6.1 ANTIDERIVATIVES

$\frac{d}{dx}(x^2+3) = 2x$ ANTIDERIVATIVE$(2x) = x^2 + c$

$\frac{d}{dx}(x^2+7) = 2x$ $\boxed{\int 2x\,dx = x^2 + c}$

$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c}$ IN GENERAL $(n \neq -1)$

EXAMPLE

$\int x^4\,dx = \frac{x^{4+1}}{4+1} + c = \frac{x^5}{5} + c$

CHECK $\rightarrow$ $\frac{d}{dx}\left(\frac{x^5}{5} + c\right) = x^4$ ✓

$\frac{d}{dx}(\sin x + 5) = \cos x$ ANTIDERIVATIVE$(\cos x) = \sin x + c$

$\frac{d}{dx}(\sin x - \sqrt{2}) = \cos x$ $\boxed{\int \cos x\,dx = \sin x + c}$

$\boxed{\int \cos kx\,dx = \frac{\sin kx}{k} + c}$ IN GENERAL

EXAMPLE $\int \cos 7x\,dx = \frac{\sin 7x}{7} + c$

CHECK $\rightarrow$ $\frac{d}{dx}\left(\frac{\sin 7x}{7} + c\right) = \frac{1}{7}\cos(7x)\,7 = \cos 7x$ ✓

EXAMPLE $\int \left(\cos \pi x - \sqrt{x} + x^7 + \frac{1}{x^2}\right) dx$

$= \int \left(\cos \pi x - x^{1/2} + x^7 + x^{-2}\right) dx = \frac{\sin \pi x}{\pi} - \frac{x^{3/2}}{3/2} + \frac{x^8}{8} + \frac{x^{-1}}{-1} + c$

$= \frac{\sin \pi x}{\pi} - \frac{2}{3}x^{3/2} + \frac{x^8}{8} - \frac{1}{x} + c$

ALL DERIVATIVE FORMULAS CAN BE REVERSED. (SEE p. 307)

$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)}$ $\boxed{\int \frac{dx}{x} = \ln|x| + c}$

$\boxed{\int e^{kx} dx = \frac{e^{kx}}{k} + c}$ $\boxed{\int \sin kx\, dx = -\frac{\cos kx}{k} + c}$

$\boxed{\int \cos kx\, dx = \frac{\sin kx}{k} + c}$ $\boxed{\int \sec^2 x\, dx = \tan x + c}$

$\boxed{\int \csc^2 x\, dx = -\cot x + c}$ $\boxed{\int \sec x \tan x\, dx = \sec x + c}$

$\boxed{\int \csc x \cot x\, dx = -\csc x + c}$ $\boxed{\int k\, dt = kt + c}$

$\boxed{\int k f(x) dx = k \int f(x) dx}$ $\boxed{\int \frac{dx}{x+k} = \ln|x+k| + c}$

EXAMPLE

$$\int \left(e^{t/3} - \frac{1}{t} + 7\right) dt = \frac{e^{\frac{1}{3}t}}{1/3} - \ln|t| + 7t + c$$

$$= \boxed{3e^{t/3} - \ln|t| + 7t + c}$$

EXAMPLE

$$\int \left(\frac{10}{\sqrt[4]{x}} - \sec x \tan x + 4\csc^2 x\right) dx$$

$$= 10 \int x^{-1/4} dx - \int \sec x \tan x\, dx + 4 \int \csc^2 x\, dx$$

$$= 10 \frac{x^{3/4}}{3/4} - \sec x + 4(-\cot x) + c$$

$$= \boxed{\frac{40}{3} x^{3/4} - \sec x - 4\cot x + c}$$

HOMEWORK

p. 312 EXERCISES → 1-23 ALL

INITIAL VALUE PROBLEMS

EXAMPLE (LIKE 31-34)

$\frac{dy}{dx} = \sin x + 5 \quad y(0) = 7$

$y = -\cos x + 5x + C \quad 7 = -\cos 0 + 5 \cdot 0 + C$

$\boxed{y = -\cos x + 5x + 8} \quad 7 = -1 + 0 + C \quad \underline{C = 8}$

ANOTHER EXAMPLE (LIKE 35-38)

$\frac{d^3y}{dt^3} = 2t + 1 \quad y(1) = 2, \quad \underline{y'(1) = 3}, \quad y''(1) = 4$

$\frac{d^2y}{dt^2} = t^2 + t + C \quad 4 = 1^2 + 1 + C \quad C = 2$

$\frac{d^2y}{dt^2} = t^2 + t + 2 \quad \frac{dy}{dt} = \frac{t^3}{3} + \frac{t^2}{2} + 2t + C$

$3 = \frac{1^3}{3} + \frac{1^2}{2} + 2 \cdot 1 + C \quad C = \frac{1}{6}$

$\frac{dy}{dt} = \frac{t^3}{3} + \frac{t^2}{2} + 2t + \frac{1}{6} \quad y = \frac{t^4}{12} + \frac{t^3}{6} + t^2 + \frac{1}{6}t + C$

$\underline{y(1) = 2} = \frac{1^4}{12} + \frac{1^3}{6} + 1^2 + \frac{1}{6}(1) + C \quad C = \frac{7}{12}$

$$\boxed{y = \frac{t^4}{12} + \frac{t^3}{6} + t^2 + \frac{1}{6}t + \frac{7}{12}}$$

DO 39-42 LIKE 31-38, BUT REMEMBER

$v = \frac{ds}{dt}$ AND $a = \frac{d^2s}{dt^2}$ OMIT 36, 38

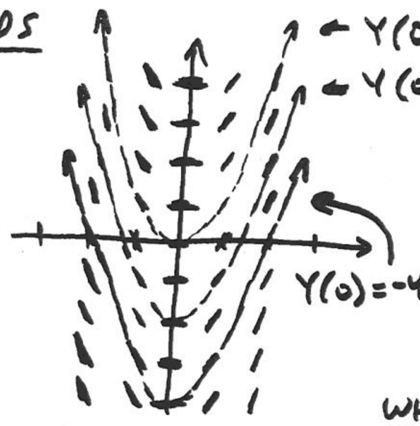
HOMEWORK p. 313 → 31-42 ALL

* DO 42 WITH CLASS IF TIME PERMITS *

SLOPE FIELDS

$\frac{dY}{dx} = 2X$

X	dY/dx
-2	-4
-1	-2
0	0
1	2
2	4



(86)

$\frac{dY}{dx} = 2X$ SUGGESTS

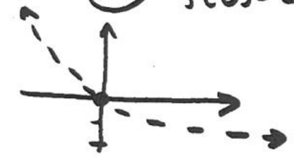
$Y = X^2 + C$

WHICH IS TRUE!

FIG. 6.1 p. 304 IS A SLOPE FIELD FOR $\frac{dY}{dt} = .056Y$

THIS SLOPE FIELD SUGGESTS AN EXPONENTIAL FUNCTION, WHICH IS TRUE! $Y = 100e^{.056t}$

p.313 (43) $f(0) = 0$ WHAT FUNCTION DOES THIS SUGGEST ??



$Y = 2e^{-x} - 2$??

EXAMPLE p.314 52 h $g(x) = \frac{d}{dx} x\sin x$

FIND $\int [g(x) - 4]\,dx = \int g(x)\,dx - \int 4\,dx$

$= \boxed{X\sin X - 4X + C}$

HOMEWORK p. 312-314 → 25-27, 29, 44, 52

AB 28, 30 SOLVE ONLY [redacted]