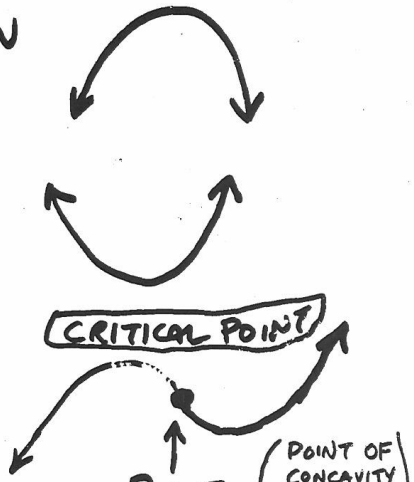
4.3 Connecting f, f' and f''

If $f(x)$ is a **twice differentiable** function, then:

1) $f(x)$ is concave down when $f''(x) < 0$
2) $f(x)$ is concave up when $f''(x) > 0$
3) $f(x)$ **may** have a point of inflection when $f''(x) = 0$



Example → analytically →

$f(x) = 20 + 2x^2 - 9x^4$ Analyze completely.

$f'(x) = 4x - 36x^3 = 4x(1 - 9x^2) = 4x(1-3x)(1+3x)$

$4x = 0 \Rightarrow x = 0$, $1 - 3x = 0 \Rightarrow x = \frac{1}{3}$, $1 + 3x = 0 \Rightarrow x = -\frac{1}{3}$

$f(0) = 20$ $\quad f\left(\frac{1}{3}\right) = 20.\overline{1}$ $\quad f\left(-\frac{1}{3}\right) = 20.\overline{1}$

(0, 20) Rel. Min. **$(.\overline{3}, 20.\overline{1})$ Max.** **$(-.\overline{3}, 20.\overline{1})$ Max.**

$f''(x) = 4 - 108x^2$

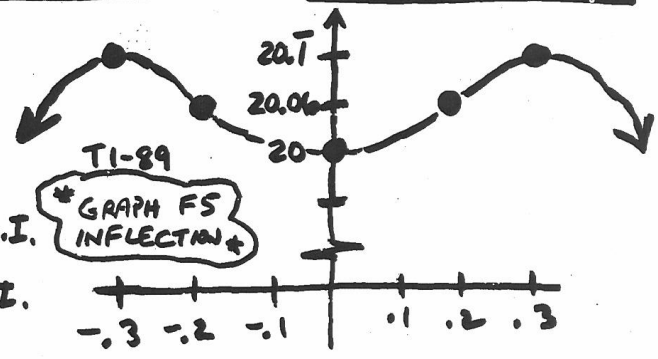
$0 = 4 - 108x^2$

$x = \pm\sqrt{\frac{4}{108}} = \pm .1925$

$f(-.1925) = 20.06$ P.O.I.

$f(.1925) = 20.06$ P.O.I.

TI-89 * Graph f5 Inflection



EXAMPLE CONTINUED

INCREASING $(-\infty, -.\bar{3}) \cup (0, .\bar{3})$
DECREASING $(-.\bar{3}, 0) \cup (.\bar{3}, \infty)$
CONCAVE DOWN $(-\infty, -.2) \cup (.2, \infty)$
CONCAVE UP $(-.2, .2)$

EXAMPLE → ANALYZE COMPLETELY. (WITH CALCULATOR)

$$f(x) = \frac{1}{.2 + 2^{-.5x}}$$

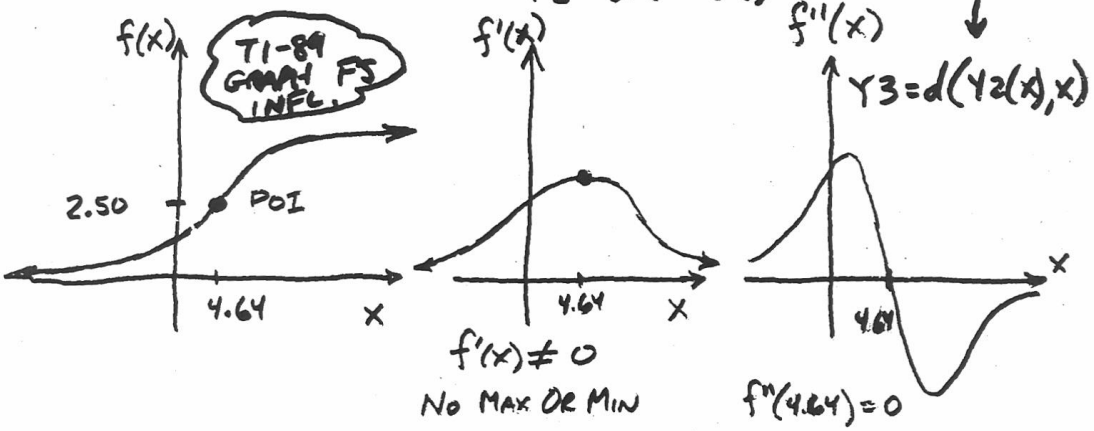
$Y1 = 1/(.2 + 2\wedge(-.5X))$

$Y2 = der1(Y1, X)$ ← TI-86
↑ 2ND CALC ↑ 2ND VARS / MORE EQU

$Y3 = der2(Y1, X)$

SET APPROPRIATE WINDOWS (SEE p. 280). DESELECT GRAPHS THAT YOU DON'T WANT TO SEE.

