# 9.2 TAYLOR SERIES



A TAYLOR SERIES IS AN INFINITE SERIES THAT IS EQUIVALENT TO A GIVEN FUNCTION

SIMPLIFIED TAYLOR SERIES $(a=0)$ ARE CALLED MACLAURIN SERIES. (p.477)

**EXAMPLE 1**

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^1 = \frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \cdots\cdots = 2.7182818\cdots\cdots$$

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**EXAMPLE 2**

$$\cos x = \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos 5x = \frac{(5x)^0}{0!} - \frac{(5x)^2}{2!} + \frac{(5x)^4}{4!} - \cdots\cdots = \sum_{n=0}^{\infty} \frac{(-1)^n (5x)^{2n}}{(2n)!}$$

$$2x \cos x = 2x\frac{x^0}{0!} - 2x\frac{x^2}{2!} + \cdots\cdots = \sum_{n=0}^{\infty} \frac{2(-1)^n x^{2n+1}}{(2n)!}$$

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SOME SERIES "WORK" FOR ALL VALUES OF X. ($e^x$, $\sin x$, $\cos x$)

WE SAY THE <u>INTERVAL OF CONVERGENCE</u> IS ALL REAL NUMBERS.

$\boxed{\text{HWORK p. 478} \rightarrow 1, 4, 5, 6, 8}$

IGNORE HINT

SOME TAYLOR SERIES ONLY CONVERGE ("WORK") FOR CERTAIN X VALUES

p. 477 $\tan^{-1}x$ $(\underline{|x| \le 1})$,

$\frac{1}{1-x}$ $(\underline{|x| < 1})$, $\ln(1+x)$ $(\underline{-1 < x \le 1})$

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\cdots \quad (|x| \le 1)$$

$$\tan^{-1}(5x) = 5x - \frac{(5x)^3}{3} + \frac{(5x)^5}{5} - \cdots\cdots \quad (\underline{|5x| \le 1})$$

$\boxed{\text{HWORK p. 478} \rightarrow \text{2, 3 AND p. 486} \rightarrow \text{6, 10}}$

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## 9.3 ESTIMATING WITH TAYLOR SERIES

ESTIMATE $\tan^{-1}(5x)$ AT $x = .2$ ORDER 4

$$\tan^{-1}(5(.2)) = (5(.2)) - \frac{(5(.2))^3}{3} + \frac{(5(.2))^5}{5} - \frac{(5(.2))^7}{7} + \frac{(5(.2))^9}{9}$$

$= \overset{.835}{\cancel{.997}}$ POOR ESTIMATE $(5x = 1)$

$\tan^{-1} 1 = \frac{\pi}{4} = .785$

HOMEWORK p. 486 → 1-4
COMPARE ESTIMATE WITH ACTUAL

(FORMULAS ON PAGE 477)

1) 5 TERMS 2) 3 TERMS 3) 2 TERMS 4) 2 TERMS

## 9.2 CONTINUED

A TAYLOR SERIES CAN BE CREATED AN INFINITE NUMBER OF WAYS FOR A FUNCTION $f(x)$ AND THE USE OF AN ARBITRARY CONSTANT $a$ BY THE FOLLOWING FORMULA:

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n}$$ PROOF NOT IN BOOK

WHERE $f^{(n)}(a)$ IS THE $n^{th}$ DERIVATIVE OF $f(x)$ EVALUATED AT $a$. ORDER 0 = 1 TERM 6 TERMS

**EXAMPLE** $f(x) = \sin x$ $a = \frac{\pi}{2}$ ORDER 5

$f^{(0)}(x) = \sin x$ $f^{(1)}(x) = \cos x$ $f^{(2)}(x) = -\sin x$ $f^{(3)}(x) = -\cos x$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = \frac{f^{(0)}(\frac{\pi}{2})}{0!}(x-\tfrac{\pi}{2})^0 + \frac{f^{(1)}(\frac{\pi}{2})}{1!}(x-\tfrac{\pi}{2})^1$$

$$+ \frac{f^{(2)}(\frac{\pi}{2})}{2!}(x-\tfrac{\pi}{2})^2 + \frac{f^{(3)}(\frac{\pi}{2})}{3!}(x-\tfrac{\pi}{2})^3 + \cdots\cdots$$

$$= \frac{\sin(\frac{\pi}{2})}{0!}(x-\tfrac{\pi}{2})^0 + \frac{\cos(\frac{\pi}{2})}{1!}(x-\tfrac{\pi}{2})^1 + \frac{-\sin(\frac{\pi}{2})}{2!}(x-\tfrac{\pi}{2})^2$$

$$+ \frac{-\cos(\frac{\pi}{2})}{3!}(x-\tfrac{\pi}{2})^3 + \frac{\sin(\frac{\pi}{2})}{4!}(x-\tfrac{\pi}{2})^4 + \frac{\cos(\frac{\pi}{2})}{5!}(x-\tfrac{\pi}{2})^5 + \cdots$$

ORDER 5

$$= 1 + 0 - \frac{1}{2!}(x-\tfrac{\pi}{2})^2 - 0 + \frac{1}{4!}(x-\tfrac{\pi}{2})^4 + 0 + \cdots$$

REMEMBER p. 477 $\sin x = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$f(x) = \sin x$ $a = 0$

WHEN $a = 0$ A TAYLOR SERIES IS CALLED A MACLURIN SERIES.

TI-89 F3 CALC TAYLOR(sin(x), x, 7, 0)

HOMEWORK p. 478 → 9-13, 15