3.8 INVERSE TRIG DERIVATIVES

→ CONTINUED

$Y = \sin^{-1} x$ $\qquad \frac{dy}{dx} = ???$

ANGLE (Y) RATIO (x)

$30^\circ = \sin^{-1} \frac{1}{2} \qquad \sin 30^\circ = \frac{1}{2}$

$\sin Y = X$

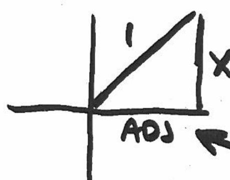
ANGLE (Y) RATIO (X)

IMPLICIT DIFFERENTIATION

$\frac{d}{dx}$

$\cos Y \frac{dy}{dx} = 1 \rightarrow \frac{dy}{dx} = \frac{1}{\cos Y} = \frac{1}{\cos[\sin^{-1} x]}$

$$\boxed{\frac{dY}{dx} = \frac{1}{\sqrt{1-X^2}}}$$



$\sin = \frac{Y}{R} = \frac{X}{1}$ (OPP: Y, HYP: R)

$ADJ^2 + X^2 = 1^2 \qquad ADJ = \sqrt{1-X^2}$

$\cos[\sin^{-1} x] = \frac{\sqrt{1-X^2}}{1} = \sqrt{1-X^2}$

SIMILAR PROOFS CAN BE DONE FOR OTHERS.

$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$ $-1 < u < 1$	$\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$ $-1 < u < 1$
$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \cdot \frac{du}{dx}$	$\frac{d}{dx} \cot^{-1} u = \frac{-1}{1+u^2} \cdot \frac{du}{dx}$
$\frac{d}{dx} \sec^{-1} u = \frac{1}{\lvert u \rvert \sqrt{u^2-1}} \frac{du}{dx}$ $\lvert u \rvert > 1$	$\frac{d}{dx} \csc^{-1} u = \frac{-1}{\lvert u \rvert \sqrt{u^2-1}} \cdot \frac{du}{dx}$ $\lvert u \rvert > 1$

Ex. 1 p.159 $\quad \frac{d}{dx} \sin^{-1} x^2 \quad u = x^2$ (44)

$$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

EXAMPLE $\quad \frac{d}{dx} \tan^{-1} \sqrt{x+1} \quad u = (x+1)^{1/2}$

$$\frac{1}{1+u^2} \frac{du}{dx} = \frac{1}{1+(\sqrt{x+1})^2} \cdot \frac{1}{2}(x+1)^{-1/2}$$

$$= \frac{1}{1+x+1} \cdot \frac{1}{2\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}\,(x+2)}$$

EXAMPLE $\quad \frac{d}{dx} \sec^{-1}(3x) \quad u = 3x$

$$\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx} = \frac{1}{|3x|\sqrt{(3x)^2-1}} \cdot 3 = \frac{1}{|x|\sqrt{9x^2-1}}$$

HOMEWORK p.162 → 1-15, 19