4.1 EXTREMA AND THE MEAN VALUE THEOREM

TYPES OF EXTREMA p.179 (SEE PICTURE)

ABSOLUTE MINIMUM - NO SMALLER f VALUE ANYWHERE.
ABSOLUTE MAXIMUM - NO LARGER f VALUE ANYWHERE.
LOCAL (RELATIVE) MINIMUM - NO SMALLER f NEARBY
LOCAL (RELATIVE) MAXIMUM - NO LARGER f NEARBY

THERE ARE 3 PLACES EXTREMA CAN OCCUR.

1) ENDPOINTS OF THE FUNCTIONS DOMAIN.

p.179 MINIMUM AT $(a, f(a))$. (ABSOLUTE)

2) INTERIOR POINTS WHERE f' IS 0.

p.179 REL. MAX AT $(c, f(c))$

3) INTERIOR POINTS WHERE f' DOES NOT EXIST

EXAMPLE $\longrightarrow$ $Y=\sqrt{1-x^2}$ $\quad Y'=\dfrac{-x}{\sqrt{1-x^2}}$

MINIMUM AT $(-1,0)$ AND $(1,0)$.

2 & 3 ARE ALSO CALLED CRITICAL POINTS.

EXAMPLE $\longrightarrow$ $f(x)=x^3-2x^2-15x+2$

FIND CRITICAL POINTS. $f'(x)=3x^2-4x-15$

$0=3x^2-4x-15 \Rightarrow 0=(3x+5)(x-3) \Rightarrow x=3, \frac{-5}{3}$

$f(3)=3^3-2\cdot3^2-15(3)+2=-34$ $\quad (3,-34)$ REL. MIN.

$f(-5/3)=\left(\frac{-5}{3}\right)^3-2\cdot\left(\frac{-5}{3}\right)^2-15\left(\frac{-5}{3}\right)+2=16\frac{22}{27}$ $\quad \left(\frac{-5}{3}, 16\frac{22}{27}\right)$ REL. MAX.

OR TI-86 FMIN, FMAX

EXAMPLE FIND CRITICAL POINTS.

$g(x)=\sqrt{5-4x-x^2} \qquad g(x)=(5-4x-x^2)^{1/2}$

NOTE $5-4x-x^2 \geq 0 \qquad (5+x)(1-x) \geq 0$

DOMAIN $-5 \leq x \leq 1$

BUT $g'(x)=\frac{1}{2}(5-4x-x^2)^{-1/2}(-4-2x) = \dfrac{-4-2x}{2\sqrt{5-4x-x^2}}$

OR $g'(x)=\dfrac{-4-2x}{2\sqrt{(5+x)(1-x)}} \rightarrow x \neq -5$ AND $x \neq 1$

(f' DOES NOT EXIST)

SO THERE ARE CRITICAL POINTS AT $x=1$ & $x=-5$

ALSO $0=\dfrac{-4-2x}{2\sqrt{5-4x-x^2}}$ SO $0=-4-2x$

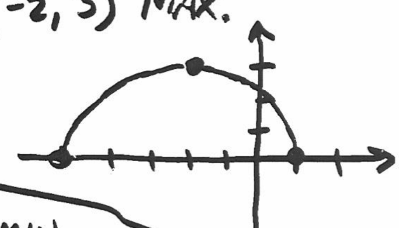
$4=-2x \Rightarrow x=-2$

$g(1)=\sqrt{5-4\cdot1-1^2}=0 \quad (1,0)$ MIN.

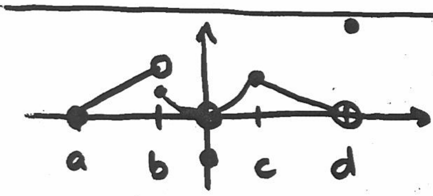
$g(-5)=\sqrt{5-4(-5)-(-5)^2}=0 \quad (-5,0)$ MIN.

$g(-2)=\sqrt{5-4(-2)-(-2)^2}=3 \quad (-2,3)$ MAX.

CRITICAL POINTS



OR TI-86 FMIN, FMAX



LOCAL MIN AT $x=a$ (1)

NOTHING AT $x=b$

ABS. MIN AT $x=0$ (3)

LOCAL MAX AT $x=c$ (3) ABS. MAX AT $x=d$ (1)

$\boxed{\text{HOMEWORK p.184} \rightarrow 1\text{-}10, 11\text{-}23\text{ odd}, 27, 41, 45\text{-}48}$