3.7 IMPLICIT DIFFERENTIATION (38)
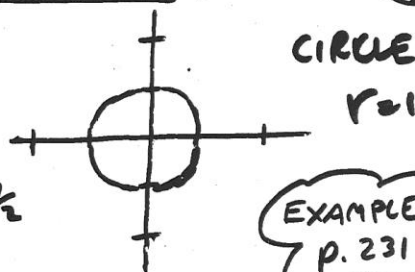
$$X^2 + Y^2 = 1$$

$$Y^2 = 1 - X^2$$

$$Y = \pm\sqrt{1-X^2} = \pm(1-X^2)^{\frac{1}{2}}$$

CIRCLE r=1

EXAMPLE 1 p. 231

$$\frac{dy}{dx} = \pm\frac{1}{2}(1-X^2)^{-\frac{1}{2}}(-2X) = \frac{-X}{\pm\sqrt{1-X^2}}$$

ANOTHER WAY $\frac{d}{dx}(X^2+Y^2=1)$

FIND $\frac{dy}{dx}$

$$\frac{d}{dx}X^2 + \frac{d}{dx}Y^2 = \frac{d}{dx}(1)$$

$$2X + 2Y\frac{dy}{dx} = 0 \Rightarrow 2Y\frac{dy}{dx} = -2X$$

$$\frac{dy}{dx} = \frac{-2X}{2Y} \qquad \boxed{\frac{dy}{dx} = \frac{-X}{Y}} \quad \text{SAME AS ABOVE}$$

ANOTHER EXAMPLE
EXAMPLE 3 p. 151

$$2Y = X^2 + \sin Y \qquad \text{FIND } \frac{dy}{dx}$$

$$\frac{d}{dx}(2Y = X^2 + \sin Y) \Rightarrow \frac{d}{dx}(2Y) = \frac{d}{dx}(X^2) + \frac{d}{dx}(\sin Y)$$

$$2\frac{dy}{dx} = 2X + \cos Y\frac{dy}{dx} \Rightarrow 2\frac{dy}{dx} - \cos Y\frac{dy}{dx} = 2X$$

$$\frac{dy}{dx}(2 - \cos Y) = 2X \Rightarrow \frac{dy}{dx} = \frac{2X}{2 - \cos Y}$$

EXAMPLE   FIND $\frac{dy}{dx}$ (39)

$$X^3 - XY + Y^3 = 1 \qquad \frac{d}{dx}(X^3 - XY + Y^3 = 1)$$

$$\frac{d}{dx}X^3 - \frac{d}{dx}XY + \frac{d}{dx}Y^3 = \frac{d}{dx}(1)$$

$$3X^2 - \left(X\frac{dy}{dx} + Y(1)\right) + 3Y^2\frac{dy}{dx} = 0$$

$$3X^2 - X\frac{dy}{dx} - Y + 3Y^2\frac{dy}{dx} = 0$$

$$3Y^2\frac{dy}{dx} - X\frac{dy}{dx} = Y - 3X^2$$

$$\frac{dy}{dx}(3Y^2 - X) = Y - 3X^2 \longrightarrow \boxed{\frac{dy}{dx} = \frac{Y - 3X^2}{3Y^2 - X}}$$

EXAMPLE   FIND Y'

$$Y^2 \cos\left(\frac{1}{Y}\right) = 2X + 2Y$$

$$\frac{d}{dx}\left(Y^2 \cos\left(\frac{1}{Y}\right)\right) = \frac{d}{dx}(2X) + \frac{d}{dx}(2Y) \qquad \boxed{\frac{d}{dx}Y^{-1} = -1Y^{-2}\frac{dY}{dx}}$$

$$\underset{u}{\underline{}}\,\underset{v}{\underline{}}$$

$$Y^2\left(-\sin\frac{1}{Y}\right)\left(-1Y^{-2}\right)\frac{dY}{dx} + \cos\frac{1}{Y}\cdot 2Y\frac{dY}{dx} = 2 + 2\frac{dY}{dx}$$

$$\sin\frac{1}{Y}\frac{dY}{dx} + 2Y\cos\frac{1}{Y}\frac{dY}{dx} - 2\frac{dY}{dx} = 2$$

$$\boxed{\frac{dY}{dx} = \frac{2}{\sin\frac{1}{Y} + 2Y\cos\frac{1}{Y} - 2}}$$

HOMEWORK p.155 $\rightarrow$ 9-12, 17-20

HARDER RADICAL & FRACTIONAL EXPONENTS

EXAMPLE $Y = \sqrt[3]{\sin(5x^2+2x)}$ FIND Y'

