room#
	Kyle S.	Yelp	04/28/18	EME102A
	Aaron B.	Yelp	04/28/18	EME102A
	Jeromy J.	Yelp	04/28/18	EME102A
	Kelly K. NULL	OODB	04/30/18	ETRL101

Deletion Anomaly:

- How to delete the fact that Kelly K. dropped out of the project without deleting information about the OODB project.
 - possible solution: use null values in the student field

Relation Decomposition



CptS451 Projects	Student	Proj title	Date	Room#	
	Kyle S.	Yelp	04/28/14	EME102A	_
	Aaron B.	Yelp	04/28/14	EME102A	
	Jeromy J.	Yelp	04/28/14	EME102A	
	Kelly K.	OODB	04/30/14	ETRL101	
	NULL	dDB	04/24/14	SLOAN123	
Student	Proj title	_	Proj title	Date	Room#
Kyle S.	Yelp	_	Yelp	04/28/14	EME102A
Aaron B.	Yelp		OODB	04/30/14	ETRL101
Jeromy J.	Yelp		dDB	04/24/14	SLOAN123
Kelly K.	OODB	Anom	alies have s	ione:	

Anomalies have gone:

- No more repeated data
- Easy to update project information



Design by decomposition – how does it work?

- Start with "mega" relations
- Decompose into smaller, better relations with same info.
- Can do decomposition automatically

Automatic decomposition

- "Mega" relations + properties of the data (functional dependencies)
- System decomposes based on properties
- Final set of relations satisfy normal form
 - No anomalies, no lost information

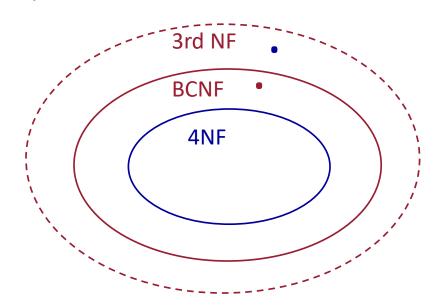


Normal Forms

Functional dependencies \Rightarrow Boyce-Codd Normal Form

+ Multivalued dependencies ⇒ Fourth Normal Form

1st NF 2nd NF 3rd NF



In CptS451, we will only cover BCNF and 3NF

What we will cover: Functional Dependencies and BCNF



Apply(SSN, SName, cName)

Redundancy: Storing SSN-sName pair once for each college

Functional Dependency SSN → sName

- Same SSN always has same sName
- Should store each SSN's sName only once

SSN	sName	cName
1111	Kyle	WSU
1111	Kyle	UW
1111	Kyle	OSU

Boyce-Codd Normal Form If $A \rightarrow B$ then A is a key

Apply(SSN, sName, cName): SSN is not a key
Apply is not in BCNF

Decompose: Student(SSN, sName) Apply(SSN, cName)
SSN is the key SSN, CName is the key

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Functional Dependency

Functional Dependency - Example



Example:

Assume there is fixed hourly pay rate for each rating.

ssn	name	dept	rating	hourly_pay	num_hours
111-11-1111	Kelly	123	5	18	40
222-11-2222	Kyle	124	4	16	40
333-11-3333	John	124	4	16	20
444-11-4444	Roseanne	123	4	16	20
555-11-5555	Ning	123	5	18	40

Problems????

Key: ssn FDs: ssn → name,dept,rating,num_hours rating → hourly_pay

Functional Dependency - Example



Suppose hourly_pay is determined by rating

```
rating = 4 \rightarrow \text{hourly_pay} = 16
rating = 5 \rightarrow \text{hourly_pay} = 18
rating = 6 \rightarrow \text{hourly_pay} = 20
```

Two tuples with same rating have same hourly_pay
 rating > hourly_pay (rating functionally determines hourly pay)

Functional Dependency - Definition



```
For all tuples t, u in R,

if t[A] = u[A] \Rightarrow t[B] = u[B]

then A \rightarrow B
```

<u>Definition</u>: Given Relation R(A1,...,An,B1,...,Bm,C1,...,Cl)

A1,...,An *functionally determine* B1,...,Bm, i.e.,

$$(A1,...,An \rightarrow B1,...,Bm)$$

When any two tuples agree on the attributes

Then they must also agree on the attributes

Functional Dependency (FD) – Some Terminology



- FDs are based on knowledge of real world. They generalize the concept of a key.
- If we know that an <u>FD holds on all tuples</u>, then we say that <u>R satisfies the FD</u>
- If we say that R satisfies an FD "F", we are stating a constraint on R

Functional Dependency (FD) - Some Terminology



Let X and Y be set of attributes from relation R=(A,B,C,...)

Trivial FD

- Those that are true for every relation
- X→Y is trivial if Y is a subset of the X, i.e., Y⊆X
- Example: AB→A

Nontrivial FD

- $X \rightarrow Y$ is called nontrivial if at least one of the attributes in Y is not among the attributes in X, i.e., $Y \subset X$
- Examples: AB→AC

Completely nontrivial FD

- Called *completely nontrivial* if none of the attributes in Y is one of the attributes in X, i.e., $Y \cap X = \emptyset$
- Example: $AB \rightarrow C$

FD - Observation



• If both of these FDs are true:

Then this FD also holds:

If we find out from application domain (real world data) that a relation satisfies some FDs, it doesn't mean that we found all the FDs that it satisfies!

