5.4 FUNDAMENTAL THEOREM OF CALCULUS

FTC PART II p. 282

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$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

REMEMBER $F(x)$ = THE ANTIDERIVATIVE OF $f(x)$

EXAMPLES

$\int_0^{\pi/2} \cos x\,dx$

$\frac{d}{dx}\sin x = \cos x$

$f(x) = \cos x$ $F(x) = \sin x$

$= \sin x \Big]_0^{\pi/2}$

$]$ MEANS EVALUATE

$\sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$

$\int_1^5 \frac{1}{x}\,dx$

$\frac{d}{dx}\ln x = \frac{1}{x}$

* $F(x) = \ln|x|$ ACTUALLY

$f(x) = \frac{1}{x}$ $F(x) = \ln x$ *

$= \ln x \Big]_1^5 = \ln 5 - \ln 1 = \ln 5 - 0 = \boxed{\ln 5}$

$\int_{\pi/2}^{\pi}\left(\frac{5}{x^2} + \sin x\right)dx = \int_{\pi/2}^{\pi}\frac{5}{x^2}\,dx + \int_{\pi/2}^{\pi}\sin x\,dx$

$= 5\int_{\pi/2}^{\pi} x^{-2}\,dx - \int_{\pi/2}^{\pi}(-\sin x)\,dx = 5\frac{x^{-1}}{-1} - \cos x\Big]_{\pi/2}^{\pi}$

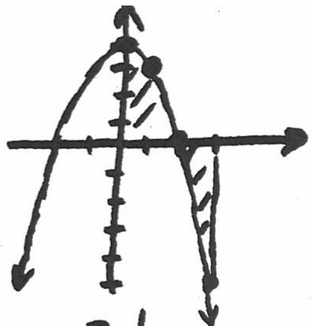
$= \left(5\frac{\pi^{-1}}{-1} - \cos\pi\right) - \left(5\frac{\left(\frac{\pi}{2}\right)^{-1}}{-1} - \cos\frac{\pi}{2}\right) = \boxed{2.59}$

HWORK
p. 286
1-13 ODD, 4, 6, 8

EXAMPLE 6 p. 283

$Y = 4 - X^2 \quad 0 \leq X \leq 3$

FIND THE AREA.



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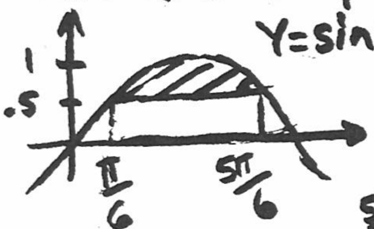
$$\int_0^2 (4-x^2)\,dx + + \left|\int_2^3 (4-x^2)\,dx\right|$$

$$= \left(4x - \frac{x^3}{3}\right)\Big]_0^2 + \left|\left(4x - \frac{x^3}{3}\right)\Big]_2^3\right|$$

$$= 4\cdot 2 - \frac{2^3}{3} - \left(4\cdot 0 - \frac{0^3}{3}\right) + \left|\left[\left(4\cdot 3 - \frac{3^3}{3}\right) - \left(4\cdot 2 - \frac{2^3}{3}\right)\right]\right|$$

$$= \frac{16}{3} + \frac{7}{3} = \frac{23}{3}$$

EXAMPLE p.286 #28



$$A = \int_{\pi/6}^{5\pi/6} \sin x\,dx - .5\left(\frac{5\pi}{6} - \frac{\pi}{6}\right)$$

$$A = -\cos x\Big]_{\pi/6}^{5\pi/6} - .5\left(\frac{4\pi}{6}\right) = -\cos\frac{5\pi}{6} - -\cos\frac{\pi}{6} - \frac{2\pi}{6}$$

$$= -\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3}$$

HOMEWORK p. 286 → 15-18, 25-27

(32, 34) $\int$

USE FNINT

THE MEAN VALUE THEOREM FOR DEFINITE INTEGRALS

p.272

SEE FIGURE 5.25

5.3 AGAIN

IF f IS CONTINUOUS ON $[a, b]$ THEN AT SOME POINT C IN THE INTERVAL

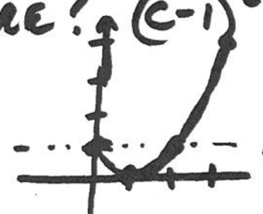
$$\boxed{f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

WHERE $f(c)$ IS THE AVERAGE VALUE IN THE INTERVAL.

EXAMPLE p. 275 # 28 $\quad Y = (x-1)^2 \quad [0, 3]$

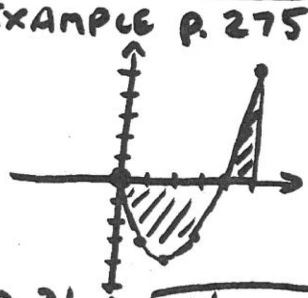
$$f(c) = \frac{1}{3-0}\int_0^3 (x-1)^2\,dx = 1$$ AVERAGE VALUE

WHERE? $(c-1)^2 = 1 \qquad c - 1 = \pm 1 \qquad \boxed{C = 2 \text{ OR } 0}$



AVERAGE VALUE = 1

EXAMPLE p. 275 # 24 $\quad Y = x^2 - 4x \quad [0, 5]$



a) $\int_0^5 (x^2 - 4x)\,dx = -25/3$

b) AREA $= \left|\int_0^4 (x^2 - 4x)\,dx\right| + \int_4^5 (x^2 - 4x)\,dx = 13$

30, 31 → $\boxed{\text{HOMEWORK p. 275-276} \to 17\text{-}29 \text{ ODD} \ \& \ 26}$

F.T.C. PART I

LET $F(x) = \int_a^x f(t)\,dt$ (80)

THEN $\dfrac{dF}{dx} = \dfrac{d}{dx}\int_a^x f(t)\,dt = f(x)$

a must be constant

EXAMPLE 3 p. 279

a) $Y = \int_x^5 3t \sin t \, dt$ FIND $\dfrac{dY}{dx}$

$$\frac{dY}{dx} = \frac{d}{dx}\int_x^5 3t\sin t\,dt = -\frac{d}{dx}\int_5^x 3t\sin t\,dt$$

$$= \boxed{-3x\sin x} \text{ BY FTC I}$$

b) $Y = \int_{2x}^{x^2} \dfrac{1}{2+e^t}\,dt$ FIND dY/dx

$$Y = \int_a^{x^2}\frac{1}{2+e^t}\,dt - \int_a^{2x}\frac{1}{2+e^t}\,dt$$

FTC I AND CHAIN RULE

$$\frac{dY}{dx} = \frac{1}{2+e^{x^2}}(2x) - \frac{1}{2+e^{2x}}(2)$$

$$\frac{dY}{dx} = \frac{dY}{du}\cdot\frac{du}{dx}$$

$$\boxed{\frac{dY}{dx} = \frac{2x}{2+e^{x^2}} - \frac{2}{2+e^{2x}}}$$

HWORK p. 287 → 37–45 ALL

$$\frac{d}{dx}\int_u^v f(t)\,dt = f(v)\frac{dv}{dx} - f(u)\frac{du}{dx}$$