

Manual

The CADET Authors

CADET Version 4.1.0

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1 Simulation

This chapter gives an overview of the simulation process and the different steps involved.

CADET uses a backward-differentiation-formula (BDF) time discretization as implemented by the IDAS solver from SUNDIALS [Hin+05]. Each time step requires the solution of a nonlinear algebraic system of equations is performed by a Newton method. Since chromatographic systems can exhibit strong nonlinearity and stiff systems, the Jacobian of the equation system is always updated (i.e., an "exact" Newton method is used).

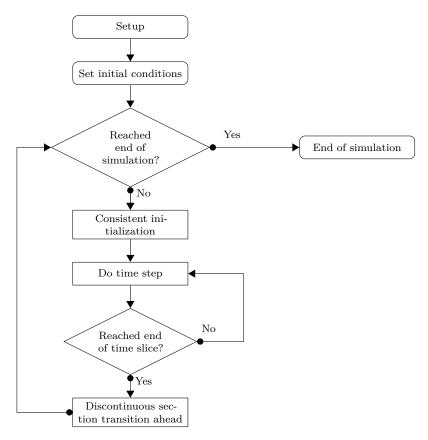


Figure 1.0.1: General time integration procedure

The general control flow of a simulation is shown in Fig. 1.0.1. Some aspects of the tasks involved are discussed below.

1.1 Time sections and transitions

The timespan $[t_0, t_{\text{end}}]$ of the simulation can be divided into multiple time sections $[t_i, t_{i+1}]$ with

$$t_0 < t_1 < \ldots < t_i < t_{i+1} < \ldots < t_{\text{end}}.$$

Time sections are used for various purposes, for example, for defining piecewise polynomials used as inlet profiles or external function, for changing operating conditions such as flow rates, or for changing the connectivity in a network of unit operations. A sequence of (one or more) time sections with smooth transitions is called a time slice (see Fig. 1.1.1).

A transition from one section to the next can either be smooth or discontinuous. On smooth transitions, the time integrator can just step over the section transition. Contrary to smooth transitions, a discontinuous transition requires some extra work in resetting the time integrator and finding consistent initial conditions for the next time slice.

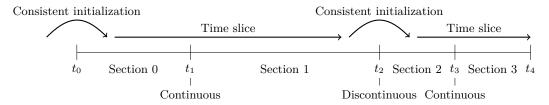


Figure 1.1.1: Time sections, time slices, and transitions between them

Time section specification Time sections are specified by an array (SECTION_TIMES, see Tab. 3.5.9) which contains the t_i that denote the start- and endpoint of a time section $[t_i, t_{i+1}]$. This means, that the array contains n+1 values if there are n sections. The smoothness of a section transition is indicated by the array SECTION_CONTINUITY, which contains n-1 elements if there are n sections.

1.2 Setup

In the setup phase, the model specification is checked and data structures are prepared for the simulation. The simulation can be run multiple times with different initial conditions, which also allows to continue a simulation. However, the model structure must not change after this point. This means, the different model and submodel types (e.g., unit operation model, binding model) as well as their discretization (i.e., number of cells) and structure (i.e., number of components, number of reactions) are fixed. On the contrary, their parameters (e.g., flow rates, porosities, dispersion coefficients) may change on different runs of a prepared simulation.

This assumption allows an accelerated simulation as there is no memory allocation performed during time integration.

1.3 Consistent initialization

At the beginning of the simulation and on discontinuous section transitions, consistent initial conditions have to be computed. In CADET, the general differential-algebraic equation (DAE)

$$F(t, y, \dot{y}, p) = 0$$

consists of purely algebraic equations that do not contain \dot{y} and dynamic equations. Let \mathcal{I}_d be the index set of dynamic equations and \mathcal{I}_a the index set of algebraic equations. The general DAE can be decomposed into

$$M\dot{y}_{\mathcal{I}_d} + G_{\mathcal{I}_d}(t, y, p) = 0, \tag{1.1}$$

$$G_{\mathcal{I}_a}(t, y, p) = 0, (1.2)$$

where M is the so called "mass matrix", which essentially consist of the state time derivative Jacobian $\partial F/\partial \dot{y}$. Furthermore, it is assumed that the Jacobian of the algebraic equations with respect to the algebraic variables $\partial G_{\mathcal{I}_a}/\partial y_{\mathcal{I}_a}$ is invertible. Hence, the original DAE is of differential index 1.

Given $y_{\mathcal{I}_d}$, consistently initialization means finding $y_{\mathcal{I}_a}$, $y_{\mathcal{I}_d}$, and \dot{y} such that the DAE holds at the initial time point t_0 and can be solved for some non-empty time span. Thus, consistent initial conditions

 y_0 and \dot{y}_0 satisfy

$$\begin{split} 0 &= M\dot{y}_{\mathcal{I}_{d}} + G_{\mathcal{I}_{d}}\left(t_{0}, y, p\right), \\ 0 &= G_{\mathcal{I}_{a}}\left(t_{0}, y, p\right), \\ 0 &= \frac{\partial G_{\mathcal{I}_{a}}}{\partial t}\left(t_{0}, y, p\right) + \frac{\partial G_{\mathcal{I}_{a}}}{\partial y}\left(t_{0}, \dot{y}, p\right). \end{split}$$

The last equation, which determines $\dot{y}_{\mathcal{I}_a}$, arises from taking the total derivative with respect to time t of the second equation.

Concluding, a nonlinear algebraic equation system has to be solved in order to compute consistent initial conditions. The error in the solution of these systems is controlled by the ALGTOL setting (see Tab. 3.5.9).

1.4 Time stepping

Given the general differential-algebraic equation

$$F(t, y, \dot{y}, p) = 0,$$
 $y(t_0) = y_0,$ $\dot{y}(t_0) = \dot{y}_0$

with consistent initial values y_0 and \dot{y}_0 , the BDF discretization uses past time steps $y(t_{\tau-i})$ to compute the current $\dot{y}(t_{\tau})$ as

$$\dot{y}\left(t_{\tau}\right) = \frac{1}{\Delta t_{\tau}} \sum_{i=0}^{q_{\tau}} \alpha_{\tau,i} y\left(t_{\tau-i}\right),\,$$

where t_{τ} denotes some time step. The IDAS time integrator adaptively changes the step size $\Delta t_{\tau} = t_{\tau} - t_{\tau-1}$ and order $1 \leq q_{\tau} \leq 5$ in each time step and automatically determines the corresponding coefficients $\alpha_{\tau,i}$. This *ansatz* is inserted into the DAE and the resulting nonlinear algebraic equation system is solved for $y_{\tau} = y(t_{\tau})$ using a Newton method.

$$H(y_{\tau}) = F\left(t_{\tau}, y_{\tau}, \frac{1}{\Delta t_{\tau}} \sum_{i=0}^{q_{\tau}} \alpha_{\tau, i} y_{\tau-i}, p\right) = 0$$
 (1.3)

This requires (possibly many) solutions of linear equation systems involving the Jacobian of H given by

$$\frac{\mathrm{d}H}{\mathrm{d}y_{\tau}} = \frac{\partial F}{\partial y} + \frac{\alpha_{\tau,0}}{\Delta t_{\tau}} \frac{\partial F}{\partial \dot{y}}.$$

The Jacobian of H, which is the linear combination of the partial derivatives $\partial F/\partial y$ and $\partial F/\partial \dot{y}$, can be calculated in two ways. The fastest way is the analytic computation, which is to be preferred if available. However, the implementation of the analytic Jacobian can be time consuming and complicated, especially when quickly testing new models. As a remedy, CADET offers to compute the complicated Jacobian $\partial F/\partial y$ automatically by automatic differentiation (AD). The much simpler Jacobian $\partial F/\partial y$ has to be implemented manually.

Adaptivity The BDF order q_{τ} and size of the time step t_{τ} are chosen such that the error bounds are satisfied with minimal work. Equation (1.3) is solved with a Newton iteration using very few iterations (MAX_NEWTON_ITER from Table 3.5.9) since the starting point is assumed to be close to the solution as the time steps are small. If the solution of the Newton iteration does not pass the convergence test, the step size t_{τ} is reduced and the Newton iteration is tried again. This may happen at most MAX_CONVTEST_FAIL times, otherwise time integration is aborted with failure. Having a solution of the Newton iteration at hand, a local truncation error test is performed which controls the error of the BDF method. If this test fails, step size t_{τ} and order q_{τ} are adapted and the process restarts with solving the nonlinear system Eq. (1.3). The error test may be failed at most MAX_ERRTEST_FAIL times before time integration is aborted with failure.

Error bounds for the local truncation error test are specified by an absolute tolerance (ABSTOL) and a relative tolerance (RELTOL). Note that the relative tolerance only works for non-zero values, whereas zero values are accounted for by the absolute tolerance. For example, a relative tolerance of 10^{-4} and absolute tolerance of 10^{-8} requests 3 significant digits (correct digits after the comma in scientific notation) and considers all numbers with magnitude smaller than 10^{-8} as 0.

The size of the first time step in a time slice is given by INIT_STEP_SIZE from Table 3.5.9. If a simulation fails on the first time step, it might help to reduce the initial step size. It can also help to increase the error tolerances, at the cost of imprecise results over the whole course of the simulation.

It can happen that due to severe stiffness or ill-posed models a simulation may exceed its computational budget given by the maximum number of time steps (per time slice) MAX_STEPS. In this case, the formulation of the model should be checked and, if necessary, the error tolerances increased. Note that the number of time steps is independent of possible USER_SOLUTION_TIMES and refers to internal steps of the time integrator.

On the other extreme, the time integrator might jump over an important change in the solution or not capture some feature of the solution because of too large time steps. This can be avoided by specifying the maximum time step size MAX_STEP_SIZE. However, such a situation is unlikely to occur and can also be alleviated by placing a discontinuous section transition at the time of the event.

Note that while the error in time integration is controlled and can (in theory) be made arbitrarily small, the spatial discretization error is not. Hence, it is important and necessary to test different spatial resolutions (number of cells) in order to find a sufficiently accurate (but minimal) number of grid cells. This is particularly relevant for problems with steep fronts as a low spatial resolution adds a substantial amount of numerical dispersion to the solution.

1.5 Parameter sensitivities

Parameter sensitivities $s = \partial y/\partial p$ of a solution y to the DAE with respect to some parameter p are required for various tasks, for example, parameter estimation, process design, and process analysis. The CADET simulator implements the forward sensitivity approach which creates a linear companion DAE for each sensitive parameter

$$\begin{split} 0 &= \frac{\mathrm{d}}{\mathrm{d}p} F(t,y,\dot{y},p) = \frac{\partial F}{\partial y}(t,y,\dot{y},p) \frac{\partial y}{\partial p} + \frac{\partial F}{\partial \dot{y}}(t,y,\dot{y},p) \frac{\partial \dot{y}}{\partial p} + \frac{\partial F}{\partial p}(t,y,\dot{y},p) \\ &= \frac{\partial F}{\partial y}(t,y,\dot{y},p)s + \frac{\partial F}{\partial \dot{y}}(t,y,\dot{y},p)\dot{s} + \frac{\partial F}{\partial p}(t,y,\dot{y},p). \end{split}$$

These linear DAEs depend on the solution y, \dot{y} of the original DAE system. Consequently, the sensitivity systems are solved together with the original DAE system in a staggered approach [FTB97].

After the nonlinear Eq. (1.3) has been successfully solved using Newton iteration (i.e., it has passed the convergence test), each sensitivity is solved using the same Newton iteration. If direct linear solvers were used, this iteration would convergence with a single iteration as there is no nonlinearity. The Newton method for the sensitivities performs at most MAX_NEWTON_ITER_SENS (see Table 3.5.9) iterations. Assuming the convergence test has passed for each sensitivity, the local truncation error test is performed for the full set of variables (i.e., original system and sensitivities). The sensitivities can be excluded from the local truncation error test by setting ERRORTEST_SENS appropriately. Note that time integration step size is always affected by sensitivities due to possible convergence test failures.

While the Jacobians $\partial F/\partial y$ and $\partial F/\partial \dot{y}$ can be computed either analytically or via AD, the last term $\partial F/\partial p$ is always computed by AD. In fact, the terms $\partial F/\partial y$ and $\partial F/\partial p$ can be computed by one AD-enabled evaluation of F using the vector mode with appropriate seed vectors [Püt+16].

A parameter sensitivity is specified by identifying the involved parameters, which can be more than one (see below). Parameters are identified by name and multiple indices, which may not all be used (see Tab. 3.5.8). Indices that are not used for identification (e.g., flow rate is independent of bound phase or component) are set to -1. All other indices are zero-based.

Note that the sensitivity systems need to be consistently initialized, too. However, since only linear systems are involved, no nonlinear equation system has to be solved and the procedure is much simpler computationally.

Joint parameters The use of AD also enables support of meta parameters or joint parameters. Consider the situation in which several parameters p_1, p_2, \ldots, p_n depend (linearly) on a single meta parameter p

$$p_i = p_i(p) = \alpha_i p$$
 for $i = 1, \dots, n$

and some coefficients $\alpha_i \in \mathbb{R}$. Hence, the DAE residual function F becomes

$$F(t, y, \dot{y}, p_1(p), \dots, p_n(p)) = 0$$

and the partial derivative $\partial F/\partial p$ can be computed automatically by AD:

$$\frac{\partial F}{\partial p} = \frac{\partial F}{\partial p_1} \alpha_1 + \ldots + \frac{\partial F}{\partial p_n} \alpha_n.$$

In order to use this functionality, all parameters involved have to be specified in a single sensitivity instance (param_XXX group, see Tables 3.5.8 and 3.5.8) by using arrays for the name and indices instead of scalar values. The coefficients α_i are provided in the SENS_FACTOR vector.

A simple example for a situation, which benefits from having a single meta parameter, would be a chain of unit operations in a network in which the sensitivity with respect to the flow rate is to be computed. Instead of computing the sensitivity with respect to each single flow rate and fusing them together in a postprocessing step, a meta parameter that maps to all flow rates can be introduced.

Note that nonlinear relationships between original parameter and meta parameters are supported by updating the coefficients α_i before each simulation:

$$\alpha_i = \frac{\partial p_i}{\partial p}.$$

2 Models

2.1 Network of unit operation models

Unit operation models can be composed into a network or graph, in which a node represents a unit operation and an edge denotes a connection between two unit operations. When utilized to full extent, this allows the simulation of complicated setups and processes (e.g., SMB, MCSGP). A more simple use case is the addition of plug flows and stirred tanks up- and downstream of a column in order to account for dead volume and additional dispersion from the tubing.

In a network, outlet ports of unit operations can be connected to any number of inlet ports of unit operations. Even direct cycles, where an outlet port of a unit operation is connected to its own inlet, are possible. A unit operation does not have to possess both inlet and outlet, but it has to have at least one of them. Pseudo unit operations such as inlet and outlet serve as sources and sinks for the network. However, the latter is not strictly required as any terminal node (i.e., a unit operation that possesses an outlet but does not have an outgoing connection) serves as a sink.

Each connection between two unit operation ports (i.e., an edge in the graph) is equipped with a volumetric flow rate that determines the mass flow from source to target port. These flow rates are used to determine the weight of the different incoming feeds at a unit operation's inlet port. Some unit operations can infer their internal flow rate (e.g., interstitial velocity) from their total incoming volumetric flow rate. In general, the mass balance at a unit operation has to be closed, except for unit operations that act as source or sink in the network and variable volume units (e.g., stirred tanks).

The network of unit operations uses "connection"-variables c_{con} to connect the different unit operation ports with each other. The inlet port variables $c_{\text{in},n,k}$ of unit operation n are attached to $c_{\text{con},n}$ via

$$c_{\text{in},n,k,i} = c_{\text{con},n,k,i}, \qquad k = 1, \dots, N_{\text{port,in},n}, \quad i = 1, \dots, N_{\text{comp},n}.$$
 (2.1)

While $N_{\text{port,in},n}$ denotes the number of inlet ports of unit operation n, the number of outlet ports is given by $N_{\text{port,out,}n}$. The connection variables $c_{\text{con},n,k,i}$ collect all inflows of component i into port k of unit operation n:

$$c_{\text{con},n,k,i} = \frac{\sum_{m=1}^{N_{\text{units}}} \sum_{\ell=1}^{N_{\text{port,out},n}} \sum_{j=1}^{N_{\text{comp},m}} S_{(n,k,i),(m,\ell,j)} F_{m,\ell} c_{\text{out},m,\ell,j}}{\sum_{m=1}^{N_{\text{units}}} \sum_{\ell=1}^{N_{\text{port,out},m}} \hat{S}_{(n,k),(m,\ell)} F_{m,\ell}},$$
(2.2)

where $F_{m,\ell}$ denotes the volumetric flow rate from outlet port ℓ of unit operation m, $S_{(n,k,i),(m,\ell,j)} \in \{0,1\}$ is a connection matrix indicating whether component i at outlet port k of unit operation n is connected to component j at inlet port ℓ of unit operation m, and $\hat{S}_{(n,k),(m,\ell)} \in \{0,1\}$ is another connection matrix indicating whether outlet port k of unit operation n is connected to inlet port ℓ of unit operation m, that is

$$\hat{S}_{(n,k),(m,\ell)} = \begin{cases} 1 & \text{if } \sum_{i=1}^{N_{\text{comp},n}} \sum_{j=1}^{N_{\text{comp},m}} S_{(n,k,i),(m,\ell,j)} \geq 1, \\ 0 & \text{otherwise}. \end{cases}$$

Note that for each unit operation the number of inlet ports may be different from the number of outlet ports. Hence, the mass balance of a single unit operation is taken with respect to all its ports combined.

Specification of network connections The connections between the different unit operations in the network are specified by a table. There are two table formats:

• The long format includes seven columns. The first two columns specify source and destination unit operation id. The next two columns give source and destination port indices. Source and

destination component indices are given by the following two columns. Finally, the seventh column specifies the volumetric flow rate of this connection.

• The short format includes five columns. The first two columns specify source and destination unit operation id. Source and destination component indices are given by the following two columns. Finally, the fifth column specifies the volumetric flow rate of this connection. Here, the omitted port indices default to -1, which connects all ports of the source unit operation to the corresponding ports of the target.

By default, the short format is used (i.e., a table with five columns is expected). However, if a unit operation with multiple ports is present, a table with seven columns is required. The default format can be overruled by setting a field (see Table 3.5.1).

With this setup, it is possible to connect single components of unit operations with each other yielding a maximum in flexibility. However, the predominant case is to connect all components of the source unit operations with their respective counterparts in the destination unit. This can easily be done by setting both component indices to -1 instead of writing a separate row for each component of the connection. The same setting (i.e., setting both port indices to -1) can be used to connect all ports of one unit operation with all corresponding ports of another one.

Note that in case of multiple rows for one connection between two unit operation ports (e.g., in case of separate component connections) the flow rate of the first row of that connection is used and all following flow rates are ignored. Consequently, there can only be one flow rate for a connection between two unit operations regardless of which components are connected.

The connection table is expected in row-major storage format (i.e., the rows are appended to one long array). See Table 3.5.1.

Valve switches The connectivity of the network can only change on a discontinuous section transition. Such a transition with changing connectivity is referred to as valve switch and the connectivity itself as valve configuration.

A list of valve configurations with at least one entry is required. Each valve configuration consists of a network connectivity table as described in Section 2.1 and a section index. The latter denotes the section in which the connectivity table becomes active. Hence, the one required (i.e., the first) entry must have a section index of 0 denoting the initial connectivity.

Note that the section index has to be monotonically increasing throughout the list of valve configurations. See Tables 3.5.1 and 3.5.1.

Dynamic flow rates The volumetric flow rates may vary over time while the valve configuration is active. The rates are assumed to be cubic polynomials,

$$Q = Q_0 + Q_1(t - t_s) + Q_2(t - t_s)^2 + Q_3(t - t_s)^3,$$

where t_s is the beginning of the time section that triggers the valve switch.

Note that the denominator in Eq. 2.2 must always be positive. That is, the flow rate coefficients have to be chosen such that the flow into every connected inlet port is strictly positive at all times.

Solution of the linear systems Each time step in the simulation requires the solution of a nonlinear system Eq. (1.3) (see Sec. 1.4). The nonlinear problem is solved by a Newton iteration, which, in turn, requires the solution of a linear system that essentially consists of the Jacobians of the unit operations and some coupling matrices from Eqs. (2.1) and (2.2).

These linear systems are either solved in parallel or sequentially. The parallel method first solves each unit operation (in parallel) to compute the solution at its outlet. Using these values, the inlets are adjusted and the unit operations are solved again. This is iterated until the system is fully solved.

In contrast, the sequential method first determines an ordering of the unit operations such that each unit only receives inflow from the previous units in the ordering. Such an ordering requires an acyclic graph of unit operations. Finally, the linear system is solved by solving the unit operations in the ordering determined above. Before a unit is solved, its inlet is calculated from the outlets of the previously solved units. This means, the system is solved from system inlets to system outlets.

The parallel method works regardless of the network topology (i.e., cycles in the graph), but requires to solve each unit operation at least twice. The sequential method solvs each unit exactly once, but is restricted to acyclic networks and works best for small graphs. By default, CADET uses a heuristic to select an appropriate solution method. This default can be overridden by a flag (see Table 3.5.1).

The solution method is selected for each valve switch individually. If some network configurations contain cycles, the parallel method is chosen for them regardless of the method used for the other configurations.

2.2 Unit operation models

A short comparison of the most prominent unit operation model features is given in Table 2.2.1.

Unit operation model	Radial diffusion	Particle diffusion	Film diffusion	Multiple- particle	Particle geome- tries
2D General rate model	✓	✓	✓	✓	✓
General rate model	X	✓	✓	✓	✓
Lumped rate model with pores	X	×	✓	✓	✓
Lumped rate model without pores	X	X	X	X	X
Continuous stirred tank	X	X	X	✓	X

Table 2.2.1: Supported features of the different unit operation models

2.2.1 Inlet

A system inlet unit operation is a pseudo unit operation since there is no physical correspondence. The inlet serves as a mass source in the network of unit operations. Consequently, it only possesses an outlet port and no inlet port. Note that an inlet unit operation can provide arbitrary many components and there can be arbitrary many inlet unit operations in a network.

An inlet unit operation provides a feed in which the concentration of each component is given by a profile. The most common profile is a piecewise cubic polynomial, which can both represent discontinuous signals (e.g., pulse or step) and smooth C^2 signals (cubic spline):

$$c_{i}(t) = \sum_{k=1}^{N_{\text{sect}}} \mathbb{1}_{\left[t_{k}, t_{k+1}\right)}(t) \left[a_{k,i} \left(t - t_{k}\right)^{3} + b_{k,i} \left(t - t_{k}\right)^{2} + d_{k,i} \left(t - t_{k}\right) + f_{k,i} \right],$$

where $0 \le t_1 < t_2 < \ldots < t_{N_{\text{sect}}+1} \le T_{\text{sim}}$ is a decomposition of the simulation time interval $[0, T_{\text{sim}}]$ into pieces $[t_k, t_{k+1})$. On each piece, the profile is given by a cubic (fourth order) polynomial shifted to the beginning t_k of the piece.