There could be more FDs implied by the ones we have.

Reasoning About FDs



- Given some FDs, we can usually infer additional FDs: $ssn \rightarrow rating \ and \ rating \rightarrow hourly_pay \ implies \ ssn \rightarrow hourly_pay$
- An FD f is <u>implied by</u> a set of FDs F, if f holds whenever all FDs in F hold.
 - $-F^{+}$ = closure of F is the set of all FDs that are implied by F.
- Armstrong's Axioms (X, Y, Z are sets of attributes):
 - Reflexivity: If $X \subseteq Y$, then $Y \rightarrow X$
 - <u>Augmentation</u>: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
 - <u>Transitivity</u>: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- These are sound and complete inference rules for FDs!

Additional Rules for FDs



Additional rules while reasoning about F⁺

1. Combining Rule (Union)

Combining Right sides of FDs

A1,...An
$$\rightarrow$$
 B1 A1,...An \rightarrow B2 is equivalent to A1,...An \rightarrow B1,...Bm A1,...An \rightarrow Bm

Example: $A \rightarrow B$ and $A \rightarrow C$ is equivalent to $A \rightarrow BC$.



Additional Rules for FDs

2. Splitting Rule (Decomposition)

Splitting <u>right</u> sides of FDs

A1,...An
$$\rightarrow$$
 B1,...Bm is equivalent to A1,...An \rightarrow B1
A1,...An \rightarrow B2
....
A1,...An \rightarrow Bm

Example: $A \rightarrow BC$ is equivalent to $A \rightarrow B$ and $A \rightarrow C$.

Can we also split left-hand-side?

No. There is no splitting rule for left sides

Example?

What we talked about so far...



- 1. Redundancy and Anomalies in Relational Schema
- 2. Design by decomposition
 - Start with "mega" relations containing everything
 - Decompose into smaller, better relations with same info.
 - Can do decomposition automatically
 - Final set of relations satisfies normal form
 - No anomalies, no lost information
 - BCNF (will cover)
 - 1NF, 2NF, 3NF, 4NF (won't cover)
- 3. Functional dependencies (FDs)
- 4. Rules for Functional Dependencies
 - Armstrong Axioms,
 - Closure of FDs
- 5. Closure of Attributes (next)

Closure of Attributes



- Given relation R, FDs F, set of attributes X={A1,A2,...An}
- Find all $Y=\{B1,...,Bm\}$ such that $X \rightarrow Y$

Closure of X denoted by $X^+ = \{A1, A2, ...An\}^+$

```
How to calculate {A1,A2,...An} † ?

Start with X={A1,A2,...An}

Repeat until no change:

For every FD rule in F X' → Y',

if X' is in the closure set

add Y' to the set
```

```
Hourly_Emps(SSN,name,dept,
rating,hourly_pay,
num_hours)

ssn → name,dept,rating,num_hours
rating → hourly pay
```

Closure Example



Hourly_Emps(SSN, name, dept, rating, hourly_pay, num_hours)

```
ssn → name,dept,rating,num_hours
rating → hourly_pay

{ssn}+= {ssn, name, dept, rating, num_hours, hourly_pay}

Hence:
ssn → ssn, name, dept, rating, num_hours, hourly_pay
```

Key: ssn

Armstrong's Axioms

<u>Reflexivity</u>: If $X \subseteq Y$, then $Y \to X$ <u>Augmentation</u>: If $X \to Y$, then $XZ \to YZ$ for any Z <u>Transitivity</u>: If $X \to Y$ and $Y \to Z$, then $X \to Z_{38}$

Closure Example



R(A,B,C,D,E,F)

A, B
$$\rightarrow$$
 C
A, D \rightarrow E
B \rightarrow D
A, F \rightarrow B

- Compute $\{A,B\}^+$ $\{A,B\}^+ = \{A,B,$
- Compute $\{A,F\}^+$ $\{A,F\}^+ = \{A,F,$





A, B
$$\rightarrow$$
 C
A, D \rightarrow E
B \rightarrow D
A, F \rightarrow B

• Compute
$$\{A,B\}^+$$
 $\{A,B\}^+ = \{A,B,C,D,E\}$

• Compute $\{A,F\}^+$ $\{A,F\}^+ = \{A, F, B, C, D, E\}$

What is the key of R?

Closure Example 2



Practice yourselves:

R(A,B,C,D)

A, B
$$\rightarrow$$
 C
A, D \rightarrow B
B \rightarrow D

• Compute
$$\{A,B\}^+ = \{A,B\}^+ = \{A,B,B\}^+$$

• Compute
$$\{A,D\}^+ = \{A,D\}^+ = \{A,D, \}$$

• Compute
$$\{B,D\}^+$$
 $\{B,D\}^+ = \{B,D,$

What is the key of R?

