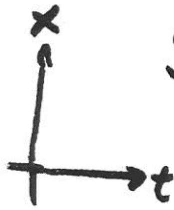
3.4 VELOCITY, SPEED AND OTHER RATES OF CHANGE

$$Y = X^3 \qquad Y' = 3X^2 \quad \text{OR} \quad \frac{dY}{dX} = 3X^2$$

$$g(x) = x^3 \qquad g'(x) = 3x^2$$



$$g(t) = t^3 \qquad g'(t) = 3t^2$$

$$x = t^3 \qquad \frac{dx}{dt} = 3t^2$$

YOU CAN PICK ANY VARIABLES YOU WANT.

S = POSITION $\qquad$ t = TIME

$$\text{AVERAGE VELOCITY} = V_{AV} = \frac{\text{DISPLACEMENT}}{\text{TRAVEL TIME}} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

$$\text{INSTANTANEOUS VELOCITY} = V \text{ OR } V(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

$$\text{ACCELERATION} = a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

EXAMPLE

$$S = 4 - 7t + 6t^2 - t^3$$

$$S(0) = 4 - 7\cdot 0 + 6\cdot 0^2 - 0^3 = 4$$

$$S(1) = 4 - 7\cdot 1 + 6\cdot 1^2 - 1^3 = 2$$

GRAPH $Y1 = 4 - 7X + 6X^2 - X^3$ → EVALUATE

$X = 2 \quad Y = 6$

$S(2) = 6$

$X = 3 \quad Y = 10$

$S(3) = 10$

WHERE DOES PARTICLE CHANGE DIRECTION? WHEN $\frac{ds}{dt}=0$ (22)

$(V=0)$

$S=4-7t+6t^2-t^3$

$\frac{ds}{dt}=-7+12t-3t^2=0$ * TI-89 F2 ALG. SOLVE*

$3t^2-12t+7=0$ $t=\frac{-(-12)\pm\sqrt{(-12)^2-4(3)7}}{2(3)}=.709$ & 3.291

OR GRAPH MORE MATH FMIN FMAX TI-86

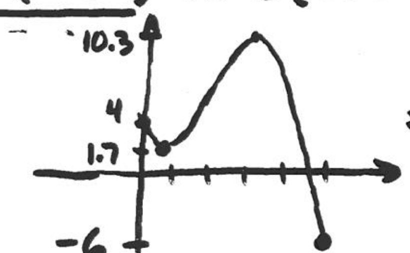
$\boxed{\begin{array}{l} V(.709)=0 \ m/s \\ V(3.291)=0 \ m/s \end{array}}$

.709 SECONDS	3.291 SECONDS
1.7 m	10.3 m

$a=\frac{d^2s}{dt^2}=12-6t$

$a(.709)=12-6(.709)=7.746 \ m/s^2$

$a(3.291)=12-6(3.291)=-7.746 \ m/s^2$



0-5 sec.

DISTANCE TRAVELED
$=(4-1.7)+(10.3-1.7)+(10.3-{}^-6)$
$=2.3+8.6+16.3$
$=27.2$ meters

SPEED = |VELOCITY|

DISPLACEMENT (0-5) = $-6-4=-10$

EXAMPLE p.129 #3

23

a) $s = 24t - .8t^2$ $\quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$V = \frac{ds}{dt} = 24 - 1.6t$ $\quad a = -1.6\ m/s^2$ CONSTANT

b) HIGH POINT = 0 SLOPE $\rightarrow 24 - 1.6t = 0$

$24 = 1.6t$ $\quad$ $t = 15$ SECONDS

c) $S(15) = 24(15) - .8(15)^2 = 180$ meters

VERIFY $Y = 24X - .8X^2$ GRAPH MORE MATH FMAX
(WINDOW X→0-20 Y→0-200)

d) $\frac{1}{2}(180) = 90$ TRACE $X = 4.4$ $Y = 90$

4.4 seconds

OR $90 = 24t - .8t^2$

$.8t^2 - 24t + 90 = 0$ $\quad t = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(.8)(90)}}{2(.8)}$

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e) $0 = 24t - .8t^2$

$0 = t(24 - .8t)$ OR

$t = 0$ $\quad 24 - .8t = 0$

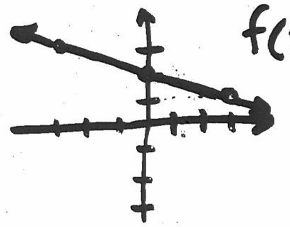
$24 = .8t$ $\quad \boxed{t = 30 \text{ sec.}}$

