# 6.6 EULER'S METHOD



SHOW THAT $y = C_1 \cos x + C_2 \sin x$
IS A SOLUTION OF $y'' + y = 0$

EXAMPLE

$y = C_1 \cos x + C_2 \sin x$

$y' = -C_1 \sin x + C_2 \cos x$

$y'' = -C_1 \cos x - C_2 \sin x$

$y'' + y = 0$

$(-C_1 \cos x - C_2 \sin x) + C_1 \cos x + C_2 \sin x = 0$ ✓

$\boxed{\text{YES}}$ $Y = C_1 \cos x + C_2 \sin x$ <u>is a solution.</u>

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<u>EXAMPLE LIKE 1-4 p. 355</u>

SHOW THAT $y = 2 - \ln \cos(x-1)$ IS A SOLUTION

FOR $y'' - (y')^2 = 1$ $\quad y' = \frac{-1}{\cos(x-1)} - \sin(x-1) = \tan(x-1)$

$y'' = \sec^2(x-1)$ $\quad \sec^2(x-1) - (\tan(x-1))^2 = 1$ ✓

<u>YES, THIS IS AN IDENTITY.</u>

ALSO
VERIFY THE INITIAL CONDITIONS. <u>$Y(1) = 2$</u> <u>$Y'(1) = 0$</u>

$Y = 2 - \ln \cos(1-1) = 2 - \ln 1 = \underline{2}$ ✓ <u>YES</u>

$Y' = \tan(1-1) = \tan 0 = \underline{0}$ ✓ <u>YES</u>

$\boxed{\text{HOMEWORK p. 355} \longrightarrow \text{1-4 ALL}}$

EXAMPLE p. 356 #5

SOLVE $Y' = 1 + Y \quad Y(0) = 1$

$$\frac{dY}{dx} = 1 + Y \Rightarrow \int \frac{dY}{Y+1} = \int dx$$

$$\ln|Y+1| = x + C \Rightarrow \ln|1+1| = 0 + C$$

$$C = \ln 2$$

$$\ln|Y+1| = x + \ln 2 \rightarrow e^{\ln|Y+1|} = e^{x+\ln 2}$$

$$|Y+1| = e^x \cdot e^{\ln 2} \rightarrow |Y+1| = 2e^x$$

$$Y + 1 = 2e^x \quad \text{OR} \quad \cancel{Y+1 = -2e^x}$$

NO DOESN'T FIT INITIAL COND.

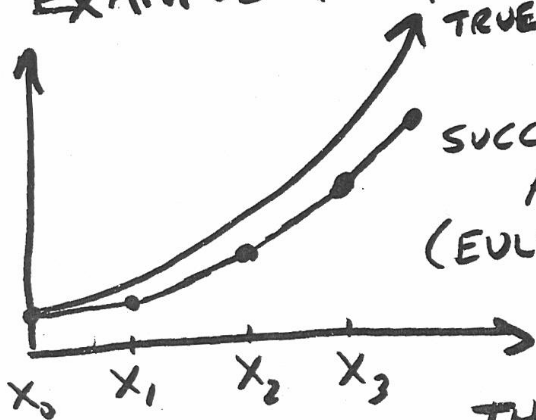
$$\rightarrow \boxed{Y = 2e^x - 1}$$

**HOMEWORK** p. 356 → 6, 7, 8
p. 349 → 27, 28, 29

EULER'S METHOD

EXAMPLE 1 p. 351 $Y' = 1 + Y \quad Y(0) = 1$

TRUE SOLUTION CURVE




SUCCESSIVE LINEAR APPROXIMATIONS (EULER APPROXIMATION)

ERROR IS THE DIFFERENCE IN THE 2 CURVES.

EX. 1 p. 351 $Y' = 1 + Y$ $Y(0) = 1$ (102)

$X_0 = 0$ $Y_0 = 1$ $dx = .1$ $X_0$ $Y_0$

$L(x) = f(a) + f'(a)(x - a)$ $Y - Y_1 = m(x - x_1)$

$L(x) = Y_0 + \underbrace{f'(x_0, Y_0)}_{m_0}\, dx$ FIRST

$= 1 + (1 + 1).1 = 1.2$ $(.1, 1.2)$ $(x_1, Y_1)$

USE $x_1\ Y_1\ m_1$

SECOND → $1.2 + (1 + 1.2).1 = 1.42$ $(.2, 1.42)$ $(x_2, Y_2)$

USE $x_2\ Y_2\ m_2$

3RD → $1.42 + (1 + 1.42).1 = 1.662$ $(.3, 1.662)$

etc....

ANALYSIS OF ERROR p. 352

| x | Y(EULER) (ABOVE | Y(EXACT) $Y = 2e^x - 1$ | ERROR |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| .1 | 1.2 | 1.2103 | .0103 |
| .2 | 1.42 | 1.4428 | .0228 |
| .3 | 1.662 | 1.6997 | .0377 |
| .4 | 1.9282 | 1.9836 | .0554 |
| .5 | etc... | etc... | etc... |

AS dx GETS SMALLER EULER BECOMES MORE EXACT. WITH COMPUTERS EXACT! THIS IS IMPORTANT, BECAUSE SOME DIFF EQ'S CANNOT BE SOLVED, THEY MUST BE EULERED.

HWORK p. 356 → 9-12

REVIEW p. 358-359 → 2, 4, 7, 12, 17, 21, 28, 38, 39, 46 p. 338 → 13, p. 347 → 2