Functional Dependencies and Keys



- Assume R=(A1,..An,B1,..Bm,C1,..Ck) is a relation with no duplicates
- ■Suppose A1,A2,...An \rightarrow all attributes
 - $-i.e., A1,A2,...An \rightarrow A1,...An,B1,...Bm,C1,...,Ck$
 - The group of attributes A1,...,An is a key that functionally determine the complete tuple.

$$R(A,B,C,D)$$

 $\{A,B\}^+ = \{A, B, C, D\}$

$$A, B \rightarrow C$$

 $A, D \rightarrow B$
 $B \rightarrow D$

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Keys

- A superkey is a set of attributes A1, ..., An such that they functionally determine the complete tuple.
 - **Ex:** R(A,B,C,D,E,F) with FDs {AB→C; AD→E; B → DA; F →B} {A,C,F} is a superkey for R

- A key is a minimal key
 - A superkey and for which no subset is a key
 - Ex: {A,F} is a minimal key for R



Computing (Super)Keys

- Given R=(A1,..,An,B1,..Bm,C1,..Ck), let X be a subset of the attributes for R, i.e.,
 X ⊆ {A1,..,An,B1,..Bm,C1,..Ck}
- For all subsets X, compute X⁺
- If X⁺ is equal to [all attributes], then X is a superkey
- Minimal X(s) is/are the key(s)
 - Can we have more than one key?
 - YES

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Computing Keys – Example 1

Product(name, price, category, color)

FDs:

```
name, category → price category → color
```

What is the key?

Key

Closures:

```
{name}+ = {name}
{price}+ = {price}
{category}+ = {category,color}
{color}+ = {color}
```

{name,price}⁺ = {name,price}

```
{name,category}+ ={name,category,price,color}

{name,color}+ = {name,color}

{price,category}+ ={price,category,color}

{price,color}+ = {price,color}
```

{category,color}+={category,color}

Superkey

{name,price,category}* = {name,price,category,color}

{name,price,color}+ ={name,price,color}

Superkey

{name,category,color}* = {name,price,category,color}

{price,category,color}+ = {price,category,color}

Superkey

{name,price,category,color}+ ={name,price,category,color}

Functional Dependency



Back to Example:

```
Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority)
```

Functional dependencies:

```
SSN → sName

SSN → address

SSN → HScode

HScode → HSname, HScity

HSname, HScity → HScode

SSN → GPA

GPA → priority
```

We assume that each student has one (current) address, and has graduated from one high schools.

From the last2 above we can derive $SSN \rightarrow priority$



Computing Keys – Example3

Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority)

• FDs:

```
SSN → sName, address,

HScode,GPA

HScode → HSname, HScity

HSname, HScity → HScode

GPA → priority
```

What is the key?

```
{SSN}+ = {SSN,sName,address,HSname,Hscity,GPA,priority}
{sName}+ = {sName}
{HScode}+ = {HScode,HSname,HScity}
{HSname}+ = {HSname}
{HScity}+ = {HScity}
{GPA}+ = {GPA,priority}
{priority}+ = {priority}

{SSN,sName}+ = {SSN,sName,address,GPA,priority}
{SSN,address}+ = {SSN,sName,address, HScode,HSname,Hscity,GPA,priority}
{SSN,HScode}+ = {SSN,sName,address,HScode,HSname,Hscity,GPA,priority}
{SSN,HSname}+ = {SSN,sName,address,HSname,GPA,priority}
{SSN,HScity}+ = {SSN,sName,address,HScity,GPA,priority}
{SSN,GPA}+ = {SSN,sName,address,GPA,priority}
{SSN,priority}+ = {SSN,sName,address,GPA,priority}
```

Computing Keys – Example3

```
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```