See Tables 3.5.2 and 3.5.2.

2.2.2 Outlet

A system outlet unit operation is a pseudo unit operation since there is no physical correspondence. The outlet serves as a sink (terminal node) in the network of unit operations. Since any terminal node in the network is a sink (see Section 2.1), outlet unit operations are not strictly necessary. However, in some applications (e.g., SMB) only a certain fraction of a unit operation's output is taken out of the system and the rest is recycled. In this case, outlet unit operations are required in order to avoid unbalanced mass flow in the other unit operations.

Outlets can also be of help if the output of multiple unit operations merges together leaving the network. Instead of manually adding the streams together in a post-processing step, the unit operations can be connected to the same outlet unit.

See Table 3.5.2.

2.2.3 General rate model (GRM)

The general rate model is the most comprehensive model of mass transfer in column liquid chromatography, when only the axial coordinate in the column and the radial coordinate in the beads are considered [Kuč65; Gu95; Gui+06; FG04].

The main assumptions are:

- 1. The cross sections of the column are homogenous in terms of interstitial volume, fluid flow, and distribution of components. Thus, only one spatial coordinate in axial direction is needed and radial transport is neglected in the column bulk volume.
- 2. The bead radii r_p are much smaller than the column radius r_c and the column length L. Therefore, the beads can be seen as continuously distributed inside the column (i.e., at each point there is interstitial and bead volume).

Variable	Domain	Description
\overline{i}	$\{0,\ldots,N_{\mathrm{comp}}-1\}$	Component index
j	$\{0,\ldots,N_{\mathrm{partype}}-1\}$	Particle type index
$m_{j,i}$	$\{0,\ldots,N_{\mathrm{bnd},j,i}-1\}$	Bound state index of i th component in j th particle type
m_{j}	$\left\{0, \dots, \sum_{i=0}^{N_{\text{comp}}-1} N_{\text{bnd}, j, i} - 1\right\}$	Total bound state index in particle type j
t	$[0, T_{\mathrm{end}}]$	Time coordinate
z	[0, L]	Axial coordinate
r	$[r_{c,j}, r_{p,j}]$	Generic bead radial coordinate
$c_i^l(t,z)$	$[0, T_{\mathrm{end}}] \times [0, L]$	Interstitial concentration of the i th component
$c_{j,i}^p(t,z,r)$	$[0, T_{\mathrm{end}}] \times [0, L] \times [r_{c,j}, r_{p,j}]$	Mobile phase concentration of the i th component in
•		the j th particle type
$c_{j,i,m_{j,i}}^s(t,z,r)$	$[0, T_{\mathrm{end}}] \times [0, L] \times [r_{c,j}, r_{p,j}]$	Solid phase concentration of the <i>i</i> th component's
$j_{f,j,i}(t,z)$	$[0,T_{\mathrm{end}}]\times[0,L]$	$m_{j,i}$ th bound state in particles of type j Flux of the i th component through stagnant film into the bead of type j

Table 2.2.2: Variables and unknowns

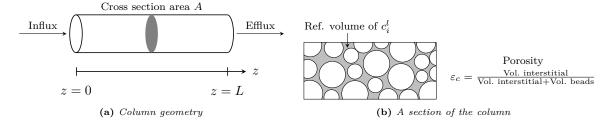


Figure 2.2.1: Column bulk model

The GRM describes transport of solute molecules through the interstitial column volume by convective flow, band broadening caused by axial dispersion, mass transfer resistance through a stagnant film around the beads, pore (and surface) diffusion in the porous beads [MWW96; SS68; Miy07], and adsorption to the inner bead surfaces.

Consider a column of length L > 0 filled with spherical beads of (possibly) multiple types with radius $r_{p,j} \ll L$ (see Fig. 2.2.1), where j is the particle type index. The mass balance in the interstitial column volume is described by

$$\frac{\partial c_i^l}{\partial t} = -u \frac{\partial c_i^l}{\partial z} + D_{\text{ax},i} \frac{\partial^2 c_i^l}{\partial z^2} - \frac{1}{\beta_c} \sum_j d_j \frac{3}{r_{p,j}} k_{f,j,i} \left[c_i^l - c_{j,i}^p(\cdot, \cdot, r_{p,j}) \right] + f_{\text{react},i}^l \left(c^l \right). \tag{2.3}$$

Here, c_i^l : $[0, T_{\text{end}}] \times [0, L] \to \mathbb{R}^{\geq 0}$ denotes the concentration in the interstitial column volume, $c_{j,i}^p$: $[0, T_{\text{end}}] \times [0, L] \times [r_{c,j}, r_{p,j}] \to \mathbb{R}^{\geq 0}$ the liquid phase concentration in the beads, $k_{f,j,i}$ the film diffusion coefficient, $D_{\text{ax},i}$ the dispersion coefficient, u the interstitial velocity, d_j the volume fraction of particle type j, and $\beta_c = \varepsilon_c/(1 - \varepsilon_c)$ the column phase ratio, where ε_c is the column porosity (ratio of interstitial volume to total column volume). If reactions are considered, the term $f_{\text{react},i}^l(c^l)$ represents the net change of concentration c_i due to reactions involving component i.

Danckwerts boundary conditions [Dan53] are applied to inlet and outlet of the column:

$$uc_{\text{in},i}(t) = uc_i^l(t,0) - D_{\text{ax},i}\frac{\partial c_i^l}{\partial z}(t,0) \qquad \forall t > 0,$$
(2.4)

$$\frac{\partial c_i^l}{\partial z}(t, L) = 0 \qquad \forall t > 0. \tag{2.5}$$

Note that the outlet boundary condition Eq. (2.5) is also known as "do nothing" or natural outflow condition.

In the liquid phase of the porous beads (see Fig. 2.2.2) the mass balance is given by

$$\frac{\partial c_{j,i}^{p}}{\partial t} + \frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} \frac{\partial}{\partial t} \sum_{m_{j,i}} c_{j,i,m_{j,i}}^{s} = \underbrace{D_{p,j,i} \left[\frac{\partial^{2}}{\partial r^{2}} + \frac{2}{r} \frac{\partial}{\partial r} \right] c_{j,i}^{p}}_{\text{Pore diffusion}} + \underbrace{\frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} D_{s,j,i} \left[\frac{\partial^{2}}{\partial r^{2}} + \frac{2}{r} \frac{\partial}{\partial r} \right] \sum_{m_{j,i}} c_{j,i,m_{j,i}}^{s}}_{\text{Surface diffusion}} + f_{\text{react},j,i}^{p} \left(c_{j}^{p}, c_{j}^{s} \right) + \frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} f_{\text{react},j,i}^{s} \left(c_{j}^{p}, c_{j}^{s} \right), \tag{2.6}$$

where $c_{j,i,m_{j,i}}^s$: $[0,T_{\mathrm{end}}] \times [0,L] \times [r_{c,j},r_{p,j}] \to \mathbb{R}^{\geq 0}$ denotes the solid phase concentration of the *i*th component's $m_{j,i}$ th bound state in the beads of *j*th type, $D_{p,j,i}$ the effective diffusion coefficient in the beads, $D_{s,j,i}$ the surface diffusion coefficient, $F_{\mathrm{acc},j,i} \in [0,1]$ the pore accessibility factor, and $\varepsilon_{p,j}$ the particle porosity (ratio of pore volume to total bead volume). The inner bead radius $r_{c,j} \in [0,r_{p,j})$ is assumed to be 0 by default, but can be positive in order to account for core-shell particles thave an impermeable core. Reaction terms in liquid and solid phase are collected in $f_{\mathrm{react},j,i}^p(c_j^p, c_j^s)$, respectively.

The GRM is used with both quasi-stationary (Eq. (2.7)) and dynamic (Eq. (2.8)) binding models.

quasi-stationary:
$$0 = f_{\text{ads},j} \left(c_j^p, c_i^s \right), \tag{2.7}$$

dynamic:
$$\frac{\partial c_j^s}{\partial t} = D_{s,j} \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right] c_j^s + f_{\text{ads},j} \left(c_j^p, c_j^s \right) + f_{\text{react},j}^s \left(c_j^p, c_j^s \right). \tag{2.8}$$

Note that c_j^p and c_j^s denote the vector of all $c_{j,i}^p$ and $c_{j,i,m_{j,i}}^s$, respectively.

The boundary conditions of the bead model the film diffusion and are given for all $t \in (0, \infty)$ and $z \in [0, L]$ by

$$k_{f,j,i} \left[c_i^l - c_{j,i}^p(\cdot, \cdot, r_{p,j}) \right] = F_{\text{acc},j,i} \varepsilon_{p,j} D_{p,j,i} \frac{\partial c_{j,i}^p}{\partial r} (\cdot, \cdot, r_{p,j}) + (1 - \varepsilon_{p,j}) D_{s,j,i} \sum_{m_{j,i}} \frac{\partial c_{j,i,m_{j,i}}^s}{\partial r} (\cdot, \cdot, r_{p,j}), \quad (2.9)$$

$$\frac{\partial c_{j,i}^p}{\partial r} (\cdot, \cdot, r_{c,j}) = 0. \quad (2.10)$$

By default, the following initial conditions are applied for all $z \in [0, L]$ and $r \in [r_{c,j}, r_{p,j}]$:

$$c_i^l(0,z) = 0,$$
 $c_{j,i}^p(0,z,r) = 0,$ $c_{j,i,m_{j,i}}^s(0,z,r) = 0.$ (2.11)

See Table 3.5.2.

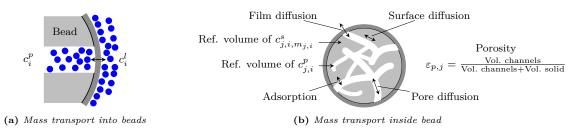


Figure 2.2.2: Column bead model

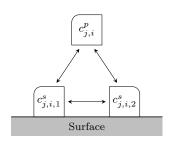


Figure 2.2.3: Binding with multiple bound states

Particle geometry In the model above, spherical particles are considered. Other supported particle forms are cylinders and slabs. For cylinders, it is assumed that molecules can only enter through the lateral surface (i.e., the caps are sealed). Slabs are assumed to have two large sides such that molecules enter through the two large faces (i.e., the remaining four small faces are sealed).

All particle forms support core-shell beads that have an impermeable core. The particles are characterized by their (outer) "radius" $r_{p,j}$ and their (inner) core "radius" $r_{c,j} \in [0, r_{p,j})$. See Fig. 2.2.4.

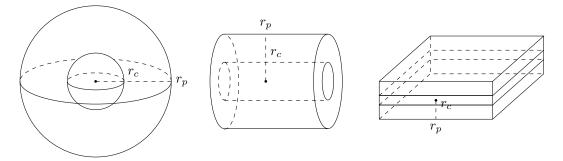


Figure 2.2.4: Particle geometries

For cylinders, the factor $3/r_{p,j}$ in (2.3) changes to $2/r_{p,j}$ and the diffusion operator in (2.6) and (2.8) changes as

$$\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right] \quad \to \quad \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right].$$

For slabs, the factor $3/r_{p,j}$ in (2.3) changes to $1/r_{p,j}$ and the diffusion operator in (2.6) and (2.8) changes

$$\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right] \quad \to \quad \frac{\partial^2}{\partial r^2}.$$

Multiple particle types A particle type has its own set of mass transfer and geometry parameters $\varepsilon_{p,j}$, $D_{p,j}$, $D_{s,j}$, etc. (see Eq. (2.6)) and its own binding model $f_{\rm ads}$ (including a possibly differing number of bound states). This allows, for example, modeling of particle size distributions or potential applications with differently functionalized beads (e.g., immobilized enzymes).

The distribution of the particle types is governed by their volume fractions d_j in Eq. (2.3). The volume fractions have to sum to 1:

$$\sum_{j=0}^{N_{\text{partype}}-1} d_j = 1.$$

The particle type volume fractions can be spatially constant throughout the column, or depend on the position inside the column bulk volume. In the latter case, the user can specify a set of volume fractions for each discretized finite volume cell. This allows, for example, the placement of smaller particles near the frits.

Size exclusion chromatography The general rate model can be used to simulate size exclusion chromatography (SEC) [Gu95]. The particle porosity $\varepsilon_{p,j}$ on the mobile phase side of the transport equations is replaced by a component-dependent accessible porosity

$$\varepsilon_{p,j,i} = F_{\text{acc},j,i}\varepsilon_{p,j},$$
(2.12)

where the pore accessibility factor $F_{\text{acc},j,i}$ ranges in (0,1].

Small molecules that can enter any pore have $F_{\text{acc},j,i} = 1$, whereas larger molecules that can enter some, but not small pores, have values $0 < F_{\text{acc},j,i} < 1$. The other extreme is given by molecules so large that they cannot enter any pore and, consequently, $F_{\text{acc},j,i} = 0$. Note that $F_{\text{acc},j,i} = 0$ is not allowed in a simulation, which can be circumvented by setting $k_{f,j,i} = 0$.

By default, $F_{\text{acc},j,i} = 1$ for all components i and all particle types j, which disables size exclusion chromatography. In order to simulate pure SEC, binding is disabled by setting $N_{\text{bnd},i} = 0$ for all components i and applying no binding model. If adsorption is present, it is important to note that any saturation capacity (e.g., q_{max} of Langmuir-type binding models) is subject to the full pore volume fraction $\varepsilon_{p,j}$.

Note that multiple particle types can also be used to aid in modeling size exclusion effects, see Section 2.2.3.

Specification of flow rate / velocity and direction Since volumetric flow rates are specified for each network connection, the unit operation can infer its interstitial velocity via

$$u = u_{\rm int} = \frac{F_{\rm in}}{A\varepsilon_c},$$

where $F_{\rm in}$ denotes the volumetric flow rate and A the cross section area. Note that without the bulk porosity ε_c , the superficial velocity would be obtained.

The direction of flow inside the unit operation is governed by the sign of the interstitial velocity u. A positive sign results in (standard) forward flow, whereas a negative sign reverses the flow direction. Note that in case of reversed flow, the chromatogram is returned at the unit operation's *inlet*, which may not be returned from simulation by default.

The final behavior is controlled by the interplay of cross section area and interstitial velocity:

- 1. If cross section area A is given and u is not, u is inferred from the volumetric flow rate.
- 2. If u is given and A is not, the volumetric flow rate is ignored and the provided interstitial velocity is used.
- 3. If both cross section area A and interstitial velocity u are given, the magnitude of the actual interstitial velocity u is inferred from the volumetric flow rate and the flow direction is given by the sign of the provided u.

2.2.4 Lumped rate model with pores (LRMP)

The lumped rate model with pores [Gui+06; FG04] deviates from the general rate model (see Section 2.2.3) by neglecting pore diffusion. The particle phase c_i^p is still there, but no mass transfer happens except

for binding and film diffusion. Hence, the model equations are given by

$$\frac{\partial c_i^l}{\partial t} = -u \frac{\partial c_i^l}{\partial z} + D_{\text{ax},i} \frac{\partial^2 c_i^l}{\partial z^2} - \frac{1}{\beta_c} \sum_j d_j \frac{3}{r_{p,j}} k_{f,j,i} \left[c_i^l - c_{j,i}^p \right] + f_{\text{react},i}^l \left(c^l \right), \tag{2.13}$$

$$\frac{\partial c_{j,i}^{p}}{\partial t} + \frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} \frac{\partial}{\partial t} \sum_{m_{j,i}} c_{j,i,m_{j,i}}^{s} = \frac{3}{F_{\text{acc},j,i}\varepsilon_{p,j}r_{p,j}} k_{f,j,i} \left[c_{i}^{l} - c_{j,i}^{p} \right] + f_{\text{react},j,i}^{p} \left(c_{j}^{p}, c_{j}^{s} \right) + \frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} f_{\text{react},j,i}^{s} \left(c_{j}^{p}, c_{j}^{s} \right)$$
(2.14)

with the same meanings of variables and parameters as in the general rate model. The equations are complemented by Danckwerts boundary conditions [Dan53]

$$uc_{\text{in},i}(t) = uc_i^l(t,0) - D_{\text{ax},i} \frac{\partial c_i^l}{\partial z}(t,0) \qquad \forall t > 0,$$

$$\frac{\partial c_i^l}{\partial z}(t,L) = 0 \qquad \forall t > 0.$$

As for the general rate model, both quasi-stationary and dynamic binding models are supported:

quasi-stationary:
$$0 = f_{\mathrm{ads},j}\left(c_{j}^{p},c_{j}^{s}\right),$$

$$\mathrm{dynamic:} \ \frac{\partial c_{j}^{s}}{\partial t} = f_{\mathrm{ads},j}\left(c_{j}^{p},c_{j}^{s}\right) + f_{\mathrm{react},j}^{s}\left(c_{j}^{p},c_{j}^{s}\right).$$

By default, the following initial conditions are applied for all $z \in [0, L]$:

$$c_i^l(0,z) = 0,$$
 $c_{i,i}^p(0,z) = 0,$ $c_{i,i,m_{i,i}}^s(0,z) = 0.$ (2.15)

Multiple particle types and all particle geometries are supported (see Section 2.2.3). This model can also be used to simulate size exclusion chromatography (see Section 2.2.3). For the specification of flow rate and direction, the same holds as for the general rate model (see Section 2.2.3). See Table 3.5.2.

2.2.5 Lumped rate model without pores (LRM)

The lumped rate model without pores [Gui+06; FG04] deviates from the lumped rate model with pores (see Section 2.2.4) by neglecting pores completely. The particle phase c^p is removed and the porosity ε_t is taken as total porosity

$$\varepsilon_t = \varepsilon_c + (1 - \varepsilon_c) \,\varepsilon_p. \tag{2.16}$$

The phase ratio is denoted by $\beta_t = \varepsilon_t/(1-\varepsilon_t)$ accordingly. The model equations are given by

$$\frac{\partial c_i^l}{\partial t} + \frac{1}{\beta_t} \frac{\partial}{\partial t} \sum_{m} c_{i,m_i}^s = -u \frac{\partial c_i^l}{\partial z} + D_{\text{ax},i} \frac{\partial^2 c_i^l}{\partial z^2} + f_{\text{react},i}^l \left(c^l, c^s \right) + \frac{1}{\beta_t} f_{\text{react},i}^s \left(c^l, c^s \right), \tag{2.17}$$

where $\beta_t = \varepsilon_t/(1-\varepsilon_t)$ denotes the (total) phase ratio. The equations are complemented by Danckwerts boundary conditions [Dan53]

$$uc_{\text{in},i}(t) = uc_i^l(t,0) - D_{\text{ax},i} \frac{\partial c_i^l}{\partial z}(t,0) \qquad \forall t > 0,$$

$$\frac{\partial c_i^l}{\partial z}(t,L) = 0 \qquad \forall t > 0.$$

Both quasi-stationary and dynamic binding models are supported:

quasi-stationary:
$$0 = f_{\text{ads}}\left(c^{l}, c^{s}\right)$$
,
dynamic: $\frac{\partial q}{\partial t} = f_{\text{ads}}\left(c^{l}, c^{s}\right) + f_{\text{react}}^{s}\left(c^{l}, c^{s}\right)$.

By default, the following initial conditions are applied for all $z \in [0, L]$:

$$c_i^l(0,z) = 0,$$
 $c_{i,m_i}^s(0,z) = 0.$ (2.18)

Note that by setting $\varepsilon_t = 1$, removing all bound states by setting $N_{\mathrm{bnd},i} = 0$ for all components i, and applying no binding model, a dispersive plug flow reactor (DPFR) is obtained.

For the specification of flow rate and direction, the same holds as for the general rate model (see Section 2.2.3). See Table 3.5.2.

2.2.6 Two Dimensional General rate model (GRM2D)

The general rate model as introduced in Section 2.2.3 assumes homogeneity in the cross sections of the column. This allows to consider transport along the axial dimension only. However, due to packing irregularity and inhomogeneous flow at the inlet (i.e., frits), this assumption may be a crude approximation. This model can be improved by introducing a radial coordinate $\rho \in [0, R]$, where R is the column radius, in the interstitial volume Eq. (2.3):

$$\varepsilon_{c} \frac{\partial c_{i}^{l}}{\partial t} = -\varepsilon_{c} u \frac{\partial c_{i}^{l}}{\partial z} + \varepsilon_{c} D_{\text{ax},i} \frac{\partial^{2} c_{i}^{l}}{\partial z^{2}} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho D_{\text{rad},i} \frac{\partial}{\partial \rho} \left(\varepsilon_{c} c_{i}^{l} \right) \right) \\
- (1 - \varepsilon_{c}) \sum_{i} d_{j} \frac{3k_{f,j,i}}{r_{p,j}} \left[c_{i}^{l} - c_{j,i}^{p}(\cdot, \cdot, \cdot, r_{p,j}) \right] + \varepsilon_{c} f_{\text{react},i}^{l} \left(c^{l} \right).$$
(2.19)

Here, $c_i^l \colon [0, T_{\mathrm{end}}] \times [0, L] \times [0, R] \to \mathbb{R}^{\geq 0}$, $c_{j,i}^p \colon [0, T_{\mathrm{end}}] \times [0, L] \times [0, R] \times [r_{c,j}, r_{p,j}] \to \mathbb{R}^{\geq 0}$, and $c_{j,i,m_{j,i}}^s \colon [0, T_{\mathrm{end}}] \times [0, L] \times [0, R] \times [r_{c,j}, r_{p,j}] \to \mathbb{R}^{\geq 0}$ depend on ρ . Additionally, the porosity ε_c , axial dispersion coefficient $D_{\mathrm{ax},i}$, radial dispersion coefficient $D_{\mathrm{rad},i}$, and interstitial velocity u may depend on ρ .

The dependence of the parameters on ρ is not arbitrary. For simplicity, it is assumed that the parameters are piecewise constant, that is, the range [0,R] is divided into disjoint zones in which all parameters are constant. These zones are used for radial discretization and can be supplied to the simulator. Continuous dependence of the parameters can be realized by piecewise constant approximation.

The Danckwerts boundary conditions at the column in- and outlet, Eq. (2.4) and (2.5), are modified to account for the radial coordinate:

$$u(\rho)c_{\mathrm{in},i}(t,\rho) = u(\rho)c_i^l(t,0,\rho) - D_{\mathrm{ax},i}(\rho)\frac{\partial c_i^l}{\partial z}(t,0,\rho) \qquad \forall t > 0, \rho \in (0,R),$$
(2.20a)

$$\frac{\partial c_i^l}{\partial z}(t, L, \rho) = 0 \qquad \forall t > 0, \rho \in (0, R). \tag{2.20b}$$

Conditions for the radial direction are added:

$$\frac{\partial c_i^l}{\partial \rho}(\cdot, \cdot, 0) = 0, \tag{2.21a}$$

$$\frac{\partial c_i^l}{\partial \rho}(\cdot, \cdot, R) = 0. \tag{2.21b}$$

While the inner condition Eq.(2.21a) represents symmetry at the column center, the outer condition Eq. (2.21b) is a no-flux condition.

