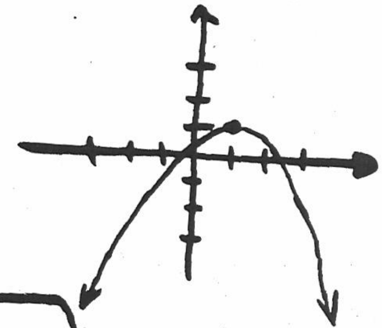
4.2 MEAN VALUE THEOREM

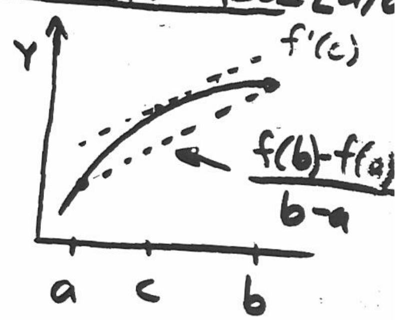
[redacted]

[redacted] $f(x)$ IF $f(1)=1$,
$f'(x)>0$ FOR $x<1$ &
$f'(x)<0$ FOR $x>1$



THE MEAN VALUE THEOREM
SEE FIGURE 4.10 p.186
IF $f(x)$ IS CONTINUOUS AND DIFFERENTIABLE [a, b]
THEN THERE IS AT LEAST
ONE NUMBER c AT WHICH

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$



EXAMPLE p.192 #16
$f(x)=x^{2/3}$ $\quad 0\le x\le 1$ $\quad$ APPLY M.V.T. [0, 1]

$f(0)=0^{2/3}=0$ $\quad f(1)=1^{2/3}=1$ $\quad m_{sec}=\frac{f(1)-f(0)}{1-0}=\frac{1}{1}=1$

$f'(x)=\frac{2}{3}x^{-1/3}$ $\quad$ M.V.T. SOMEWHERE $1=\frac{2}{3}x^{-1/3}$

$\frac{3}{2}=x^{-1/3}\Rightarrow x^{1/3}=\frac{2}{3}\Rightarrow x=\left(\frac{2}{3}\right)^3$ $\quad \boxed{x=\frac{8}{27}}$

AT $x=\frac{8}{27}$ $\quad f'(x)=1$

HOMEWORK
p.192 → 15, 17, 18, 35-39

EXAMPLE $\longrightarrow$ FIND EXTREMA

4.2 CONTINUED

$g(x) = -x^4 + 5x^2 - 4$ ON $[-3, 3]$

$g'(x) = -4x^3 + 10x = 2x(-2x^2 + 5) = 0$ (FOR EXTREMA)

$2x = 0 \Rightarrow x = 0 \quad -2x^2 + 5 = 0 \Rightarrow x^2 = \frac{5}{2} \quad x = \pm\sqrt{2.5}$

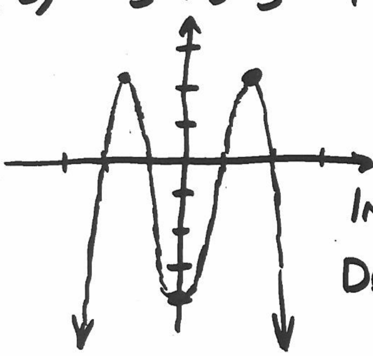
$g(0) = -0^4 + 5 \cdot 0^2 - 4 = -4$ $(0, -4)$ REL. MIN.

$g(\sqrt{2.5}) = -\sqrt{2.5}^4 + 5 \cdot \sqrt{2.5}^2 - 4 = 2.25$ $(\sqrt{2.5}, 2.25)$ MAX.

$g(-\sqrt{2.5}) = -(-\sqrt{2.5})^4 + 5(-\sqrt{2.5})^2 - 4 = 2.25$ $(-\sqrt{2.5}, 2.25)$ MAX.

$g(-3) = -(-3)^4 + 5(-3)^2 - 4 = -40$ $(-3, -40)$ MIN.

$g(3) = -3^4 + 5 \cdot 3^2 - 4 = -40$ $(3, -40)$ MIN.

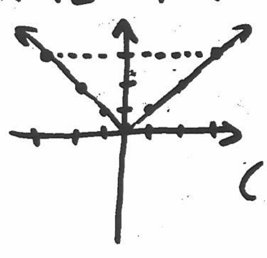


WHERE IS THE FUNCTION INCREASING & DECREASING?

INCREASING $[-3, -\sqrt{2.5}) \cup (0, \sqrt{2.5})$

DECREASING $(-\sqrt{2.5}, 0) \cup (\sqrt{2.5}, 3]$

NOTE: $y = |x|$ DOES NOT SATISFY THE MEAN VALUE THEOREM, BECAUSE IT IS NOT DIFFERENTIABLE AT $x = 0$. (NO WHERE ON $[-3, 3]$ DOES THE SLOPE $= 0$)



HOMEWORK p. 192 → 1–11 ODD, 21–24 ALL