```
{sName,address}+={sName,address}
{sName,HScode}+ = {sName,Hscode,HSname,Hscity}
{sName,HSname}<sup>+</sup> = {sName,HSname}
{sName,HScity}+ = {sName,HScity}
{sName,GPA}+ = {sName,GPA,priority}
{sName,priority}+ = {sName,priority}
{address, HScode}+ = {address, HScode, HSname, Hscity}
{address,HSname}* = {address,HSname}
{address, HScity}* = {address, HScity}
{address,GPA}<sup>+</sup> = {address,GPA,priority}
{address,priority}* = {address,priority}
{HScode, HSname}+={HScode, HSname, Hscity}
{HScode, HScity}+ = {HScode, HSname, Hscity}
{HScode,GPA}+ = {HScode,HSname,Hscity,GPA,priority}
{HScode,priority}+ = {HScode,HSname,Hscity,priority}
{HSname, HScity}+ = {HScode, HSname, Hscity}
{HSname,GPA}+ = {Hsname,GPA,priority}
{HSname,priority}* = {Hsname,priority}
{HScity,GPA}+ = {HScity,GPA,priority}
{HScity,priority}* = {HScity,priority}
{GPA,priority}+ = {GPA,priority}
```

Complete the rest:

- Closures for subsets with three attributes
- Closures for subsets with four attributes
- •Closures for subsets with five attributes
- Closures for subsets with six attributes
- Closures for subsets with seven attributes
- Closure for the complete set



Key or Keys?

- Can a relation have more than one key?
 - Yes.
- Examples:

For R(A,B,C)

A→B,

 $B \rightarrow C$

 $C \rightarrow A$

 $AB \rightarrow C$

 $BC \rightarrow A$

 $A \rightarrow BC$

 $B \rightarrow AC$

3 Keys:

A or B or C

2 Keys:

AB or BC

2 Keys:

A or B

Back to the main problem...



- Given a relation schema, we need to decide whether it is a good design
 - Such a decision must be guided by an understanding of what problems, if any, arise from the current schema.
 - Redundancy, Update/Insert/Delete anomalies
 - FDs that hold on the relation may cause such problems.
 - Example: hourly_emp relation

ssn	name	dept	rating	hourly_pay	num_hours
111-11-1111	Kelly	123	5	18	40
222-11-2222	Kyle	124	4	16	40
333-11-3333	John	124	4	16	20
444-11-4444	Roseanne	123	4	16	20
555-11-5555	Min	123	5	18	40

Key : ssn FDs: $ssn \rightarrow name, dept, rating, num_hours$ rating $\rightarrow hourly_pay$

Back to the main problem...



- Several "Normal Forms" have been proposed. If a relation is in one of these normal forms, we know that certain kinds of problems does not exist.
 - BCNF (most important normal forms from DB design standpoint)
 - BCNF Main Idea: We want all attributes in every tuple to be determined by the tuple's key attributes, i.e. part of a superkey
 - X → A is OK if X is a (super)key
 - X → A is not OK otherwise

What does this say about redundancy?

ssn	name	dept	rating	hourly _pay	num_ hours
111-11-1111	Kelly	123	5	18	40
222-11-2222	Kyle	124	4	16	40
333-11-3333	John	124	4	16	20
444-11-4444	Roseanne	123	4	16	20
555-11-5555	Min	123	5	18	40

Key: ssn **FDs**: ssn \rightarrow name,dept,rating, num_hours rating \rightarrow hourly_pay

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Design by Decomposition – Eliminate Anomalies

- Main idea:
 - $-X \rightarrow A$ is OK if X is a (super)key
 - $-X \rightarrow A$ is not OK otherwise
 - Need to decompose the table, but how?

Decompositions in General



Lossless Decomposition