Using the inlet boundary condition Eq. (2.20a), each radial zone is equipped with its own inlet and outlet port. That is, this unit operation has as many inlet and outlet ports as it has radial zones (parameter NRAD in the discretization group). This allows each radial zone to have its own inlet profile.

which enables modeling of flow distribution in the frits by sending the feed through varying hold-up volumes before injecting it into a radial zone.

See Table 3.5.2.

Specification of flow rate / velocity and direction Since the column radius R and the zones (ρ_k, ρ_{k+1}) , $k = 0, \ldots, N_{\text{rad}} - 1$, are known, the interstitial velocities u_k are inferred from the volumetric flow rates via

$$u_k = u_{\text{int},k} = \frac{F_{\text{in},k}}{\pi \left(\rho_{k+1}^2 - \rho_k^2\right) \varepsilon_{c,k}},$$

where $F_{\text{in},k}$ denotes the volumetric flow rate into zone k.

The direction of flow inside the radial zone of the unit operation is governed by the sign of the interstitial velocity u_k . A positive sign results in (standard) forward flow, whereas a negative sign reverses the flow direction. Note that in case of reversed flow, the chromatogram is returned at the unit operation's *inlet* port, which may not be returned from simulation by default.

Note that, contrary to the standard general rate model as presented in Section 2.2.3, the interstitial flow rate is always given by the volumetric flow rate. The velocity parameter only determines the flow direction.

2.2.7 Continuous stirred tank reactor model (CSTR)

The continuous stirred tank reactor model is a basic building block in unit operation networks and often used to model holdup volume. When combined with a binding model, it can be used to model batch uptake experiments.

Assuming that the fluid inside the tank is well-mixed and that the volume can vary, the governing equations are given by

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\left[c_i + \frac{1-\varepsilon}{\varepsilon} \sum_j d_j \sum_{m_{j,i}} c_{j,i,m_{j,i}}^s \right] V \right) = F_{\mathrm{in}} c_{\mathrm{in},i} - F_{\mathrm{out}} c_i + V f_{\mathrm{react},i}^l \left(c \right) + V \frac{1-\varepsilon}{\varepsilon} \sum_j d_j f_{\mathrm{react},j,i}^s \left(c, c_j^s \right),$$

which balances the mass, the binding equation

See Table 3.5.2.

quasi-stationary:
$$0 = f_{\mathrm{ads},j}\left(c,c_{j}^{s}\right),$$

$$\mathrm{dynamic:} \ \frac{\partial c_{j}^{s}}{\partial t} = f_{\mathrm{ads},j}\left(c,c_{j}^{s}\right) + f_{\mathrm{react},j}^{s}\left(c,c_{j}^{s}\right),$$

depending on whether quasi-stationary or dynamic binding is used, and the evolution of volume

$$\frac{\mathrm{d}V}{\mathrm{d}t} = F_{\mathrm{in}} - F_{\mathrm{out}} - F_{\mathrm{filter}}.$$

The porosity ε denotes the ratio of liquid phase volume to total tank volume. Thus, setting $\varepsilon = 1$, removing all bound states by setting $N_{\mathrm{bnd},j,i} = 0$ for all components i and particle types j, and applying no binding model results in a simple tank. The additional parameter F_{filter} , which denotes the flow rate of pure liquid (without any components) out of the tank, can be used to model a filtering unit.

Note that it is the user's duty to make sure that the volume of the CSTR does not fall below 0 m³. If it does, the simulation may fail to run or may produce unreasonable (e.g., unphysical) results.

2.3 Binding models

The following binding models are presented in dynamic binding mode. By replacing all occurrences of dq/dt with 0, quasi-stationary (rapid-equlibrium) binding mode is achieved. In quasi-stationary binding it is assumed that ad- and desorption take place on a much faster time scale than the other transport processes such that bead liquid phase $c_{p,i}$ (or bulk liquid phase c_i for certain unit operation models) are always in equilibrium with the solid phase q_i .

Equilibrium constants For the quasi-stationary binding mode, adsorption and desorption rate are no longer separate entities. Instead, the quotient $k_{\rm eq} = k_a/k_d$ of adsorption and desorption coefficient is the relevant parameter as shown for the linear binding model (see Section 2.3.1):

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} - k_{d,i}q_i \qquad \Rightarrow 0 = k_{a,i}c_{p,i} - k_{d,i}q_i \qquad \Leftrightarrow q_i = \frac{k_{a,i}}{k_{d,i}}c_{p,i} = k_{\mathrm{eq},i}c_{p,i}.$$

The equilibrium constant $k_{eq,i}$ is used in CADET by setting $k_{d,i} = 1$ and $k_{a,i} = k_{eq,i}$.

Correlation of ad- and desorption rates Note that adsorption rate $k_{a,i}$ and desorption rate $k_{d,i}$ are linearly correlated in both binding modes due to the form of the equilibrium constant k_{eq} :

$$k_{a,i} = k_{eq} k_{d,i}$$
.

This correlation can potentially degrade performance of some optimization algorithms. While in quasistationary binding mode this is prevented by using the technique above, a dynamic binding model has to be reparameterized in order to decouple parameters:

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} - k_{d,i}q_i = k_{d,i}\left[k_{\mathrm{eq},i}c_{p,i} - q_i\right] = k_{a,i}\left[c_{p,i} - \frac{1}{k_{\mathrm{eq},i}}q_i\right].$$

This can be achieved by a (nonlinear) parameter transform

$$F\left(k_{\mathrm{eq},i},k_{d,i}\right) = \begin{pmatrix} k_{\mathrm{eq},i}k_{d,i} \\ k_{d,i} \end{pmatrix} \text{ with Jacobian } J_F\left(k_{\mathrm{eq},i},k_{d,i}\right) = \begin{pmatrix} k_{d,i} & k_{\mathrm{eq},i} \\ 0 & 1 \end{pmatrix}.$$

Dependence on external function A binding model may depend on an external function or profile $T: [0, T_{\text{end}}] \times [0, L] \to \mathbb{R}$, where L denotes the physical length of the unit operation, or $T: [0, T_{\text{end}}] \to \mathbb{R}$ if the unit operation model has no axial length. By using an external profile, it is possible to account for effects that are not directly modeled in CADET (e.g., temperature). The dependence of each parameter is modeled by a polynomial of third degree. For example, the adsorption rate k_a is really given by

$$k_a(T) = k_{a,3}T^3 + k_{a,2}T^2 + k_{a,1}T + k_{a,0}.$$

While $k_{a,0}$ is set by the original parameter XXX_KA of the file format (XXX being a placeholder for the binding model), the parameters $k_{a,3}$, $k_{a,2}$, and $k_{a,1}$ are given by XXX_KA_TTT, XXX_KA_TT, and XXX_KA_T, respectively. The identifier of the externally dependent binding model is constructed from the original identifier by prepending EXT_ (e.g., MULTI_COMPONENT_LANGMUIR) is changed into EXT_MULTI_COMPONENT_LANGMUIR). This pattern applies to all parameters and supporting binding models (see Table 2.3.1). Note that the parameter units have to be adapted to the unit of the external profile by dividing with an appropriate power.

Each parameter of the externally dependent binding model can depend on a different external source. The 0-based indices of the external source for each parameter is given in the dataset EXTFUN. By assigning only one index to EXTFUN, all parameters use the same source. The ordering of the parameters in EXTFUN is given by the ordering in the file format specification in Section 3.5.5.

Binding model feature matrix A short comparison of the most prominent binding model features is given in Table 2.3.1. The implemented binding models can be divided into two main classes: Single-state and multi-state binding. While single-state models only have one bound state per component (or less),

multi-state models provide multiple (possibly different) bound states for each component, which may correspond to different binding orientations or binding site types. The models also differ in whether a mobile phase modifier (e.g., salt) is supported to modulate the binding behavior.

Binding model	Competitive	Mobile phase modifier	External function	Multi-state
Linear	Х	Х	✓	Х
Multi component Langmuir	✓	X	✓	×
Multi component Anti-Langmuir	✓	X	✓	×
Steric mass action	✓	✓	✓	×
Self association	✓	✓	✓	×
Mobile phase modulator Langmuir	✓	✓	✓	×
Extended Mobile phase modulator Langmuir	✓	✓	✓	×
Kumar-Langmuir	✓	✓	✓	×
Saska	X	X	✓	×
Multi component Bi-Langmuir	✓	X	✓	✓
Multi component spreading	✓	X	✓	✓
Multi-state steric mass action	✓	✓	✓	✓
Simplified multi-state steric mass action	✓	✓	×	✓
Bi steric mass action	✓	✓	✓	✓

Table 2.3.1: Supported features of the different binding models

Reference concentrations Some binding models use reference concentrations $c_{\rm ref}$ and $q_{\rm ref}$ of the mobile phase modulator (e.g., salt) in the particle liquid and solid phase, respectively. The reference values are mainly used for normalizing adsorption and desorption rates, but also for other parameters that appear with those concentrations. They amount to a simple parameter transformation that is exemplified at one equation of the steric mass action binding model

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i}\bar{q}_0^{\nu_i} - k_{d,i}q_ic_{p,0}^{\nu_i},$$

where $c_{p,0}$ denotes the mobile phase salt concentration and

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}} - 1} (\nu_j + \sigma_j) q_j$$

is the number of available binding sites which is related to the number of bound salt ions. Using the parameter transformation

$$k_{a,i} = \tilde{k}_{a,i} q_{\text{ref}}^{-\nu_i},$$

$$k_{d,i} = \tilde{k}_{d,i} c_{\text{ref}}^{-\nu_i},$$

we obtain the modified model equation

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = \tilde{k}_{a,i} c_{p,i} \left(\frac{\bar{q}_0}{q_{\mathrm{ref}}}\right)^{\nu_i} - \tilde{k}_{d,i} q_i \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{\nu_i}.$$

This transformation serves as a (partial) nondimensionalization of the adsorption and desorption rates and, by properly choosing the reference concentrations c_{ref} and q_{ref} , may improve the optimizer performance.

Recommended choices for c_{ref} are the average or maximum inlet concentration of the mobile phase modifier c_0 , and for q_{ref} the ionic capacity Λ . Note that setting the reference concentrations to 1.0 each results in the original binding model.

2.3.1 Linear

A linear binding model, which is often employed for low concentrations or in analytic settings [Gui+06].

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} - k_{d,i}q_i \qquad i = 0,\dots, N_{\mathrm{comp}} - 1.$$

See Table 3.5.5.

2.3.2 Multi Component Langmuir

The Langmuir binding model includes a saturation term and takes into account the capacity of the resin [Lan16; Gui+06]. All components compete for the same binding sites.

$$\frac{dq_i}{dt} = k_{a,i} c_{p,i} q_{\max,i} \left(1 - \sum_{j=0}^{N_{\text{comp}} - 1} \frac{q_j}{q_{\max,j}} \right) - k_{d,i} q_i \qquad i = 0, \dots, N_{\text{comp}} - 1.$$

See Table 3.5.5.

2.3.3 Multi Component Anti-Langmuir

The Anti-Langmuir (or generalized Langmuir) binding model extends the Langmuir model (see Section 2.3.2). The factor $p_j \in \{-1,1\}$ determines the shape of the isotherm. For $p_j = 1$ (standard Langmuir) the chromatograms have sharp fronts and a dispersed tail (isotherm is concave). In case of the Anti-Langmuir $(p_j = -1)$ it is the other way around (isotherm is convex).

$$\frac{dq_i}{dt} = k_{a,i} c_{p,i} q_{\max,i} \left(1 - \sum_{j=0}^{N_{\text{comp}} - 1} p_j \frac{q_j}{q_{\max,j}} \right) - k_{d,i} q_i \qquad i = 0, \dots, N_{\text{comp}} - 1.$$

See Table 3.5.5.

2.3.4 Steric Mass Action

The steric mass action model takes charges of the molecules into account [BC92] and is, thus, often used in ion-exchange chromatography. Each component has a characteristic charge ν that determines the number of available binding sites Λ (ionic capacity) used up by a molecule. Due to the molecule's shape, some additional binding sites (steric shielding factor σ) may be shielded from other molecules and are not available for binding.

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} \left(\frac{\bar{q}_0}{q_{\mathrm{ref}}}\right)^{\nu_i} - k_{d,i} q_i \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{\nu_i} \qquad i = 1,\dots, N_{\mathrm{comp}} - 1,$$

where $c_{p,0}$ and q_0 denote the salt concentrations in the liquid and solid phase of the beads, respectively. The number of free binding sites

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} (\nu_j + \sigma_j) q_j = q_0 - \sum_{j=1}^{N_{\text{comp}}-1} \sigma_j q_j$$

is calculated from the number of bound counter ions q_0 by taking steric shielding into account. In turn, the number of bound counter ions q_0 (electro-neutrality condition) is given by

$$q_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} \nu_j q_j,$$

which also compensates for the missing equation for $\frac{dq_0}{dt}$. See Table 3.5.5.

The concept of reference concentrations (c_{ref} and q_{ref}) is explained in the respective paragraph in Section 2.3.

2.3.5 Self Association

This binding model is similar to the steric mass action model (see Section 2.3.4) but is also capable of describing dimerization [Mol08; Wes+12]. The dimerization, which is the immobilization of protein at some already bound protein, is also termed "self-association". It is modeled by adding a quadratic (in $c_{p,i}$) term to the adsorption part of the equation.

$$\frac{dq_i}{dt} = c_{p,i} \left(\frac{\bar{q}_0}{q_{\text{ref}}}\right)^{\nu_i} [k_{a,i,1} + k_{a,i,2}c_{p,i}] - k_{d,i} q_i \left(\frac{c_{p,0}}{c_{\text{ref}}}\right)^{\nu_i} \qquad i = 1, \dots, N_{\text{comp}} - 1,$$

$$q_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}} - 1} \nu_j q_j,$$

where the number of available binding sites is given by

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} (\nu_j + \sigma_j) q_j = q_0 - \sum_{j=1}^{N_{\text{comp}}-1} \sigma_j q_j.$$

See Table 3.5.5.

The concept of reference concentrations (c_{ref} and q_{ref}) is explained in the respective paragraph in Section 2.3.

2.3.6 Mobile Phase Modulator Langmuir

This model is a modified Langmuir model (see Section 2.3.2) which can be used to describe hydrophobic interaction chromatography [MEH89; Kar+04]. A modulator component (termed "salt", $c_{p,0}$ and q_0) influences ad- and desorption processes:

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}e^{\gamma_i c_{p,0}} c_{p,i} \, q_{\max,i} \left(1 - \sum_{j=1}^{N_{\text{comp}}-1} \frac{q_j}{q_{\max,j}} \right) - k_{d,i} \, c_{p,0}^{\beta_i} \, q_i \qquad i = 1, \dots, N_{\text{comp}} - 1.$$

where $c_{p,0}$ and q_0 denote the salt concentrations in the liquid and solid phase of the beads respectively. Salt is considered to be inert, therefore either

$$\frac{\mathrm{d}q_0}{\mathrm{d}t} = 0$$

is used if salt has one bound state, or salt can be used without a bound state. The parameter γ describes the hydrophobicity and β the ion-exchange characteristics. See Table 3.5.5.

2.3.7 Extended Mobile Phase Modulator Langmuir

This model is an extension of the mobile phase modulator Langmuir model (see Section 2.3.6), which allows linear binding of some selected components. A modifier component $c_{p,\text{mod}}$ is selected and the remaining components are divided into the index sets \mathcal{I}_{lin} and $\mathcal{I}_{\text{lang}}$.

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i} e^{\gamma_i c_{p,\mathrm{mod}}} c_{p,i} \, q_{\mathrm{max},i} \left(1 - \sum_{j=1}^{N_{\mathrm{comp}}-1} \frac{q_j}{q_{\mathrm{max},j}} \right) - k_{d,i} \, c_{p,\mathrm{mod}}^{\beta_i} \, q_i \qquad i \in \mathcal{I}_{\mathrm{lang}},$$

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i} c_{p,i} - k_{d,i} \, q_i \qquad i \in \mathcal{I}_{\mathrm{lin}}.$$

The modifier component is considered to be inert, therefore either

$$\frac{\mathrm{d}q_{\mathrm{mod}}}{\mathrm{d}t} = 0$$

is used if the modifier component has a bound state, or it can be used without a bound state.

The model can also be used without a modifier component. In this case, the equations are given by

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} \, q_{\max,i} \left(1 - \sum_{j=1}^{N_{\text{comp}}-1} \frac{q_j}{q_{\max,j}} \right) - k_{d,i} \, q_i \qquad \qquad i \in \mathcal{I}_{\text{lang}}$$

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} - k_{d,i} \, q_i \qquad \qquad i \in \mathcal{I}_{\text{lin}}.$$

See Table 3.5.5.

2.3.8 Kumar-Langmuir

This extension of the Langmuir isotherm (see Section 2.3.2) developed in [Kum+15] was used to model charge variants of monoclonal antibodies in ion-exchange chromatography. A non-binding salt component $c_{p,0}$ is added to modulate the ad- and desorption process.

$$\frac{dq_i}{dt} = k_{a,i} \exp\left(\frac{k_{\text{act},i}}{T}\right) c_{p,i} q_{\text{max},i} \left(1 - \sum_{j=1}^{N_{\text{comp}}-1} \frac{q_j}{q_{\text{max},j}}\right) - k_{d,i} \left(c_{p,0}\right)^{\nu_i} q_i \qquad i = 1, \dots, N_{\text{comp}} - 1$$

In this model, the true adsorption rate $k_{a,i,\text{true}}$ is governed by the Arrhenius law in order to take temperature into account

$$k_{a,i,\text{true}} = k_{a,i} \exp\left(\frac{k_{\text{act},i}}{T}\right).$$

Here, $k_{a,i}$ is the frequency or pre-exponential factor, $k_{\text{act},i} = E/R$ is the activation temperature (E denotes the activation energy and R the Boltzmann gas constant), and T is the temperature. The characteristic charge ν of the protein is taken into account by the power law. See Table 3.5.5.

2.3.9 Saska

In this binding model an additional quadratic term is added to the linear model [Sas+92]. The quadratic term allows to take interactions of liquid phase components into account.

$$\frac{dq_i}{dt} = H_i c_{p,i} + \sum_{j=0}^{N_{\text{comp}}-1} k_{ij} c_{p,i} c_{p,j} - q_i \qquad i = 0, \dots, N_{\text{comp}} - 1$$

See Table 3.5.5.

2.3.10 Multi Component Bi-Langmuir

The multi component Bi-Langmuir model [Gui+06] adds M-1 additional types of binding sites $q_{i,j}$ $(0 \le j \le M-1)$ to the Langmuir model (see Section 2.3.2) without allowing an exchange between the different sites $q_{i,j}$ and $q_{i,k}$ $(k \ne j)$. Therefore, there are no competitivity effects between the different types of binding sites and they have independent capacities.

$$\frac{\mathrm{d}q_{i,j}}{\mathrm{d}t} = k_{a,i}^{(j)} c_{p,i} q_{\max,i}^{(j)} \left(1 - \sum_{k=0}^{N_{\mathrm{comp}}-1} \frac{q_{k,j}}{q_{\max,k}^{(j)}} \right) - k_{d,i}^{(j)} q_{i,j} \qquad i = 0, \dots, N_{\mathrm{comp}} - 1, \ j = 0, \dots, M - 1.$$

Note that all binding components must have exactly the same number of binding site types $M \ge 1$. See the Langmuir isotherm in Section 2.3.2 and Table 3.5.5.

Originally, the Bi-Langmuir model is limited to two different binding site types. Here, the model has been extended to arbitrary many binding site types.

2.3.11 Multi Component Spreading

The multi component spreading model adds a second bound state $q_{i,2}$ to the Langmuir model (see Section 2.3.2) and allows the exchange between the two bound states $q_{i,1}$ and $q_{i,2}$. In the spreading model a second state of the bound molecule (e.g., a different orientation on the surface or a different folding state) is added. The exchange of molecules between the two states is allowed and, since the molecules can potentially bind in both states at the same binding site, competitivity effects are present. This is different to the Bi-Langmuir model in which another type of binding sites is added and no exchange between the different bound states is considered (see Section 2.3.10). For all components $i = 0, \ldots, N_{\text{comp}} - 1$ the equations are given by

$$\frac{\mathrm{d}q_{i,1}}{\mathrm{d}t} = \left(k_a^A c_{p,i} - k_{12}q_{i,1}\right) q_{\max,i}^A \left(1 - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^A}{q_{\max,j}^A} - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^B}{q_{\max,j}^B}\right) - k_d^A q_{i,1} + k_{21}q_{i,2},$$

$$\frac{\mathrm{d}q_{i,2}}{\mathrm{d}t} = \left(k_a^B c_{p,i} + k_{12}q_{i,1}\right) q_{\max,i}^A \left(1 - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^A}{q_{\max,j}^A} - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^B}{q_{\max,j}^B}\right) - \left(k_d^B + k_{21}\right) q_{i,2}.$$

See Table 3.5.5.

2.3.12 Multi-State Steric Mass Action

The multi-state steric mass action model adds $M_i - 1$ additional bound states $q_{i,j}$ $(j = 0, ..., M_i - 1)$ for each component i to the steric mass action model (see Section 2.3.4) and allows the exchange between the different bound states $q_{i,0}, ..., q_{i,M-1}$ of each component. In the multi-state SMA model a variable number of states of the bound molecule (e.g., different orientations on the surface, binding strength of tentacle adsorbers) is added which are more and more strongly bound, i.e.,

$$\nu_{i,j} \le \nu_{i,j+1}$$
 $i = 1, \dots, N_{\text{comp}} - 1, \quad j = 0, \dots, M_i - 1.$

The exchange between the different states of each component is allowed and, since the molecules can potentially bind in all states at the same binding site, competitive effects are present.

$$\begin{split} \frac{\mathrm{d}q_{i,j}}{\mathrm{d}t} &= \ k_{a,i}^{(j)}c_{p,i}\left(\frac{\bar{q}_{0}}{q_{\mathrm{ref}}}\right)^{\nu_{i,j}} - k_{d,i}^{(j)} \ q_{i,j} \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{\nu_{i,j}} \\ &- \sum_{\ell=0}^{j-1} \underbrace{k_{j\ell}^{(i)} \ q_{i,j} \ \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{(\nu_{i,j}-\nu_{i,\ell})}}_{\text{to weak state}} - \sum_{\ell=j+1}^{M_{i}-1} \underbrace{k_{j\ell}^{(i)} \ q_{i,j} \ \left(\frac{\bar{q}_{0}}{q_{\mathrm{ref}}}\right)^{(\nu_{i,\ell}-\nu_{i,j})}}_{\text{to strong state}} \\ &+ \sum_{\ell=0}^{j-1} \underbrace{k_{\ell j}^{(i)} \ q_{i,\ell} \ \left(\frac{\bar{q}_{0}}{q_{\mathrm{ref}}}\right)^{(\nu_{i,j}-\nu_{i,\ell})}}_{\text{from weak state}} + \sum_{\ell=j+1}^{M_{i}-1} \underbrace{k_{\ell j}^{(i)} \ q_{i,\ell} \ \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{(\nu_{i,\ell}-\nu_{i,j})}}_{j=0,\ldots,M_{i}-1,} \end{split}$$

where $c_{p,0}$ and q_0 denote the salt concentrations in the liquid and solid phase of the beads respectively. The number of available salt ions \bar{q}_0 is given by

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} \sum_{\ell=0}^{M_j-1} (\nu_{j,\ell} + \sigma_{j,\ell}) q_{j,\ell}.$$

A neutrality condition compensating for the missing equation for $\frac{dq_0}{dt}$ is required:

$$q_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} \sum_{\ell=0}^{M_j-1} \nu_{j,\ell} q_{j,\ell}.$$

See Table 3.5.5.

