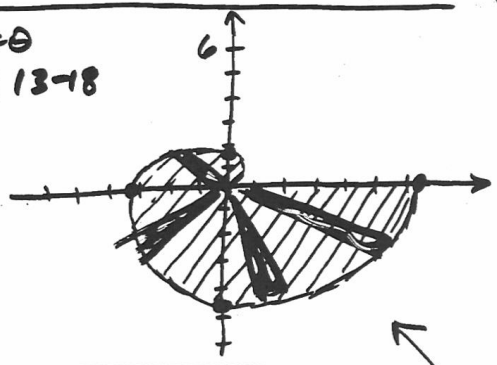
10.6 Area Computation In Polar Coordinates

EXAMPLE SKETCH $r=\theta$ $(0 \le \theta \le 2\pi)$ LIKE 13-18

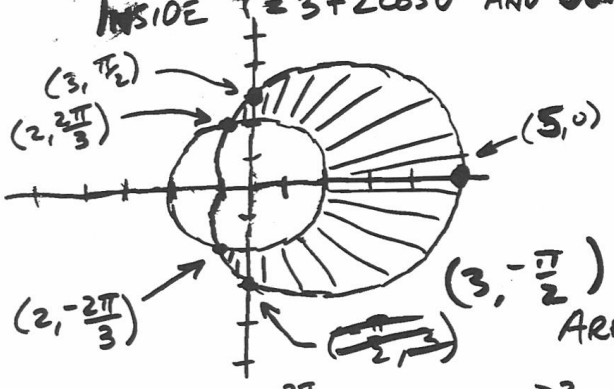
θ	r
0	0
$\pi/2$	$\pi/2 = 1.57$
π	$\pi = 3.14$
$3\pi/2$	$3\pi/2 = 4.71$
2π	$2\pi = 6.28$



AREA UNDER A POLAR CURVE.

$$A = \int_{\alpha}^{\beta} \frac{1}{2}\left[f(\theta)\right]^2 d\theta = \int_{0}^{2\pi} \frac{1}{2}\theta^2 d\theta = 41.34 \left(\frac{4\pi^3}{3}\right)$$

EXAMPLE FIND THE AREA OF THE REGION INSIDE $r = 3 + 2\cos\theta$ AND OUTSIDE $r = 2$



FIND INTERSECTIONS

$$2 = 3 + 2\cos\theta$$
$$-1 = 2\cos\theta$$
$$-\frac{1}{2} = \cos\theta \quad \theta = \frac{2\pi}{3} \text{ \& } \frac{-2\pi}{3}$$

AREA = OUTSIDE AREA − INSIDE AREA

$$\text{AREA} = \int_{-2\pi/3}^{2\pi/3} \frac{1}{2}\left[3 + 2\cos\theta\right]^2 d\theta - \int_{-2\pi/3}^{2\pi/3} \frac{1}{2} 2^2 d\theta$$

$$= 32.565 - 8.378 = 24.187$$

HOMEWORK p. 566 → 13, 16, 19, 21, 25

p.559

SLOPE OF A POLAR CURVE $\frac{dy}{dx}$

(165)

$y = r\sin\theta = \underset{u}{f(\theta)}\underset{v}{\sin\theta}$ $\quad \frac{dy}{d\theta} = uv' + vu'$

$\frac{dy}{d\theta} = f(\theta)\cos\theta + \sin\theta\, f'(\theta)$

DERIVATION

$x = r\cos\theta = \underset{u}{f(\theta)}\underset{v}{\cos\theta}$ $\quad \frac{dx}{d\theta} = uv' + vu'$

$\frac{dx}{d\theta} = f(\theta)(-\sin\theta) + \cos\theta \cdot f'(\theta)$

$r = f(\theta)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

p. 566 #4 SEE PICTURE IN BOOK

$r = 3(1-\cos\theta)$ FIND SLOPE AT $\left(3, \frac{3\pi}{2}\right)$ ← θ

$$\frac{dy}{dx} = \frac{3\sin\theta\cdot\sin\theta + 3(1-\cos\theta)\cdot\cos\theta}{3\sin\theta\cos\theta - 3(1-\cos\theta)\cdot\sin\theta}$$

$$\frac{dy}{dx} = \frac{3\sin\frac{3\pi}{2}\sin\frac{3\pi}{2} + 3\left(1-\cos\frac{3\pi}{2}\right)\cos\frac{3\pi}{2}}{3\sin\frac{3\pi}{2}\cos\frac{3\pi}{2} - 3\left(1-\cos\frac{3\pi}{2}\right)\sin\frac{3\pi}{2}} = \frac{3}{3} = 1$$

HOMEWORK p. 566 → 1, 3, 5

POLE ($R=0$)

CHAPTER REVIEW

p. 518 → 5, 9, 13 p. 527-528 → 5, 23

p. 537 → 5, 7, 9, 15 p. 545 → EXAMPLE 4a

p. 549 → 3 p. 558 → 25, 53 p. 566 → 1, 33