$Y2 = d(Y1(X), X)$ ← TI-89



$$f(4.64) = \frac{1}{.2 + 2^{-.5(4.64)}} = 2.50 \qquad \text{POI} = (4.64, 2.50)$$

NO MAX, NO MIN, ALWAYS INCREASING FUNCTION

HOMEWORK p. 204 → 7-27 ODD

THERE ARE 4 BASIC GRAPH SHAPES FOR RATIONAL FUNCTIONS.

INCREASING, CONCAVE UP $f' \underset{+}{>} 0, f'' \underset{+}{>} 0$

INCREASING, CONCAVE DOWN $f' \underset{+}{>} 0, f'' \underset{-}{<} 0$

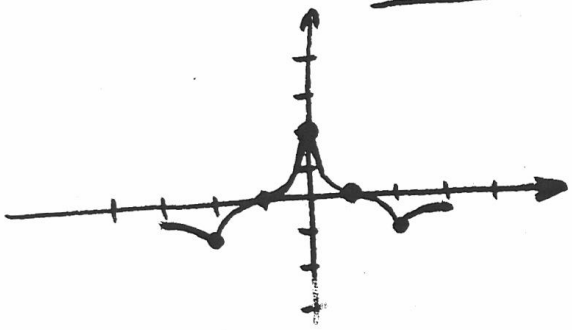
DECREASING, CONCAVE UP $f' \underset{-}{<} 0, f'' \underset{+}{>} 0$

DECREASING, CONCAVE DOWN $f' \underset{-}{<} 0, f'' \underset{-}{<} 0$

(34) p. 205 EXAMPLE EVEN FUNCTION (Y-AXIS SYM.)

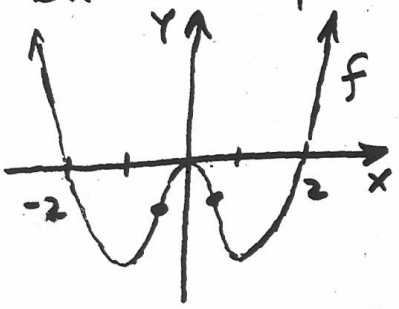
X	0	1	2
f	2	0	-1
f'	DNE	0	DNE
f''	DNE	0	DNE

X	$0<x<1$	$1<x<2$	$2<x<3$
f	+	−	−
f'	−	−	+
f''	+	−	−



4.3 CONTINUED

EXAMPLE p. 203 #2 WHERE IS $f'(x)$ 0, POS., NEG.

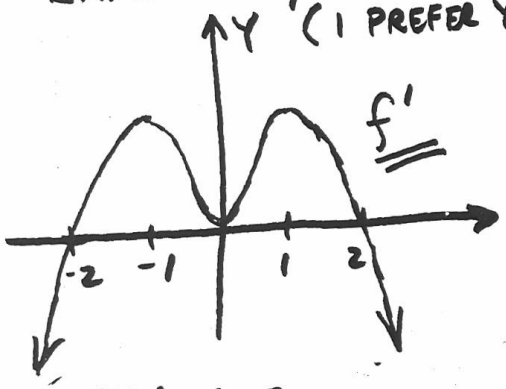


f' IS 0 AT $-1, 0, 1$

f' IS POS. $(-1,0) \cup (1,\infty)$

f' IS NEG. $(-\infty,-1) \cup (0,1)$

EXAMPLE p. 203 #4 (I PREFER Y') WHERE IS f FALLING OR RISING? (f' NEG) (f' POS)



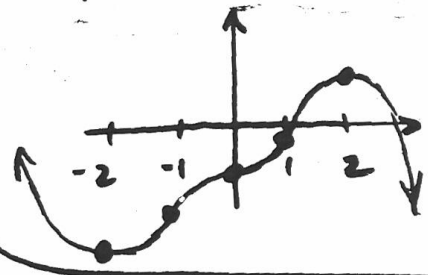
f FALLING $(-\infty,-2) \cup (2,\infty)$

f RISING $(-2,2)$

P.O.I's AT $X = -1, 0, 1$

LOCAL MIN $X=-2$

LOCAL MAX $X=2$



f COULD LOOK LIKE THIS.

EXAMPLE

$$Y' = X(X+2)^2(X-1)^3(X-3)^5(X-5)^8$$

MIN OR MAX AT 0, 1, 3 POI AT -2, 5

$Y'(.5) = .5(2.5)^2(-.5)^3(-2.5)^5(-4.5)^8$ = POSITIVE

$0 \to .5 \to 1$ INCREASING 0 MIN 1 MAX

HOMEWORK p. 203-205 → 1, 3, 5, 6, 29-33, 45-48, 37, 39, 41, 42