The concept of reference concentrations (c_{ref} and q_{ref}) is explained in the respective paragraph in Section 2.3

2.3.13 Simplified Multi-State Steric Mass Action

The simplified multi-state steric mass action is the same as the multi-state SMA model described above (see Section 2.3.12), but with additional assumptions:

- Molecules are only exchanged between two adjacent states, that is, no transfer from state $q_{i,1}$ to state $q_{i,3}$ is allowed.
- Characteristic charge $\nu_{i,j}$ and shielding factor $\sigma_{i,j}$ only depend on the index of the state j.

Thus, the exchange parameters $k_{j\ell}^{(i)}$, the characteristic charge $\nu_{i,j}$, and the shielding $\sigma_{i,j}$ can be parameterized with few degrees of freedom. For all $i=1,\ldots,N_{\text{comp}}-1$ and $j,\ell=0,\ldots,M_i-1$ let

$$k_{j\ell}^{(i)} = \begin{cases} 0, & \text{for } |j-\ell| \neq 1 \\ K_{ws}^{(i)} + jK_{ws,\text{lin}}^{(i)} - K_{ws,\text{quad}}^{(i)} j(j-M_i+2), & \text{for } \ell = j+1 \\ K_{sw}^{(i)} + \ell K_{sw,\text{lin}}^{(i)} - K_{sw,\text{quad}}^{(i)} \ell(\ell-M_i+2), & \text{for } \ell = j-1, \end{cases}$$

$$\nu_{i,j} = \nu_{\min,i} + \frac{j}{M_i - 1} \left(\nu_{\max,i} - \nu_{\min,i}\right) - \nu_{\text{quad},i} j(j-M_i+1),$$

$$\sigma_{i,j} = \sigma_{\min,i} + \frac{j}{M_i - 1} \left(\sigma_{\max,i} - \sigma_{\min,i}\right) - \sigma_{\text{quad},i} j(j-M_i+1).$$

Note that the characteristic charge $\nu_{i,j}$ has to be monotonically non-decreasing in the second index j and all other rates and the steric factor $\sigma_{i,j}$ have to be non-negative. See Table 3.5.5.

2.3.14 Bi Steric Mass Action

Similar to the Bi-Langmuir model (see Section 2.3.10), the Bi-SMA model adds M-1 additional types of binding sites $q_{i,j}$ ($0 \le j \le M-1$) to the SMA model (see Section 2.3.4) without allowing an exchange between the different sites $q_{i,j}$ and $q_{i,k}$ ($k \ne j$). Therefore, there are no competitivity effects between the two types of binding sites and they have independent capacities.

$$\frac{\mathrm{d}q_{i,j}}{\mathrm{d}t} = k_{a,i,j}c_{p,i} \left(\frac{\bar{q}_{0,j}}{q_{\mathrm{ref},j}}\right)^{\nu_{i,j}} - k_{d,i,j} \ q_{i,j} \ \left(\frac{c_{p,0}}{c_{\mathrm{ref},j}}\right)^{\nu_{i,j}} \qquad i = 1, \dots, N_{\mathrm{comp}} - 1, \quad j = 0, \dots, M - 1,$$

where $c_{p,0}$ and $q_{0,j}$ $(0 \le j \le M-1)$ denote the salt concentrations in the liquid and solid phases of the beads respectively. The number of available salt ions $\bar{q}_{0,j}$ for each binding site type $0 \le j \le M-1$ is given by

$$\bar{q}_{0,j} = \Lambda_j - \sum_{k=1}^{N_{\text{comp}}-1} (\nu_{k,j} + \sigma_{k,j}) \, q_{k,j}.$$

Electro-neutrality conditions compensating for the missing equations for $\frac{dq_{0,j}}{dt}$ are required:

$$q_{0,j} = \Lambda_j - \sum_{k=1}^{N_{\text{comp}}-1} \nu_{k,j} q_{k,j}$$
 $j = 0, \dots, M-1.$

Note that all binding components must have exactly the same number of binding site types $M \ge 1$. See Table 3.5.5.

The reference concentrations $c_{\text{ref},j}$ and $q_{\text{ref},j}$ can be specified for each binding site type $0 \le j \le M-1$. The concept of reference concentrations is explained in the respective paragraph in Section 2.3.

Originally, the Bi-SMA model is limited to two different binding site types. Here, the model has been extended to arbitrary many binding site types.

2.3.15 Generalized Ion Exchange

The generalized ion exchange model is based on the steric mass action model [Huu+17; Mol08]. In addition to the first component $c_{p,0}$, which represents salt, the second component $c_{p,1}$ represents another non-binding modifier (e.g., pH). In comparison to the SMA model, the characteristic charge ν and the adsorption and desorption rate constants are modified:

$$q_0 = \Lambda - \sum_{j=2}^{N_{\text{comp}}-1} \nu_j(c_{p,1}) q_j$$

$$\frac{\partial q_i}{\partial t} = k_{a,i}(c_{p,0}, c_{p,1}) c_{p,i} \left(\frac{\bar{q}_0}{q_{\text{ref}}}\right)^{\nu_i(c_{p,1})} - k_{d,i}(c_{p,0}, c_{p,1}) q_i \left(\frac{c_{p,0}}{c_{\text{ref}}}\right)^{\nu_i(c_{p,1})} \qquad i = 2, \dots, N_{\text{comp}} - 1,$$

where

$$\bar{q}_0 = \Lambda - \sum_{j=2}^{N_{\text{comp}}-1} (\nu_j(c_{p,1}) + \sigma_j) q_j = q_0 - \sum_{j=2}^{N_{\text{comp}}-1} \sigma_j q_j$$

The dependence of the parameters on $c_{p,0}$ and $c_{p,1}$ is given for $i=2,\ldots,N_{\text{comp}}-1$ by

$$\begin{split} \nu_i(c_{p,1}) &= \nu_{i,\text{base}} + c_{p,1}\nu_{i,\text{lin}} + c_{p,1}^2\nu_{i,\text{quad}} \\ k_{a,i}\left(c_{p,0},c_{p,1}\right) &= k_{a,i,\text{base}} \exp\left(k_{a,i,\text{lin}}c_{p,1} + k_{a,i,\text{quad}}c_{p,1}^2 + k_{a,i,\text{salt}}\frac{c_{p,0}}{c_{\text{ref}}} + k_{a,i,\text{prot}}c_{p,i}\right) \\ k_{d,i}\left(c_{p,0},c_{p,1}\right) &= k_{d,i,\text{base}} \exp\left(k_{d,i,\text{lin}}c_{p,1} + k_{d,i,\text{quad}}c_{p,1}^2 + k_{d,i,\text{salt}}\frac{c_{p,0}}{c_{\text{ref}}} + k_{d,i,\text{prot}}c_{p,i}\right) \end{split}$$

See Table 3.5.5

The concept of reference concentrations (c_{ref} and q_{ref}) is explained in the respective paragraph in Section 2.3.

2.4 Reaction models

Reaction models describe the (net) fluxes f_{react} of a reaction mechanism. The most common mechanism is the mass action law.

Correlation of forward- and backward rate constants Note that forward rate constant $k_{\text{fwd},i}$ and backward rate constant $k_{\text{bwd},i}$ of reaction i are linearly correlated due to the form of the equilibrium constant $k_{\text{eq},i}$:

$$k_{\text{fwd},i} = k_{\text{eq},i} k_{\text{bwd},i}$$
.

This correlation can potentially degrade performance of some optimization algorithms. The parameters can be decoupled by reparameterization:

$$r_{\text{net},i} = k_{\text{fwd},i} f_{\text{fwd},i} - k_{\text{bwd},i} f_{\text{bwd},i} = k_{\text{bwd},i} \left[k_{\text{eq},i} f_{\text{fwd},i} - f_{\text{bwd},i} \right] = k_{\text{fwd},i} \left[f_{\text{fwd},i} - \frac{1}{k_{\text{eq},i}} f_{\text{bwd},i} \right].$$

This can be achieved by a (nonlinear) parameter transform

$$F\left(k_{\mathrm{eq},i},k_{\mathrm{bwd},i}\right) = \begin{pmatrix} k_{\mathrm{eq},i}k_{\mathrm{bwd},i} \\ k_{\mathrm{bwd},i} \end{pmatrix} \text{ with Jacobian } J_F\left(k_{\mathrm{eq},i},k_{\mathrm{bwd},i}\right) = \begin{pmatrix} k_{\mathrm{bwd},i} & k_{\mathrm{eq},i} \\ 0 & 1 \end{pmatrix}.$$

Dependence on external function A reaction model may depend on an external function or profile $T: [0, T_{\text{end}}] \times [0, L] \to \mathbb{R}$, where L denotes the physical length of the unit operation, or $T: [0, T_{\text{end}}] \to \mathbb{R}$ if the unit operation model has no axial length. By using an external profile, it is possible to account for effects that are not directly modeled in CADET (e.g., temperature). The dependence of each parameter is modeled by a polynomial of third degree. For example, the forward rate constant k_{fwd} is really given by

$$k_{\text{fwd}}(T) = k_{\text{fwd},3}T^3 + k_{\text{fwd},2}T^2 + k_{\text{fwd},1}T + k_{\text{fwd},0}.$$

While $k_{\text{fwd},0}$ is set by the original parameter XXX_KFWD of the file format (XXX being a placeholder for the reaction model), the parameters $k_{\text{fwd},3}$, $k_{\text{fwd},2}$, and $k_{\text{fwd},1}$ are given by XXX_KFWD_TT, XXX_KFWD_TT, and XXX_KFWD_T, respectively. The identifier of the externally dependent reaction model is constructed from the original identifier by prepending EXT_ (e.g., MASS_ACTION_LAW is changed into EXT_MASS_ACTION_LAW). This pattern applies to all parameters and supporting reaction models. Note that the parameter units have to be adapted to the unit of the external profile by dividing with an appropriate power.

Each parameter of the externally dependent reaction model can depend on a different external source. The 0-based indices of the external source for each parameter is given in the dataset EXTFUN. By assigning only one index to EXTFUN, all parameters use the same source. The ordering of the parameters in EXTFUN is given by the ordering in the file format specification in Section 3.5.6.

2.4.1 Mass action law

The mass action law reaction model is suitable for most reactions. Note that the concentrations are directly used for calculating the fluxes. Hence, the model only holds for dilute solutions under the assumption of a well-stirred reaction vessel. These assumptions can be weakened by passing to the generalized mass action law, which uses chemical activities instead of concentrations.

The mass action law states that the speed of a reaction is proportional to the product of the concentrations of their reactants. The net flux for component i is given by

$$f_{\mathrm{react},i}^{l}\left(\boldsymbol{c}^{l}\right) = \sum_{j=0}^{N_{\mathrm{react}}-1} s_{i,j}^{l} \varphi_{j}^{l}\left(\boldsymbol{c}^{l}\right), \qquad \varphi_{j}^{l}(\boldsymbol{c}^{l}) = k_{\mathrm{fwd},j}^{l} \prod_{\ell=0}^{N_{\mathrm{comp}}-1} \left(\boldsymbol{c}_{\ell}^{l}\right)^{e_{\mathrm{fwd},\ell,j}^{l}} - k_{\mathrm{bwd},j}^{l} \prod_{\ell=0}^{N_{\mathrm{comp}}-1} \left(\boldsymbol{c}_{\ell}^{l}\right)^{e_{\mathrm{bwd},\ell,j}^{l}},$$

where $S^l=(s^l_{i,j})\in\mathbb{R}^{N_{\text{comp}}\times N_{\text{react}}}$ is the stoichiometric matrix, $\varphi^l_j(c)$ is the net flux of reaction j, and $k^l_{\text{fwd},j}$ and $k^l_{\text{bwd},j}$ are the rate constants. The matrices $E^l_{\text{fwd}}=(e^l_{\text{fwd},\ell,j})\in\mathbb{R}^{N_{\text{comp}}\times N_{\text{react}}}$ and

 $E^l_{\text{bwd}} = (e^l_{\text{bwd},\ell,j}) \in \mathbb{R}^{N_{\text{comp}} \times N_{\text{react}}} \text{ are usually derived by the order of the reaction, that is,}$

$$e_{\text{fwd},\ell,j}^l = -\min(0, s_{\ell,j}^l), \qquad e_{\text{bwd},\ell,j}^l = \max(0, -s_{\ell,j}^l).$$
 (2.22)

However, these defaults can be changed by providing those matrices.

In situations where both liquid and solid phase are present (e.g., in a bead), the respective other phase may act as a modifier in the net flux equation. For example, consider reactions in the liquid phase of a particle given by

$$f_{\mathrm{react},i}^{p}\left(\boldsymbol{c}^{p},\boldsymbol{c}^{s}\right) = \sum_{i=0}^{N_{\mathrm{react}}-1} s_{i,j}^{p} \varphi_{j}^{p}\left(\boldsymbol{c}^{p},\boldsymbol{c}^{s}\right),$$

where

$$\begin{split} \varphi_{j}^{p}(c^{p},c^{s}) &= k_{\text{fwd},j}^{p} \left[\prod_{\ell=0}^{N_{\text{comp}}-1} (c_{\ell}^{p})^{e_{\text{fwd},\ell,j}^{p}} \right] \left[\prod_{m=0}^{\sum_{i=0}^{N_{\text{comp}}-1} N_{\text{bnd},i}-1} (c_{m}^{s})^{e_{\text{fwd},m,j}^{ps}} \right] \\ &- k_{\text{bwd},j}^{p} \left[\prod_{\ell=0}^{N_{\text{comp}}-1} (c_{\ell}^{p})^{e_{\text{bwd},\ell,j}^{p}} \right] \left[\prod_{m=0}^{\sum_{i=0}^{N_{\text{comp}}-1} N_{\text{bnd},i}-1} (c_{m}^{s})^{e_{\text{bwd},m,j}^{ps}} \right]. \end{split}$$

The forward and backward rates of the liquid phase particle reactions can be modified by a power of every bound state in the solid phase of the particle. The exponents of these powers are given by the matrices $E^{ps}_{\mathrm{fwd}} = (e^{ps}_{\mathrm{fwd},m,j})$ and $E^{ps}_{\mathrm{bwd}} = (e^{ps}_{\mathrm{bwd},m,j})$, which are both of size $(\sum_i N_{\mathrm{bnd},i}) \times N_{\mathrm{react}}$. Whereas the exponent matrices E^{p}_{fwd} , $E^{p}_{\mathrm{bwd}} \in \mathbb{R}^{N_{\mathrm{comp}} \times N_{\mathrm{react}}}$ are initialized based on the stoichiometric matrix $S^p \in \mathbb{R}^{N_{\mathrm{comp}} \times N_{\mathrm{react}}}$, see Eq. (2.22), the exponent matrices E^{ps}_{fwd} , E^{ps}_{bwd} of the modifier terms default to 0. Vice versa, the rates of solid phase reactions can be modified by liquid phase concentrations. The corresponding exponent matrices $E^{sp}_{\mathrm{fwd}} = (e^{sp}_{\mathrm{fwd},\ell,j})$ and $E^{sp}_{\mathrm{bwd}} = (e^{sp}_{\mathrm{bwd},\ell,j})$ are both of size $N_{\mathrm{comp}} \times N_{\mathrm{react}}$.

$$f_{\mathrm{react},i}^{s}\left(c^{s},c^{p}\right) = \sum_{j=0}^{N_{\mathrm{react}}-1} s_{i,j}^{s} \varphi_{j}^{s}\left(c^{s},c^{p}\right),$$

where

$$\begin{split} \varphi_{j}^{s}(c^{s},c^{p}) &= k_{\text{fwd},j}^{s} \begin{bmatrix} \sum_{i=0}^{N_{\text{comp}}-1} N_{\text{bnd},i} - 1 \\ \prod_{m=0}^{N_{\text{comp}}-1} (c_{m}^{s})^{e_{\text{fwd},m,j}^{s}} \end{bmatrix} \begin{bmatrix} N_{\text{comp}}-1 \\ \prod_{\ell=0}^{N_{\text{comp}}-1} (c_{\ell}^{p})^{e_{\text{fwd},\ell,j}^{sp}} \end{bmatrix} \\ &- k_{\text{bwd},j}^{p} \begin{bmatrix} \sum_{i=0}^{N_{\text{comp}}-1} N_{\text{bnd},i} - 1 \\ \prod_{m=0}^{N_{\text{comp}}-1} (c_{m}^{s})^{e_{\text{bwd},m,j}^{s}} \end{bmatrix} \begin{bmatrix} N_{\text{comp}}-1 \\ \prod_{\ell=0}^{N_{\text{comp}}-1} (c_{\ell}^{p})^{e_{\text{bwd},\ell,j}^{sp}} \end{bmatrix}. \end{split}$$

Whereas the exponent matrices E^s_{fwd} , $E^s_{\text{bwd}} \in \mathbb{R}^{(\sum_i N_{\text{bnd},i}) \times N_{\text{react}}}$ are initialized based on the stoichiometric matrix $S^s \in \mathbb{R}^{(\sum_i N_{\text{bnd},i}) \times N_{\text{react}}}$, see Eq. (2.22), the exponent matrices E^{sp}_{fwd} , E^{sp}_{bwd} of the modifier terms default to 0.

The layout of the matrices in the file format is presented in Table 3.5.6.

3 CADET File Format Specifications

The CADET framework is designed to work on a file format structured into groups and datasets. This concept may be implemented by different file formats. At the moment, CADET natively supports HDF5 and XML as file formats. The choice is not limited to those two formats but can be extended as needed. In this section the general layout and structure of the file format is described.

File format versions The file format may change and evolve over time as new features are added to the simulator. This manual describes the most recent file format version that is also set as default value in /meta/FILE_FORMAT (see Tab. 3.7). The simulator assumes that the input file uses the most recent format version and does not update old files to the current standard.

3.1 Global structure

The global structure (see Fig. 3.1.1) is divided into three parts: input, output, and meta. Every valid CADET file needs an input group (see Fig. 3.1.2) which contains all relevant information for simulating a model. It does not need an output (see Fig. 3.1.4) or meta (see Fig. 3.1.1) group, since those are created when results are written. Whereas the output group is solely used as output and holds the results of the simulation, the meta group is used for input and output. Details such as file format version and simulator version are read from and written to the meta group.

If not explicitly stated otherwise, all datasets are mandatory. By convention all group names are lowercase, whereas all dataset names are uppercase. Note that this is just a description of the file format and not a detailed explanation of the meaning of the parameters. For the latter, please refer to the corresponding sections in the previous chapter.

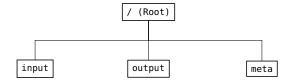


Figure 3.1.1: Structure of the groups in the root group of the file format

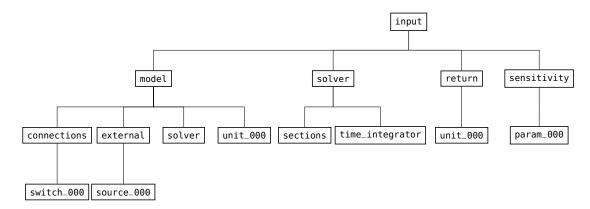


Figure 3.1.2: High-level structure of the groups in the input part of the file format

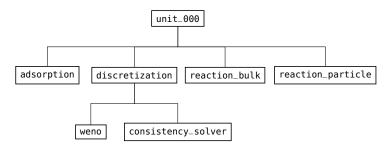


Figure 3.1.3: Structure of the groups in a column unit operation (/input/model group)

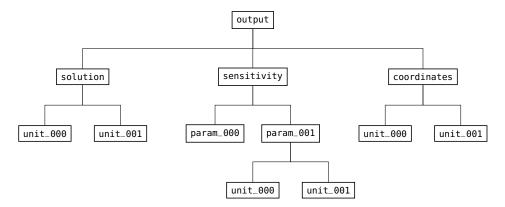


Figure 3.1.4: Structure of the groups in the output part of the file format

3.2 Notation and identifiers

Reference volumes are denoted by subscripts:

 m_{IV}^3 Interstitial volume

 $\rm m_{MP}^3$ Bead mobile phase volume

m_{SP}³ Bead solid phase volume

Common notation and identifiers that are used in the subsequent description are listed in Table 3.2.1.

Identifier	Meaning
NCOMP Number of components of a unit operation	
NTOTALCOMP	Total number of components in the system (sum of all unit operation components)
NPARTYPE	Number of particle types of a unit operation
$NBND_i$	Number of bound states of component i of the current particle type
NTOTALBND	Total number of bound states of the current particle type (sum of all bound states of all components)
NSTATES	Maximum of the number of bound states for each component of a particle type
NREACT	Number of reactions (in bulk volume or in the current particle type)
NDOF	Total number of degrees of freedom of the current unit operation model or system of unit operations
NSEC	Number of time integration sections
PARAM_VALUE	Value of a generic unspecified parameter

Table 3.2.1: Common notation and identifiers used in the file format description

3.3 Ordering of multi dimensional data

Some model parameters, especially in certain binding models, require multi dimensional data. Since CADET only reads one dimensional arrays, the layout of the data has to be specified (i.e., the way how the data is linearized in memory). The term "xyz-major" means that the index corresponding to xyz changes the slowest.

For instance, suppose a model with 2 components and 3 bound states has a "component-major" dataset. Then, the requested matrix is stored in memory such that all bound states are listed for each component (i.e., the component index changes the slowest and the bound state index the fastest):

```
comp0bnd0, comp0bnd1, comp0bnd2, comp1bnd0, comp1bnd1, comp1bnd2.
```

This linear array can be represented as a 2×3 matrix in "row-major" storage format, or a 3×2 matrix in "column-major" ordering.

3.4 Section dependent model parameters

Some model parameters (see Table 3.4.1) can be assigned different values for each section. For example, the velocity a column is operated with could differ in the load, wash, and elution phases. Section dependency is recognized by specifying the appropriate number of values for the parameters (see *Length* column in the following tables). If a parameter depends on both the component and the section, the ordering is section-major.

For instance, the *Length* field of the parameter VELOCITY reads "1 / NSEC" which means that it is not recognized as section dependent if only 1 value (scalar) is passed. However, if NSEC many values (vector) are present, it will be treated as section dependent.