$Y = (\sin(5x^2+2x))^{1/3}$ $Y = u^{1/3}$ $Y' = \frac{1}{3}u^{-\frac{2}{3}}\frac{du}{dx}$

$Y' = \frac{1}{3}(\sin(5x^2+2x))^{-\frac{2}{3}}\cos(5x^2+2x)(10x+2)$

$Y' = \dfrac{(10x+2)\cos(5x^2+2x)}{3(\sin(5x^2+2x))^{\frac{2}{3}}}$

EXAMPLE p. 238 → #34

$\overset{u}{X}\overset{v}{Y} + Y^2 = 1$ FIND $\dfrac{d^2y}{dx^2}$

$X\frac{dy}{dx} + Y(1) + 2Y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(X+2Y) = -Y$ $\dfrac{dy}{dx} = \dfrac{-Y}{X+2Y}$ ← u ← v

$\dfrac{d^2Y}{dx^2} = \dfrac{(X+2Y)(-\frac{dy}{dx}) - (-Y)(1+2\frac{dy}{dx})}{(X+2Y)^2}$

$= \dfrac{(X+2Y)(-\frac{-Y}{X+2Y}) - (-Y)(1+2(\frac{-Y}{X+2Y}))}{(X+2Y)^2}$

$= \dfrac{Y + Y - \frac{2Y^2}{X+2Y}}{(X+2Y)^2} \cdot \dfrac{X+2Y}{X+2Y} = \boxed{\dfrac{2Y(X+2Y) - 2Y^2}{(X+2Y)^3}}$

P. 155 EXAMPLE # 30   ❸

$Y^2 - 2X - 4Y - 1 = 0$ FIND TANLINE
                              AND NORMAL LINE
$2Y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$ AT $(-2, 1)$

$\frac{dy}{dx}(2Y - 4) = 2$ $\frac{dy}{dx} = \frac{2}{2Y - 4}$

$M = \frac{2}{2(1) - 4} = \frac{2}{-2}$ $M = -1$

$M_L = \frac{-1}{-1} = 1$

$Y - Y_1 = m(X - X_1)$

$\boxed{Y - 1 = -1(X - -2)}$ ⟹ $Y = -X - 1$ TANLINE

$\boxed{Y - 1 = 1(X - -2)}$ ⟹ $Y = 1X + 3$ NORMAL

HOMEWORK

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↳ 13-16, 21-26, 27-35 ODD

BRAD WESTOVER TI-89 IMPLICIT SHORTCUT

$Y^2 - 2X - 4Y - 1 = 0$ (MUST BE SET = 0)

$- d(Y^2 - 2X - 4Y - 1, X) \div d(Y^2 - 2X - 4Y - 1, Y)$

PRESTO! $\boxed{\frac{1}{Y - 2}}$ SAME AS $\frac{2}{2Y - 4}$

3.8 INVERSE TRIG FUNCTIONS

$\sin 30° = \frac{1}{2}$ $\sin \frac{\pi}{6} = \frac{1}{2}$

$\sin^{-1} \frac{1}{2} = ?$

I HAVE THE RATIO.
WHAT'S THE ANGLE?

$\sin^{-1} \frac{1}{2} = 30°$

$\boxed{\sin^{-1} \frac{1}{2} = \frac{\pi}{6}}$

$30° = \frac{\pi}{6}$ RADIANS

$180° = \pi$ RADIANS

$2^{nd} \sin \quad \underline{\sin^{-1}} \quad \underline{arcsin} \quad \underline{invsin}$

ALL MEAN THE SAME THING.

ON TI-86 $2^{ND} \sin (1/2)$ ENTER .523599

$S = \frac{O}{H}$

$C = \frac{A}{H}$

$T = \frac{O}{A}$

$CSC = \frac{H}{O}$

$sec = \frac{H}{A}$

$cot = \frac{A}{O}$
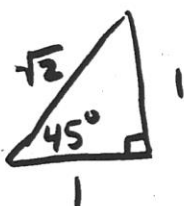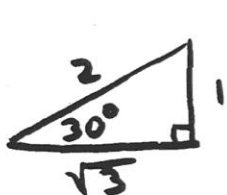
$\cos^{-1} \frac{\sqrt{3}}{2} = 30° = \frac{\pi}{6}$

$\tan^{-1} 1 = \tan^{-1} \frac{1}{1} = 45° = \frac{\pi}{4}$

$\csc^{-1} \frac{2}{\sqrt{3}} = 60° = \frac{\pi}{3}$

$\cot^{-1} \sqrt{3} = \cot^{-1} \frac{\sqrt{3}}{1} = 30° = \frac{\pi}{6}$

HOMEWORK WORKSHEET