8-1 INDETERMINATE FORMS AND L'HÔPITAL'S RULE

$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$ IS AN INDETERMINATE FORM, BECAUSE $x=0$ CANNOT BE SUBSTITUTED DIRECTLY. WE MUST TRACE NEAR $x=0$.

L'HOPITAL'S RULE PROOF p. 418

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}}$$

FIRST FORM $\boxed{f(a) = g(a) = 0}$ $\frac{0}{0}$

EXAMPLE

$$\lim_{x \to 0} \frac{\sin x}{x} = \frac{\frac{d}{dx}\sin x}{\frac{d}{dx} x} = \frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = 1$$

ANOTHER EXAMPLE

$$\lim_{x \to 0} \frac{3x - \sin x}{x} = \frac{\frac{d}{dx}(3x - \sin x)}{\frac{d}{dx} x} = \frac{3 - \cos x}{1} = \frac{3 - \cos 0}{1} = \frac{3-1}{1} = 2$$

AB → 1, 2, 5, 10, 11, 15, 16

L'HOPITALS RULE STRONGER FORM

$$\boxed{\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}}$$

$\boxed{f(a) = g(a) = 0}$

WHAT THIS IMPLIES IS WE CAN APPLY L'HOPITALS RULE OVER AND OVER UNTIL THE LIMIT CAN BE DETERMINED.

EXAMPLE

$$\lim_{x\to 0} \frac{x-\sin x}{x^3} = \lim_{x\to 0} \frac{1-\cos x}{3x^2} = \lim_{x\to 0} \frac{\sin x}{6x}$$

FIRST DERIV. SECOND DERIVATIVE

$$= \lim_{x\to 0} \frac{\cos x}{6} = \frac{\cos 0}{6} = \frac{1}{6} \qquad \frac{0}{0} \to \frac{0}{0} \to \frac{1}{6}$$

THIRD DERIVATIVE

EXAMPLE 3 p. 419

$$\lim_{x\to 0} \frac{(1+x)^{1/2}-1-x/2}{x^2} = \lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-1/2}-\frac{1}{2}}{2x}$$

FIRST DERIVATIVE

$$= \lim_{x\to 0} \frac{-\frac{1}{2}\cdot\frac{1}{2}(1+x)^{-3/2}}{2} = \frac{-\frac{1}{2}\cdot\frac{1}{2}(1+0)^{-3/2}}{2} = -\frac{1}{8}$$

SECOND DERIVATIVE $\frac{0}{0} \to -\frac{1}{8}$

EXAMPLE 4 p. 419

$$\lim_{x\to 0^+} \frac{\sin x}{x^2} \quad \frac{0}{0} = \lim_{x\to 0^+} \frac{\cos x}{2x} \quad \frac{1}{0} \quad \underline{\text{STOP!}}$$

$$\frac{\cos 0^+}{2\cdot 0^+} = \boxed{\infty}$$

~~$\frac{-\sin x}{2}$ NO, BAD~~

L'HÔPITAL'S RULE ALSO WORKS FOR $\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{5x^2-1}{2x^2+x} \quad \frac{\infty}{\infty} \to \frac{10x}{4x+1} \quad \frac{\infty}{\infty} \to \frac{10}{4} \text{ or } \frac{5}{2}$$

FIRST DER. 2ND DER.

HOMEWORK p. 423-424 → 1, 2, 4, 5, 6, 8-11, 14-21

8.1 CONTINUED, L'HOPITAL TRICKS

(WHAT IF THE INDETERMINATE FORM IS NOT $\frac{0}{0}$ OR $\frac{\infty}{\infty}$?)

EXAMPLE 6 p.420 MULT TO DIV TRICK (MY WAY)

$$\lim_{x \to \pm\infty} x \sin\frac{1}{x} \qquad \text{FORM } \infty \cdot 0$$

$$\lim_{x \to \pm\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} \quad \frac{0}{0} \xrightarrow{\text{L'HOP}} = \lim_{x \to \pm\infty} \frac{\cos\frac{1}{x} \cdot \frac{-1}{x^2}}{-\frac{1}{x^2}}$$

$$= \lim_{x \to \pm\infty} \cos\frac{1}{x} = \cos\frac{1}{\pm\infty} = \cos 0 = \boxed{1}$$

EXAMPLE 7 p.421 COMMON DENOMINATOR TRICK

$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right) \qquad \text{FORM } \infty - \infty$$

$$\lim_{x \to 1} \left(\frac{1(x-1)}{\ln x(x-1)} - \frac{1 \ln x}{(x-1)\ln x}\right) = \lim_{x \to 1} \frac{x-1-\ln x}{\underset{u}{(x-1)}\underset{v}{\ln x}} \qquad \text{FORM } \frac{0}{0}$$

L'HOPITAL #1 SIMPLIFY

$$\lim_{x \to 1} \frac{1-0-\frac{1}{x}}{(x-1)\frac{1}{x}+\ln x \cdot 1} = \lim_{x \to 1} \left(\frac{1-\frac{1}{x}}{(x-1)\frac{1}{x}+\ln x}\right)\frac{x}{x}$$

$$= \lim_{x \to 1} \frac{x-1}{x-1+\underset{u}{x}\underset{v}{\ln x}} \xrightarrow{\text{L'HOPITAL \#2}} = \lim_{x \to 1} \frac{1}{1-0+x\cdot\frac{1}{x}+\ln x \cdot 1}$$

$$= \lim_{x \to 1} \frac{1}{2+\ln x} = \frac{1}{2}$$

FOR EXPRESSIONS IN THE FORM 1^{∞}, 0^{0} & ∞^{0} SOMETIMES THE LOG OF THE LIMIT CAN BE FOUND.

$$\boxed{\begin{array}{l} \text{IF } \displaystyle\lim_{x\to a} \ln f(x) = L \\ \text{THEN } \displaystyle\lim_{x\to a} f(x) = \lim_{x\to a} e^{\ln f(x)} = e^{L} \end{array}}$$

EXAMPLE 8 p. 421

ALSO SEE EX. 9 & EX. 10

$$L = \lim_{x\to\pm\infty} \left(1+\frac{1}{x}\right)^{x}$$

FIRST FIND $\displaystyle\lim_{x\to\pm\infty} \ln\left(1+\frac{1}{x}\right)^{x} = \ln L$

$$= \lim_{x\to\pm\infty} x\ln\left(1+\frac{1}{x}\right) = \lim_{x\to\pm\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

FORM $\frac{0}{0}$

L'HOPITAL

$$\lim_{x\to\pm\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot -1x^{-2}}{-1x^{-2}} = \lim_{x\to\pm\infty} \frac{1}{1+\frac{1}{x}} = 1 = \underline{\ln L}$$

SO $\ln L = 1$ $\quad e^{\ln L} = e^{1}$ $\quad L = e$

AB ASSIGN #1 → 12, <u>23–28</u>, 31, 32 THEN ASSIGN #2

$$\boxed{\text{HWORK p. 423-424} \to 3, 7, 12, 13, 23\text{-}41 \ \underline{\text{ODD}}}$$

(24, 26, 28, 32)