```
R(A1, ..., An, B1, ..., Bm, C1, ..., Cp)
R1(A1, ..., An, B1, ..., Bm)
R2(A1, ..., An, C1, ..., Cp)
```

- Decomposition of R into R1 and R2 is lossless if,
 - 1. {A1, ..., An, B1, ..., Bm} U {A1, ..., An, C1, ..., Cp} = {A1, ..., An, B1, ..., Bm, C1, ..., Cp}
 - 2. $R1 \bowtie R2 = R$

Lossless Decomposition



ssn	name	dept	rating	hourly_ pay	num_ hours
111-11-1111	Kelly	123	5	18	40
222-11-2222	Kyle	124	4	16	40
333-11-3333	John	124	4	16	20
444-11-4444	Roseanne	123	4	16	20
555-11-5555	Min	123	5	18	40

Key: ssn

FDs: $ssn \rightarrow name, dept, rating, num_hours$

rating → hourly_pay

Hourly_Emps

lack lac						
ssn	name	dept	rating	num_hours		
111-11-1111	Kelly	123	5	40		
222-11-2222	Kyle	124	4	40		

124

123

123

H2

rating	hourly_pay
4	16
5	18

1. {ssn, name,dept,rating,num_hours} U {rating_hourly_pay} =

20

20

40

{ssn, name,dept,rating,num_hours,rating_hourly_pay }

2. $H1\bowtie H2 = Hourly_Emps$

John

Roseanne

Min

H1

333-11-3333

444-11-4444

555-11-5555



Boyce-Codd Normal Form

Definition. A relation R is in BCNF if: Whenever $X \rightarrow B$ is a <u>non-trivial</u> dependency, then X is a <u>key</u> or <u>superkey</u>.

- i.e., X+={all attributes}

There are no "bad" FDs

Boyce-Codd Normal Form



Definition. A relation R is in BCNF if:

Whenever $X \rightarrow B$ is a <u>non-trivial</u> dependency, then X is a <u>key</u> or <u>superkey</u>.

i.e., X⁺={all attributes}

hourly_emp (SSN, name, dept, rating, hourly_pay, num_hours)

Key: {SSN}

hourly_emp is not in BCNF.
Not every FD has a key on the left hand side

ssn → name,dept,rating, num_hours rating → hourly pay

Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority)

Key: {SSN, HScode}

Student is not in BCNF.

Not every FD has a key on the left hand side

SSN → sName, address, GPA HScode → HSname, HScity HSname, HScity → Hscode GPA → priority



Boyce-Codd Normal Form – Example 1

Given R(A,B,C,D)

 $AB \rightarrow C$

 $BC \rightarrow D$

 $CD \rightarrow A$

 $AD \rightarrow B$

Calculate:

$${A,B}^+ = {A,B,C,D}$$
 key

 $\{B,C\}^+ = \{B,C,D,A\}$ key

 $\{C,D\}^+ = \{C,D,A,B\}$ key

 ${A,D}^+ = {A,D,B,C} key$

In BCNF



Boyce-Codd Normal Form – Example 2

Given R(A,B,C,D,E)

$$AB \rightarrow C$$

 $DE \rightarrow C$
 $B \rightarrow D$
 $BC \rightarrow B$ trivial FD

Calculate:

$${A,B}^+ = {A,B,C,D} X$$

 ${D,E}^+ = {D,E,C} X$
 ${B}^+ = {B,D} X$

Not in BCNF



Boyce-Codd Normal Form – Example3

Given R(A,B,C,D)

$$AB \rightarrow C$$

$$AB \rightarrow D$$

$$C \rightarrow A$$

$$D \rightarrow B$$

Calculate:

$${A,B}^+ = {A,B,C,D}$$
 key

$$\{C\}^+ = \{C,A\} X$$

$$\{D\}^+ = \{D,B\} X$$

Not in BCNF

Lossless BCNF Decomposition



- Next we will study algorithm to decompose a relational schema into sub-schemas which are in BCNF such that the decomposition is lossless
- Setting:

Given relation R, and

F, FDs for R

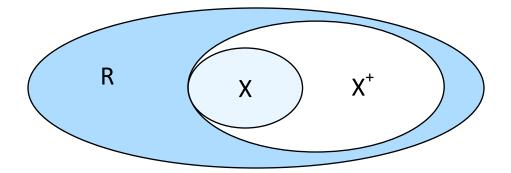
Suppose relation R has BCNF violation $X \rightarrow Y$.

Decomposition to Reach BCNF (II)



The FD $X \rightarrow Y$ violates BCNF.

- 1. Expand X to include X⁺. (Cannot be all attributes why?)
- Decompose R into R1(X⁺) and R2(X,rest), i.e., X U (R-X⁺).



- 3. Find the FD's for the decomposed relations.
 - Project the FD's from F⁺ on R1 and R2:
 - calculate all consequents of F that involve only attributes from R1 and R2
- 4. Iterate over all the resulting sub schemes until all in BCNF

Note: Any table with only 2 attributes is always in BCNF!!!



BCNF Decomposition Algorithm

Input: relation R and F (FDs for R)

Output: decomposition of R into BCNF relations with "lossless join"

- Compute keys for R
- Repeat until all relations are in BCNF:

Pick any R' with $X \rightarrow Y$ that violates BCNF

Decompose R' into $R_1(X^+)$ and $R_2(X, rest)$

Compute FDs for R₁ and R₂

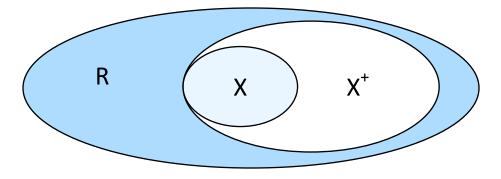
Compute keys for R₁ and R₂

Decomposition to Reach BCNF (II)



The FD X -> Y violates BCNF.

- Expand X to include X⁺.
 - Cannot be all attributes why?
- 2. Decompose R into R1(X^{+}) and R2(X, rest), i.e., $X \cup (R-X^{+})$.



How do we calculate this?

- 3. Find the FD's for the decomposed relations.
 - Project the FD's from F on R1 and R2:
 - calculate all consequents of F that involve only attributes from R1 and R2
- 4. Iterate over all the resulting sub schemes until all in BCNF





Let R have a schema R(A,B,C,D)

