6.2 INTEGRATION BY SUBSTITUTION (87)

LET u BE A FUNCTION OF x.

$\int \cos u\, du = \sin u + C$	$\int \sin u = -\cos u + C$
$\int \sec^2 u\, du = \tan u + C$	$\int \csc^2 u\, du = -\cot u + C$
$\int \sec u \tan u\, du = \sec u + C$	
$\int \csc u \cot u\, du = -\csc u + C$	

$$\boxed{\int u^n du = \frac{u^{n+1}}{n+1} + C}$$

EX 1. p. 316 $\int (x+2)^5 dx$

$u = x+2$

$\frac{du}{dx} = 1$

$du = 1dx$

$du = dx$

$$= \int u^5 du = \frac{u^6}{6} + C = \boxed{\frac{(x+2)^6}{6} + C}$$

EX. 2 p. 316 $\int \sqrt{4x-1}\, dx = \int (4x-1)^{1/2} dx$

$$= \frac{1}{4}\int (4x-1)^{1/2} 4\, dx$$

$u = 4x-1$

$du = 4dx$

$$= \frac{1}{4}\int u^{1/2} du = \frac{1}{4}\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{4}\cdot\frac{2}{3}(4x-1)^{3/2} + C = \boxed{\frac{1}{6}(4x-1)^{3/2} + C}$$

EX. 3 p. 317

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$$\int \cos(7x+5)\,dx \qquad u = 7x+5 \quad du = 7dx$$

$$\frac{1}{7}\int \cos(7x+5)\,7dx = \frac{1}{7}\int \cos u\,du$$

$$= \frac{1}{7}(\sin u) + C = \boxed{\frac{1}{7}\sin(7x+5) + C}$$

EX. 4 p. 317

$$u = 2x$$

$$\int \frac{1}{\cos^2(2x)}\,dx = \int \sec^2(2x)\,dx \qquad du = 2dx$$

$$= \frac{1}{2}\int \sec^2(2x)\,2dx = \frac{1}{2}\int \sec^2 u\,du$$

$$= \frac{1}{2}\tan u + C = \boxed{\frac{1}{2}\tan(2x) + C}$$

ANOTHER EXAMPLE

$$u = 11 + \tan x$$

$$\int \frac{5\sec^2 x\,dx}{\sqrt{11+\tan x}} \qquad du = \sec^2 x\,dx$$

$$= 5\int (11+\tan x)^{-\frac{1}{2}}\sec^2 x\,dx$$

$$= 5\int u^{-\frac{1}{2}}\,du = \frac{5u^{\frac{1}{2}}}{\frac{1}{2}} + C = 10\sqrt{u} + C$$

$$= \boxed{10\sqrt{11+\tan x} + C}$$

HOMEWORK

p. 321 → 1-4, 6-12, 15, 16

6.2 CONTINUED

$$\boxed{\int \frac{du}{1+u^2} = \tan^{-1} u + c}$$

$$\boxed{\int e^u\,du = e^u + c}$$

$$\boxed{\int \frac{du}{u} = \ln|u| + c}$$

EX. 5 p. 318

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$= -\int \frac{-\sin x\,dx}{\cos x} = -\int \frac{du}{u}$$

$$= -\ln|u| + c = \boxed{-\ln|\cos x| + c}$$

ANOTHER EXAMPLE

$$\int \frac{8\,dx}{x^2+16} = 8\int \frac{dx}{\frac{x^2+16}{16}\,16}$$

$u^2 = \frac{x^2}{16}$

$u = \frac{1}{4}x$

$du = \frac{1}{4}dx$

$$= 8\int \frac{dx}{\left(\frac{x^2}{16}+1\right)16} = \frac{8}{16}\int \frac{dx}{1+\left(\frac{x}{4}\right)^2}$$

$$= \frac{1}{2}\,4\int \frac{\frac{1}{4}\,dx}{1+\left(\frac{x}{4}\right)^2} = 2\int \frac{du}{1+u^2} = 2\tan^{-1} u + c$$

$$= \boxed{2\tan^{-1}\left(\frac{x}{4}\right) + c}$$

HWORK p. 321-322 → 5, 18-22, 27, 28

EXAMPLE

$$\int_e^{10} \frac{dx}{x \ln^4 x} \qquad u = \ln x \qquad du = \frac{1}{x}dx$$

$$= \int_e^{10} (\ln x)^{-4} \cdot \frac{1}{x} dx = \int_{x=e}^{x=10} u^{-4}\, du$$

$$= \frac{u^{-3}}{-3}\Bigg]_{x=e}^{x=10} = \frac{-1}{3(\ln x)^3}\Bigg]_e^{10} = \frac{-1}{3(\ln 10)^3} - \frac{-1}{3(\ln e)^3}$$

$$= \boxed{.306}$$

EXAMPLE

$$\int_0^{\pi/6} \cos^3(3\theta)\sin(3\theta)\,d\theta \qquad u = \cos(3\theta) \qquad du = -\sin(3\theta)\,3\,d\theta$$

$$= -\frac{1}{3}\int_0^{\pi/6} \cos^3(3\theta)(-3\sin(3\theta)\,d\theta)$$

$$= -\frac{1}{3}\int u^3\,du = -\frac{1}{3}\,\frac{u^4}{4} = -\frac{[\cos(3\theta)]^4}{12}\Bigg]_0^{\pi/6}$$

$$= -\frac{\left[\cos\left(\frac{3\pi}{6}\right)\right]^4}{12} - -\frac{[\cos(3\cdot 0)]^4}{12} = -\frac{0^4}{12} - -\frac{1^4}{12}$$

$$= \frac{1}{12}$$

HWORK

p. 322 → 13, 14, 31–38 <u>ALL</u>

6.2 STILL DIFFERENTIAL EQUATIONS

EXAMPLE 9 p. 320

$$\frac{dY}{dx} = 2X(1+Y^2)e^{X^2}$$

PUT Y's ON LEFT

X's ON RIGHT

$$\int \frac{dY}{1+Y^2} = \int e^{X^2} 2X\,dx$$

$u = X^2 \quad du = 2X\,dx$

INTEGRATE BOTH SIDES

$$\tan^{-1} Y = e^{u} + C$$

$$\tan^{-1} Y = e^{X^2} + C$$

$\xrightarrow{\text{TAKE tan OF BOTH SIDES}}$ $Y = \tan(e^{X^2} + c)$

INITIAL VALUE EXAMPLE

$$\frac{dY}{dx} = Y(X+2) \qquad Y(2) = 1$$

$$\int \frac{dY}{Y} = \int (X+2)\,dx$$

$$\ln|Y| = \frac{X^2}{2} + 2X + C$$

$$\ln|1| = \frac{2^2}{2} + 2(2) + c \rightarrow 0 = 2 + 4 + C \rightarrow C = -6$$

EXPONENTIATE

$$e^{\ln|Y|} = e^{\frac{X^2}{2} + 2X - 6}$$

$$|Y| = e^{\frac{X^2}{2} + 2X - 6} \rightarrow Y = e^{\frac{X^2}{2} + 2X - 6}$$

FOR $Y > 0$

HOMEWORK

p. 322 → 39-44 ALL & 23-26