# 7.1 INTEGRAL AS NET CHANGE 

S = POSITION   V = VELOCITY   a = acceleration

$$\frac{ds}{dt} = V \Rightarrow \int ds = \int V\,dt \quad \boxed{s = vt + c}$$

$\Delta S$ = CHANGE IN POSITION (DISPLACEMENT)

$$\boxed{\Delta S = \int_{t_1}^{t_2} V\,dt}$$

$t_1$ = time 1
$t_2$ = time 2

$$\boxed{\text{DISTANCE TRAVELED} = \int_{t_1}^{t_2} |v|\,dt}$$

LIKE 1-8

Ex. 1 p. 363   $V = t^2 - \frac{8}{(t+1)^2}$   $0 \le t \le 5$

| t | v |
|---|---|
| 0 | -8 |
| 1.25 | 0 |
| 5 | 24.8 |

THE PARTICLE TRAVELS ← a
BACKWARDS $0 \le t < 1.25$ (LEFT)
FORWARDS $1.25 < t \le 5$ (RIGHT)

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Ex. 2 p. 364   $\frac{ds}{dt} = t^2 - \frac{8}{(t+1)^2}$   $s(0) = 9$

$s(1) = ?$

$$\Delta S = \int_0^1 V\,dt = \int_0^1 \left(t^2 - \frac{8}{(t+1)^2}\right)dt = \frac{t^3}{3} + \frac{8}{t+1}\Big]_0^1$$

$$\Delta S = -\frac{11}{3} \quad s(1) = 9 + \left(-\tfrac{11}{3}\right) = \frac{16}{3}$$

$0 \le t \le 5$

$$\Delta S = \int_0^5 V\,dt = \int_0^5 \left(t^2 - \frac{8}{(t+1)^2}\right)dt = \underline{\underline{35}} \leftarrow b$$

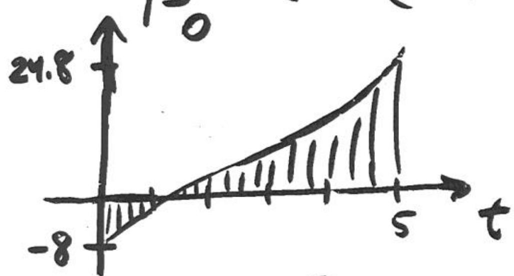
TI-89

EX. 3 p. 366 $\quad V = t^2 - \dfrac{8}{(t+1)^2}$ $\quad$ 104

$0 \le t \le 5$

$$\text{DISTANCE TRAVELED} = \int_0^5 |v|\,dt = \int_0^5 \left| t^2 - \frac{8}{(t+1)^2} \right| dt$$

$$= \left| \int_0^{1.25} \left( t^2 - \frac{8}{(t+1)^2} \right) dt \right| + \left| \int_{1.25}^{5} \left( t^2 - \frac{8}{(t+1)^2} \right) dt \right|$$




OR $\displaystyle\int_0^5 \left| t^2 - \frac{8}{(t+1)^2} \right| dt$

ON TI-89

$V = t^2 - \dfrac{8}{(t+1)^2}$

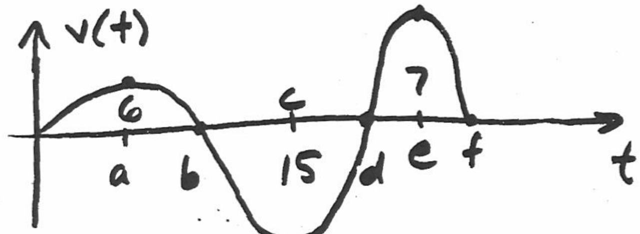
$3.7934 + 38.7934$

$= 42.59$

$\underline{\underline{C}} \longrightarrow$

HWORK p. 371 → 1-6, 9, 10

EXAMPLE LIKE p. 371 → 12-16




$s(0) = 5$

DISTANCE TRAVELED 0-f $= 6 + 15 + 7 = 28$

6 RIGHT, 15 LEFT, 7 RIGHT

$\Delta s = 6 - 15 + 7 = -2$ (2 LEFT)

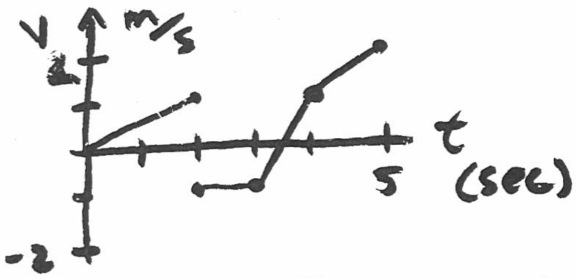
$s(b) = s(0) + 6 = 5 + 6 = 11$

$a = \frac{dv}{dt}$ $\quad \underline{a = 0}$ AT a AND c AND e

$\boxed{a > 0 \quad 0 \le t < a,\ c < t < e}$

ACCELERATION

EXAMPLE LIKE 17-20. GIVEN $x(0) = 2$




a) END OF TRIP

$s(5) = 2 + 1 - 1 - \frac{1}{4} + \frac{1}{4} + 1\frac{1}{2}$

$\boxed{s(5) = 3\tfrac{1}{2} \text{ meters}}$

b) DISTANCE TRAVELED $= 1 + 1 + \frac{1}{4} + \frac{1}{4} + 1\frac{1}{2} = \underline{4\ m}$

HOMEWORK p. 371-372 → 12-20 ALL

ANY RATE CAN BE INTEGRATED TO OBTAIN A QUANTITY. 

p. 368 EX. 5 C = POTATO CONSUMPTION

$C(t) = 2.2 + 1.1^t$ (MILLIONS OF BUSHELS / YR.)

$$\int_2^4 (2.2 + 1.1^t)\,dt$$ TI-89

$= 7.066$ (MILLION)

$t=0$ 1970
$t=2$ 1972
$t=4$ 1974

THEREFORE 7,066,000 BUSHELS OF POTATOS WERE EATEN FROM JAN. 1 1972 TO DEC 31 1973

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EX. 6 p. 369 $G = \int_0^{60} R(t)\,dt$

G = GALLONS PUMPED R = PUMP RATE

| MIN $t$ | GAL/MIN $R$ |
|---|---|
| 0 | 58 |
| 5 | 60 |
| 10 | 65 |
| 15 | 64 |
| 20 | 58 |
| 25 | 57 |
| 30 | 55 |
| 35 | 55 |
| 40 | 59 |
| 45 | 60 |
| 50 | 60 |
| 55 | 63 |
| 60 | 63 |

TRAPEZOIDAL ESTIMATE

$$\frac{h}{2}(R_0 + 2R_1 + 2R_2 + \cdots + R_N)$$

$$= \frac{(60/12)}{2}(58 + 2(60) + 2(65) + \cdots + 2(63) + 63)$$

$$= 2.5(1433) = \boxed{3580 \text{ GALLONS}}$$

HOMEWORK

p. 372-373 → 21-22, 26a, [redacted], 28

27a → $B(x) = 1.6x^2 + 2.3x + 5$

p. 413 → 1, 3, 5