4.5 LINEAR APPROXIMATIONS

THE TANGENT LINE OF A FUNCTION AT SOME POINT $(a, f(a))$ IS CALLED THE LINEARIZATION OF THE FUNCTION $L(x)$.

$$Y - Y_1 = m(X - X_1)$$

$$L(x) - f(a) = f'(a)(x - a)$$

OR $\boxed{L(x) = f(a) + f'(a)(x - a)}$

EXAMPLE p.229 #1

$f(x) = x^3 - 2x + 3$ FIND $L(x)$ AT $x = 2$ ← $a = 2$

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = f(2) + f'(2)(x - 2)$$

$$L(x) = 7 + 10(x - 2)$$

OR $\boxed{L(x) = 10x - 13}$

$f(2) = 2^3 - 2\cdot 2 + 3 = 7$

$f'(x) = 3x^2 - 2$

$f'(2) = 3\cdot 2^2 - 2 = 10$

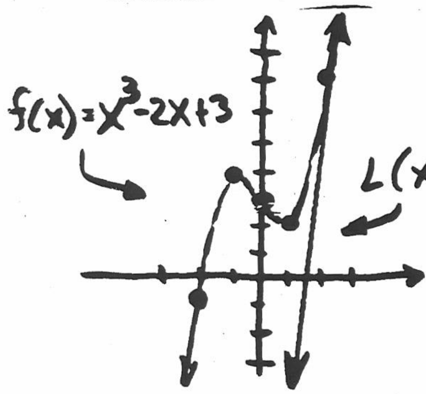
$f(a + .1) = L(a + .1)$?

$f(2.1) = 2.1^3 - 2(2.1) + 3$

$= 8.061$

$L(2.1) = 10(2.1) - 13$

$= 8$ ← CLOSE



HOMEWORK
p.229 EX. 2-6

ERROR = 8.061 − 8

ERROR = .061

4.5 CONTINUED - LINEAR APPROXIMATIONS

NOTE:
NEAR $(a, f(a))$ $\quad f(x) \approx L(x)$
(APPROXIMATELY EQUAL)

IN OTHER WORDS, NEAR THE POINT OF TANGENCY $(a, f(a))$, THE FUNCTION $f(x)$ AND THE TANGENT LINE $(L(x))$ YIELD APPROXIMATELY EQUAL Y VALUES.
THE FARTHER YOU GET FROM $(a, f(a))$, THE WORSE IS YOUR ESTIMATE.

$(a, f(a))$

LIKE p.230 → 27-30 EXAMPLE SEE PICTURE p.230

$f(x) = x^4 \qquad a = 1 \qquad dx = .1$

(FIND 1.1^4)

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = f(1) + f'(1)(x-1)$

$f(1) = 1^4 = 1$

$f'(x) = 4x^3 \quad f'(1) = 4 \cdot 1^3 = 4$

$L(x) = 1 + 4(x-1)$

$L(x) = 4x - 3 \qquad L(1.1) = 4(1.1) - 3 = 1.4$

BUT $1.1^4 = 1.4641$

A) $\Delta f = 1.4641 - 1 = \underline{.4641}$ B) $df = 1.4 - 1 = .4$

c) ERROR $= |\Delta f - df| = |.4641 - .4| = \underline{.0641}$

$\boxed{\text{HOMEWORK p. 230} \rightarrow \text{27-30}}$

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EXAMPLE p.230 #34

$S = 6x^2$ ESTIMATE CHANGE IN S. FROM a TO a + dx

$$\frac{dS}{dx} = 12x \quad dS = 12x\,dx$$

$$\boxed{dS = 12a \cdot dx}$$

EXAMPLE p.230 #39

$C = \pi D \qquad D = 10 \qquad dC = 2$

$$\frac{dC}{dD} = \pi \qquad dD = \frac{dC}{\pi} \qquad \boxed{dD = \frac{2}{\pi}\text{ IN.}}$$

$$A = \pi r^2$$

$$A = \pi\left(\frac{D}{2}\right)^2$$

$$A = \pi\frac{D^2}{4} \quad \frac{dA}{dD} = 2\frac{\pi D}{4} = \frac{\pi D}{2}$$

$$dA = \frac{\pi D}{2}\,dD$$

$$dA = \frac{\pi \cdot 10}{2} \cdot \frac{2}{\pi} = \underline{10 \text{ inches}^2}$$

EXAMPLE p.230 #41

HOW ACCURATELY SHOULD YOU MEASURE THE SIDE OF A SQUARE TO MAKE SURE AREA IS WITHIN 2%.

$$A = S^2 \quad \frac{dA}{ds} = 2S \quad dA = 2S\,ds \rightarrow \underline{\pm.02A = 2S\,dS}$$

$$\pm.02S^2 = 2S\,dS \quad dS = \frac{\pm.02S^2}{2S} \quad \boxed{dS = \pm.01S}$$

HOMEWORK p. 230-231 → 31-33, 35, 36, 38, 40

OVERHEAD 65 OPTIONAL