GRAPH MORE MATH ROOT TI-86
WINDOW X→0-40

* TI-89 GRAPH F5 ZERO *

HWORK p.129-132 → 2, 4-6, 12, 13, 20, 27

3.4 CONTINUED - ESTIMATING THE DERIVATIVE FROM GRAPHS & TABLES OF f(x)



f(x)

$f(x) = -\frac{1}{3}x + 2$

$f'(x) = -\frac{1}{3}$

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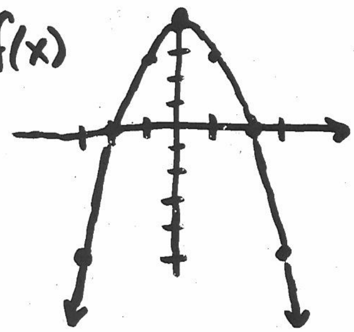
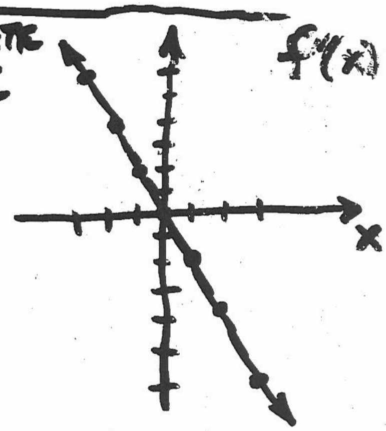
f(x)



LIKE #26, 28, 29

X	ESTIMATE SLOPE
-3	6
-2	4
-1	2
0	0
1	-2
2	-4
3	-6

f'(x)



APPLICATION p.129 EX.6 LIKE 10, 11, 15 → 10# a) $\frac{c(100)}{100}$

$c(x) = x^3 - 6x^2 + 15x$ COST

$c'(x) = 3x^2 - 12x + 15$ MARGINAL

$c'(10) = 3\cdot 10^2 - 12\cdot 10 + 15 = 195$ \$

$r(x) = x^3 - 3x^2 + 12x$ revenue

$r'(x) = 3x^2 - 6x + 12$ marginal

$r'(10) = 3\cdot 10^2 - 6\cdot 10 + 12 =$ \$252

EXAMPLE (LIKE 21, 22, 36)

t (sec)	0	.5	1	1.5	2	2.5
s (ft)	1	4	9	16	21	24

ESTIMATE V(1) & V(2)

PICK POINTS BEFORE & AFTER ← MORE ACCURATE SYMMETRIC DIFFERENCE QUOTIENT p.108

$$V_{AV} = \frac{\Delta s}{\Delta t}$$

V AT $t=1$

CHOOSE $t = .5$ & 1.5

BEFORE

AFTER

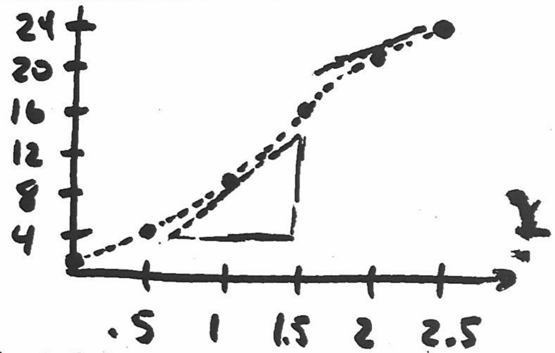
$$V(1) = \frac{\overset{\text{AFTER}}{s(1.5)} - \overset{\text{BEFORE}}{s(.5)}}{1.5 - .5} = \frac{16 \text{ ft} - 4 \text{ ft}}{1 \text{ SECOND}}$$

$V(1) = 12$ ft/second ESTIMATE

$$V(2) = \frac{\overset{\text{AFTER}}{s(2.5)} - \overset{\text{BEFORE}}{s(1.5)}}{2.5 - 1.5} = \frac{24 \text{ ft} - 16 \text{ ft}}{1 \text{ SECOND}}$$

$\boxed{V(2) = 8 \text{ ft/SECOND ESTIMATE}}$

GRAPH POINTS



NOTICE THE SLOPE AT $s=1$ ($m=12$) IS STEEPER THAN AT $s=2$ ($m=8$). THE VELOCITY IS THE SLOPE.

* OTHER RATES p.130 - 133 → 8, 9, 30, 31

HWORK p. 130 - 133 → 8-11, 15, 16, 18, 21-26, 28-31, 36

15 Y1 = 10/(1+50·2^(5-.1x))

Y2 = d(Y1(x), x)

GRAPH SET WINDOW