# 7.3 VOLUMES OF SOLIDS OF REVOLUTION



IF $Y = f(x)$ IS ROTATED AROUND THE X-AXIS, THE VOLUME GENERATED FROM $x = a$ TO $x = b$ IS

$$V = \underbrace{\int_a^b}_{\text{SUM FROM } a \text{ to } b} \underbrace{\pi r^2}_{\text{AREA OF A CIRCULAR SLICE.}} \underbrace{dx}_{\text{WIDTH OF EACH SLICE.}} \quad \text{OR} \quad \boxed{V = \int_a^b \pi \left[f(x)\right]^2 dx}$$

EXAMPLE 3 p. 385 ESTIMATE = $4 \cdot 4 \cdot 4 = 64$

REVOLVE $Y = 2 + x\cos x$ AROUND THE X-AXIS. WHAT IS THE VOLUME FROM $-2 \leq x \leq 2$

SEE FIG. 7.19 & 7.20 p. 385

$$V = \int_{-2}^{2} \pi (2 + x\cos x)^2 dx$$

TI-89

∫(π(2+x*cos(x))^2, x, -2, 2)

= 52.43 UNITS CUBED

$\boxed{\text{HOMEWORK p. } 392 \rightarrow 13, 16-20}$

IF $X=f(y)$ IS ROTATED AROUND THE Y-AXIS, THE VOLUME GENERATED FROM $Y=c$ TO $Y=d$ IS

$$V=\underbrace{\int_c^d}_{\substack{\text{SUM} \\ \text{FROM} \\ c \text{ to } d \\ \text{(UP)}}} \underbrace{\pi r^2}_{\substack{\text{AREA} \\ \text{OF A} \\ \text{CIRCULAR} \\ \text{SLICE}}} \underbrace{dy}_{\substack{\text{WIDTH OF EACH SLICE} \\ \text{(STACKING THEM IN THE} \\ \text{Y DIRECTION.)}}} \quad \text{OR} \quad \boxed{V=\int_c^d \pi\left[f(y)\right]^2 dy}$$

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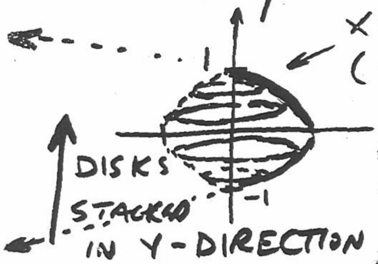
EXAMPLE

FIND THE VOLUME GENERATED BY REVOLVING $x=1-y^2$ AROUND THE Y-AXIS. ($X=0$ BOUND)




$x=1-y^2$
$(y=\pm\sqrt{1-x})$

$$V=\int_{-1}^{1}\pi(1-y^2)^2 dy$$

$$V=\int_{-1}^{1}\pi(1-2y^2+y^4)dy=\left(\pi y-2\pi\frac{y^3}{3}+\frac{\pi y^5}{5}\right)\Big]_{-1}^{1}$$

$$=\pi-\frac{2\pi}{3}+\frac{\pi}{5}-\left(-\pi+\frac{2\pi}{3}-\frac{\pi}{5}\right) \qquad V=\frac{16\pi}{15}$$

OR $\int(\pi(1-y^2)^2, y, -1, 1)$ ENTER TI-89

OR $\text{fnint}(\pi(1-x^2)^2, x, -1, 1)$ ENTER TI-86

OR SWITCH VARIABLES CHANGE PROBLEM

$X=1-Y^2$, $X=0$ REVOLVE AROUND Y-AXIS
$Y=1-X^2$, $Y=0$ REVOLVE AROUND X-AXIS
} SAME

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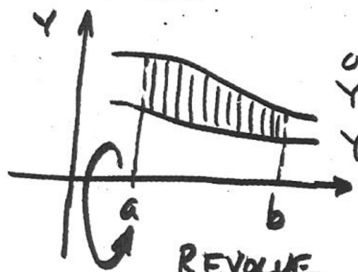
HWORK p. 392 → 14, 15, 29, 30

# 7.3 CONTINUED — WASHERS = DISKS − HOLES


AREA = $\pi R^2 - \pi r^2$ PICTURES p. 447

$V = \text{SUM}(\text{DISKS} - \text{HOLES})$ OR

$V = \text{SUM}(\text{DISKS}) - \text{SUM}(\text{HOLES})$




OUTER RADIUS $Y = f(x)$

$Y = g(x)$ INNER RADIUS

$$V = \int_a^b \pi [R(x)]^2 dx - \int_a^b [r(x)]^2 dx$$

$V =$ <u>WHOLE</u> − <u>HOLE</u>

REVOLVE $f(x)$ & $g(x)$ AROUND X-AXIS

VOLUME OF SOLID →

$$\boxed{V = \int_a^b \pi \overbrace{[f(x)]^2}^{\text{TOP}^2} dx - \int_a^b \pi \overbrace{[g(x)]^2}^{\text{BOTTOM}^2} dx}$$

