DERIVATIVES OF EXPONENTIALS

$$\boxed{\frac{d}{dx}e^{x} = e^{[illegible]}}$$

PROOF p. 164

$$\boxed{\frac{d}{dx}e^{u} = e^{u}\frac{du}{dx}}$$

EXAMPLE   FIND $\frac{d}{dx}e^{4x}$ $u = 4x$ $\frac{du}{dx} = 4$

$$\frac{d}{dx}e^{4x} = e^{u}\frac{du}{dx} = e^{4x}(4) = 4e^{4x}$$

EXAMPLE   FIND $\frac{d}{dx}e^{\sin x}$ $u = \sin x$ $\frac{du}{dx} = [illegible]$

$$\boxed{\frac{d}{dx}e^{\sin x} = e^{\sin x}\cdot\cos x}$$

EXAMPLE

FIND $\frac{d}{dx}\, e^{\sqrt{x}}x^{3}$ $u = e^{\sqrt{x}}$ $v = x^{3}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{du}{dx} = e^{\sqrt{x}}\cdot\frac{1}{2}x^{-\frac{1}{2}}$

$e^{\sqrt{x}}\cdot 3x^{2} + x^{3}\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

$\frac{dv}{dx} = 3x^{2}$

OR $\boxed{x^{2}e^{[illegible]}\left(3 + \frac{\sqrt{x}}{2}\right)}$

HOMEWORK EXER.
p. 170 → 1–10

OTHER EXPONENTIALS

REMEMBER

VARIABLE BASE $\boxed{\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}}$ $(u > 0)$

EXAMPLE $\longrightarrow$ FIND $\frac{d}{dx} x^{\sqrt{3}}$ $\sqrt{3}\, x^{\sqrt{3}-1}$

EXAMPLE $\longrightarrow$ FIND $\frac{d}{dx} (\sin x)^{\pi}$

$u = \sin x$

$\pi (\sin x)^{(\pi - 1)} \cos x$

$\frac{du}{dx} = \cos x$

$\boxed{\text{NOTE: } e^{x \ln a} = a^x}$

CHAIN RULE

$$\frac{d}{dx} a^x = \frac{d}{dx} e^{x \ln a} = e^{x \ln a} \cdot \frac{d}{dx}(x \ln a)$$

OR $\boxed{\frac{d}{dx} a^x = a^x \ln a}$ VARIABLE EXPONENT

AND $\boxed{\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}}$

EXAMPLE $\longrightarrow$ FIND $\frac{d}{dx} 3^x$ $3^x \ln 3$

EXAMPLE $\longrightarrow$ FIND $\frac{d}{dx} 2^{\csc x}$

$u = \csc x$

$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$

$\frac{du}{dx} = -\csc x \cot x$

$= 2^{\csc x} \cdot \ln 2 (-\csc x \cot x)$

HOMEWORK p. 170 $\rightarrow$ 11–18

$$\boxed{\frac{d}{dx}\log_a x = \frac{1}{\ln a}\cdot\frac{1}{x}}$$

SEE PROOF p. ~~552~~ 167

CHAIN RULE → $\boxed{\frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{1}{u}\cdot\frac{du}{dx}}$

EXAMPLE

FIND $\frac{d}{dx}\log_{10}(3x+1)$

$$\frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{1}{u}\cdot\frac{du}{dx}$$

$u = 3x+1$

$\frac{du}{dx} = 3 \quad a = 10$

$$= \frac{1}{\ln 10}\cdot\frac{1}{3x+1}\cdot 3 = \frac{3}{(\ln 10)(3x+1)}$$

IF VARIABLE IS IN BASE AND EXPONENT, TO TAKE THE DERIVATIVE:

1) ln BOTH SIDES 2) SIMPLIFY
3) IMPLICITLY DIFFERENTIATE 4) SOLVE FOR $\frac{dy}{dx}$

$Y = (x+4)^{\ln x}$ FIND $\frac{dY}{dx}$

1) $\ln Y = \ln(x+4)^{\ln x}$ 2) $\ln Y = \underbrace{\ln x}_{u}\cdot\underbrace{\ln(x+4)}_{v}$

3) $\frac{1}{Y}\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = (\ln x)\cdot\frac{1}{x+4} + \ln(x+4)\cdot\frac{1}{x}$

$$\boxed{4)\ \frac{dY}{dx} = (x+4)^{\ln x}\left(\frac{\ln x}{x+4} + \frac{\ln(x+4)}{x}\right)}$$

REVIEW p. 172-174 5, 10, 17, 20, 23, 25, 26, 30, 35, 41, 47, 49, 51, 64, 68

HWORK p. 170 → 19-20, 31-38, 43