8.4 PARTIAL FRACTIONS BC

REMEMBER HOW TO COMBINE FRACTIONS:

$$\frac{3}{x-5}+\frac{2}{x+1}=\frac{3(x+1)}{(x-5)(x+1)}+\frac{2(x-5)}{(x+1)(x-5)}$$

$$=\frac{3x+3+2x-10}{(x+1)(x-5)}=\boxed{\frac{5x-7}{x^2-4x-5}}$$

SUPPOSE I WANT TO DO THIS BACKWARDS?

SPLIT $\frac{5x-7}{x^2-4x-5}$ INTO PARTIAL FRACTIONS.

$$\frac{5x-7}{x^2-4x-5}=\boxed{\frac{5x-7}{(x-5)(x+1)}}=\frac{A}{x-5}+\frac{B}{x+1}$$

$$\frac{A(x+1)}{(x-5)(x+1)}+\frac{B(x-5)}{(x+1)(x-5)}=\boxed{\frac{Ax+A+Bx-5B}{(x+1)(x-5)}}$$

$$Ax+Bx=5x \xrightarrow{\div x} A+B=5$$

$$A-5B=-7 \xrightarrow{x-1} -A+5B=7$$

$$6B=12 \rightarrow B=2$$

$$A+2=5 \rightarrow A=3$$

HOMEWORK
p.452 → 1, 7-10

SO $\frac{5x-7}{x^2-4x-5}=\frac{A}{x-5}+\frac{B}{x+1}=\boxed{\frac{3}{x-5}+\frac{2}{x+1}}$

EXAMPLE EVALUATE $\int\frac{5x-7}{x^2-4x-5}dx$

$$\int\frac{5x-7}{x^2-4x-5}dx=\int\frac{3}{x-5}dx+\int\frac{2}{x+1}dx=\boxed{3\ln|x-5|+2\ln|x+1|+C}$$

REMEMBER $\int\frac{1}{u}du=\ln|u|+C$

8.4 CONTINUED BC

DEG(NUM) ≥ DEG(DENOM)

REMEMBER

$\frac{x^3-6x^2+2x+1}{x+3}$ MEANS $x+3\overline{)x^3-6x^2+2x+1}$

$$\begin{array}{r} x^2-9x+29 \\ x+3\overline{)\,x^3-6x^2+2x+1} \\ -(x^3+3x^2) \\ \hline -9x^2+2x \\ -(-9x^2-27x) \\ \hline 29x+1 \\ -(29x+87) \\ \hline -86 \end{array}$$

$x^2-9x+29+\frac{-86}{x+3}$

EXAMPLE $\int \frac{x^3-6x^2+2x+1}{x+3}\,dx$

DEG(NUM) ≥ DEG(DENOM)

$= \int \left(x^2-9x+29-\frac{86}{x+3}\right)dx$

$= \frac{x^3}{3}-\frac{9x^2}{2}+29x-86\ln|x+3|+C$

HOMEWORK p. 452 → 6, 13, 14, 25, 26

NOTE: WE SKIPPED REPEAT FACTORS Ex. 2, 3 p. 446 AND IRREDUCIBLE QUADRATIC FACTORS Ex. 5 p. 447

8.4 STILL INITIAL VALUE PROBLEMS

BC

EXAMPLE LIKE #27

$$\frac{dY}{dx} = \sqrt{x}(Y^2+2Y) \quad Y(0)=1$$

$$\frac{dY}{Y^2+2Y} = \sqrt{x}\,dx \qquad \frac{dY}{Y(Y+2)} = x^{\frac{1}{2}}dx$$

$$\frac{1}{Y(Y+2)} = \frac{A}{Y} + \frac{B}{Y+2} = \frac{A(Y+2)+BY}{Y(Y+2)}$$

$$AY+BY=0 \quad 2A=1 \quad A=\tfrac{1}{2} \quad B=-\tfrac{1}{2}$$

$$\int \frac{\frac{1}{2}dY}{Y} + \frac{-\frac{1}{2}dY}{Y+2} = \int x^{\frac{1}{2}}dx$$

$$\frac{1}{2}\ln|Y| - \frac{1}{2}\ln|Y+2| = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$(0,1) \rightarrow \frac{1}{2}\ln|1| - \frac{1}{2}\ln|1+2| = \frac{0^{\frac{3}{2}}}{\frac{3}{2}} + C \quad C = -\frac{1}{2}\ln 3$

$$\left[\frac{1}{2}\ln\left|\frac{Y}{Y+2}\right| = \frac{2}{3}x^{\frac{3}{2}} - \frac{\ln 3}{2}\right] \times 2$$

$$e^{\left[\ln\frac{Y}{Y+2} = \frac{4}{3}x^{\frac{3}{2}} - \ln 3\right]} \rightarrow \frac{Y}{Y+2} = e^{\frac{4}{3}x^{\frac{3}{2}} + \ln 3^{-1}}$$

$$Y = (Y+2)\left(e^{\frac{4}{3}x^{\frac{3}{2}}} \cdot e^{\ln\frac{1}{3}}\right)$$

$$Y = (Y+2)e^{\frac{4}{3}x^{\frac{3}{2}}} \cdot \frac{1}{3}$$

CONTINUED →

$$3Y = Ye^{\frac{4}{3}x^{\frac{3}{2}}} + 2e^{\frac{4}{3}x^{\frac{3}{2}}}$$

$$3Y - Ye^{\frac{4}{3}x^{\frac{3}{2}}} = 2e^{\frac{4}{3}x^{\frac{3}{2}}}$$

$$Y\left(3 - e^{\frac{4}{3}x^{\frac{3}{2}}}\right) = 2e^{\frac{4}{3}x^{\frac{3}{2}}}$$

$$Y = \frac{2e^{\frac{4}{3}x^{\frac{3}{2}}}}{3 - e^{\frac{4}{3}x^{\frac{3}{2}}}}$$

HOMEWORK p. 452 → 27, 29, 30

CHAPTER REVIEW

p. 454-455 → 1, 3, 6, 15, 23, 24, 31, 37, 42, 43, 55, 61, 68

AND p. 452 → 1