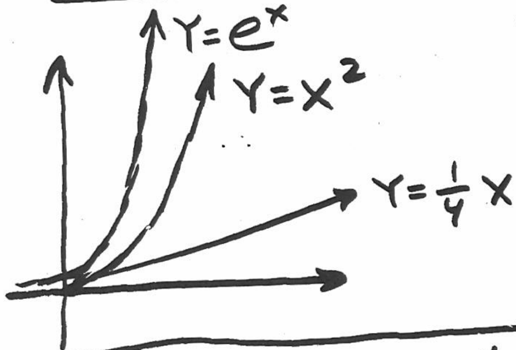
8.2 THE RATES AT WHICH FUNCTIONS GROW



$Y=e^x$ "GROWS FASTER" THAN $Y=X^2$

$Y=X^2$ "GROWS FASTER" THAN $Y=\frac{1}{4}X$

$$\boxed{f \text{ "grows faster" than } g \text{ (as } x\to\infty) \text{ IF } \lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty}$$

EXAMPLE 1 p. 427 COMPARE e^x AND X^2.

$$\lim_{x\to\infty}\frac{e^x}{X^2} \quad \frac{\infty}{\infty} \;\xrightarrow{\text{L'HOPITAL}}\; \lim_{x\to\infty}\frac{e^x}{2X} \;\xrightarrow{\text{L'HOPITAL}}\; = \lim_{x\to\infty}\frac{e^x}{2} = \frac{\infty}{2} = \infty$$

$$\boxed{\text{THEREFORE } Y=e^x \text{ grows faster than } Y=X^2 \text{ (as } x\to\infty)}$$

NOTE: $Y=2X$, $Y=3X$, $Y=4X$ etc. . . . . ARE ALL CONSIDERED TO "GROW" AT THE SAME RATE. (BY DEFINITION)

$$\lim_{x\to\infty}\frac{3X}{2X} = \frac{3}{2} \quad (\text{NOT } \infty)$$

p.431 → AB 1-27 odd

"FASTER GROWTH" IN THIS SENSE REALLY MEANS HIGHER ORDER

X^3 "HAS HIGHER ORDER" THAN X^2

f is of smaller order than g (as $x \to \infty$)

IF $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

LITTLE-O

NOTATION $f = o(g)$

f is smaller order than g

$\text{Deg}(f) < \text{Deg}(g)$

SMALLER ORDER MEANS GROWS SLOWER

x^2 is smaller order than x^3 $\rightarrow x^2 = o(x^3)$

BECAUSE $\lim_{x\to\infty} \frac{x^2}{x^3} = 0$ ZERO

EXAMPLE 5 p. 429 COMPARE $\ln x$ & x

$\lim_{x\to\infty} \frac{\ln x}{x} \underset{\text{L'HOP}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$ So $\boxed{\ln x = o(x)}$

$\text{Deg}(f) \leq \text{Deg}(g)$

f is at most order of g (as $x \to \infty$)

IF $\lim_{x\to\infty} \frac{f(x)}{g(x)} \leq M$ NOTATION $f = O(g)$

ANY POSITIVE CONSTANT (f is big-O of g)

EXAMPLE 6 p. 429 COMPARE x AND $x + \sin x$

$\lim_{x\to\infty} \frac{x+\sin x}{x} \underset{\text{L'HOP}}{=} \frac{1+\cos x}{1} \leq 2$

THEREFORE $x + \sin x = O(x)$

$\ln x < 3x < x^2 < x^3 < 2^x < 3^x < 4^x$

HOMEWORK p. 431-432 $\rightarrow$ 1-37 ODD EC

\#3 HINT $\rightarrow$ $4^x = \left(\frac{4e}{e}\right)^x = \left(\frac{4}{e}\right)^x e^x$

log SLOWER THAN POLYNOMIAL SLOWER THAN e^x