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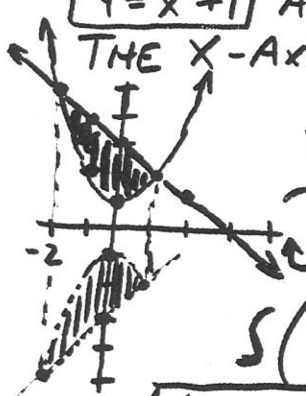
EXAMPLE

THE REGION BOUNDED BY $\boxed{Y = x^2 + 1}$ AND $\boxed{Y = -X + 3}$ IS REVOLVED AROUND THE X-AXIS TO GENERATE A SOLID. FIND V.




$$V = \int_{-2}^{1} \pi(-x+3)^2 dx - \int_{-2}^{1} \pi(x^2+1)^2 dx \quad V = \frac{117\pi}{5}$$

← SUM(DISKS) − SUM(HOLES)

TI-89

← SUM(DISKS − HOLES)

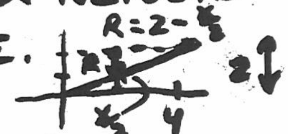
$\int(\pi((-X+3)^2 - (X^2+1)^2), X, -2, 1)$ ENTER

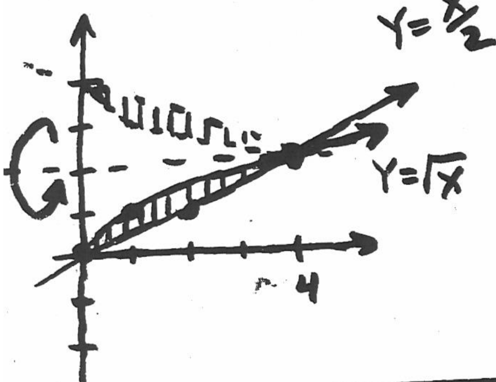
HWORK p. 392 → 22-26 | $\boxed{V = 73.5}$

NOTE: FOR REVOLVING AROUND THE Y-AXIS EITHER SWITCH X & Y OR USE CYLINDRICAL SHELLS (FUTURE METHOD)

## REVOLVING AREA AROUND SOMETHING OTHER THAN THE X AXIS

EXAMPLE
REGION BETWEEN $Y=\frac{x}{2}$ & $Y=\sqrt{x}$ REVOLVED ABOUT $\underline{Y=2}$ FIND THE VOLUME.




$$V=\int_0^4 \pi[R(x)]^2dx-\int_0^4\pi[r(x)]^2dx$$

WHOLE - HOLE

$$V=\int_0^4 \pi\left(2-\frac{x}{2}\right)^2dx-\int_0^4\pi\left(2-\sqrt{x}\right)^2dx$$

$$=\frac{8\pi}{3} \quad \text{(TI-89)}$$

**HWORK p. 392 37a, 37b, 37c**

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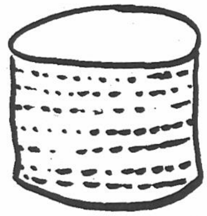
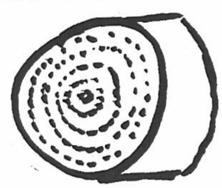
## [redacted] THE SHELL METHOD



WASHER METHOD
(SUM OF WASHERS)
EACH WASHER
dx THICK

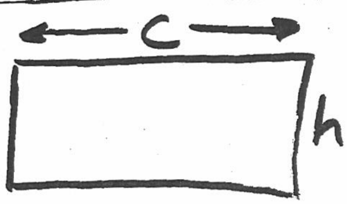
SHELL METHOD
(SUM OF SHELLS)



EACH SHELL
dx THICK

$C=2\pi R$

$$\boxed{V=\text{sum}(C\cdot h)}$$



REVOLVE $f(x)$ OVER $g(x)$ ABOUT THE Y-AXIS

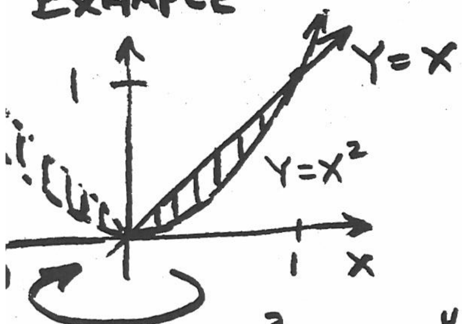
THE VOLUME GENERATED FROM $a$ to $b$ IS: (IN THE x-direction)

