9.3 CONTINUED

REMANDER ESTIMATE THEOREM

WHEN ESTIMATING WITH TAYLOR SERIES POLYNOMIALS, THERE IS SOME ERROR

$$\text{ERROR} = |R_n(x)| \le M \frac{r^{n+1}|x-a|^{n+1}}{(n+1)!}$$

$r \le 1$ (CHOOSE $r=1$)

$M = |f^{(n+1)}(x)|$ CHOOSE x FOR LARGEST M

EX. 5 p. 484 $\ln(1+x) \approx x - \frac{x^2}{2}$ $\underline{n=2}$

FIND MAX ERROR $|x| \le .1$ $\underline{a=0}$

$f(x) = \ln(1+x)$ $f'(x) = \frac{1}{1+x}$ $f''(x) = \frac{-1}{(1+x)^2}$

$f^{(3)}(x) = \frac{2}{(1+x)^3}$ $\text{ERROR} \le \frac{2}{(1-.1)^3} \times \frac{|.1|^{2+1}}{(2+1)!}$

$\text{ERROR} \le \frac{2}{.9^3} \times \frac{.1^3}{6}$ $\boxed{\text{ERROR} \le .00046}$

THAT'S PRETTY DARN ACCURATE!

EULER'S FORMULA $\boxed{e^{ix} = \cos x + i \sin x}$

$\boxed{\text{HOMEWORK p. 486-487} \rightarrow 14, 15, 32}$

p. 482 LAGRANGE ERROR BOUND $x = \theta$ WORK BACKWARDS

$$f(x) = f(a) + f'(a)(x-a) + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

$$\boxed{R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}} \leftarrow \text{LAGRANGE ERROR}$$