5.2 CALCULUS AND AREA

SIGMA (Σ) NOTATION Σ = ADD

$$\sum_{k=1}^{5} k = 1+2+3+4+5 = 15$$

$$\sum_{k=1}^{4} k^2-1 = \underline{1^2-1} + \underline{2^2-1} + \underline{3^2-1} + \underline{4^2-1} = 26$$

EXAMPLE

WRITE $-\frac{1}{5}+\frac{2}{5}-\frac{3}{5}+\frac{4}{5}-\frac{5}{5}$ IN Σ NOTATION

ANSWER: $\sum_{k=1}^{5} \frac{k}{5}\cdot(-1)^k$

$$\sum c \cdot f(k) = c \sum f(k)$$

$$\sum f(x) \pm g(x) = \sum f(x) \pm \sum g(x)$$

EXAMPLE

EVALUATE $\sum_{k=1}^{7}(2k-8) = \sum_{k=1}^{7} 2k - \sum_{k=1}^{7} 8$

$$= 2\sum_{k=1}^{7} k - 8\sum_{k=1}^{7} 1 = 2\cdot(1+2+3+4+5+6+7) - 8\cdot(1+1+1+1+1+1+1) = 0$$

HWORK p. 267 QUICK REV. 1–10

p. 260 Riemann Sum Notation

A = Area (72)

$$A = \lim_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x = I$$

- $\sum_{k=1}^{n}$: ADD
- $f(c_k)$: HEIGTH
- Δx: WIDTH
- $\|P\| \to 0$: MAXIMUM PARTITION SIZE APPROACHES 0.

INTEGRAL NOTATION

$$* \text{ "Area"} = I = \int_{a}^{b} f(x)\,dx$$

- a: LEFT X-VALUE (LOWER LIMIT)
- b: RIGHT X-VALUE (UPPER LIMIT)
- $f(x)$: HEIGTH
- dx: WIDTH (INFINITESSIMALLY SMALL)

ADD AN INFINITE NUMBER OF RECTANGLES.

* $(f(x) > 0)$

EXAMPLE #6 p. 267

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} (\sin^3 c_k)\,\Delta x_k \quad [-\pi, \pi]$$

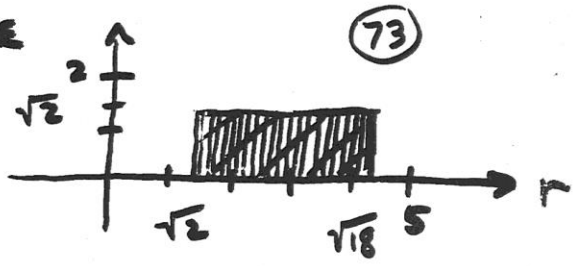
$$\int_{-\pi}^{\pi} \sin^3 x\,dx$$

HWORK p. 267 → 1-5

p.267 #12 EXAMPLE

$$\int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2}\, dr$$

$f(r) = \sqrt{2}$



$$\int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2}\, dr = \text{AREA} = \sqrt{2}(\sqrt{18} - \sqrt{2})$$
$$= \sqrt{36} - \sqrt{4} = 6 - 2 = \boxed{4}$$

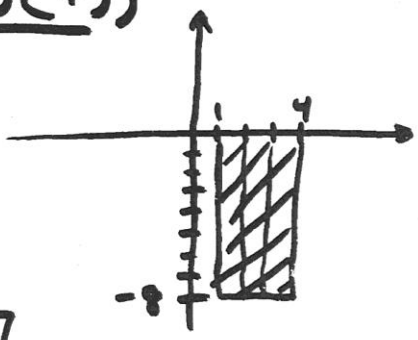
IF $f(x) < 0$ $\quad \int f(x)\,dx = -\text{AREA}$

$(\Sigma (-)(+))$

EXAMPLE

$$\int_1^4 (-8)\, dx$$

$f(x) = -8$



$$\int_1^4 (-8)\,dx = -8(4-1)$$
$$= \boxed{-24}$$

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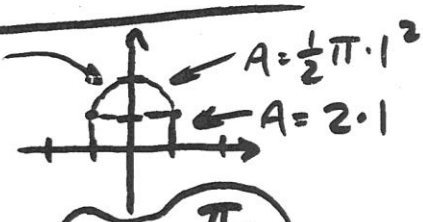
p.267 #20 EXAMPLE

$Y = 1 + \sqrt{1 - x^2}$

$$\int_{-1}^{1} \left(1 + \sqrt{1 - x^2}\right) dx$$



$$= 2 \cdot 1 + \tfrac{1}{2}\pi \cdot 1^2 = \boxed{2 + \frac{\pi}{2}}$$

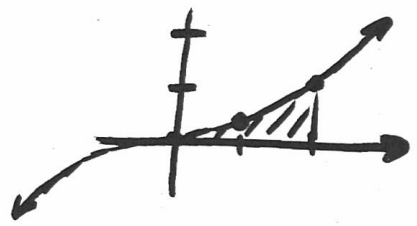
HOMEWORK p. 267 → 7-11 ALL, 13-27 ODD

DO #26 FOR EXAMPLE

INTEGRAL BY TI-89

p. 267 #35

$$\int_0^2 \left(\frac{x}{2}\right)^3 dx$$



2ND $\frac{S}{7}$

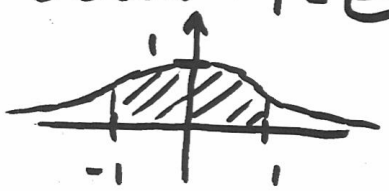
$\int((x \div 2) \wedge 3, X, 0, 2)$

ENTER

$\frac{1}{2}$

EXAMPLE

FIND THE AREA BETWEEN -1 AND 1 BELOW $Y = e^{-X^2}$ AND ABOVE THE X-AXIS.



$\int(e \wedge (-X \wedge 2), X, -1, 1)$

1.49365

OR

USE TI-89 ◇ GRAPH F5 MATH → $\int f(x)\,dx$

p. 268 → 37-46 ALL

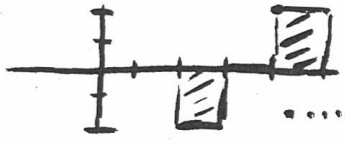
TI-86
fnint( )

$Y = e \wedge (-X \wedge 2)$

CHECK THIS WAY TOO.

(p. 267 → 18, 22, 24, 26)

#44 →



etc....

..... + − 4 + − 2 + 0 + 2