Note that all components of component dependent datasets have to be section dependent (e.g., you cannot have a section dependency on component 2 only while the other components are not section dependent).

Dataset	Component dependent	Section dependent
COL_DISPERSION	✓	✓
FILM_DIFFUSION	✓	\checkmark
PAR_DIFFUSION	✓	\checkmark
PAR_SURFDIFFUSION	✓	\checkmark
VELOCITY		\checkmark

Table 3.4.1: Section dependent datasets in the 1D unit operation models (/input/model/unit_XXX group)

3.5 Input group

3.5.1 System of unit operations

```
Group /input/model
```

NUNITS Number of unit operations in the system

Type: int Range: ≥ 1 Length: 1

INIT_STATE_Y Initial full state vector (optional, unit operation specific initial data is ignored)

Type: int Range: ≥ 1 Length: 1

INIT_STATE_YDOT Initial full time derivative state vector (optional, unit operation specific initial data is ignored)

Type: double Length: NDOF

INIT_STATE_SENSY_XXX Number of unit operations in the system

Type: double Length: NDOF

INIT_STATE_SENSYDOT_XXX Initial full state vector of the XXXth sensitivity system (optional, unit operation specific initial data is ignored)

Type: double Length: NDOF

NUNITS Initial full time derivative state vector of the XXXth sensitivity system (optional, unit operation specific initial data is ignored)

Type: double Length: NDOF

Group /input/model/connections

NSWITCHES Number of valve switches

Type: int Range: ≥ 1 Length: 1

CONNECTIONS_INCLUDE_PORTS Determines whether the **CONNECTIONS** table includes ports (1) or not (0). Optional, defaults to 0 unless a unit operation model with multiple ports is present.

Type: int Range: $\{0,1\}$ Length: 1

CONNECTIONS_INCLUDE_DYNAMIC_FLOW Determines whether the **CONNECTIONS** table includes linear, quadratic, and cubic flow rate coefficients (1) or not (0). Optional, defaults to 0.

Type: int Range: $\{0,1\}$ Length: 1

Group /input/model/connections/switch_XXX

SECTION Index of the section that activates this connection set

Type: int Range: ≥ 0 Length: 1

CONNECTIONS Matrix with list of connections in row-major storage. Columns are UnitOpID from, UnitOpID to, Port from, Port to, Component from, Component to, volumetric flow rate, linear flow rate coefficient, quadratic flow rate coefficient, cubic flow rate coefficient. If both port indices are -1, all ports are connected. If both component indices are -1, all components are connected.

The flow rate is a cubic function of time,

$$Q = Q_0 + Q_1(t - t_s) + Q_2(t - t_s)^2 + Q_3(t - t_s)^3,$$

where t_s is the beginning of the section that activates the switch (i.e., SECTION_TIMES at index SECTION).

The port indices are left out if CONNECTIONS_INCLUDE_PORTS is set to 0 and no unit operation with multiple ports is present in the system. If a unit operation with multiple ports is present, CONNECTIONS_INCLUDE_PORTS is ignored and port indices are mandatory.

The last three flow rate coefficients are left out if CONNECTIONS_INCLUDE_DYNAMIC_FLOW is set to 0. Contrary to the constant coefficient, which has the parameter name CONNECTION, the other coefficients are named CONNECTION_LIN, CONNECTION_QUAD, and CONNECTION_CUB, respectively.

For addressing the flow rates as a parameter sensitivity, the mapping is as follows:

SENS_UNIT Unused, always set to -1

SENS_BOUNDPHASE $UnitOpID\ from$

SENS_REACTION UnitOpID to

SENS_COMP Port from

SENS_PARTYPE Port to

 $\textbf{SENS_SECTION} \ \ \textbf{SECTION} \ \ \textbf{that} \ \ \textbf{activates} \ \ \textbf{the} \ \ \textbf{valve} \ \ \textbf{switch}$

Type: double Range: ≥ -1 Length: $\{5, 7, 8, 10\}$ · NCONNECTIONS

Group /input/model/external/source_XXX - EXTFUN_TYPE = LINEAR_INTERP_DATA

VELOCITY Velocity of the external profile in positive column axial direction

Unit: s^{-1} Type: double Range: ≥ 0 Length: 1

DATA Function values T at the data points

Unit: [T] Type: double Range: \mathbb{R} Length: Arbitrary

TIME Time of the data points

Unit: s Type: double Range: ≥ 0.0 Length: Same as DATA

Group /input/model/external/source_XXX - EXTFUN_TYPE = PIECEWISE_CUBIC_POLY

VELOCITY Velocity of the external profile in positive column axial direction

Unit: s^{-1} Type: double Range: ≥ 0 Length: 1

CONST_COEFF Constant coefficients of piecewise cubic polynomial

Unit: [T] Type: double Range: \mathbb{R} Length: Arbitrary

LIN_COEFF Linear coefficients of piecewise cubic polynomial

Unit: $[T] s^{-1}$ Type: double Range: \mathbb{R} Length: Same as CONST_COEFF

CONST_COEFF Quadratic coefficients of piecewise cubic polynomial

Unit: $[T] s^{-2}$ Type: double Range: \mathbb{R} Length: Same as CONST_COEFF

QUAD_COEFF Cubic coefficients of piecewise cubic polynomial

Unit: $[T] s^{-3}$ Type: double Range: \mathbb{R} Length: Same as CONST_COEFF

SECTION_TIMES Simulation times at which a new piece begins (breaks of the piecewise polynomial)

Unit: s Type: double Range: ≥ 0.0 Length: CONST_COEFF+1

Group /input/model/solver

GS_TYPE Type of Gram-Schmidt orthogonalization, see IDAS guide Section 4.5.7.3, p. 41f. A value of 0 enables classical Gram-Schmidt, a value of 1 uses modified Gram-Schmidt.

Type: int Range: $\{0,1\}$ Length: 1

MAX_KRYLOV Defines the size of the Krylov subspace in the iterative linear GMRES solver (0: MAX_KRYLOV = NDOF)

Type: int Range: $\{0, \dots, NDOF\}$ Length: 1

MAX_RESTARTS Maximum number of restarts in the GMRES algorithm. If lack of memory is not an issue, better use a larger Krylov space than restarts.

Type: int Range: ≥ 0 Length: 1

SCHUR_SAFETY Schur safety factor; Influences the tradeoff between linear iterations and nonlinear error control; see IDAS guide Section 2.1 and 5.

Type: double Range: ≥ 0 Length: 1

LINEAR_SOLUTION_MODE Determines whether the system of models is solved in parallel (1) or sequentially (2). A sequential solution is only possible for systems without cyclic connections. The setting can be chosen automatically (0) based on a heuristic (less than 6 unit operations and acyclic network selects sequential mode). Optional, defaults to automatic (0).

Type: int Range: $\{0,1,2\}$ Length: 1

3.5.2 Unit operation models

Inlet

Group /input/model/unit_XXX - UNIT_TYPE = INLET

UNIT_TYPE Specifies the type of unit operation model

Type: string Range: INLET Length: 1

NCOMP Number of chemical components in the chromatographic medium

Type: int Range: ≥ 1 Length: 1

INLET_TYPE Specifies the type of inlet profile

Type: string Range: PIECEWISE_CUBIC_POLY Length: 1

Group /input/model/unit_XXX/sec_XXX

CONST_COEFF Constant coefficients for inlet concentrations

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3}$ Type: double Range: \mathbb{R} Length: NCOMP

LIN_COEFF Linear coefficients for inlet concentrations

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3} \operatorname{s}^{-1}$ Type: double Range: \mathbb{R} Length: NCOMP

QUAD_COEFF Quadratic coefficients for inlet concentrations

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3} \operatorname{s}^{-2}$ Type: double Range: $\mathbb R$ Length: NCOMP

CUBE_COEFF Cubic coefficients for inlet concentrations

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3} \operatorname{s}^{-3}$ Type: double Range: $\mathbb R$ Length: NCOMP

Outlet

Group /input/model/unit_XXX - UNIT_TYPE = OUTLET

UNIT_TYPE Specifies the type of unit operation model

Type: string Range: OUTLET Length: 1

NCOMP Number of chemical components in the chromatographic medium

Type: int Range: ≥ 1 Length: 1

General rate model

Group /input/model/unit_XXX - UNIT_TYPE = GENERAL_RATE_MODEL

 $\ensuremath{\textbf{UNIT_TYPE}}$ Specifies the type of unit operation model

Type: string Range: GENERAL_RATE_MODEL Length: 1

NCOMP Number of chemical components in the chromatographic medium

Type: int Range: ≥ 1 Length: 1

ADSORPTION_MODEL Specifies the type of binding model of each particle type (or of all particle types if

length is 1)

Type: string Range: See Section 3.5.5 Length: 1 / NPARTYPE

ADSORPTION_MODEL_MULTIPLEX Multiplexing mode of ADSORPTION_MODEL. If set to 0, each particle type has a different binding model and the length of ADSORPTION_MODEL is NPARTYPE. If set to 1, all particle types share the same binding model and the length of ADSORPTION_MODEL is 1.

This field is optional and inferred from the length of ADSORPTION_MODEL if left out.

Unit: - Type: int Range: $\{0,1\}$ Length: 1

REACTION_MODEL_BULK Specifies the type of reaction model of the bulk volume. The model is configured in the subgroup reaction_bulk.

Type: string Range: See Section 3.5.6 Length: 1

REACTION_MODEL_PARTICLES Specifies the type of reaction model of each particle type (or of all particle types if length is 1). The model is configured in the subgroup reaction_particle, or reaction_particle_XXX in case of disabled multiplexing.

Type: string Range: See Section 3.5.6 Length: 1 / NPARTYPE

REACTION_MODEL_PARTICLES_MULTIPLEX Multiplexing mode of REACTION_MODEL_PARTICLES. If set to 0, each particle type has a different reaction model and the length of REACTION_MODEL_PARTICLES is NPARTYPE. If set to 1, all particle types share the same reaction model and the length of REACTION_MODEL_PARTICLES is 1.

This field is optional and inferred from the length of REACTION_MODEL_PARTICLES if left out.

Unit: -Type: int **Range:** $\{0,1\}$ Length: 1

INIT_C Initial concentrations for each component in the bulk mobile phase

Unit: $mol m_{IV}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

INIT_CP Initial concentrations for each component in the bead liquid phase (optional, INIT_C is used if left out). The length of this field can be NCOMP (same values for each particle type) or $NPARTYPE \cdot NCOMP$ Values for each particle type can only be given when ADSORPTION_MODEL_MULTIPLEX is 0. The ordering is type-major.

Unit: $mol m_{MP}^{-3}$ Type: double Range: > 0Length: NCOMP / NPARTYPE · NCOMP

INIT_Q Initial concentrations for each bound state of each component in the bead solid phase. If ADSORPTION_MODEL_MULTIPLEX is 0, values for each particle type are required in type-componentmajor ordering (length is NTOTALBND). If ADSORPTION_MODEL_MULTIPLEX is 1, values for one particle type are required in component-major ordering (length is $\sum_{i=0}^{\mathsf{NCOMP}-1} \mathsf{NBND}_i$).

 $\mathbf{Length}: \mathtt{NTOTALBND} \ / \ \sum_{i=0}^{\mathtt{NCOMP}-1} \mathtt{NBND}_i$ Unit: $mol m_{SP}^{-3}$ **Type:** double **Range:** ≥ 0

Length: see COL_DISPERSION_MULTIPLEX

INIT_STATE Full state vector for initialization (optional, INIT_C, INIT_CP, and INIT_Q will be ignored; if length is 2NDOF, then the second half is used for time derivatives)

Type: double Range: \mathbb{R} Length: NDOF / 2NDOF Unit: various

COL_DISPERSION Axial dispersion coefficient

COL_DISPERSION_MULTIPLEX Multiplexing mode of COL_DISPERSION. Determines whether COL_DISPERSION is treated as component- and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of COL_DISPERSION.

Valid modes are:

Unit: $m_{IV}^2 s^{-1}$

0 Component-independent, section-independent; length of COL_DISPERSION is 1

Type: double Range: > 0

- ${\bf 1} \ \ {\bf Component\text{-}dependent}, \ {\bf section\text{-}independent}; \ {\bf length} \ \ {\bf of} \ \ {\bf COL_DISPERSION} \ \ {\bf is} \ \ {\bf NCOMPORTION COMPONENTS}$
- 2 Component-independent, section-dependent; length of COL_DISPERSION is NSEC
- 3 Component-dependent, section-dependent; length of COL_DISPERSION is NCOMP · NSEC; ordering is section-major

Range: $\{0, ..., 3\}$ Unit: -Type: int Length: 1

COL_LENGTH Column length

Unit: m **Type:** double **Range:** > 0Length: 1

COL_POROSITY Column porosity

Type: double Range: (0,1]Unit: -Length: 1 FILM_DIFFUSION Film diffusion coefficients for each component of each particle type Unit: $m s^{-1}$ **Type:** double **Range:** ≥ 0 Length: see FILM_DIFFUSION_MULTIPLEX FILM_DIFFUSION_MULTIPLEX Multiplexing mode of FILM_DIFFUSION. Determines whether FILM_DIFFUSION is treated as component-, type-, and/or section-independent. This field is optional. When left out, multiplexing behavior is inferred from the length of FILM_DIFFUSION. Valid modes are: **0** Component-dependent, type-independent, section-independent; length of FILM_DIFFUSION is 1 Component-dependent, type-independent, section-dependent; length of FILM_DIFFUSION is NCOMP-NSEC; ordering is section-major 2 Component-dependent, type-dependent, section-independent; length of FILM_DIFFUSION is NCOMP-NPARTYPE; ordering is type-major 3 Component-dependent, type-dependent, section-dependent; length of FILM_DIFFUSION is NCOMP. NPARTYPE · NSEC; ordering is section-type-major **Range:** $\{0, ..., 3\}$ Length: 1 Unit: -Type: int **PAR_POROSITY** Particle porosity of all particle types or for each particle type Length: 1 / NPARTYPEUnit: -Type: double Range: (0,1]**PAR_RADIUS** Particle radius of all particle types or for each particle type Length: 1 / NPARTYPE Unit: m **Type:** double **Range:** > 0PAR_CORERADIUS Particle core radius of all particle types or for each particle type (optional, defaults to $0\,\mathrm{m}$ **Type:** double **Range:** [0, PAR_RADIUS) Length: 1 / NPARTYPE Unit: mPORE_ACCESSIBILITY Pore accessibility factor of each component in each particle type (optional, defaults to 1) Unit: -Type: double Range: (0,1]Length: see PORE_ACCESSIBILITY_MULTIPLEX PORE_ACCESSIBILITY_MULTIPLEX Multiplexing mode of PORE_ACCESSIBILITY. Determines whether PORE_ACCESSIBILITY is treated as component-, type-, and/or section-independent. This field is optional. When left out, multiplexing behavior is inferred from the length of PORE_ACCESSIBILITY. Valid modes are: **0** Component-dependent, type-independent, section-independent; length of PORE_ACCESSIBILITY 1 Component-dependent, type-independent, section-dependent; length of PORE_ACCESSIBILITY is NCOMP · NSEC: ordering is section-major 2 Component-dependent, type-dependent, section-independent; length of PORE_ACCESSIBILITY is NCOMP · NPARTYPE; ordering is type-major 3 Component-dependent, type-dependent, section-dependent; length of PORE_ACCESSIBILITY is NCOMP · NPARTYPE · NSEC; ordering is section-type-major Unit: -Type: int **Range:** $\{0, ..., 3\}$ Length: 1 PAR_DIFFUSION Effective particle diffusion coefficients of each component in each particle type

Unit: $m_{MP}^2 s^{-1}$ Type: double Range: ≥ 0 Length: see PAR_DIFFUSION_MULTIPLEX

PAR_DIFFUSION_MULTIPLEX Multiplexing mode of PAR_DIFFUSION. Determines whether PAR_DIFFUSION is treated as component-, type-, and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of PAR_DIFFUSION. Valid modes are:

- $\textbf{0} \ \, \text{Component-dependent}, \ \, \text{type-independent}, \ \, \text{section-independent}; \ \, \text{length of PAR_DIFFUSION} \ \, \text{is NCOMP}$
- 1 Component-dependent, type-independent, section-dependent; length of PAR_DIFFUSION is NCOMP·NSEC; ordering is section-major
- 2 Component-dependent, type-dependent, section-independent; length of PAR_DIFFUSION is NCOMP· NPARTYPE; ordering is type-major
- $\textbf{3} \ \, \text{Component-dependent, type-dependent, section-dependent; length of PAR_DIFFUSION is NCOMP} \cdot \\ \text{NPARTYPE} \cdot \text{NSEC; ordering is section-type-major}$

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

PAR_SURFDIFFUSION Particle surface diffusion coefficients of each bound state of each component in each particle type (optional, defaults to all $0\,\mathrm{m_{SP}^2\,s^{-1}})$

Unit: $m_{SP}^2 s^{-1}$ Type: double Range: ≥ 0 Length: see PAR_SURFDIFFUSION_MULTIP

PAR_SURFDIFFUSION_MULTIPLEX Multiplexing mode of PAR_SURFDIFFUSION. Determines whether PAR_SURFDIFFUSION is treated as component-, type-, and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of PAR_SURFDIFFUSION. Valid modes are:

- $\textbf{0} \ \, \text{Component-dependent, type-independent, section-independent; length of PAR_SURFDIFFUSION is NBND; ordering is component-major } \\$
- $\textbf{1} \ \, \text{Component-dependent, type-independent, section-dependent; length of PAR_SURFDIFFUSION is } \\ \text{NBND} \cdot \text{NSEC; ordering is section-component-major}$
- 2 Component-dependent, type-dependent, section-independent; length of PAR_SURFDIFFUSION is NTOTALBND; ordering is type-component-major
- **3** Component-dependent, type-dependent, section-dependent; length of PAR_SURFDIFFUSION is NTOTALBND · NSEC; ordering is section-type-component-major

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

VELOCITY Interstitial velocity of the mobile phase (optional if CROSS_SECTION_AREA is present, see Section 2.2.3)

Unit: $m s^{-1}$ Type: double Range: \mathbb{R} Length: 1 / NSEC

CROSS_SECTION_AREA Cross section area of the column (optional if **VELOCITY** is present, see Section 2.2.3)

Unit: m^2 Type: double Range: > 0 Length: 1

PAR_TYPE_VOLFRAC Volume fractions of the particle types. The volume fractions can be set for all axial cells together or for each individual axial cell. For each cell, the volume fractions have to sum to 1. In case of a spatially inhomogeneous setting, the data is expected in cell-major ordering and the SENS_SECTION field is used for indexing the axial cell when specifying parameter sensitivities.

This field is optional in case of only one particle type.

Unit: - Type: double Range: [0,1] Length: NPARTYPE / NCOL · NPARTYPE

Group /input/model/unit_XXX/discretization - UNIT_TYPE = GENERAL_RATE_MODEL

NCOL Number of axial column discretization cells

Type: int Range: ≥ 1 Length: 1

NPARTYPE Number of particle types. Optional, inferred from the length of NPAR or NBOUND if left out.

Type: int Range: ≥ 1 Length: 1

NPAR Number of particle (radial) discretization cells for each particle type

Type: int Range: ≥ 1 Length: 1 / NPARTYPE

NBOUND Number of bound states for each component in each particle type in type-major ordering Length: NCOMP / NPARTYPE · NCOMP Type: int Range: ≥ 0 PAR_GEOM Specifies the particle geometry for all or each particle type. Valid values are SPHERE, CYLINDER, and SLAB. Optional, defaults to SPHERE. Type: string Range: {SPHERE, CYLINDER, SLAB} Length: 1 / NPARTYPE PAR_DISC_TYPE Specifies the discretization scheme inside the particles for all or each particle type. Valid values are EQUIDISTANT_PAR, EQUIVOLUME_PAR, and USER_DEFINED_PAR. Length: 1 / NPARTYPEType: string **PAR_DISC_VECTOR** Node coordinates for the cell boundaries (ignored if PAR_DISC_TYPE \neq USER_DEFINED_PAR). The coordinates are relative and have to include the endpoints 0 and 1. They are later linearly mapped to the true radial range $[r_{c,i}, r_{p,i}]$. The coordinates for each particle type are appended to one long vector in type-major ordering. Unit: -Type: double Range: [0,1] Length: $\sum_{i} (NPAR_i + 1)$ PAR_BOUNDARY_ORDER Order of accuracy of outer particle boundary condition. Optional, defaults to 2. **Range:** $\{1, 2\}$ Type: int Length: 1 USE_ANALYTIC_JACOBIAN Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0) **Range:** $\{0,1\}$ Length: 1 Type: int **RECONSTRUCTION** Type of reconstruction method for fluxes Type: string Range: WEN0 Length: 1 **GS_TYPE** Type of Gram-Schmidt orthogonalization, see IDAS guide Section 4.5.7.3, p. 41f. A value of 0 enables classical Gram-Schmidt, a value of 1 uses modified Gram-Schmidt. Type: int **Range:** $\{0,1\}$ Length: 1 MAX_KRYLOV Defines the size of the Krylov subspace in the iterative linear GMRES solver (0: MAX_KRYLOV = NCOL · NCOMP · NPARTYPE) Range: $\{0,\ldots,\mathsf{NCOL}\cdot\mathsf{NCOMP}\cdot\mathsf{NPAR}\mathbf{T}\}$ Type: int MAX_RESTARTS Maximum number of restarts in the GMRES algorithm. If lack of memory is not an issue, better use a larger Krylov space than restarts. Type: int Range: ≥ 0 Length: 1 SCHUR_SAFETY Schur safety factor; Influences the tradeoff between linear iterations and nonlinear error control; see IDAS guide Section 2.1 and 5. Type: double Range: ≥ 0 Length: 1 $\textbf{FIX_ZERO_SURFACE_DIFFUSION} \ \ \text{Determines whether the surface diffusion parameters PAR_SURFDIFFUSION}$ are fixed if the parameters are zero. If the parameters are fixed to zero ($FIX_ZERO_SURFACE_DIFFUSION =$ 1, PAR_SURFDIFFUSION = 0), the parameters must not become non-zero during this or subsequent simulation runs. The internal data structures are optimized for a more efficient simulation. This field is optional and defaults to 0 (optimization disabled in favor of flexibility). Length: 1 Type: int **Range:** $\{0,1\}$

Lumped rate model with pores

Group /input/model/unit_XXX - UNIT_TYPE = LUMPED_RATE_MODEL_WITH_PORES

UNIT_TYPE Specifies the type of unit operation model

Type: string Range: LUMPED_RATE_MODEL_WITH_IP@RESh: 1

NCOMP Number of chemical components in the chromatographic medium

Type: int Range: ≥ 1 Length: 1

ADSORPTION_MODEL Specifies the type of binding model of each particle type (or of all particle types if length is 1)

Type: string Range: See Section 3.5.5 Length: 1 / NPARTYPE

ADSORPTION_MODEL_MULTIPLEX Multiplexing mode of ADSORPTION_MODEL. If set to 0, each particle type has a different binding model and the length of ADSORPTION_MODEL is NPARTYPE. If set to 1, all particle types share the same binding model and the length of ADSORPTION_MODEL is 1.