$$V = \text{SUM}\left(\underbrace{\text{CIRCUMFERENCE} \cdot \text{HEIGTH}}_{2\pi r \cdot h}\right)$$

$$\boxed{V = \int_a^b \underline{2\pi x} \cdot \underline{(f(x) - g(x))}\, dx}$$

SOMETIMES $g(x) = 0$

EXAMPLE




$$V = \int_{x=0}^{x=1} 2\pi x(x - x^2)\, dx$$

$x=1$ ← OUTSIDE RADIUS

$x=0$ ← INSIDE RADIUS

$$= \int_0^1 (2\pi x^2 - 2\pi x^3)\, dx$$

$$= \left(2\pi \frac{x^3}{3} - 2\pi \frac{x^4}{4}\right)\Big]_0^1 = \frac{2\pi}{3} - \frac{2\pi}{4}$$

$$\boxed{V = \frac{\pi}{6}}$$

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REVOLVE AROUND X-AXIS OR Y=C USE DISC METHOD (OR WASHER)

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REVOLVE AROUND Y-AXIS USE SHELL.

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IF REVOLVING AROUND X-AXIS, ALL VARIABLES CAN BE SWITCHED SO THAT ITS REVOLVING AROUND THE Y-AXIS, THEN SHELL CAN BE USED.

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p.392-393 → 39-42, 43a, 44a, 48b,

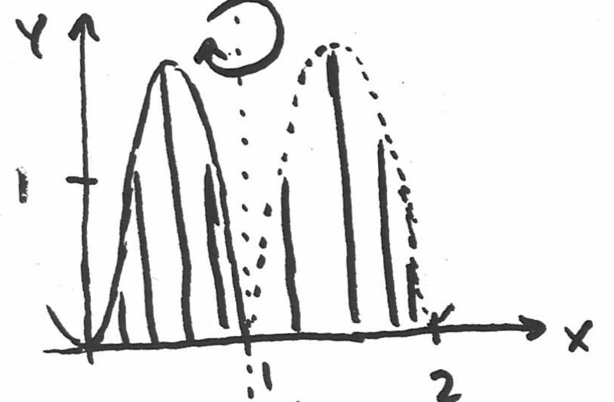
HWORK SWITCH VARIABLES $\int_{.0001}^{\pi}$

# GENERALIZING SHELL FOR MORE COMPLICATED PROBLEMS

P.392 → 43b REVOLVE $X = 12(Y^2 - Y^3)$ ABOUT $Y = 1$

SWITCH VARIABLES!

REVOLVE $Y = 12(X^2 - X^3)$ ABOUT $X = 1$




$$V = \int_a^b 2\pi R\big(f(x) - g(x)\big)\,dx$$

$$V = \int_0^1 2\pi(1-x)\left[12(x^2 - x^3) - 0\right]dx$$

TI-89 → $\frac{4\pi}{5}$

HWORK

P. 392-393 → 43c, 43d, 44b, 44c, 44d

# 7.3 STILL!



## MORE ON VOLUME (IN GENERAL)

$$\boxed{V=\int_a^b A(x)\,dx} \quad \text{OR} \quad V=\int_c^d A(y)\,dy$$

EXAMPLE 2 p.385 (SEE PICTURE p.385)

A 3m HIGH PYRAMID HAS A SQUARE BASE 3m ON EACH SIDE. FIND THE VOLUME (WITH CALCULUS)

$$V=\int_0^3 A(x)\,dx \qquad \underline{\underline{A(x)=x^2}}$$

$$V=\int_0^3 x^2\,dx = \frac{x^3}{3}\Big]_0^3 = \frac{27}{3}-0 \quad \boxed{V=9\,m^3}$$

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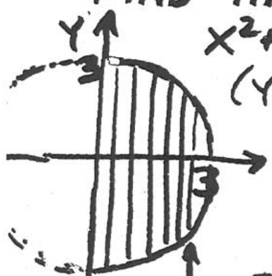
EXAMPLE p.508 OLD BOOK

A 45° WEDGE IS CUT FROM A CYLINDER WITH THE SLICE ENDING EXACTLY AT THE DIAMETER OF THE BOTTOM OF THE CYLINDER (DIAMETER = 6). FIND THE VOLUME OF THE WEDGE.




$x^2+y^2=3^2$

$(y=\pm\sqrt{9-x^2})$

$$V=\int_a^b A(x)\,dx \qquad \boxed{A=b\cdot h}$$

TI-89

$$V=\int_0^3 \underbrace{2\sqrt{9-x^2}}_{\text{base}}\ \underbrace{x}_{h}\ \underbrace{dx}_{\text{WIDTH}} = \boxed{18}$$

THIN TALL RECTANGLE b·h

WIDE SHORT RECTANGLE b·h

Do #1 WITH ~~THE~~

HOMEWORK p.390-391 1-7 ALL