3.5 Derivatives Of Trig Functions

Remember From Trigonometry

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\sin^2\theta + \cos^2\theta = 1$

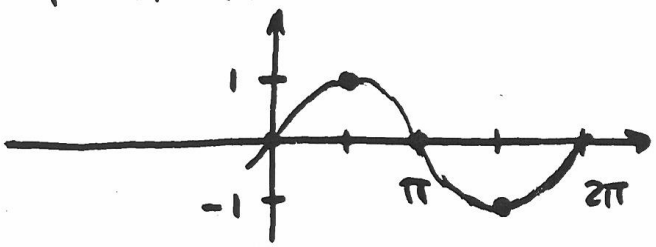
$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

etc..... SEE p.41-51

$Y = \sin X$ $Y' = ???$

RADIAN MODE



Y1 = SIN X

Y2 = d(Y1(X), X)

GRAPH

ZTRIG

So, IF $Y = \sin X$ $Y' = \cos X$

$$\frac{d}{dx}\sin x = \cos x$$ SEE PROOF p. 135 & SIN(A+B) p. 51

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

EXAMPLE

$Y = 2x + \cot x$ $\quad Y' = 2 + -\csc^2 x$

$$Y' = 2 - \csc^2 x$$

CHECK ON TI-89

$d(2x + 1/\tan x, x)$ ENTER

$$-\frac{(2\cos^2 x - 1)}{\sin^2 x}$$

SAME (BELIEVE IT OR NOT!)

REMEMBER $\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$

EXAMPLE

$Y = x \csc x \quad Y = uv \quad u = x \quad \frac{du}{dx} = 1$

$v = \csc x \quad \frac{dv}{dx} = -\csc x \cot x$

$Y' = x(-\csc x \cot x) + \csc x (1)$

OR $\boxed{Y' = \csc x (1 - x \cot x)}$

EXAMPLE p.140 #10

$Y = \frac{\cos x}{1 + \sin x} \quad \leftarrow u \quad \leftarrow v$

REMEMBER $\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}}$

$Y' = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$

$Y' = \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1+\sin x)^2} = \frac{-\sin x - 1}{(1+\sin x)^2}$

$Y' = \frac{-(1+\sin x)}{(1+\sin x)^2}$ OR $\boxed{Y' = \frac{-1}{1+\sin x}}$

EXAMPLE

$Y = \frac{\sin x + \cos x}{\cos x}$ SIMPLIFY $Y = \frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}$

$Y = \tan x + 1 \quad \boxed{Y' = \sec^2 x}$

HOMEWORK p. 140 → 1-9, 26

3.5 CONTINUED

EXAMPLE

$Y = \frac{2+\cot x}{x}$ $\begin{matrix} u \\ v \end{matrix}$ FIND tanline EQUATION AT $X=1$ $(x_1=1)$

$Y' = \frac{vu' - uv'}{v^2} = \frac{x(-\csc^2 x) - (2+\cot x)1}{x^2}$

$Y = (2+\cot 1)+1 \quad Y_1 = 2.6421$

$= \frac{1(-\csc^2 1) - (2+\cot 1)1}{1^2} = -4.0544$

$\cot x = \frac{1}{\tan}$

$\csc x = \frac{1}{\sin x}$

$Y - Y_1 = m(x - x_1)$ $\quad Y(1) = 2.6421$ $\quad (Y_1 = 2.6421)$

$Y - 2.6421 = -4.0544(X-1)$

$Y = -4.0544X + 6.6965$

ALSO FIND EQUATION OF THE NORMAL AT $X=1$ ($\perp$ TO THE TANGENT) $\quad m_\perp = \frac{-1}{m_{tan}}$

$Y - Y_1 = m(x - x_1)$

$Y - 2.6421 = \frac{-1}{-4.0544}(x-1)$

$Y = .2466X + 2.3955$

CHECK TANGENT LINE ON TI-89

GRAPH F5 A TANGENT

$Y = (2 + 1/\tan(x)) \div x$

EXAMPLE p.140 #16c

SHOW $\frac{d}{dx}\csc x = -\csc x \cot x$

$Y = \csc x$ OR $Y = \frac{1}{\sin x}$ ← u, ← v

$$Y' = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2} = \frac{\sin x(0) - 1\cdot\cos x}{\sin^2 x}$$

OR $$Y' = -\frac{\cos x}{\sin^2 x} = -\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} = -\csc x \cot x$$

DONE!

EXAMPLE p.140 #17b

SHOW THAT $Y = \cos x$ HAS HORIZONTAL TANGENT AT $x = 0$.

$\frac{dy}{dx} = -\sin x$ $\frac{dy}{dx} = -\sin 0 = 0$ ✓

HOMEWORK p.140 → 11-13, 15, 16a, 17, 19, 20, 23, 24, 29, 30

→ JERK: $\frac{d^3s}{dt^3}$

REVIEW p.172-174

→ 1, 3, 4, 32, 43, 46, 53, 59, 60, 65, 67a, 67e, 71, 74