R1 have a schema R1(A,C)

FD over R be:

 $A \rightarrow B$ and $B \rightarrow C$

How to find the FD's for the decomposed relations?

Algorithm to compute the set of FD's that hold on R1

Input: Relation R R1, a sub-schema of R. Set of FDs , **F** that hold in R

Output: The set of FDs that hold in R1.

Method:

Let **T** be the set of FDs that hold in **R1** (initially empty).

- For each X that is a subset of R1, do
 - Compute X⁺
 - For each attribute B in X⁺ in such that:

 \Rightarrow B is in R1 \Rightarrow B is not in X

- Add X \rightarrow B to **T** (i.e., the functional dependency **X** \rightarrow **B** holds in **R1**)
- Eliminate trivial dependencies from T

Example - 1



Let R have a schema R(A,B,C,D)
R1 have a schema R1(A,C)
FD over R be:

 $A \rightarrow B$ and $B \rightarrow C$

- Compute {A}+ = {A,B,C}
 - hence dependency A → C holds in R1
- Compute {C}⁺ = {C}
 - no new dependency gets added.
- Compute {AC} + = {ABC}
 - AC → AC holds, but it is a trivial dependency. Therefore, no new dependency gets added.

In general you can limit search as follows:

- 1. It is not necessary to consider the closure of the set of all attributes
 - For example, {AC}+ need not have been considered in the above example
- 2. Not necessary to consider a set of attributes that does not contain the "left hand side" of any dependency.
 - {C}+ need not have been considered in the above example
- 3. Not necessary to consider a set that contains an attribute that is not in the "left hand side" of any functional dependency
 - {AC}+ need not have been considered in the above example.

Example - 2



Consider R(A,B,C,D,E) and R1(A,B,C) FD on R be $A \rightarrow D$, $B \rightarrow E$, $DE \rightarrow C$

- Compute {A}⁺ == {A,D}
 - no dependency gets added.
- Compute {B}+ == {B,E}
 - no dependency gets added
- {C} + does not need to be considered since {C} not in the left hand side of any FD
- Compute {AB}+ == {A,B,C,D,E}
 - add dependency AB → C
- {AC}+ and {BC}+ do not need to be considered since {C} not in the left hand side
 of any FD
- Since {AB}+ == all attributes in R, {ABC} need not be considered.
- Hence, the only dependency on R1 is: AB → C

Example - 2



Consider R(A,B,C,D,E) and R1(A,B,C) FD on R be A \rightarrow D, B \rightarrow E, DE \rightarrow C



BCNF Decomposition – Example 1

Given R(A,B,C,D,E) with functional dependencies: $D \rightarrow B$ CE $\rightarrow A$

a. In BCNF?
 {D}+ = {D,B} D is not a key and D → B violates BCNF.
 {CE}+ = {C,E,A} CE is not a key and CE → A violates BCNF.
 The Key for R is {CDE}

BCNF Decomposition – Example 1 (cont.) Cpts 45

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Given R(A,B,C,D,E) with functional dependencies:

 $D \rightarrow B$ CE $\rightarrow A$

R(A,B,C,D,E)

decompose R (apply $D \rightarrow B$)

R1(B,D)

Compute FDs for R1: D \rightarrow B

Compute key for R1: D

(see next slide)

(in BCNF)

Any table with only 2 attributes is always in BCNF!!!

(R2(A,C,D,E)

Compute FDs for R2: $CE \rightarrow A$, $CDE \rightarrow A$ (see next slide)

Compute keys for R2: CDE (see next slide)

(not in BCNF): $CE \rightarrow A$ violates BCNF,

CE is not a key

decompose R2

(apply $CE \rightarrow A$)

(R3(A,C,E)

Compute FDs for R3: $CE \rightarrow A$

Compute keys for R3: CE

(see next-next slide)

(in BCNF)

(R4(C,D,E)

Compute FDs for R4: None

Compute keys for R3: CDE

(see next-next slide)

(in BCNF)

R(A,B,C,D,E) is decomposed into R1(B,D), R3(A,C,E), and R4(C,D, E) The decomposition is **dependency-preserving. Why?**



BCNF Decomposition - Example1(cont.)

Given R(A,B,C,D,E)

$$D \rightarrow B$$

$$CE \rightarrow A$$

Compute FD's for R1(B,D):

{B}+		
{D}+		
{B,D}+		

{D} is the key

Compute FD's for R2(A,C,D,E):

{A}+	{A,C,D}+
	{A,C,E}+
{D} +	{A,D,E}+
{E}+	{C,D,E}+
{A,C}+	{A,C,D,E} +
{A,D}+	
{A,E}+	
{C,D}+	
{C,E}+	
{D,E}+	

{CDE} is the key



BCNF Decomposition – Example1(cont.)

Given R(A,B,C,D,E)

$$D \rightarrow B$$

$$CE \rightarrow A$$

Compute FD's for R1(B,D):

$${B}^+$$

 ${D}^+={B,D}$ D \rightarrow B added ${B,D}^+$

{D} is the key

Compute FD's for R2(A,C,D,E):

{A}+ {C}+= {C} {D}+= {B,D} {E}+= {E}		{A,C,D} + {A,C,E} + {A,D,E} + {C,D,E} += {C,D,E,B,A}	CDF → A added
{A,C}+ {A,D}+ {A,E}+		{ A,C,D,E} +	CDE 7 A added
{C,D} += {C,D,B} {C,E} += {C,E,A} {D,E} += {D,E,B}	CE → A added		

{CDE} is the key



BCNF Decomposition – Example1(cont.)