This field is optional and inferred from the length of ADSORPTION_MODEL if left out.

Unit: – Type: int Range: $\{0,1\}$ Length: 1

REACTION_MODEL_BULK Specifies the type of reaction model of the bulk volume. The model is configured in the subgroup reaction_bulk.

Type: string Range: See Section 3.5.6 Length: 1

REACTION_MODEL_PARTICLES Specifies the type of reaction model of each particle type (or of all particle types if length is 1). The model is configured in the subgroup reaction_particle, or reaction_particle_XXX in case of disabled multiplexing.

Type: string Range: See Section 3.5.6 Length: 1 / NPARTYPE

REACTION_MODEL_PARTICLES_MULTIPLEX Multiplexing mode of REACTION_MODEL_PARTICLES. If set to 0, each particle type has a different reaction model and the length of REACTION_MODEL_PARTICLES is NPARTYPE. If set to 1, all particle types share the same reaction model and the length of REACTION_MODEL_PARTICLES is 1.

This field is optional and inferred from the length of REACTION_MODEL_PARTICLES if left out.

Unit: – Type: int Range: $\{0,1\}$ Length: 1

INIT_C Initial concentrations for each component in the bulk mobile phase

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

INIT_CP Initial concentrations for each component in the bead liquid phase (optional, INIT_C is used if left out). The length of this field can be NCOMP (same values for each particle type) or NPARTYPE·NCOMP Values for each particle type can only be given when ADSORPTION_MODEL_MULTIPLEX is 0. The ordering is type-major.

Unit: $mol \, m_{MP}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP / NPARTYPE · NCOMP

INIT_Q Initial concentrations for each bound state of each component in the bead solid phase. If ADSORPTION_MODEL_MULTIPLEX is 0, values for each particle type are required in type-component-major ordering (length is NTOTALBND). If ADSORPTION_MODEL_MULTIPLEX is 1, values for one particle type are required in component-major ordering (length is $\sum_{i=0}^{\text{NCOMP}-1} \text{NBND}_i$).

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{SP}}^{-3}$ Type: double Range: ≥ 0 Length: NTOTALBND $/\sum_{i=0}^{\operatorname{NCOMP}-1} \operatorname{NBND}_i$

INIT_STATE Full state vector for initialization (optional, INIT_C, INIT_CP, and INIT_Q will be ignored;
 if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range: \mathbb{R} Length: NDOF / 2NDOF

 $\textbf{COL_DISPERSION} \ \ Axial \ dispersion \ coefficient$

Unit: $m_{\mathrm{IV}}^2 \, \mathrm{s}^{-1}$ Type: double Range: ≥ 0 Length: see COL_DISPERSION_MULTIPLEX

 $\label{lem:col_dispersion_multiplex} \textbf{Col_Dispersion}. \ \ \textbf{Determines whether Col_Dispersion} \ \ \textbf{is treated as component- and/or section-independent}.$

This field is optional. When left out, multiplexing behavior is inferred from the length of COL_DISPERSION. Valid modes are:

- **0** Component-independent, section-independent; length of COL_DISPERSION is 1
- 1 Component-dependent, section-independent; length of COL_DISPERSION is NCOMP
- 2 Component-independent, section-dependent; length of COL_DISPERSION is NSEC
- $\textbf{3} \ \, \text{Component-dependent}, \, \text{section-dependent}; \, \text{length of COL_DISPERSION} \, \, \text{is NCOMP} \cdot \, \text{NSEC}; \, \text{ordering is section-major}$

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

COL_LENGTH Column length

Unit: m Type: double Range: > 0 Length: 1

COL_POROSITY Column porosity

Unit: - Type: double Range: (0,1] Length: 1

FILM_DIFFUSION Film diffusion coefficients for each component of each particle type

Unit: $m s^{-1}$ Type: double Range: ≥ 0 Length: see FILM_DIFFUSION_MULTIPLEX

FILM_DIFFUSION_MULTIPLEX Multiplexing mode of FILM_DIFFUSION. Determines whether FILM_DIFFUSION is treated as component-, type-, and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of FILM_DIFFUSION.

Valid modes are:

- $\textbf{0} \ \, \text{Component-dependent, type-independent, section-independent; length of FILM_DIFFUSION is } \\ \text{NCOMP}$
- 1 Component-dependent, type-independent, section-dependent; length of FILM_DIFFUSION is NCOMP-NSEC; ordering is section-major
- ${\bf 2} \ \ {\bf Component\text{-}dependent, type\text{-}dependent, section\text{-}independent; length of FILM_DIFFUSION is NCOMP-NPARTYPE; ordering is type-major}$
- **3** Component-dependent, type-dependent, section-dependent; length of FILM_DIFFUSION is NCOMP · NPARTYPE · NSEC; ordering is section-type-major

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

PAR_POROSITY Particle porosity of all particle types or for each particle type

Unit: - Type: double Range: (0,1] Length: 1 / NPARTYPE

PAR_RADIUS Particle radius of all particle types or for each particle type

Unit: m Type: double Range: > 0 Length: 1 / NPARTYPE

PORE_ACCESSIBILITY Pore accessibility factor of each component in each particle type (optional, defaults to 1)

Unit: - Type: double Range: (0,1] Length: see PORE_ACCESSIBILITY_MULTIPLEX

PORE_ACCESSIBILITY_MULTIPLEX Multiplexing mode of PORE_ACCESSIBILITY. Determines whether PORE_ACCESSIBILITY is treated as component-, type-, and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of PORE_ACCESSIBILITY.

Valid modes are:

- 0 Component-dependent, type-independent, section-independent; length of PORE_ACCESSIBILITY is NCOMP
- $\begin{tabular}{ll} {\bf 1} & {\bf Component-dependent, type-independent, section-dependent; length of PORE_ACCESSIBILITY is NCOMP \cdot NSEC; ordering is section-major \\ \end{tabular}$
- 2 Component-dependent, type-dependent, section-independent; length of PORE_ACCESSIBILITY is NCOMP · NPARTYPE; ordering is type-major

3 Component-dependent, type-dependent, section-dependent; length of PORE_ACCESSIBILITY is NCOMP · NPARTYPE · NSEC; ordering is section-type-major Unit: -Type: int **Range:** $\{0, ..., 3\}$ Length: 1 **VELOCITY** Interstitial velocity of the mobile phase (optional if CROSS_SECTION_AREA is present, see Section 2.2.3Unit: $m s^{-1}$ Type: double Range: \mathbb{R} Length: 1 / NSEC $\textbf{CROSS_SECTION_AREA} \ \ Cross\ section\ area\ of\ the\ column\ (optional\ if\ \textbf{VELOCITY}\ is\ present,\ see\ Section\ 2.2.3)$ Unit: m² Type: double Range: > 0Length: 1 PAR_TYPE_VOLFRAC Volume fractions of the particle types. The volume fractions can be set for all axial cells together or for each individual axial cell. For each cell, the volume fractions have to sum to 1. In case of a spatially inhomogeneous setting, the data is expected in cell-major ordering and the SENS_SECTION field is used for indexing the axial cell when specifying parameter sensitivities. This field is optional in case of only one particle type. Unit: -Length: NPARTYPE / NCOL · NPARTYPE Type: double Range: [0,1]Group /input/model/unit_XXX/discretization - UNIT_TYPE = LUMPED_RATE_MODEL_WITH_PORES NCOL Number of axial column discretization cells Type: int Range: > 1Length: 1 NPARTYPE Number of particle types. Optional, inferred from the length of NBOUND if left out. Type: int Range: ≥ 1 Length: 1 **NBOUND** Number of bound states for each component in each particle type in type-major ordering $\mathbf{Length:} \ \mathsf{NCOMP} \ / \ \mathsf{NPARTYPE} \cdot \mathsf{NCOMP}$ Type: int Range: > 0PAR_GEOM Specifies the particle geometry for all or each particle type. Valid values are SPHERE, CYLINDER, and SLAB. Optional, defaults to SPHERE. Type: string Range: {SPHERE, CYLINDER, SLAB} Length: 1 / NPARTYPE USE_ANALYTIC_JACOBIAN Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0) Type: int **Range:** {0,1} Length: 1 **RECONSTRUCTION** Type of reconstruction method for fluxes Type: string Range: WEN0 Length: 1 **GS_TYPE** Type of Gram-Schmidt orthogonalization, see IDAS guide Section 4.5.7.3, p. 41f. A value of 0 enables classical Gram-Schmidt, a value of 1 uses modified Gram-Schmidt. Type: int **Range:** $\{0,1\}$ Length: 1 MAX_KRYLOV Defines the size of the Krylov subspace in the iterative linear GMRES solver (0: MAX_KRYLOV = NCOL · NCOMP · NPARTYPE) Range: $\{0, \dots, NCOL \cdot NCOMP \cdot NPAR \mathbf{T})$ Type: int MAX_RESTARTS Maximum number of restarts in the GMRES algorithm. If lack of memory is not an issue, better use a larger Krylov space than restarts. Type: int Range: > 0Length: 1 SCHUR_SAFETY Schur safety factor; Influences the tradeoff between linear iterations and nonlinear error control; see IDAS guide Section 2.1 and 5. **Type:** double **Range:** ≥ 0 Length: 1

Lumped rate model without pores

Group /input/model/unit_XXX - UNIT_TYPE = LUMPED_RATE_MODEL_WITHOUT_PORES

UNIT_TYPE Specifies the type of unit operation model

Type: string Range: LUMPED_RATE_MODEL_WITHOUTINEDERS

NCOMP Number of chemical components in the chromatographic medium

Type: int Range: ≥ 1 Length: 1

ADSORPTION_MODEL Specifies the type of binding model

Type: string Range: See Section 3.5.5 Length: 1

REACTION_MODEL Specifies the type of reaction model of the combined bulk and particle volume. The model is configured in the subgroup reaction.

Type: string Range: See Section 3.5.6 Length: 1

INIT_C Initial concentrations for each component in the bulk mobile phase

Unit: $mol m_{IV}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

INIT_Q Initial concentrations for each bound state of each component in the bead solid phase in component-major ordering

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: NTOTALBND

INIT_STATE Full state vector for initialization (optional, INIT_C and INIT_Q will be ignored; if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range: R Length: NDOF / 2NDOF

COL_DISPERSION Axial dispersion coefficient

Unit: $m_{IV}^2 s^{-1}$ Type: double Range: ≥ 0 Length: see COL_DISPERSION_MULTIPLEX

COL_DISPERSION_MULTIPLEX Multiplexing mode of COL_DISPERSION. Determines whether COL_DISPERSION is treated as component- and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of COL_DISPERSION.

Valid modes are:

- **0** Component-independent, section-independent; length of COL_DISPERSION is 1
- ${\bf 1} \ \ {\bf Component\text{-}dependent}, \ {\bf section\text{-}independent}; \ {\bf length} \ \ {\bf of} \ \ {\bf COL_DISPERSION} \ \ {\bf is} \ \ {\bf NCOMP}$
- 2 Component-independent, section-dependent; length of COL_DISPERSION is NSEC
- **3** Component-dependent, section-dependent; length of $COL_DISPERSION$ is $NCOMP \cdot NSEC$; ordering is section-major

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

COL_LENGTH Column length

Unit: m Type: double Range: > 0 Length: 1

TOTAL_POROSITY Total porosity

Unit: - Type: double Range: [0,1] Length: 1

VELOCITY Interstitial velocity of the mobile phase (optional if CROSS_SECTION_AREA is present, see Section 2.2.3)

Unit: $m s^{-1}$ Type: double Range: \mathbb{R} Length: 1 / NSEC

CROSS_SECTION_AREA Cross section area of the column (optional if VELOCITY is present, see Section 2.2.3)

Unit: m^2 Type: double Range: > 0 Length: 1

Group /input/model/unit_XXX/discretization - UNIT_TYPE = LUMPED_RATE_MODEL_WITHOUT_PORES

NCOL Number of axial column discretization cells

Type: int Range: > 1 Length: 1

NBOUND Number of bound states for each component

Type: int Range: ≥ 0 Length: NCOMP

USE_ANALYTIC_JACOBIAN Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0)

Type: int Range: $\{0,1\}$ Length: 1

RECONSTRUCTION Type of reconstruction method for fluxes

Type: string Range: WEN0 Length: 1

Two dimensional general rate model

Group /input/model/unit_XXX - UNIT_TYPE = GENERAL_RATE_MODEL_2D

UNIT_TYPE Specifies the type of unit operation model

Type: string Range: GENERAL_RATE_MODEL_2D Length: 1

NCOMP Number of chemical components in the chromatographic medium

Type: int Range: ≥ 1 Length: 1

ADSORPTION_MODEL Specifies the type of binding model of each particle type (or of all particle types if length is 1)

Type: string Range: See Section 3.5.5 Length: 1 / NPARTYPE

ADSORPTION_MODEL_MULTIPLEX Multiplexing mode of ADSORPTION_MODEL. If set to 0, each particle type has a different binding model and the length of ADSORPTION_MODEL is NPARTYPE. If set to 1, all particle types share the same binding model and the length of ADSORPTION_MODEL is 1.

This field is optional and inferred from the length of ADSORPTION_MODEL if left out.

Unit: – Type: int Range: $\{0,1\}$ Length: 1

REACTION_MODEL_BULK Specifies the type of reaction model of the bulk volume. The model is configured in the subgroup reaction_bulk.

Type: string Range: See Section 3.5.6 Length: 1

REACTION_MODEL_PARTICLES Specifies the type of reaction model of each particle type (or of all particle types if length is 1). The model is configured in the subgroup reaction_particle, or reaction_particle_XXX in case of disabled multiplexing.

Type: string Range: See Section 3.5.6 Length: 1 / NPARTYPE

REACTION_MODEL_PARTICLES_MULTIPLEX Multiplexing mode of REACTION_MODEL_PARTICLES. If set to 0, each particle type has a different reaction model and the length of REACTION_MODEL_PARTICLES is NPARTYPE. If set to 1, all particle types share the same reaction model and the length of REACTION_MODEL_PARTICLES is 1.

This field is optional and inferred from the length of REACTION_MODEL_PARTICLES if left out.

Unit: – Type: int Range: $\{0,1\}$ Length: 1

INIT_C Initial concentrations for each component in all radial zones the bulk mobile phase (length NCOMP), or for each component in each radial zone (length $NCOMP \cdot NRAD$, ordering radial-major)

Unit: $mol \, m_V^{-3}$ Type: double Range: ≥ 0 Length: NCOMP / NCOMP · NRAD

INIT_CP Initial concentrations for each component in the bead liquid phase (optional, INIT_C is used if left out). The length of this field can be NCOMP (same values for each radial zone and particle type), NPARTYPE·NCOMP (same values for each radial zone), RAD·NCOMP (same values for each particle type), or NRAD·NPARTYPE·NCOMP. The ordering is radial-type-major.

Values for each particle type can only be given when ADSORPTION_MODEL_MULTIPLEX is 0. In the radial-inhomogeneous case, the SENS_REACTION field is used for indexing the radial zone when specifying parameter sensitivities.

Unit: $mol \, m_{MP}^{-3}$ Type: double Range: ≥ 0

INIT_Q Initial concentrations for each bound state of each component in the bead solid phase. If ADSORPTION_MODEL_MULTIPLEX is 0, values for each particle type are required in type-component-major ordering (length is NTOTALBND). If ADSORPTION_MODEL_MULTIPLEX is 1, values for one particle type are required in component-major ordering (length is $\sum_{i=0}^{\text{NCOMP}-1} \text{NBND}_i$).

Alternatively, values for each radial zone can be supplied. If ADSORPTION_MODEL_MULTIPLEX is 0, values for each radial zone and each particle type are required in radial-type-component-major ordering (length is NRAD·NTOTALBND). If ADSORPTION_MODEL_MULTIPLEX is 1, values for each radial zone and all particle types are required in radial-component-major ordering (length is NRAD· $\sum_{i=0}^{\mathsf{NCOMP}-1}\mathsf{NBND}_i).$ In the radial-inhomogeneous case, the SENS_REACTION field is used for indexing the radial zone when specifying parameter sensitivities.

Unit: $mol m_{SP}^{-3}$ Type: double Range: ≥ 0

INIT_STATE Full state vector for initialization (optional, INIT_C, INIT_CP, and INIT_Q will be ignored;
 if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range: \mathbb{R} Length: NDOF / 2NDOF

COL_DISPERSION Axial dispersion coefficient.

In case of a spatially inhomogeneous setting, the SENS_PARTYPE field is used for indexing the radial zone when specifying parameter sensitivities.

Unit: $m_{IV}^2 s^{-1}$ Type: double Range: ≥ 0

Length: see COL_DISPERSION_MULTIPLEX

COL_DISPERSION_MULTIPLEX Multiplexing mode of COL_DISPERSION. Determines whether COL_DISPERSION is treated as component-, radial-, and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of COL_DISPERSION. Valid modes are:

- ${\bf 1} \ \ {\bf Component\text{-}independent}, \ {\bf radial\text{-}dependent}, \ {\bf section\text{-}independent}; \ {\bf length} \ \ {\bf of} \ \ {\bf COL_DISPERSION} \ \ {\bf is} \ \ \\ {\bf NRAD}$
- ${\bf 2} \ \ {\bf Component\text{-}dependent}, \ {\bf radial\text{-}independent}, \ {\bf section\text{-}independent}; \ {\bf length} \ \ {\bf of} \ \ {\bf COL_DISPERSION} \ is \\ {\bf NCOMP}$
- ${\bf 3} \ \, {\bf Component\text{-}dependent}, \ \, {\bf radial\text{-}dependent}, \ \, {\bf section\text{-}independent}; \ \, {\bf length} \ \, {\bf of} \ \, {\bf COL_DISPERSION} \ \, {\bf is} \\ {\bf NCOMP\cdot NRAD}; \ \, {\bf ordering} \ \, {\bf is} \ \, {\bf radial\text{-}major} \\$
- $\begin{tabular}{ll} \bf 5 & {\bf Component-independent, radial-dependent, section-dependent; length of COL_DISPERSION is NRAD \cdot NSEC; ordering is section-major \\ \end{tabular}$
- **6** Component-dependent, radial-independent, section-independent; length of COL_DISPERSION is $NCOMP \cdot NSEC$; ordering is section-major
- 7 Component-dependent, radial-dependent, section-dependent; length of $COL_DISPERSION$ is $NCOMP \cdot NRAD \cdot NSEC$; ordering is section-radial-major

Unit: – Type: int Range: $\{0, \dots, 7\}$ Length: 1

COL_DISPERSION_RADIAL Radial dispersion coefficient.

In case of a spatially inhomogeneous setting, the SENS_PARTYPE field is used for indexing the radial zone when specifying parameter sensitivities.

zone when specifying parameter sensitivities.

Unit: $m_{\text{TV}}^2 \text{ s}^{-1}$ Type: double Range: ≥ 0 Length: see COL_DISPERSION_RADIAL_MU

COL_DISPERSION_RADIAL_MULTIPLEX Multiplexing mode of COL_DISPERSION_RADIAL. Determines whether COL_DISPERSION_RADIAL is treated as component-, radial-, and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of COL_DISPERSION_RADIAL. Valid modes are:

- O Component-independent, radial-independent, section-independent; length of COL_DISPERSION_RADIAL is 1
- 1 Component-independent, radial-dependent, section-independent; length of COL_DISPERSION_RADIAL is NRAD
- ${\bf 2} \ \ {\bf Component-dependent}, \ {\bf radial-independent}, \ {\bf section-independent}; \ {\bf length} \ \ {\bf of} \ \ {\bf COL_DISPERSION_RADIAL} \\ \ is \ \ {\bf NCOMP}$
- **3** Component-dependent, radial-dependent, section-independent; length of COL_DISPERSION_RADIAL is NCOMP · NRAD; ordering is radial-major
- 4 Component-independent, radial-independent, section-dependent; length of COL_DISPERSION_RADIAL is NSEC
- $\begin{tabular}{ll} \bf 5 & {\bf Component-independent, radial-dependent, section-dependent; length of COL_DISPERSION_RADIAL is NRAD \cdot NSEC; ordering is section-major \\ \end{tabular}$
- $\label{eq:component-dependent} \textbf{6} \ \ \text{Component-dependent}, \ \text{radial-independent}, \ \text{section-independent}; \ \text{length of COL_DISPERSION_RADIAL} \\ \ \ \text{is NCOMP} \cdot \text{NSEC}; \ \text{ordering is section-major} \\$
- 7 Component-dependent, radial-dependent, section-dependent; length of COL_DISPERSION_RADIAL is NCOMP · NRAD · NSEC; ordering is section-radial-major

Unit: – Type: int Range: $\{0, \dots, 7\}$ Length: 1

COL_LENGTH Column length

Unit: m Type: double Range: > 0 Length: 1

COL_RADIUS Column radius

Unit: m Type: double Range: > 0 Length: 1

COL_POROSITY Column porosity, either constant (length is 1) or for each radial zone (length is NRAD).

In case of a spatially inhomogeneous setting, the SENS_PARTYPE field is used for indexing the radial zone when specifying parameter sensitivities.

Unit: - Type: double Range: (0,1] Length: 1 / NRAD

FILM_DIFFUSION Film diffusion coefficients for each component of each particle type

Unit: $m s^{-1}$ Type: double Range: ≥ 0 Length: see FILM_DIFFUSION_MULTIPLEX

 $\label{lem:piffusion_multiplex} \textbf{Multiplex} \ \ \text{Multiplex} \ \ \text{mode of FILM_DIFFUSION}. \ \ \text{Determines whether FILM_DIFFUSION} \\ \ \ \text{is treated as component-, type-, and/or section-independent.}$

This field is optional. When left out, multiplexing behavior is inferred from the length of FILM_DIFFUSION.

