6.3 INTEGRATION BY PARTS

$$\frac{d}{dx}(UV) = U\frac{dv}{dx} + V\frac{du}{dx}$$

IN DIFFERENTIAL FORM $\quad d(UV) = U\,dv + V\,du$

OR $\quad U\,dv = d(UV) - V\,du$

INTEGRATE BOTH SIDES $\quad \int U\,dv = \int d(UV) - \int V\,du$

$$\boxed{\int U\,dv = UV - \int V\,du}$$

INTEGRATION BY PARTS FORMULA

EX.1 p. 324

$\int \underline{x}\, \underline{\cos x\,dx}$

$\boxed{u = x \quad dV = \cos x\,dx}$

$du = dx \quad V = \sin x$

p. 324 SHOWS OTHER U, dV POSSI-BILITIES

$$\int u\,dV = UV - \int V\,du$$

$$= x\cdot\sin x - \int \sin x\,dx$$

$$= x\cdot\sin x - -\cos x \quad + C$$

$$= \boxed{x\sin x + \cos x + C}$$

CHECK BY TAKING DERIVATIVE

$$\frac{d}{dx}(x\sin x + \cos x + C)$$

$$= \underbrace{x\cos x}_{u\frac{dv}{dx}} + \underbrace{\sin x(1)}_{V\frac{du}{dx}} + -\sin x + 0 = \underline{\underline{x\cos x}}$$

6.3 CONTINUED (INTEGRATION BY PARTS)

REMEMBER $\boxed{\int u\,dv = uv - \int v\,du}$

EXAMPLE

FIND $\int \sin^{-1}x\,dx$ $\quad u = \sin^{-1}x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx \quad v = x$

$$\int u\,dv = uv - \int v\,du$$

$$= (\sin^{-1}x)x - \int x \frac{1}{\sqrt{1-x^2}}dx$$

$$= x\sin^{-1}x + \frac{1}{2}\int \frac{-2x}{(1-x^2)^{\frac{1}{2}}}dx$$

"NEW u"

$u = 1-x^2$

$du = -2x\,dx$

$$= x\sin^{-1}x + \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x\,dx)$$

$$= x\sin^{-1}x + \frac{1}{2}\int u^{-\frac{1}{2}}du = x\sin^{-1}x + \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$$

HWORK p.328
1,3,4,5,8

Ex. 4 p.326

TABULAR INTEGRATION

$\int x^2e^x\,dx \quad u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

$$\int u\,dv = uv - \int v\,du$$

$$= x^2e^x - \int e^x(2x\,dx)$$

$$= x^2e^x - 2\int xe^x\,dx$$

NEW u, v etc...

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$$x^2e^x-2\int u\,dv$$
$$=x^2e^x-2\left(uv-\int v\,du\right)$$
$$=x^2e^x-2\left(xe^x-\int e^x\,dx\right)$$
$$=\boxed{x^2e^x-2xe^x+2e^x+C}$$

↓ DER u		INT dv ↓
x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

ANOTHER "TABULAR" EXAMPLE

$$\int x^3\sin x\,dx \qquad u=x^3 \quad dv=\sin x\,dx$$

↓ DER u		INT dv ↓
x^3	$+$	$\sin x$
$3x^2$	$-$	$-\cos x$
$6x$	$+$	$-\sin x$
6	$-$	$\cos x$
0		$\sin x$

$x^3(-\cos x)-3x^2(-\sin x)$
$+6x\cos x-6\sin x+C$

$$\boxed{-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+C}$$

TO CHECK IT, TAKE THE DERIVATIVE AND YOU WILL GET $x^3\sin x$.

HOMEWORK p. 328–329
↳ 2, 7, 9–12, 15

NOTE: I SKIPPED THOSE LIKE EX. #5

POPULATION 3.5

CURRENTLY POPULATION DOUBLES EVERY 50 YEARS. THE CURRENT POPULATION IS ABOUT 6.3 BILLION.

6,300,000,000 t = JAN. 1, 2004

HOW LONG WOULD IT TAKE 8 PEOPLE, DOUBLING EVERY 50 YEARS TO REACH 6,300,000,000 ?

$$P = P_0 e^{kt} \qquad 2 = e^{50k} \qquad k = \frac{\ln 2}{50}$$

$$16 = 8e^{k50} \qquad \ln 2 = 50k \qquad k = .013863$$

$$6{,}300{,}000{,}000 = 8e^{.013863t} \leftarrow P = P_0 e^{kt}$$

$$\frac{6{,}300{,}000{,}000}{8} = e^{.013863t}$$

$$787{,}500{,}000 = e^{.013863t}$$

$$\ln 787{,}500{,}000 = .013863t$$

$$\frac{\ln 787{,}500{,}000}{.013863} = t \longrightarrow t = 1477.6 \text{ YEARS}$$

(NOT VERY LONG)

AT CURRENT RATES, WHEN WILL THE POPULATION HIT 1,000,000,000,000? 1 TRILLION!

$$P = P_0 e^{kt} \rightarrow 1{,}000{,}000{,}000{,}000 = 6{,}300{,}000{,}000 e^{.013863t}$$

$$\frac{1{,}000{,}000{,}000{,}000}{6{,}300{,}000{,}000} = e^{.013863t} \longrightarrow 158.73 = e^{.013863t}$$

$$\ln 158.73 = .013863t \rightarrow t = \frac{\ln 158.73}{.013863} = 365 \text{ YEARS}$$

IN THE YEAR 2369 (2004 + 365)
OUR PLANET'S POPULATION WILL BE 1 TRILLION!
1,000,000,000,000 !?!?!?!