```
Given R2(A,C,D,E)
CE \rightarrow A
CDE \rightarrow A
```

Compute FD's for R3(A,C,E):

```
{A}<sup>+</sup>
{C} +
{E} +
{A,C} +
{A,E} +
{C,E} +
```

{CE} is the key

Compute FD's for R4(C,D,E):

```
{C}+
{D}+
{E}+
{C,D}+
{C,E}+
{D,E}+
{C,D,E}+
```



BCNF Decomposition – Example1(cont.)

```
Given R2(A,C,D,E)
CE \rightarrow A
CDE \rightarrow A
```

Compute FD's for R3(A,C,E):

```
{A}^{+}
{C}^{+}={C}
{E}^{+}={E}
{A,C}^{+}
{A,E}^{+}
{C,E}^{+}={C,E,A}
{A,C,E}^{+}
```

{CE} is the key

Compute FD's for R4(C,D,E):

{CDE} is the key

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BCNF Decomposition – Example3

Consider a relation with schema R(A,B,C,D) and FD's:

$$BC \rightarrow D$$
,
 $BC \rightarrow A$,
 $D \rightarrow B$,
 $A \rightarrow C$

- a. Find the minimal key(s) for this relation.
- b. Is R in BCNF?
- c. If not in BCNF, decompose the relation into collections of relations that are in BCNF.
- d. Are the functional dependencies preserved in the BCNF decomposition?

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BCNF Decomposition – Example 2

Given **Hourly_Emps**(ssn, name,dept,rating,num_hours,rating_hourly_pay) with functional dependencies:

ssn → name,dept,rating,num_hours

rating → hourly_pay

a. In BCNF?
 {ssn}+ = {ssn,name,dept,rating,num_hours}
 ssn is a key and first functional dependency doesn't violate BCNF.
 {rating}+ = {rating,hourly_pay}
 rating is not a key and second functional dependency violates BCNF.

Therefore, Hourly_Emps is not in BCNF.

BCNF Decomposition – Example2 (cont.)



```
Hourly_Emps(ssn,name,dept,rating,num_hours,rating,hourly_pay)
ssn → name,dept,rating,num_hours
rating → hourly_pay

Hourly_Emps(ssn,name,dept,rating,num_hours,rating,hourly_pay)

decompose Hourly_Emps
(rating → hourly_pay)

H2(rating,hourly_pay)

H1(ssn,name,dept,rating,num_hours,rating)
mpute FDs for H2: rating → hourly pay

Compute FDs for H1: ssn → name,dept,rating,num_hours
```

Compute FDs for H2: rating → hourly_pay
Compute key for H2: rating
(in BCNF)

Compute FDs for H1: ssn → name,dept,rating,num_hours
Compute key for H1: ssn

(in BCNF)

Hourly_Emps is decomposed into: H1(ssn,name,dept,rating,num_hours,rating) and H2(rating,hourly_pay) The BCNF decomposition is dependency-preserving.

BCNF Decomposition – Example4



Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority) FDs:

```
SSN \rightarrow sName, address, GPA \rightarrow priority \rightarrow HScode \rightarrow HSname, HScity \rightarrow Hscode
```

Key: SSN, HScode

a. In BCNF?

No. Several violations.

```
For example: {HSCode}+ = {HScode, HSname, HScity} HSCode is not a key, HScode → HSname, Hscity violates BCNF.
```

```
\{SSN\}^+ = \{SSN,sName,address,GPA,priority\}
SSN is not a key , SSN \rightarrow sName and SSN \rightarrow address violates BCNF.
```

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BCNF Decomposition – Example 2 (cont.)

Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority)

```
SSN \rightarrow sName, address, GPA \rightarrow priority \rightarrow HScode \rightarrow HSname, HScity \rightarrow Hscode
```

b. Decompose Student (use HScode \rightarrow HSname, HScity)

```
S1(HScode, HSname, HScity)
```

```
FDs: HScode → HSname, Hscity ; HSname, HScity → Hscode
Keys: {HScode} and {HSname, Hscity}
(in BCNF)
```

S2(SSN, sName, address, HScode, GPA, priority)

```
FDs: SSN → sName,address,GPA,priority ; GPA→ priority
Key: {SSN,HScode}
(not in BCNF)

Decompose S2 (use GPA → priority)

S3(GPA, priority) (in BCNF)

