2.2 LIMITS INVOLVING INFINITY

EXAMPLE A

$$\lim_{x \to \infty} x^3 = \infty$$

$Y = X^3$

X	Y
1	1
10	1000
100	1,000,000
∞	∞^3

EXAMPLE B

$$\lim_{x \to -\infty} x^3 = -\infty$$

EXAMPLE C

$$\lim_{x \to \infty} \operatorname{Tan}^{-1} x = \frac{\pi}{2}$$

EXAMPLE D

$$\lim_{x \to -\infty} \operatorname{Tan}^{-1} x = -\frac{\pi}{2}$$

IF f IS A CONSTANT FUNCTION $f(x) = k$, THEN $\displaystyle\lim_{x \to c} f(x) = k$

EXAMPLE

$$f(x) = 8 \qquad \lim_{x \to 3} f(x) = 8$$

THEOREM 5 p. 67

SUM RULE

1. $$\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$$

Ex. 1

$$\lim_{x \to 3} \left[\frac{x^2-9}{x-3} + \frac{2}{x}\right] = \lim_{x \to 3} \frac{x^2-9}{x-3} + \lim_{x \to 3} \frac{2}{x}$$

$$\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} \qquad = [3+3] + \frac{2}{3} = 6\frac{2}{3}$$

(2) $$\lim_{x\to c}[f(x)-g(x)] = \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$$ (6)

EX.2 $$\lim_{x\to \frac{a}{10}}\left[\frac{100x^2-a^2}{10x-a} - \frac{4a}{5}\right]$$ DIFF. RULE

$$= \lim_{x\to \frac{a}{10}}\left[\frac{100x^2-a^2}{10x-a}\right] - \lim_{x\to \frac{a}{10}} \frac{4a}{5}$$

$$= \lim_{x\to \frac{a}{10}}\left[\frac{(10x-a)(10x+a)}{10x-a}\right] - \lim_{x\to \frac{a}{10}} \frac{4a}{5}$$

$$= 10\left(\frac{a}{10}\right) + a - \frac{4a}{5}$$

$$= 2a - \frac{4a}{5}$$

$$= \frac{6a}{5}$$

(3) $$\lim_{x\to c}[f(x)\cdot g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$$

EX3. $$\lim_{x\to \infty}\left[3^{1/x}\cdot a^2\right] = \lim_{x\to\infty} 3^{1/x} \cdot \lim_{x\to\infty} a^2$$

PRODUCT RULE

$$= 1\cdot a^2 = a^2$$

(4) EASY

(5) $$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$ QUOTIENT RULE

EX4 $$\lim_{x\to\infty}\frac{3a}{2^{1/x}} = \left[\lim_{x\to\infty} 3a\right] \Big/ \left[\lim_{x\to\infty} 2^{1/x}\right]$$

$$= 3a/1 = \boxed{3a}$$

ASSIGNMENT: LIMIT THEOREMS DITTO

END BEHAVIOR FOR RATIONAL FUNCTIONS

AS $x \to \pm\infty$

$$\boxed{f(x) = \frac{p(x)}{h(x)}}$$

ONLY THE HIGHEST ORDER TERM IN THE NUMERATOR AND DENOMINATOR ARE IMPORTANT IN DETERMINING END BEHAVIOR

EX. 8 a

LIKE 29-38

$$f(x) = \frac{2x^5 + x^4 - x^2 + 1}{3x^2 - 5x + 7} \quad \text{END BEHAVIOR} = \frac{2x^5}{3x^2} = \frac{2}{3}x^3$$

EX. 8 b

$$f(x) = \frac{2x^3 - x^2 + x - 1}{5x^3 + x^2 + x - 5} \quad \text{END BEHAVIOR} = \frac{2x^3}{5x^3} = \frac{2}{5}$$

THIS GRAPH WILL HAVE A HORIZ. ASYMP. $y = 2/5$

(GRAPH 8a WILL NOT HAVE A HORIZ. ASYMP.)

VERT. ASYMP. OCCUR WHEN DENOM. = 0.

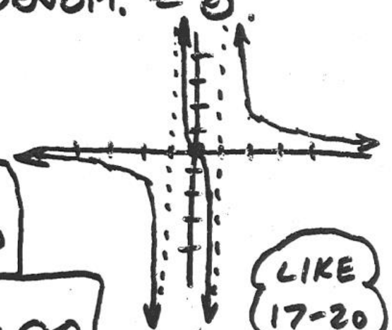
$$f(x) = \frac{2x}{x^2 - 1}$$

$$\boxed{\begin{gathered} x^2 - 1 = 0 \\ x = \pm 1 \\ \text{VERT. ASYMP.} \end{gathered}}$$

$$\boxed{\text{END BEHAVIOR} \quad \frac{2x}{x^2} = \frac{2}{x}}$$

$$\boxed{\text{HOR. ASYMP.} \quad \frac{2}{\infty} = 0 \quad y = 0}$$

$$\boxed{\lim_{x \to -1^-} f(x) = -\infty} \quad \boxed{\lim_{x \to -1^+} f(x) = \infty}$$



LIKE 17-20

HWORK P. 71-72 → 29-33, 35, 37, 17, 19, 20, 9, 11 & 39-42

P. 70 EX. 7

RIGHT END $\lim_{x \to \infty} x + \cancel{e^{-x}} = x$

LEFT END $\lim_{x \to -\infty} \cancel{x} + e^{-x} = e^{-x}$