6.5 POPULATION GROWTH

FROM PREVIOUS PAGE

$$\frac{dY}{dt} = kY \Rightarrow Y = Y_0 e^{kt}$$

FOR POPULATION

$$\frac{dP}{dt} = kP \Rightarrow P = P_0 e^{kt}$$

P_0 = ORIGINAL POPULATION

P = POPULATION AT TIME t

k = RELATIVE GROWTH RATE

EX. 1 p. 342 $P(0) = 4936$ MILLION (1986)

$P = P_0 e^{kt} \Rightarrow 5023 = 4936 e^{k(1)}$ $t=1$ 1987

$\ln \frac{5023}{4936} = \ln e^k$ $k = .0175$

FOR BRIEF TIME PERIODS, A GOOD ESTIMATE FOR k IS $\frac{5023}{4936} - 1 = .0176$

AVERAGING SEVERAL RATIOS $k = .0178$

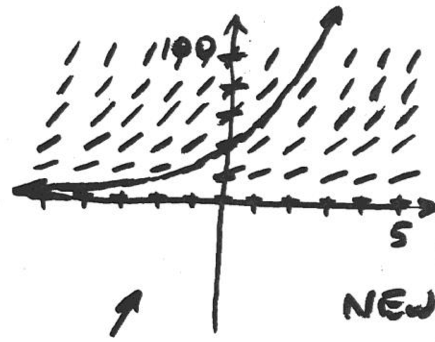
SO $\frac{dP}{dt} = .0178P$ AND $P = 4936e^{.0178t}$

(P IN MILLIONS)

$t=24$ 2010 $P = 4936e^{.0178(24)} = 7567$ MILLION

$$\frac{dP}{dt} = .0178P$$

P	$\frac{dP}{dt}$
20	.356
40	.712
60	1.068
80	1.424
100	1.78



NEW EXAMPLE

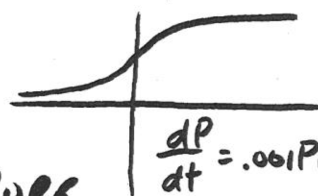
SLOPE FIELD FOR $\frac{dP}{dt} = .0178P$

GRAPH SHOWN IS $P = 40e^{.0178t}$ $\quad P(0) = 40$

FOR ANOTHER SLOPE FIELD GRAPH

<u>SEE FIG. 6.13</u> p. 344 LOGISTIC GROWTH

$$P = \frac{100}{1 + 9e^{-.1t}}$$



$\frac{dP}{dt} = .001P(100-P)$

WE WILL SAVE A MORE DETAILED ANALYSIS OF LOGISTIC GROWTH TO PERHAPS LATER IN THE YEAR.

HWORK p. 347

→ 1, 2, 5, 6, 9, 10, 11, 15