S4(SSN, sName, address, HScode, GPA) (not in BCNF)
(next slide)
```

BCNF Decomposition – Example 2 (cont.)



Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority)

SSN \rightarrow sName, address, GPA

 $GPA \rightarrow priority$

 \rightarrow HSname, Hscity

 HSname , $\mathsf{HScity} \rightarrow \mathsf{Hscode}$

S4(SSN, sName, address, HScode, GPA) (not in BCNF)

FDs: SSN → sName,address,GPA,priority

Key: {SSN,HScode}

Decompose (use SSN → sName,address,GPA,priority)

S5(SSN, sName, address, GPA, priority) (in BCNF)

FDs: SSN → sName,address,GPA,priority

Key: {SSN,HScode}

S6(SSN, HScode) (in BCNF)

Student is decomposed into S1, S3, S5, and S6





_	SSN	sName	address	HSCode	HSname	HScity	GPA	Priority
	1111	Kyle	Everett	PHS	Pullman High School	Pullman	3.4	2
Student	1111	Kyle	Everett	EHS	Everett High School	Everett	3.4	2
	2222	John	Pullman	POHS	Potlatch High School	Potlatch	3.0	3
	2222	John	Pullman	MHS	Moscow High School	Moscow	3.0	3
	2222	John	Pullman	PHS	Pullman High School	Pullman	3.0	3

S1

HSCode	HSname	HScity	
PHS	Pullman High School	Pullman	
EHS	Everett High School	Everett	
POHS	Potlatch High School	Potlatch	
MHS	Moscow High School	Moscow	

S2

SSN	sName	address	HSCode	GPA	Priority
1111	Kyle	Everett	PHS	3.4	2
1111	Kyle	Everett	EHS	3.4	2
2222	John	Pullman	POHS	3.0	3
2222	John	Pullman	MHS	3.0	3
2222	John	Pullman	PHS	3.0	3

In BCNF

Not in BCNF

BCNF Decomposition for Student



	SSN	sName	address	HSCode	GPA	Priority
	1111	Kyle	Everett	PHS	3.4	2
S2	1111	Kyle	Everett	EHS	3.4	2
	2222	John	Pullman	POHS	3.0	3
	2222	John	Pullman	MHS	3.0	3
	2222	John	Pullman	PHS	3.0	3

S3

GPA

3.4

3.0

K

Priority

2

3

In BCNF

SSN	sName	address	HSCode	GPA
1111	Kyle	Everett	PHS	3.4
1111	Kyle	Everett	EHS	3.4
2222	John	Pullman	POHS	3.0
2222	John	Pullman	MHS	3.0
2222	John	Pullman	PHS	3.0

S4

Not in BCNF

BCNF Decomposition for Student



	SSN	sName	address	HSCode	GPA
	1111	Kyle	Everett	PHS	3.4
S4	1111	Kyle	Everett	EHS	3.4
	2222	John	Pullman	POHS	3.0
	2222	John	Pullman	MHS	3.0
	2222	John	Pullman	PHS	3.0

S5

K	
1	C D

SSN	sName	address	GPA
1111	Kyle	Everett	3.4
2222	John	Pullman	3.0

In BCNF

•	1	S6
	SSN	HSCode
	1111	PHS
	1111	EHS
	2222	POHS
	2222	MHS
	2222	PHS

In BCNF

BCNF Decomposition for Student



S1

HSCode	HSname	HScity
PHS	Pullman High School	Pullman
EHS	Everett High School	Everett
POHS	Potlatch High School	Potlatch
MHS	Moscow High School	Moscow

S3

GPA	Priority
3.4	2
3.0	3

S5

SSN	sName	address	GPA
1111	Kyle	Everett	3.4
2222	John	Pullman	3.0

S6

SSN	HSCode
1111	PHS
1111	EHS
2222	POHS
2222	MHS
2222	PHS

BCNF Decomposition



- Claim: The BCNF decomposition algorithm described results in lossless decompositions (i.e., the original can be reconstructed by joining decomposed relations).
- **Proof**. We will not cover. Check the book (section 19.5) for a detailed discussion on lossless decompositions.

Schema Normalization



- So we have learnt that, if a relation R contains redundancy, we need to decompose it into sub-relations R1, R2, ..., Rn such that
 - each Ri is in BCNF, and
 - the decomposition of R into R1, R2, ..., Rn is a lossless decomposition.
- We also learnt an algorithm for BCNF decomposition that is guaranteed to be lossless.
- Does this also guarantee dependency preservation???
- Not exactly....
 - In some cases it is not possible to decompose a relation into BCNF relations that have both the lossless-join and the dependency preservation.
 - Example: see next-next slide

Example of a Non-Dependency Preserving Decomposition R1 R2

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zip

91163

99164

street	city	zip	
1025 S Main	Pullman	91163	
925 S Main	Pullman	99163	
925 N Main	Pullman	99164	

FD: street, city \rightarrow zip $zip \rightarrow city$

There are 2 keys: {street,city} and {street,zip}

 $zip \rightarrow city$ is a BCNF vialotion, so we must decompose into:

R1(street,zip)	and R2	(city,zip)

	street	zip	city
	1025 S Main	91163	Pullman
_	925 S Main	99163	Pullman
	925 N Main	99164	

FD: none FD: $zip \rightarrow city$

Key: street,zip Key: zip

- Decomposition of address into R1 and R2 is lossless.
- Furthermore, R1 and R2 are in BCNF and doesn't contain any redundancy
- However, the decomposition does not preserve FD street, city \rightarrow zip.
 - we cannot enforce the FD street, city \rightarrow zip by checking FD's in these decomposed relations.

Third Normal Form (3NF)



 The solution to the problem in the previous slide is to relax the BCNF requirement slightly and allow for some redundancy.

- This relaxed condition is called Third Normal Form -3NF.
 - Disadvantage: storage overhead, anomalies.
 - Advantage: preserve dependencies

Third Normal Form (3NF)



- Relation R with FDs F is in 3NF if, for all $X \rightarrow A$ in F, one of the following is true:
 - A ∈ X (it is a *trivial* FD), or
 - X is a superkey, or
 - A is part of some (minimal candidate) key for R.
- Minimality of a key is crucial in third condition above!
- If R is in BCNF, obviously in 3NF.
- If R is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no ``good'' decomposition, or performance considerations).
 - Lossless-join, dependency-preserving decomposition of R into a collection of 3NF relations always possible.



Third Normal Form (3NF) - Example

street	city	zip
1025 S Main	Pullman	91163
925 S Main	Pullman	99163
925 N Main	Pullman	99164

FD:

street, city \rightarrow zip zip \rightarrow city

There are 2 keys: {street,city} and {street,zip}

Not in BCNF But in 3NF

Summary of Schema Refinement



- If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.
- If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
 - Must consider whether all FDs are preserved. If a losslessjoin, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
 - Decompositions should be carried out and/or re-examined while keeping performance requirements in mind.