Valid modes are:

- $\textbf{0} \ \, \text{Component-dependent, type-independent, section-independent; length of FILM_DIFFUSION is } \\ \text{NCOMP}$
- 1 Component-dependent, type-independent, section-dependent; length of FILM_DIFFUSION is NCOMP-NSEC; ordering is section-major
- 2 Component-dependent, type-dependent, section-independent; length of FILM_DIFFUSION is NCOMP-NPARTYPE; ordering is type-major

	NSEC ; ordering is	V 1 V	
$\mathbf{Unit:} -$	Type: int	Range: $\{0,, 3\}$	Length: 1
PAR_POROSITY Partic	le porosity of all 1	particle types or for each part	icle type
$\mathbf{Unit:} -$	Type: doub!	le Range: $(0,1]$	$\textbf{Length:} \ 1 \ / \ \texttt{NPARTYPE}$
PAR_RADIUS Particle	radius of all parti	cle types or for each particle	type
$\mathbf{Unit:}\ \mathbf{m}$	Type: doub	le Range: > 0	$\textbf{Length:} \ 1 \ / \ \texttt{NPARTYPE}$
PAR_CORERADIUS Part 0 m)	ticle core radius o	f all particle types or for each	particle type (optional, defaults to
$\mathbf{Unit:}\ \mathbf{m}$	Type: doub	le $\mathbf{Range:} [0, PAR_RADIUS)$	$\textbf{Length:} \ 1 \ / \ \texttt{NPARTYPE}$
PORE_ACCESSIBILITY to 1)	Pore accessibility	factor of each component in e	ach particle type (optional, defaults
Unit: -	Type: doub!	le Range: $(0,1]$	${f Length: see}$ PORE_ACCESSIBILITY_MULTIPLEX
		iplexing mode of PORE_ACCESS and/or section-independent.	IBILITY. Determines whether PORE_ACCESSIBILITY
This field is opti-	onal. When left or	nt, multiplexing behavior is infe	erred from the length of ${\tt PORE_ACCESSIBILITY}$.
Valid modes are	; :		
O Component-dis NCOMP	lependent, type-in	ndependent, section-independent	ent; length of PORE_ACCESSIBILITY
_	lependent, type-in EC; ordering is sect		e; length of PORE_ACCESSIBILITY is
	lependent, type-de ARTYPE; ordering is		t; length of PORE_ACCESSIBILITY is
_		ependent, section-dependent ering is section-type-major	length of PORE_ACCESSIBILITY is
$\mathbf{Unit:} -$	Type: int	Range: $\{0,, 3\}$	Length: 1
PAR_DIFFUSION Effec	tive particle diffus	sion coefficients of each comp	onent in each particle type
Unit: $m_{MP}^2 s^{-1}$	Type: doub!	le Range: ≥ 0	${f Length: see}$ PAR_DIFFUSION_MULTIPLEX
		ng mode of PAR_DIFFUSION. I and/or section-independent.	Determines whether PAR_DIFFUSION
is treated as con	mponent-, type-, a	and/or section-independent.	Determines whether PAR_DIFFUSION erred from the length of PAR_DIFFUSION.
is treated as con	mponent-, type-, a onal. When left ou	and/or section-independent.	
is treated as con This field is optic Valid modes are	mponent-, type-, a onal. When left ou e:	and/or section-independent. at, multiplexing behavior is infe	
is treated as con This field is optic Valid modes are O Component-d NCOMP 1 Component-d	mponent-, type-, a onal. When left ou e: lependent, type-i	and/or section-independent. at, multiplexing behavior is information in the independent, section-independent dependent, section-dependent	erred from the length of ${\tt PAR_DIFFUSION}$.
is treated as con This field is optic Valid modes are O Component-o NCOMP Component-d NSEC; order Component-d	mponent-, type-, a onal. When left ou e: dependent, type-in lependent, type-in ring is section-ma	and/or section-independent. at, multiplexing behavior is inference independent, section-independent dependent, section-dependent jor sependent, section-independent	erred from the length of PAR_DIFFUSION. dent; length of PAR_DIFFUSION is
is treated as con This field is optic Valid modes are O Component-o NCOMP Component-d NSEC; orde: Component-d NPARTYPE; Component-d	mponent-, type-, a conal. When left out a: dependent, type-in ring is section-may lependent, type-defordering is type-n dependent, type-defordering is type-n dependent, type-defordering is type-n	and/or section-independent. at, multiplexing behavior is informate independent, section-independent dependent, section-dependent jor ependent, section-independent major	erred from the length of PAR_DIFFUSION. dent; length of PAR_DIFFUSION is length of PAR_DIFFUSION is NCOMP.
is treated as con This field is optic Valid modes are O Component-o NCOMP Component-d NSEC; orde: Component-d NPARTYPE; Component-d	mponent-, type-, a conal. When left out a: dependent, type-in ring is section-may lependent, type-defordering is type-n dependent, type-defordering is type-n dependent, type-defordering is type-n	and/or section-independent. at, multiplexing behavior is information in the section of the sect	erred from the length of PAR_DIFFUSION. dent; length of PAR_DIFFUSION is glength of PAR_DIFFUSION is NCOMP.
is treated as con This field is optic Valid modes are O Component-o NCOMP Component-d NSEC; orde: Component-d NPARTYPE; Component-d NPARTYPE: Unit: - PAR_SURFDIFFUSION	mponent-, type-, a conal. When left oute: lependent, type-intring is section-malependent, type-defordering is type-nelependent, type-defordering is type-nelependent, type-deformed is Type: int	and/or section-independent. It, multiplexing behavior is information in the mode pendent, section-independent dependent, section-dependent jor ependent, section-independent major ependent, section-dependent; section-type-major Range: {0,,3} Iffusion coefficients of each bout	erred from the length of PAR_DIFFUSION. dent; length of PAR_DIFFUSION is glength of PAR_DIFFUSION is NCOMP. glength of PAR_DIFFUSION is NCOMP. length of PAR_DIFFUSION is NCOMP.

 $\textbf{3} \ \ \text{Component-dependent, type-dependent, section-dependent; length of FILM_DIFFUSION} \ \text{is NCOMP} \cdot \\$

PAR_SURFDIFFUSION_MULTIPLEX Multiplexing mode of PAR_SURFDIFFUSION. Determines whether PAR_SURFDIFFUSION is treated as component-, type-, and/or section-independent.

This field is optional. When left out, multiplexing behavior is inferred from the length of PAR_SURFDIFFUSION. Valid modes are:

- ${f 0}$ Component-dependent, type-independent, section-independent; length of PAR_SURFDIFFUSION is NBND; ordering is component-major
- $\textbf{1} \ \, \text{Component-dependent, type-independent, section-dependent; length of PAR_SURFDIFFUSION is } \\ \text{NBND} \cdot \text{NSEC; ordering is section-component-major}$
- 2 Component-dependent, type-dependent, section-independent; length of PAR_SURFDIFFUSION is NTOTALBND; ordering is type-component-major
- **3** Component-dependent, type-dependent, section-dependent; length of PAR_SURFDIFFUSION is NTOTALBND · NSEC; ordering is section-type-component-major

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

VELOCITY Indicates flow direction in each radial zone (forward if value is positive, backward if value is negative), see Section 2.2.6).

In case of a spatially inhomogeneous setting, the SENS_PARTYPE field is used for indexing the radial cell when specifying parameter sensitivities.

Unit: - Type: double Range: \mathbb{R} Length: see VELOCITY_MULTIPLEX

 $\label{lem:velocity_multiplex} \textbf{Wultiplexing mode of VELOCITY}. \ \ Determines \ whether \ \textbf{VELOCITY} \ is \ treated \ as \ radial and/or \ section-independent.$

This field is optional. When left out, multiplexing behavior is inferred from the length of VELOCITY. Valid modes are:

0 Radial-independent, section-independent; length of VELOCITY is 1

- 1 Radial-dependent, section-independent; length of VELOCITY is NRAD
- 2 Section-dependent; length of VELOCITY is NSEC
- 3 Radial-dependent, section-dependent; length of VELOCITY is NRAD·NSEC; ordering is section-major

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

PAR_TYPE_VOLFRAC Volume fractions of the particle types. The volume fractions can be set homogeneous or individually along both axes. For each cell, the volume fractions have to sum to 1.

In case of a spatially inhomogeneous setting, the SENS_SECTION field is used for indexing the axial cell and the SENS_REACTION field is used for indexing the radial cell when specifying parameter sensitivities.

This field is optional in case of only one particle type.

Unit: - Type: double Range: [0,1] Length: see PAR_TYPE_VOLFRAC_MULTIPLE

PAR_TYPE_VOLFRAC_MULTIPLEX Multiplexing mode of PAR_TYPE_VOLFRAC. Determines whether PAR_TYPE_VOLFRAC is treated as radial- and/or section-independent.

 $This field is optional. \ When left out, multiplexing behavior is inferred from the length of {\tt PAR_TYPE_VOLFRAC}.$

Valid modes are:

- 0 Radial-independent, axial-independent; length of PAR_TYPE_VOLFRAC is NPARTYPE
- 1 Radial-dependent, axial-independent; length of PAR_TYPE_VOLFRAC is NRAD · NPARTYPE; ordering is radial-major
- 2 Axial-dependent; length of PAR_TYPE_VOLFRAC is NCOL · NPARTYPE; ordering is axial-major
- **3** Radial-dependent, axial-dependent; length of PAR_TYPE_VOLFRAC is NCOL·NRAD·NPARTYPE; ordering is axial-radial-major

Unit: – Type: int Range: $\{0, \dots, 3\}$ Length: 1

Group /input/model/unit_XXX/discretization - UNIT_TYPE = GENERAL_RATE_MODEL_2D

NCOL Number of axial column discretization cells

Type: int Range: ≥ 1 Length: 1

NRAD Number of radial column discretization cells

Type: int Range: ≥ 1 Length: 1

NPARTYPE Number of particle types. Optional, inferred from the length of NPAR or NBOUND if left out.

Type: int Range: ≥ 1 Length: 1

NPAR Number of particle (radial) discretization cells for each particle type

Type: int Range: ≥ 1 Length: 1 / NPARTYPE

NBOUND Number of bound states for each component in each particle type in type-major ordering

Type: int Range: ≥ 0 Length: NCOMP / NPARTYPE · NCOMP

PAR_GEOM Specifies the particle geometry for all or each particle type. Valid values are SPHERE, CYLINDER, and SLAB. Optional, defaults to SPHERE.

Type: string Range: {SPHERE, CYLINDER, SLAB} Length: 1 / NPARTYPE

RADIAL_DISC_TYPE Specifies the radial discretization scheme. Valid values are EQUIDISTANT, EQUIVOLUME, and USER_DEFINED.

Type: string Length: 1

RADIAL_COMPARTMENTS Coordinates for the radial compartment boundaries (ignored if RADIAL_DISC_TYPE \neq USER_DEFINED). The coordinates are absolute and have to include the endpoints 0 and COLUMN_RADIUS. The values are expected in ascending order (i.e., 0 is the first and COLUMN_RADIUS the last value in the array).

Unit: m Type: double Range: [0, COLUMN_RADIUS] Length: NRAD + 1

PAR_DISC_TYPE Specifies the discretization scheme inside the particles for all or each particle type. Valid values are EQUIDISTANT_PAR, EQUIVOLUME_PAR, and USER_DEFINED_PAR.

Type: string Length: 1 / NPARTYPE

PAR_DISC_VECTOR Node coordinates for the cell boundaries (ignored if PAR_DISC_TYPE \neq USER_DEFINED_PAR). The coordinates are relative and have to include the endpoints 0 and 1. They are later linearly mapped to the true radial range $[r_{c,j}, r_{p,j}]$. The coordinates for each particle type are appended to one long vector in type-major ordering.

Unit: – Type: double Range: [0,1] Length: $\sum_{i} (NPAR_i + 1)$

PAR_BOUNDARY_ORDER Order of accuracy of outer particle boundary condition. Optional, defaults to 2.

Type: int Range: $\{1,2\}$ Length: 1

USE_ANALYTIC_JACOBIAN Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0)

Type: int Range: $\{0,1\}$ Length: 1

LINEAR_SOLVER_BULK Linear solver used for the sparse column bulk block. This field is optional, the best available method is selected (i.e., sparse direct solver if possible).

Valid values are:

DENSE Converts the sparse matrix into a banded matrix and uses regular LAPACK. Slow and memory intensive, but always available.

UMFPACK Uses the UMFPACK sparse direct solver (LU decomposition) from SuiteSparse. Fast, but has to be enabled when compiling and requires UMFPACK library.

SUPERLU Uses the SuperLU sparse direct solver (LU decomposition). Fast, but has to be enabled when compiling and requires SuperLU library.

Type: string Range: {DENSE, UMFPACK, SUPERLU}Length: 1

RECONSTRUCTION Type of reconstruction method for fluxes

Type: string Range: WEN0 Length: 1

GS_TYPE Type of Gram-Schmidt orthogonalization, see IDAS guide Section 4.5.7.3, p. 41f. A value of 0 enables classical Gram-Schmidt, a value of 1 uses modified Gram-Schmidt.

Type: int Range: $\{0,1\}$ Length: 1

MAX_KRYLOV Defines the size of the Krylov subspace in the iterative linear GMRES solver (0: MAX_KRYLOV = $NCOL \cdot NRAD \cdot NCOMP \cdot NPARTYPE$)

Type: int Range: $\{0, \dots, NCOL \cdot NRAD \cdot NCOMP LAMPAGETYPE\}$

MAX_RESTARTS Maximum number of restarts in the GMRES algorithm. If lack of memory is not an issue, better use a larger Krylov space than restarts.

Type: int Range: ≥ 0 Length: 1

SCHUR_SAFETY Schur safety factor; Influences the tradeoff between linear iterations and nonlinear error control; see IDAS guide Section 2.1 and 5.

Type: double Range: ≥ 0 Length: 1

Continuous stirred tank reactor model

Group /input/model/unit_XXX - UNIT_TYPE = CSTR

UNIT_TYPE Specifies the type of unit operation model

Type: string Range: CSTR Length: 1

NCOMP Number of chemical components in the chromatographic medium

Type: int Range: ≥ 1 Length: 1

NBOUND Number of bound states for each component in each particle type in type-major ordering (optional, defaults to all 0)

Type: int Range: ≥ 0 Length: NPARTYPE · NCOMP

USE_ANALYTIC_JACOBIAN Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0)

Type: int Range: $\{0,1\}$ Length: 1

ADSORPTION_MODEL Specifies the type of binding model of each particle type

Type: string Range: See Section 3.5.5 Length: NPARTYPE

REACTION_MODEL_BULK Specifies the type of reaction model of the bulk volume. The model is configured in the subgroup reaction_bulk.

Type: string Range: See Section 3.5.6 Length: 1

REACTION_MODEL_PARTICLES Specifies the type of reaction model of each particle type (or of all particle types if length is 1). The model is configured in the subgroup reaction_particle, or reaction_particle_XXX in case of disabled multiplexing.

Type: string Range: See Section 3.5.6 Length: 1 / NPARTYPE

REACTION_MODEL_PARTICLES_MULTIPLEX Multiplexing mode of REACTION_MODEL_PARTICLES. If set to 0, each particle type has a different reaction model and the length of REACTION_MODEL_PARTICLES is NPARTYPE. If set to 1, all particle types share the same reaction model and the length of REACTION_MODEL_PARTICLES is 1.

This field is optional and inferred from the length of REACTION_MODEL_PARTICLES if left out.

Unit: – Type: int Range: $\{0,1\}$ Length: 1

INIT_C Initial concentrations for each component in the mobile phase

Unit: $mol m_{IV}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

INIT_VOLUME Initial tank volume

Unit: m^3 Type: double Range: ≥ 0 Length: 1

INIT_Q Initial concentrations for each bound state of each component in the bead solid phase of each particle type in type-component-major ordering. This field is optional and defaults to all 0.

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: NTOTALBND

INIT_STATE Full state vector for initialization (optional, INIT_C, INIT_Q, and INIT_VOLUME will be ignored; if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range: \mathbb{R} Length: NDOF / 2NDOF

POROSITY Porosity ε (defaults to 1)

Unit: - Type: double Range: (0,1] Length: 1

FLOWRATE_FILTER Flow rate of pure liquid without components (optional, defaults to 0 m³ s⁻¹)

Unit: $m^3 s^{-1}$ Type: double Range: ≥ 0 Length: 1 / NSEC

PAR_TYPE_VOLFRAC Volume fractions of the particle types, have to sum to 1

Unit: - Type: double Range: [0, 1] Length: NPARTYPE

3.5.3 Flux reconstruction methods

Group /input/model/unit_XXX/discretization/weno — WENO parameters

BOUNDARY_MODEL Boundary model type:

0 Lower WENO order (stable)

- **1** Zero weights (unstable for small $D_{\rm ax}$)
- **2** Zero weights for $p \neq 0$ (stable?)
- **3** Large ghost points

Type: int Range: $\{0,1,2,3\}$ Length: 1

WENO_EPS WENO ε

Type: double Range: ≥ 0 Length: 1

WENO_ORDER WENO order, also called WENO k:

1 Standard upwind scheme (order 1)

2 WENO 2 (order 3)

3 WENO 3 (order 5)

Type: int Range: $\{1, 2, 3\}$ Length: 1

3.5.4 Nonlinear solver for consistent initialization

Group /input/model/unit_XXX/discretization/consistency_solver - Nonlinear consistency solver parameters

SOLVER_NAME Name of the solver. Available solvers are LEVMAR, ATRN_RES, ATRN_ERR, and COMPOSITE.

Type: string Length: 1

INIT_DAMPING Initial damping factor (default is 0.01)

Type: double Range: ≥ 0 Length: 1

MIN_DAMPING Minimal damping factor (default is 0.0001; ignored by LEVMAR)

Type: double Range: ≥ 0 Length: 1

SUBSOLVERS Vector with names of solvers for the composite solver (only required for composite solver).

See $SOLVER_NAME$ for available solvers.

Type: string Length: > 1

3.5.5 Binding models

Externally dependent binding models Some binding models have a variant that can use external sources as specified in Section 3.5.1 (also see Section 2.3 and Table 2.3.1 on which binding models support this feature). For the sake of brevity, only the standard variant of those binding models is specified below. In order to obtain the format for the externally dependent variant, first replace the binding model name XXX by EXT_XXX. Each parameter p (except for reference concentrations XXX_REFC0 and XXX_REFQ) depends on a (possibly distinct) external source in a polynomial way:

$$p(T) = p_{\mathsf{TTT}}T^3 + p_{\mathsf{TT}}T^2 + p_{\mathsf{T}}T + p.$$

Thus, a parameter XXX_YYY of the standard binding model variant is replaced by the four parameters EXT_XXX_YYY, EXT_XXX_YYY_T, EXT_XXX_YYY_TT, and EXT_XXX_YYY_TTT. Since each parameter can depend on a different external source, the dataset EXTFUN (not listed in the standard variants below) should contain a vector of 0-based integer indices of the external source of each parameter. The ordering of the parameters in EXTFUN is given by the ordering in the standard variant. However, if only one index is passed in EXTFUN, this external source is used for all parameters.

Note that parameter sensitivities with respect to column radius, column length, particle core radius, and particle radius may be wrong when using externally dependent binding models. This is caused by not taking into account the derivative of the external profile with respect to column position.

Non-binding components For binding models that do not support multiple bound states, many parameters can vary per component and their length is taken as NCOMP. However, these models still support non-binding components. In this case, the entries in their parameters that correspond to non-binding components are simply ignored.

Multiple particle types The group that contains the parameters of a binding model in unit operation with index XXX reads /input/model/unit_XXX/adsorption. This is valid for models with a single particle type. If a model has multiple particle types, it may have a different binding model in each type. The parameters are then placed in the group /input/model/unit_XXX/adsorption_YYY instead, where YYY denotes the index of the particle type.

Note that, in any case, /input/model/unit_XXX/adsorption_000 contains the parameters of the first (and possibly sole) particle type. This group also takes precedence over a possibly existing /input/model/unit_XXX/adsorption group.

Group /input/model/unit_XXX/adsorption — ADSORPTION_MODEL = LINEAR

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

LIN_KA Adsorption rate constants for each component

Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: ≥ 0 Length: NCOMP

LIN_KD Desorption rate constants for each component

Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = MULTI_COMPONENT_LANGMUIR

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

MCL_KA Adsorption rate constants

Type: double Range: ≥ 0 Length: NCOMP

 MCL_KD Desorption rate constants

Unit: s^{-1} Type: double Range: > 0 Length: NCOMP

MCL_QMAX Maximum adsorption capacities

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: > 0 Length: NCOMP

Group /input/model/unit_XXX/adsorption — ADSORPTION_MODEL = MULTI_COMPONENT_ANTILANGMUIR

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

MCAL_KA Adsorption rate constants

Unit: $m_{MP}^3 \mod^{-1} s$ Type: double Range: ≥ 0 Length: NCOMP

MCAL_KD Desorption rate constants

Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP

 ${f MCAL_QMAX}$ Maximum adsorption capacities

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: > 0 Length: NCOMP

MCAL_ANTILANGMUIR Anti-Langmuir coefficients (optional)

Unit: $mol m_{SP}^{-3}$ Type: double Range: $\{-1,1\}$ Length: NCOMP

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = MOBILE_PHASE_MODULATOR

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

 MPM_KA Adsorption rate constants

Unit: $m_{MP}^3 \mod^{-1} s^-$ Type: double Range: ≥ 0 Length: NCOMP

 MPM_KD Desorption rate constants

Unit: $m_{MP}^{3\beta} \mod^{-\beta} s$ -Type: double Range: ≥ 0 Length: NCOMP

 $\ensuremath{\mathsf{MPM_QMAX}}$ Maximum adsorption capacities

Unit: $mol m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

MPM_BETA Parameters describing the ion-exchange characteristics (IEX)

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

MPM_GAMMA Parameters describing the hydrophobicity (HIC)

Unit: $m_{MP}^3 \mod^{-1}$ Type: double Range: ≥ 0 Length: NCOMP

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = EXTENDED_MOBILE_PHASE_MODULATOR

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

EMPM_COMP_MODE Determines the mode of each component (0 denotes the modifier component, 1 is linear binding, 2 is modified Langmuir binding). At most one modifier component is allowed, that is, a modifier is not required.

Note that this field has the same name for the externally dependent variant of the model.

