3.3 DIFFERENTIATION RULES

$\boxed{\text{RULE 1} \quad \frac{d}{dx}(c) = 0}$

EXAMPLE: $f(x) = 8 \quad f'(x) = 0$ (SLOPE = 0)

$\boxed{\text{RULE 2} \quad \frac{d}{dx}(x^n) = nx^{n-1}}$

EXAMPLE $f(x) = x^6 \quad f'(x) = 6x^5$

$\boxed{\text{RULE 3} \quad \frac{d}{dx}(cu) = c\frac{du}{dx}}$

EXAMPLE $f(x) = 4x^7 = 4(7x^6) = 28x^6$

$\boxed{\text{RULE 4} \quad \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}}$

THE DERIVATIVE OF THE SUM = THE SUM OF THE DERIVATIVE.

EXAMPLE:

$$f(x) = 4x^3 - 3x^2 - 1x^1 + 7$$
$$f'(x) = 4 \cdot 3x^2 - 3 \cdot 2x^1 - 1x^0 + 0 = 12x^2 - 6x - 1$$

$\frac{dy}{dx}$ IS THE DERIVATIVE

$\frac{d^2y}{dx^2}$ IS THE DERIVATIVE OF THE DERIVATIVE (THE SECOND DERIVATIVE)

$$\frac{d^2y}{dx^2} = f''(x) = y''$$

2ND 8
TI-89
d(5X³-3X⁵, X) ENTER

EXAMPLE

$$y = 5x^3 - 3x^5$$

$$y' = 5 \cdot 3x^2 - 3 \cdot 5x^4 = 15x^2 - 15x^4$$

$$y'' = 15 \cdot 2x^1 - 15 \cdot 4x^3$$

$$\boxed{y'' = 30x - 60x^3}$$

THE SECOND DERIVATIVE

HOMEWORK p.120 → 1-10 ALL

ALSO HOMEWORK
QUICK REVIEW 3.1 p.101 → 1-10 ALL
NOT EXERCISES

3.3 STILL TANGENTS & NORMALS (AND 3.1) 17

p. 120 # 28 $\quad Y = X^3 + X$

FIND WHERE TANGENT SLOPE = 4

$$Y' = 3X^2 + 1 \rightarrow 3X^2 + 1 = 4 \rightarrow 3X^2 = 3$$

$$X^2 = 1 \quad X = \pm 1$$

$X = 1 \quad Y = 1^3 + 1 = 2 \quad (1, 2)$

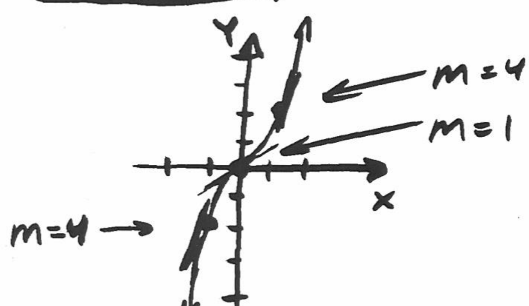
$X = -1 \quad Y = (-1)^3 + (-1) = -2 \quad (-1, -2)$

WHAT IS THE SMALLEST SLOPE?

$Y' = 3x^2 + 1 \qquad m = 0 + 1 = \underline{1}$

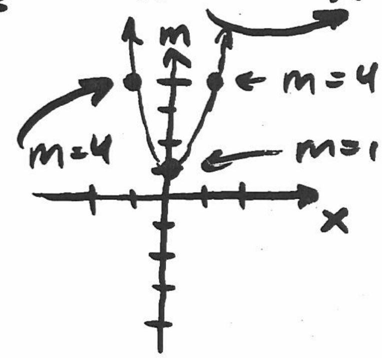
≥ 0

WHERE? $\rightarrow$ $3x^2 + 1 = \underline{1} \quad 3x^2 = 0 \quad X = 0$



$Y = X^3 + X$

THIRD DEGREE



$Y' = 3X^2 + 1$

SECOND DEGREE

$$\boxed{\begin{array}{l}\text{HOMEWORK p. 120} \rightarrow 27, 29, 30 \\ \text{p. 101–102} \rightarrow 4\text{–}14 \text{ ALL}\end{array}}$$

NOTE: AN n^{th} DEGREE FUNCTION HAS AN $n-1^{th}$ DEGREE DERIVATIVE

3.3 CONTINUED

RULE 5 $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

THE PRODUCT RULE

p. 115

SEE PROOF p. 116

EXAMPLE

LIKE #11 FIND $\frac{dy}{dx}$

$y = (x+1)(3-x^2)$

$y = u \cdot v$

$u = x+1$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{du}{dx} = 1$

$= (x+1)(-2x) + (3-x^2)1$

$v = 3-x^2 \quad \frac{dv}{dx} = -2x$

OR $\frac{dy}{dx} = -2x^2 - 2x + 3 - x^2$

$\boxed{\frac{dy}{dx} = -3x^2 - 2x + 3}$

CHECK YOUR ANSWER.

$(x+1)(3-x^2)$

$\underset{F}{3x} - \underset{O}{x^3} + \underset{I}{3} - \underset{L}{x^2}$

$\frac{dy}{dx} = 3 - 3x^2 - 2x$

EXAMPLE 5
p. 116

$u(2) = 3 \quad u'(2) = -4 \quad v(2) = 1 \quad v'(2) = 2$

FIND $y'(2)$

$y = uv \quad y' = uv' + vu'$

$y'(2) = 3(2) + 1(-4) = \underline{\underline{2}}$

RULE 6 $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

THE QUOTIENT RULE

SEE PROOF p.117

EXAMPLE

LIKE 13-18

$Y = (2x-7)^{-1}(x+5)$

OR $Y = \dfrac{1}{2x-7}(x+5)$

OR $\boxed{Y = \dfrac{x+5}{2x-7}}$

$Y = \dfrac{u}{v}$

$u = x+5 \quad \dfrac{du}{dx} = 1$

$v = 2x-7 \quad \dfrac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(2x-7)(1)-(x+5)(2)}{(2x-7)^2}$$

$$\frac{dy}{dx} = \frac{2x-7-(2x+10)}{(2x-7)^2} \qquad \frac{dy}{dx} = \frac{-17}{(2x-7)^2}$$

TI-89 d((x+5)/(2x-7), x) ENTER

HOMEWORK

p.120 → 11-14, 16, 17, 23-26