NATURAL LOGARITHMS (ln) 45

LOGARITHM REVIEW

LOG MEANS "WHAT IS THE EXPONENT?"

$10^3 = 1000$ $\log 1000 = 3$

$10^2 = 100$ $\log 100 = 2$ $10^1 = 10$ $\log 10 = 1$

$10^0 = 1$ $\log 1 = 0$ $10^{-1} = \frac{1}{10^1} = .1$ $\log .1 = -1$

$\pi = 3.14159265359\ldots\ldots\ldots$

$e = 2.718281828 46\ldots\ldots\ldots$

$e^1 = 2.71828\cdots$ so $\ln 2.71828\cdots = 1$

$e^2 = 7.389\cdots$ so $\ln 7.389\cdots = 2$

$e^3 = 20.085\cdots$ so $\ln 20.085\cdots = 3$

$\ln 10 = ?$ $e^{?} = 10$ $\boxed{2.303}$

PROPERTIES OF LOGS

I. $\log_b MN = \log_b M + \log_b N$ $\boxed{\text{PRODUCT RULE}}$

$\log 100 \cdot 1000 = \log 100 + \log 1000$

$\log 100{,}000 = 2 + 3$

$5 = 2 + 3$ ✓

$$\boxed{\ln MN = \ln M + \ln N}$$

II. $\log_b \frac{M}{N} = \log_b M - \log_b N$ QUOTIENT RULE

$\log \frac{100{,}000}{100} = \log 100{,}000 - \log 100$

$\log 1000 = 5 - 2 \qquad 3 = 5 - 2$ ✓

$$\boxed{\ln \frac{M}{N} = \ln M - \ln N}$$

III. $\log_b M^r = r \log_b M$ $\boxed{\text{POWER RULE}}$

$\log 100^3 = 3 \log 100$

$\log 1{,}000{,}000 = 3 \cdot 2$

$6 = 3 \cdot 2$ ✓

$$\boxed{\ln M^r = r \ln M}$$

IV. $\boxed{\log_a b = \frac{\log b}{\log a} = \frac{\ln b}{\ln a}}$

$\log_6 20 = \frac{\ln 20}{\ln 6}$

$\log_a a^x = x \qquad \ln e^x = x$

$a^{\log_a x} = x \qquad e^{\ln x} = x$

NOTE: $(e^{\ln 3})^x = 3^x$

$\log_3 (81^{x+2}) = (x+2) \log_3 81 = (x+2) \frac{\log 81}{\log 3}$

$= (x+2) 4$

$5\ln x - \ln 2x$

$\ln x^5 - \ln 2x \rightarrow \ln \frac{x^5}{2x} = \ln \frac{x^4}{2}$

SOLVE: $5^x = 2^{x+3}$

$\ln 5^x = \ln 2^{x+3}$

$x \ln 5 = (x+3)\ln 2$

$x \ln 5 = x \ln 2 + 3 \ln 2$

$x \ln 5 - x \ln 2 = 3 \ln 2$

$x(\ln 5 - \ln 2) = \ln 2^3$

$x \ln \frac{5}{2} = \ln 8$

$x = \ln 8 / \ln 2.5$

HOMEWORK p.170 QUICK REVIEW 1-10

3.9 CONTINUED NATURAL LOG (ln)

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\boxed{\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}}$$

CHAIN RULE

EXAMPLE

FIND THE DERIVATIVE OF $\ln(x^2+3)$

$\ln u \quad u = x^2+3 \quad \frac{du}{dx} = 2x$

$$\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx} = \frac{1}{x^2+3} \cdot 2x = \frac{2x}{x^2+3}$$

FIND $\frac{d}{dx}\sqrt{\ln x}$ (ANOTHER EXAMPLE)

$$= \frac{d}{dx}(\ln x)^{1/2} = \frac{d}{dx}u^{1/2} \qquad u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$$

$$= \frac{1}{2}u^{-1/2}\frac{du}{dx} = \frac{1}{2}(\ln x)^{-1/2}\cdot\frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$$

FIND $\frac{d}{dx}\left((\sin x)\ln x\right)$ $\quad u = \sin x \quad v = \ln x$

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx} = \boxed{\sin x\cdot\frac{1}{x} + \ln x\cdot\cos x}$$

HOMEWORK p.170 → 21-30 ALL

---

* USING ln TO TAKE DERIVATIVES

1) ln THE EQUATION 2) SIMPLIFY WITH ln RULES 3) IMPLICITLY DIFFERENTIATE 4) SOLVE FOR $\frac{dy}{dx}$

EXAMPLE #30 p.527

$$Y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}} \qquad \text{FIND } \frac{dy}{dx}$$

1) $\ln y = \ln\sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$

2) $\ln y = \ln\left(\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}\right)^{1/3} = \frac{1}{3}\ln\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}$

$$\ln y = \frac{1}{3}\left(\ln x + \ln(x+1) + \ln(x-2) - \ln(x^2+1) - \ln(2x+3)\right)$$

$$\ln y = \frac{1}{3}\ln x + \frac{1}{3}\ln(x+1) + \frac{1}{3}\ln(x-2) - \frac{1}{3}\ln(x^2+1) - \frac{1}{3}\ln(2x+3)$$

49

3) IMPLICITLY DIFFERENTIATE

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}\cdot\frac{1}{x} + \frac{1}{3}\cdot\frac{1}{x+1} + \frac{1}{3}\cdot\frac{1}{x-2} - \frac{1}{3}\cdot\frac{1}{x^2+1}\cdot 2x - \frac{1}{3}\cdot\frac{1}{2x+3}\cdot 2$$

4) SOLVE FOR $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{y}{3}\left(\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3}\right)$$

BUT $y = \sqrt[3]{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$

SO $\boxed{\dfrac{dy}{dx} = \dfrac{1}{3}\sqrt[3]{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}\left(\dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{1}{x-2} - \dfrac{2x}{x^2+1} - \dfrac{2}{2x+3}\right)}$

HWORK
p.170 → 45, 46