7.2 AREA BETWEEN CURVES

AREA BETWEEN 2 CURVES $f(x)$ & $g(x)$ ON THE INTERVAL $a \le x \le b$

$$\text{AREA} = \int_a^b \big(f(x) - g(x)\big)\,dx$$

TOP − BOTTOM

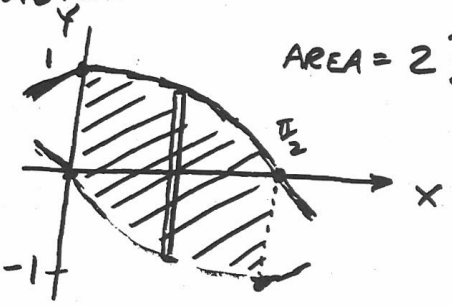
~~HEIGHT • WIDTH~~

SEE p. 374-375 FOR ILLUSTRATION & DISCUSSION

EXAMPLE

FIND AREA BETWEEN $Y = \cos X$ & $Y = -\sin X$ FROM 0 TO $\pi/2$.

$(a=0,\ b=\pi/2)$



AREA = 2?

$$\text{AREA} = \int_0^{\pi/2} (\cos x - -\sin x)\,dx$$

$$\text{AREA} = \int_0^{\pi/2} (\cos x + \sin x)\,dx = (\sin x - \cos x)\Big|_0^{\pi/2}$$

$$\text{AREA} = \left(\sin\tfrac{\pi}{2} - \cos\tfrac{\pi}{2}\right) - (\sin 0 - \cos 0)$$

$$= (1-0) - (0-1) = 2$$

AREA = 2

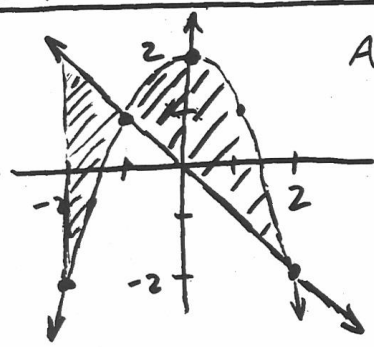
OR fnint$(\cos x + \sin x, X, 0, \pi/2)$ ENTER $=$ 2

(2ND CALC) TI-89 $\int$

HWORK p. 380-381 → 2, 5, 6, 11-13, 33

EXAMPLE

FIND THE AREA OF THE REGION ENCLOSED BY $Y = 2 - X^2$ AND $Y = -X$, BETWEEN -2 AND 2



AREA = 2 + 4 ?

$$\text{AREA} = \text{AREA 1} + \text{AREA 2}$$

$$= \int_{-2}^{-1} (-X - (2 - X^2))\,dX + \int_{-1}^{2} ((2 - X^2) - (-X))\,dX$$

USE TI-89

$$= 11/6 + 4\tfrac{1}{2}$$

$$= 6.\overline{33}$$

$2 - X^2 = -X$

$X^2 - X - 2 = 0$

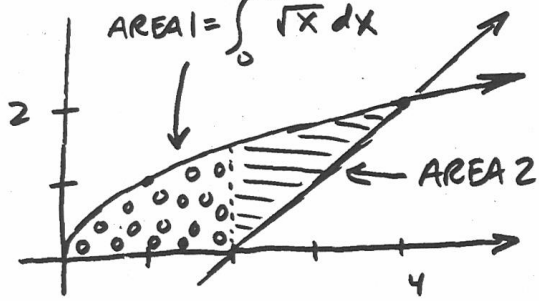
$(X+1)(X-2)$

$X = -1, \; X = 2$

OR TI-86 ISECT

EXAMPLE 4 p. 377

FIND THE AREA BOUNDED IN QUADRANT I BY $Y = \sqrt{X}$, THE X-AXIS AND $Y = X - 2$.



$\text{AREA 1} = \int_0^2 \sqrt{X}\,dX$

$\text{AREA} = \text{AREA 1} + \text{AREA 2}$

$\text{AREA 2} = \int_2^4 (\sqrt{X} - (X-2))\,dX$

$\int$ TI-89

$\text{AREA} = \text{fnint}(\sqrt{X}, X, 0, 2) + \text{fnint}(\sqrt{X} - (X-2), X, 2, 4) = 10/3$

HOMEWORK p. 381 → 7-10, 14, 16

INTEGRATING WITH RESPECT TO Y.

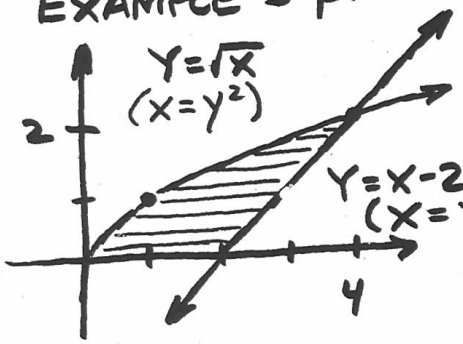
AREA BETWEEN 2 CURVES $f(y)$ & $g(y)$ ON THE INTERVAL $a \le y \le b$

$$\text{AREA} = \int_a^b (f(y) - g(y))\,dy$$

RIGHT LEFT

SEE p.378 FOR ILLUSTRATION & DISCUSSION. (NOTE EXAMPLE 5)

EXAMPLE 5 p. 378



$$A = \int_0^2 [(y+2) - y^2]\,dy$$

$$= \left(\frac{y^2}{2} + 2y - \frac{y^3}{3}\right)\Bigg|_0^2$$

$$A = \left(\frac{2^2}{2} + 2\cdot 2 - \frac{2^3}{3}\right) - \left(\frac{0^2}{2} + 2\cdot 0 - \frac{0^3}{3}\right)$$

AREA = $10/3$

OR $\int(Y+2-Y^2, Y, 0, 2)$ ENTER $10/3$

OR fnint$(X+2-X^2, X, 0, 2)$ ENTER $10/3$

SWITCHING VARIABLES

EXAMPLE

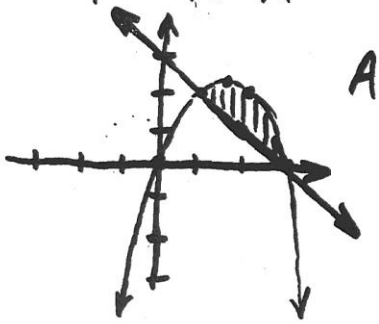
FIND THE AREA BETWEEN

$X = 3Y - Y^2$ AND $X + Y = 3$

SWITCH THE VARIABLES →

$X = 3Y - Y^2$ $\quad X + Y = 3$ (110)

$Y = 3X - X^2$ $\quad Y + X = 3 \quad (Y = -X + 3)$



$$A = \int_1^3 \left((3X - X^2) - (-X + 3)\right) dx$$

$$= \int_1^3 (-X^2 + 4X - 3)\, dx$$

$$= \left. -\frac{X^3}{3} + \frac{4X^2}{2} - 3X \right|_1^3$$

$$A = \left(-\frac{3^3}{3} + \frac{4 \cdot 3^2}{2} - 3 \cdot 3\right) - \left(\frac{-1^3}{3} + \frac{4 \cdot 1^2}{2} - 3 \cdot 1\right) = \frac{4}{3}$$

OR fnint($-X^2 + 4X - 3, X, 1, 3$) ENTER 1.333

(OR S) TI-89

HOMEWORK p. 380-381

→ 3, 4, 18, 29, 30, 35* ← HARD!