10.3 VECTOR VALUED FUNCTIONS

$r(t)$ IS A POSITION VECTOR

$v(t) = \frac{dr}{dt}$ IS THE VELOCITY VECTOR

$\|v(t)\|$ = THE SPEED

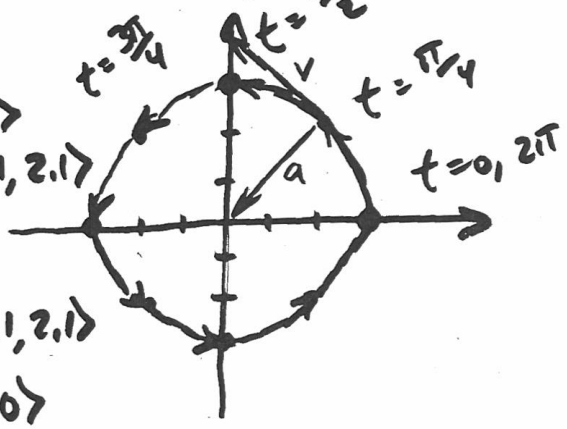
$\frac{v}{\|v\|}$ IS THE DIRECTION OF MOTION

$a(t) = \frac{d^2r}{dt^2}$ IS THE ACCELERATION VECTOR.

EXAMPLE 5 p. 533

$r(t) = (3\cos t)i + (3\sin t)j$

t	$r(t)$	
0	$3i$	$\langle 3, 0\rangle$
$\pi/4$	$2.1i + 2.1j$	$\langle 2.1, 2.1\rangle$
$\pi/2$	$3j$	$\langle 0, 3\rangle$
$3\pi/4$	$-2.1i + 2.1j$	$\langle -2.1, 2.1\rangle$
π	$-3i$	$\langle -3, 0\rangle$



$v(t) = \frac{dr}{dt} = -3\sin t\, i + 3\cos t\, j$

$a(t) = \frac{dv}{dt} = -3\cos t\, i - 3\sin t\, j$

EX.5 CONTINUED p.533

b) $v\left(\frac{\pi}{4}\right) = -3\sin\frac{\pi}{4}i + 3\cos\frac{\pi}{4}j$

$v\left(\frac{\pi}{4}\right) = \langle -2.1, 2.1\rangle$ SLOPE $= \frac{2.1}{-2.1}$

$a\left(\frac{\pi}{4}\right) = -3\cos\frac{\pi}{4}i - 3\sin\frac{\pi}{4}j = \langle -2.1, -2.1\rangle$

AT $t=\pi/4$

SPEED $= \|v\| = \sqrt{(-2.1)^2 + 2.1^2} = 3$

DIRECTION OF MOTION $= \frac{v}{\|v\|} = \frac{\langle -2.1, 2.1\rangle}{3}$

$= \langle -.7, .7\rangle$ AT $t=\pi/4$

c) $v \cdot a = \langle -2.1, 2.1\rangle \cdot \langle -2.1, -2.1\rangle$

$= (-2.1)(-2.1) + (2.1)(-2.1) = 0$

WHEN 2 VECTORS ARE NORMAL($\perp$) THEIR DOT PRODUCT IS ZERO.

NOTE: VELOCITY = SPEED * DIRECTION

REMEMBER $y - y_1 = m(x - x_1)$

AT $t=\pi/4$ $y - 2.1 = \frac{2.1}{-2.1}(x - 2.1)$ TANLINE

AT $t=\pi/4$ $y - 2.1 = \frac{-2.1}{-2.1}(x - 2.1)$ NORMAL LINE

HWORK p. 537 → 5-10 ALL

EXAMPLE 7 p. 535

EVALUATE $\int ((\cos t)i - 2tj)\,dt$

ANSWER: $(\sin t)i - t^2 j + C$

EX. 8 p. 536 $\int_0^{\pi} (\cos t\, i - 2tj)\,dt$

$= \sin t\, i - t^2 j \Big]_0^{\pi} = \sin\pi\, i - \pi^2 j - 0 = -\pi^2 j$

EX. 9 p. 536 INITIAL VALUE PROBLEM a)

$\frac{dr}{dt} = \frac{1}{t+1} i + 2t j \qquad r = \ln 2\, i$ AT $t = 1$

$r = \ln|t+1|\, i + t^2 j + C$

$\ln 2\, i = \ln 2\, i + 1^2 j + C \qquad C = -j$

$r = \ln|t+1|\, i + t^2 j - j \qquad 0 \le t \le 2$

b) DISTANCE TRAVELED $L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$L = \int_0^2 \sqrt{\left(\frac{1}{t+1}\right)^2 + (2t)^2}\,dt = 4.34$ meters

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HOMEWORK p. 537 → 11-18, 27, 28a, 28b

p.537 # 20 $r(t) = (\sin t)i + tj$

FIND WHEN $v(t)$ & $a(t)$ ARE $\perp$.

$$(v(t) \cdot a(t)) = 0$$

$v(t) = \cos t\, i + 1\, j \quad a(t) = -\sin t\, i$

$\langle \cos t, 1 \rangle \bullet \langle -\sin t, 0 \rangle = 0$

$-\cos t \cdot \sin t = 0 \quad t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ etc.....

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p.537 # 24 $r(t) = (3t+1)i + t^2 j \quad t=0$

$v(t) = 3i + 2tj \quad a(t) = 2j \quad v(0) = 3i$

ANGLE BETWEEN 2 VECTORS θ

$$\boxed{\cos\theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}} = \frac{a \cdot v}{\|a\| \cdot \|v\|}$$

$$\cos\theta = \frac{\langle 0,2 \rangle \cdot \langle 3,0 \rangle}{\sqrt{0^2+2^2} \cdot \sqrt{3^2+0^2}} = \frac{0}{6} = 0$$

$\cos^{-1} 0 = \boxed{90^\circ}$

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p.537 → #26 $\lim_{t \to 0} \left[\frac{\sin 2t}{t} i + \ln(t+1) j \right] = \boxed{2i + 0j}$

$\frac{\sin 2t}{t} \xrightarrow{\text{L'HOP}} \frac{2\cos 2t}{1} = \frac{2 \cdot 1}{1}$

HOMEWORK p. 537 → 19, 21, 22, 23, 25