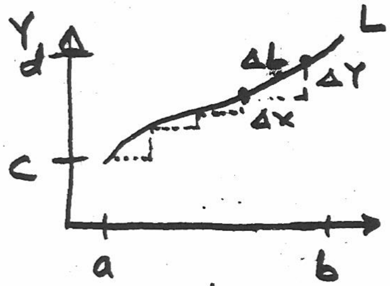
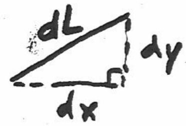
7.4 CURVE LENGTH (L)





$$(dx)^2+(dy)^2=(dL)^2$$

$$dL=\sqrt{(dx)^2+(dy)^2}$$

$$\int_a^b dL=\int_a^b \sqrt{(dx)^2+(dy)^2}\,\frac{dx}{dx}$$

$$L=\int_a^b \frac{\sqrt{(dx)^2+(dy)^2}\,dx}{\sqrt{(dx)^2}}$$

$$L=\int_a^b \sqrt{\frac{(dx)^2}{(dx)^2}+\frac{(dy)^2}{(dx)^2}}\,dx$$

$$\boxed{L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}$$

EXAMPLE 2 p. 397

FIND THE LENGTH OF $Y=\frac{4\sqrt{2}}{3}X^{3/2}-1 \quad \underline{0\leq X\leq 1}$

$$\frac{dY}{dx}=\frac{3}{2}\,\frac{4\sqrt{2}}{3}X^{1/2} \qquad \frac{dY}{dx}=2\sqrt{2}X^{1/2}$$

$$L=\int_0^1 \sqrt{1+\left(2\sqrt{2}X^{1/2}\right)^2}\,dx=\int_0^1 \sqrt{1+8X}\,dx$$

$$=\frac{1}{8}\int_0^1 (1+8X)^{1/2}\,8\,dx=\frac{1}{8}\int_0^1 u^{1/2}\,du \qquad \boxed{\begin{aligned}u&=1+8X\\du&=8dx\end{aligned}}$$

$$=\frac{1}{8}\,\frac{u^{3/2}}{3/2}\Bigg]_{X=0}^{X=1}=\frac{1}{12}(1+8X)^{3/2}\Bigg]_0^1=\frac{1}{12}9^{3/2}-\frac{1}{12}1^{3/2}=\boxed{\frac{13}{6}}$$

IF $\frac{dy}{dx}$ DOES NOT EXIST (AT SOME POINT ON INTERVAL LENGTH)

INTEGRATION CAN BE DONE WITH RESPECT TO Y.

$$\boxed{L=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy}$$

EXAMPLE 3, p. 469

FIND LENGTH OF $Y=\left(\frac{X}{2}\right)^{2/3}$ $0 \le X \le 2$

$\frac{dy}{dx} = \frac{2}{3}\left(\frac{X}{2}\right)^{-1/3}\left(\frac{1}{2}\right) = \frac{1}{3}\left(\frac{2}{X}\right)^{1/3}$ NOT DEFINED AT $X=0$

$Y^{3/2} = \left(\left(\frac{X}{2}\right)^{2/3}\right)^{3/2} \Rightarrow Y^{3/2} = \frac{X}{2} \Rightarrow X = 2Y^{3/2}$

$\frac{dx}{dy} = \frac{3}{2}\cdot 2Y^{1/2} = 3\sqrt{y}$ $L=\int_0^1 \sqrt{1+(3\sqrt{y})^2}\,dy$

WHEN $X=0$ $Y=0$

WHEN $X=2$ $Y=\left(\frac{2}{2}\right)^{2/3}=1$ $L=\int_0^1 \sqrt{1+9y}\,dy$

BY HAND OR fnint OR arc $\boxed{2.268}$

NOT ON QUIZ OR TEST

$$\boxed{\text{HOMEWORK p. 399 EX} \rightarrow 1\text{-}17 \text{ ODD}}$$