Type: int Range: $\{0,1,2\}$ Length: NCOMP

EMPM_KA Adsorption rate constants

Unit: $m_{MP}^3 \mod^{-1} s$ -Type: double Range: ≥ 0 Length: NCOMP

 $\textbf{EMPM_KD} \ \ Desorption \ rate \ constants$

Unit: $m_{MP}^{3\beta} \mod^{-\beta} s$ Type: double Range: ≥ 0 Length: NCOMP

EMPM_QMAX Maximum adsorption capacities

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

EMPM_BETA Parameters describing the ion-exchange characteristics (IEX)

Unit: $mol \, m_{\rm SP}^{-3}$ Type: double Range: ≥ 0 Length: NCOMP

EMPM_GAMMA Parameters describing the hydrophobicity (HIC)

Unit: $m_{MP}^3 \mod^{-1}$ Type: double Range: ≥ 0 Length: NCOMP

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = STERIC_MASS_ACTION

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

SMA_KA Adsorption rate constants

Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: ≥ 0 Length: NCOMP

 $\textbf{SMA_KD}$ Desorption rate constants

Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP

SMA_NU Characteristic charges of the protein; The number of sites ν that the protein interacts with on the resin surface

Type: double Range: ≥ 0 Length: NCOMP

SMA_SIGMA Steric factors of the protein; The number of sites σ on the surface that are shielded by the protein and prevented from exchange with the salt counterions in solution

Type: double Range: ≥ 0 Length: NCOMP

SMA_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites available on the resin surface

Unit: $mol m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: 1

SMA_REFCO Reference liquid phase concentration (optional, defaults to 1.0)

Unit: $mol m_{MP}^{-3}$ Type: double Range: > 0 Length: 1

SMA_REFQ Reference solid phase concentration (optional, defaults to 1.0)

Unit: $mol m_{SP}^{-3}$ Type: double Range: > 0 Length: 1

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = SELF_ASSOCIATION

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

SAI_KA1 Adsorption rate constants

Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: ≥ 0 Length: NCOMP

SAI_KA2 Adsorption rate constants

Unit: $m_{MP}^6 m_{SP}^{-6} s^{-1}$ Type: double Range: ≥ 0 Length: NCOMP

SAI_KD Desorption rate constants

Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP

SAI_NU Characteristic charges ν of the protein

Type: double Range: ≥ 0 Length: NCOMP

SAI_SIGMA Steric factors σ of the protein

Type: double Range: ≥ 0 Length: NCOMP

SAI_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites available on the resin surface

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: 1

SAI_REFCO Reference liquid phase concentration (optional, defaults to 1.0)

Unit: $mol m_{MP}^{-3}$ Type: double Range: > 0 Length: 1

SAI_REFQ Reference solid phase concentration (optional, defaults to 1.0)

Unit: $mol m_{SP}^{-3}$ Type: double Range: > 0 Length: 1

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = SASKA

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

SASKA_H Henry coefficient

Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: \mathbb{R} Length: NCOMP

SASKA_K Quadratic factors

 $\mathbf{Unit:}\ \mathrm{m_{MP}^6\,m_{SP}^{-3}\,mol}\mathbf{T_{yp}e} \ \mathrm{double}\ \mathbf{Range:}\ \mathbb{R} \\ \mathbf{Length:}\ \mathsf{NCOMP^2}$

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = MULTI_COMPONENT_BILANGMUIR

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

MCBL_KA Adsorption rate constants in state-major ordering

Unit: $m_{MP}^3 \mod^{-1} s^-$ Type: double Range: ≥ 0 Length: NSTATES · NCOMP

MCBL_KD Desorption rate constants in state-major ordering

Unit: s^{-1} Type: double Range: ≥ 0 Length: NSTATES · NCOMP

 ${f MCBL_QMAX}$ Maximum adsorption capacities in state-major ordering

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: > 0.0 Length: NSTATES · NCOMP

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = KUMAR_MULTI_COMPONENT_LANGMUIR

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

 $\ensuremath{\mathsf{KMCL_KA}}$ Adsorption pre-exponential factors

Unit: $m_{MP}^3 \mod^{-1} s^-$ Type: double Range: ≥ 0 Length: NCOMP

 $KMCL_KD$ Desorption rate

Unit: $m_{MP}^{3\nu_i} \mod^{-\nu_i} s$ Type: double Range: ≥ 0 Length: NCOMP

KMCL_KACT Activation temperatures

Unit: K Type: double Range: ≥ 0 Length: NCOMP

 ${f KMCL_QMAX}$ Maximum adsorption capacities

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: > 0 Length: NCOMP

KMCL_NU Salt exponents / characteristic charges

Type: double Range: > 0 Length: NCOMP

KMCL_TEMP Temperature

Unit: K Type: double Range: ≥ 0 Length: 1

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = MULTI_COMPONENT_SPREADING

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

MCSPR_KA Adsorption rate constants in state-major ordering

Unit: $m_{MP}^3 \mod^{-1} s$ -Type: double Range: ≥ 0 Length: NTOTALBND

MCSPR_KD Desorption rate constants in state-major ordering

Unit: s^{-1} Type: double Range: ≥ 0 Length: NTOTALBND

MCSPR_QMAX Maximum adsorption capacities in state-major ordering

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: > 0 Length: NTOTALBND

MCSPR_K12 Exchange rates from the first to the second bound state

Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP

MCSPR_K21 Exchange rates from the second to the first bound state

Unit: s^{-1} Type: double Range: > 0 Length: NCOMP

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = MULTISTATE_STERIC_MASS_ACTION

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

MSSMA_KA Adsorption rate constants of the components to the different bound states in component-major ordering

Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: ≥ 0 Length: NTOTALBND

MSSMA_KD Desorption rate constants of the components in the different bound states in component-major ordering

Unit: s^{-1} Type: double Range: ≥ 0 Length: NTOTALBND

MSSMA_NU Characteristic charges of the components in the different bound states in component-major ordering

Type: double Range: ≥ 0 Length: NTOTALBND

MSSMA_SIGMA Steric factors of the components in the different bound states in component-major ordering

Type: double Range: ≥ 0 Length: NTOTALBND

MSSMA_RATES Conversion rates between different bound states in component-row-major ordering

Unit: s^{-1} Type: double Range: ≥ 0 Length: $\sum_{i=0}^{\mathsf{NCOMP}-1} \mathsf{NBND}_i^2$

MSSMA_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites available on the resin surface

Unit: $mol m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: 1

MSSMA_REFCO Reference liquid phase concentration (optional, defaults to 1.0)

Unit: $mol m_{MP}^{-3}$ Type: double Range: > 0 Length: 1

MSSMA_REFQ Reference solid phase concentration (optional, defaults to 1.0)

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: > 0 Length: 1

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = SIMPLE_MULTISTATE_STERIC_MASS_ACTION

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

SMSSMA_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites available on the resin surface

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: 1

SMSSMA_KA Adsorption rate constants of the components to the different bound states in componentmajor ordering Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: ≥ 0 Length: NTOTALBND SMSSMA_KD Desorption rate constants of the components to the different bound states in componentmajor ordering Unit: s^{-1} Type: double Range: ≥ 0 Length: NTOTALBND SMSSMA_NU_MIN Characteristic charges of the components in the first (weakest) bound state Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP SMSSMA_NU_MAX Characteristic charges of the components in the last (strongest) bound state Length: NCOMP Type: double Range: ≥ 0 SMSSMA_NU_QUAD Quadratic modifiers of the characteristic charges of the different components depending on the index of the bound state Type: double Range: \mathbb{R} Length: NCOMP SMSSMA_SIGMA_MIN Steric factors of the components in the first (weakest) bound state Type: double Range: ≥ 0 Length: NCOMP SMSSMA_SIGMA_MAX Steric factors of the components in the last (strongest) bound state Type: double Range: ≥ 0 Length: NCOMP SMSSMA_SIGMA_QUAD Quadratic modifiers of steric factors of the different components depending on the index of the bound state Type: double Range: \mathbb{R} Length: NCOMP SMSSMA_KWS Exchange rates from a weakly bound state to the next stronger bound state Type: double Range: > 0Length: NCOMP SMSSMA_KWS_LIN Linear exchange rate coefficients from a weakly bound state to the next stronger bound state Unit: s^{-1} Type: double Range: \mathbb{R} Length: NCOMP SMSSMA_KWS_QUAD Quadratic exchange rate coefficients from a weakly bound state to the next stronger bound state Unit: s^{-1} Type: double Range: \mathbb{R} Length: NCOMP SMSSMA_KSW Exchange rates from a strongly bound state to the next weaker bound state Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP SMSSMA_KSW_LIN Linear exchange rate coefficients from a strongly bound state to the next weaker bound state Unit: s^{-1} Type: double Range: \mathbb{R} Length: NCOMP SMSSMA_KSW_QUAD Quadratic exchange rate coefficients from a strongly bound state to the next weaker bound state

Type: double Range: \mathbb{R}

SMSSMA_REFCO Reference liquid phase concentration (optional, defaults to 1.0) Type: double Range: > 0

SMSSMA_REFQ Reference solid phase concentration (optional, defaults to 1.0)

Type: double Range: > 0

Length: NCOMP

Length: 1

Length: 1

Unit: s^{-1}

Unit: $mol m_{MP}^{-3}$

Unit: $mol m_{SP}^{-3}$

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = BI_STERIC_MASS_ACTION

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

BISMA_KA Adsorption rate constants in state-major ordering

Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: ≥ 0.0 Length: NSTATES · NCOMP

BISMA_KD Desorption rate constants in state-major ordering

Unit: s^{-1} Type: double Range: ≥ 0.0 Length: NSTATES · NCOMP

BISMA_NU Characteristic charges $\nu_{i,j}$ of the *i*th protein with respect to the *j*th binding site type in state-major ordering

Unit: s^{-1} Type: double Range: ≥ 0.0 Length: NSTATES · NCOMP

BISMA_SIGMA Steric factors $\sigma_{i,j}$ of the *i*th protein with respect to the *j*th binding site type in state-major ordering

Unit: s^{-1} Type: double Range: ≥ 0.0 Length: NSTATES · NCOMP

BISMA_LAMBDA Stationary phase capacity (monovalent salt counterions) of the different binding site types λ_j

Unit: $mol \, m_{SP}^{-3}$ Type: double Range: ≥ 0.0 Length: NSTATES

BISMA_REFCO Reference liquid phase concentration for each binding site type or one value for all types (optional, defaults to 1.0)

Unit: $mol m_{MP}^{-3}$ Type: double Range: > 0 Length: $\{1, NSTATES\}$

BISMA_REFQ Reference solid phase concentration for each binding site type or one value for all types (optional, defaults to 1.0)

Unit: $mol m_{SP}^{-3}$ Type: double Range: > 0 Length: $\{1, NSTATES\}$

Group /input/model/unit_XXX/adsorption - ADSORPTION_MODEL = GENERALIZED_ION_EXCHANGE

IS_KINETIC Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary. If a single value is given, the mode is set for all bound states. Otherwise, the adsorption mode is set for each bound state separately.

Type: int Range: $\{0,1\}$ Length: 1 / NTOTALBND

 ${\tt GIEX_KA}$ Base value of adsorption rate constant

Unit: $m_{MP}^3 m_{SP}^{-3} s^{-1}$ Type: double Range: ≥ 0 Length: NCOMP

GIEX_KA_LIN Coefficient of linear dependence of adsorption rate constant on modifier component

Unit: $[Mod]^{-1}$ Type: double Length: NCOMP

GIEX_KA_QUAD Coefficient of quadratic dependence of adsorption rate constant on modifier component

Unit: $[Mod]^{-2}$ Type: double Length: NCOMP

GIEX_KA_SALT Salt coefficient of adsorption rate constants; difference of water-protein and salt-protein interactions

Type: double Length: NCOMP

GIEX_KA_PROT Protein coefficient of adsorption rate constants; difference of water-protein and protein-protein interactions

Unit: $m_{MP}^3 \text{ mol}^{-1}$ Type: double Length: NCOMP

 $\mbox{\bf GIEX_KD}$ Base value of desorption rate constant

Unit: s^{-1} Type: double Range: ≥ 0 Length: NCOMP

 $\textbf{GIEX_KD_LIN} \ \ \text{Coefficient of linear dependence of desorption rate constant on modifier component}$

Unit: $[Mod]^{-1}$ Type: double Length: NCOMP

GIEX_KD_QUAD Coefficient of quadratic dependence of desorption rate constant on modifier component

Unit: $[Mod]^{-2}$ Type: double Length: NCOMP

GIEX_KD_SALT Salt coefficient of desorption rate constants; difference of water-protein and salt-protein

interactions

Type: double Length: NCOMP

GIEX_KD_PROT Protein coefficient of desorption rate constants; difference of water-protein and protein-

protein interactions

Unit: $m_{MP}^3 \text{ mol}^{-1}$ Type: double Length: NCOMP

<code>GIEX_NU</code> Base value for characteristic charges of the protein; The number of sites ν that the protein

interacts with on the resin surface

 $\mathbf{Type:}\ \mathrm{double}\quad \mathbf{Length:}\ \mathsf{NCOMP}$

GIEX_NU_LIN Coefficient of linear dependence of characteristic charge on modifier component

Unit: $[Mod]^{-1}$ Type: double Length: NCOMP

GIEX_NU_QUAD Coefficient of quadratic dependence of characteristic charge on modifier component

Unit: $[Mod]^{-2}$ Type: double Length: NCOMP

<code>GIEX_SIGMA</code> Steric factors of the protein; The number of sites σ on the surface that are shielded by the

protein and prevented from exchange with the salt counterions in solution

Type: double Range: ≥ 0 Length: NCOMP

 $\textbf{GIEX_LAMBDA} \ \ \text{Stationary phase capacity (monovalent salt counterions)}; \ \ \text{The total number of binding sites}$

available on the resin surface $\,$

Unit: $mol m_{SP}^{-3}$ Type: double Range: ≥ 0 Length: 1

GIEX_REFCO Reference liquid phase concentration (optional, defaults to 1.0)

Unit: $mol m_{MP}^{-3}$ Type: double Range: > 0 Length: 1

 ${f GIEX_REFQ}$ Reference solid phase concentration (optional, defaults to 1.0)

Unit: mol m_{SP}^{-3} Type: double Range: > 0 Length: 1

3.5.6 Reaction models

Externally dependent reaction models Some reaction models have a variant that can use external sources as specified in Section 3.5.1 (also see Section 2.4). For the sake of brevity, only the standard variant of those reaction models is specified below. In order to obtain the format for the externally dependent variant, first replace the reaction model name XXX by EXT_XXX. Each parameter p (except for stoichiometric and exponent matrices) depends on a (possibly distinct) external source in a polynomial way:

$$p(T) = p_{TTT}T^3 + p_{TT}T^2 + p_{T}T + p.$$

Thus, a parameter XXX_YYY of the standard reaction model variant is replaced by the four parameters EXT_XXX_YYY, EXT_XXX_YYY_T, EXT_XXX_YYY_TT, and EXT_XXX_YYY_TTT. Since each parameter can depend on a different external source, the dataset EXTFUN (not listed in the standard variants below) should contain a vector of 0-based integer indices of the external source of each parameter. The ordering of the

parameters in EXTFUN is given by the ordering in the standard variant. However, if only one index is passed in EXTFUN, this external source is used for all parameters.

Note that parameter sensitivities with respect to column radius, column length, particle core radius, and particle radius may be wrong when using externally dependent reaction models. This is caused by not taking into account the derivative of the external profile with respect to column position.

Multiple particle types The group that contains the parameters of a reaction model in unit operation with index XXX reads /input/model/unit_XXX/reaction_particle. This is valid for models with a single particle type. If a model has multiple particle types, it may have a different reaction model in each type. The parameters are then placed in the group /input/model/unit_XXX/reaction_particle_YYY instead, where YYY denotes the index of the particle type.

Note that, in any case, /input/model/unit_XXX/reaction_particle_000 contains the parameters of the first (and possibly sole) particle type. This group also takes precedence over a possibly existing /input/model/unit_XXX/adsorption_particle group.

Group /input/model/unit_XXX/reaction - REACTION_MODEL = MASS_ACTION_LAW

MAL_KFWD_BULK Forward rate constants for bulk volume reactions (available for external functions)

Type: double Range: ≥ 0 Length: NREACT

MAL_KBWD_BULK Backward rate constants for bulk volume reactions (available for external functions)

Type: double Range: ≥ 0 Length: NREACT

MAL_KFWD_LIQUID Forward rate constants for particle liquid phase reactions (available for external functions)

Type: double Range: ≥ 0 Length: NREACT

MAL_KBWD_LIQUID Backward rate constants for particle liquid phase reactions (available for external functions)

Type: double Range: ≥ 0 Length: NREACT

MAL_KFWD_SOLID Forward rate constants for particle solid phase reactions (available for external functions)

Type: double Range: ≥ 0 Length: NREACT

MAL_KBWD_SOLID Backward rate constants for particle solid phase reactions (available for external functions)

Type: double Range: ≥ 0 Length: NREACT

 $\label{eq:mall_stoichiometric} \textbf{MAL_STOICHIOMETRY_BULK} \ \ \text{Stoichiometric matrix of bulk volume reactions as } \ \textbf{NCOMP} \times \textbf{NREACT matrix in row-major storage}$

 $\mathbf{Type:} \ \mathrm{double} \quad \mathbf{Length:} \ \mathsf{NCOMP} \cdot \mathsf{NREACT}$

 $\label{eq:mal_exponent_bulk_fwd} \begin{minipage}{ll} MAL_EXPONENTS_BULK_FWD & Forward exponent matrix of bulk volume reactions as NCOMP \times NREACT matrix in row-major storage (optional, calculated from MAL_STOICHIOMETRY_BULK by default) \\ \end{minipage}$

Type: double Length: NCOMP · NREACT

 $\label{eq:mal_exponent_bulk_bwd} \begin{minipage}{ll} \textbf{MAL_EXPONENTS_BULK_BWD} & \textbf{Backward exponent matrix of bulk volume reactions as NCOMP} \times \textbf{NREACT matrix in row-major storage (optional, calculated from MAL_STOICHIOMETRY_BULK by default)} \\ \end{minipage}$

 $\mathbf{Type:} \ \mathrm{double} \quad \mathbf{Length:} \ \mathsf{NCOMP} \cdot \mathsf{NREACT}$

 $\label{eq:mal_stoichiometric} \textbf{MAL_STOICHIOMETRY_LIQUID} \ \ Stoichiometric \ matrix \ of particle \ liquid \ phase \ reactions \ as \ \mbox{NCOMP} \times \mbox{NREACT} \\ matrix \ in \ row-major \ storage$

 $\mathbf{Type:} \ \mathrm{double} \quad \mathbf{Length:} \ \mathsf{NCOMP} \cdot \mathsf{NREACT}$

 $\label{eq:mallexponents} \begin{aligned} \textbf{MAL_EXPONENTS_LIQUID_FWD} & \mbox{Forward exponent matrix of particle liquid phase reactions as } \textbf{NCOMP} \times \textbf{NREACT} \\ & \mbox{matrix in row-major storage (optional, calculated from MAL_STOICHIOMETRY_LIQUID by default)} \end{aligned}$

 $\mathbf{Type:} \ \mathrm{double} \quad \mathbf{Length:} \ \mathsf{NCOMP} \cdot \mathsf{NREACT}$

 $\label{eq:mall_exponent_bound} \begin{array}{l} \textbf{MAL_EXPONENTS_LIQUID_BWD} \ \ \, \text{Backward exponent matrix of particle liquid phase reactions as NCOMP} \times \\ \textbf{NREACT matrix in row-major storage (optional, calculated from MAL_STOICHIOMETRY_LIQUID by default)} \end{array}$

Type: double Length: $NCOMP \cdot NREACT$

 $\label{eq:mal_exponent_substitution} \begin{tabular}{ll} \textbf{MAL_EXPONENTS_LIQUID_FWD_MODSOLID} & Forward solid phase modifier exponent matrix of particle liquid phase reactions as $NTOTALBND \times NREACT matrix in row-major storage (optional, defaults to all 0) \\ \end{tabular}$

Type: double Length: $NTOTALBND \cdot NREACT$

 $\label{eq:mal_exponent} \begin{tabular}{ll} \textbf{MAL_EXPONENTS_LIQUID_BWD_MODSOLID} & Backward solid phase modifier exponent matrix of particle liquid phase reactions as $NTOTALBND \times NREACT matrix in row-major storage (optional, defaults to all 0) \\ \end{tabular}$

Type: double Length: $NTOTALBND \cdot NREACT$

 $\label{eq:mallstoichiometric} \begin{subarrate}{ll} \textbf{MAL_STOICHIOMETRY_SOLID} & \textbf{Stoichiometric matrix of particle solid phase reactions as $NTOTALBND \times NREACT$ \\ matrix in row-major storage \\ \end{subarray}$

Type: double Length: NTOTALBND · NREACT

 $\label{eq:mal_exponent_solid} \mbox{MAL_EXPONENTS_SOLID_FWD} \mbox{ Forward exponent matrix of particle solid phase reactions as $NTOTALBND \times NREACT matrix in row-major storage (optional, calculated from MAL_STOICHIOMETRY_SOLID by default)}$

Type: double Length: $NTOTALBND \cdot NREACT$

 $\label{eq:mallexponents} \begin{aligned} \textbf{MAL_EXPONENTS_SOLID_BWD} & \mbox{ Backward exponent matrix of particle solid phase reactions as $NTOTALBND \times NREACT matrix in row-major storage (optional, calculated from MAL_STOICHIOMETRY_SOLID by default) \end{aligned}$

Type: double Length: $NTOTALBND \cdot NREACT$

 $\label{eq:mal_exponent_solid_final} \begin{tabular}{ll} \textbf{MAL_EXPONENTS_SOLID_FWD_MODLIQUID} & Forward liquid phase modifier exponent matrix of particle solid phase reactions as $NCOMP \times NREACT$ matrix in row-major storage (optional, defaults to all 0) \\ \end{tabular}$

Type: double Length: $NCOMP \cdot NREACT$

Type: double Length: NCOMP · NREACT

3.5.7 Return data

Group /input/return

WRITE_SOLUTION_TIMES Write times at which a solution was produced (optional, defaults to 1)

Type: int Range: $\{0,1\}$

WRITE_SOLUTION_LAST Write full solution state vector at last time point (optional, defaults to 0)

Type: int Range: $\{0,1\}$

WRITE_SENS_LAST Write full sensitivity state vectors at last time point (optional, defaults to 0)

Type: int Range: $\{0,1\}$

SPLIT_COMPONENTS_DATA Determines whether a joint dataset (matrix or tensor) for all components is created or if each component is put in a separate dataset (XXX_COMP_000, XXX_COMP_001, etc.) (optional, defaults to 1)

Type: int Range: $\{0,1\}$

```
SPLIT_PORTS_DATA Determines whether a joint dataset (matrix or tensor) for all inlet/outlet ports is
      created or if each port is put in a separate dataset (XXX_PORT_000, XXX_PORT_001, etc.) (optional,
     defaults to 1)
     Type: int
                      Range: \{0,1\}
SINGLE_AS_MULTI_PORT Determines whether single port unit operations are treated as multi port unit
     operations in the output naming scheme (i.e., _PORT_XYZ_ is added to the name) (optional, defaults
      to 0)
     Type: int
                      Range: \{0,1\}
Group /input/return/unit_XXX
WRITE_COORDINATES Write coordinates of discretization nodes
                      Range: \{0,1\}
     Type: int
WRITE_SOLUTION_INLET Write solutions at unit operation inlet c_i^l(t,0)
      Type: int
                      Range: \{0,1\}
WRITE_SOLUTION_OUTLET Write solutions at unit operation outlet (chromatograms) c_i^l(t,L)
                      Range: \{0,1\}
      Type: int
WRITE_SOLUTION_BULK Write solutions of the bulk volume c_i^l
     Type: int
                      Range: \{0,1\}
WRITE_SOLUTION_PARTICLE Write solutions of the particle mobile phase c_{i,i}^p
                      Range: \{0,1\}
     Type: int
WRITE_SOLUTION_SOLID Write solutions of the solid phase c_{j,i,m_{j,i}}^s
      Type: int
                      Range: {0,1}
WRITE_SOLUTION_FLUX Write solutions of the bead fluxes j_{f,i}
      Type: int
                      Range: \{0,1\}
WRITE_SOLUTION_VOLUME Write solutions of the volume V
                      Range: \{0,1\}
      Type: int
WRITE_SOLDOT_INLET Write solution time derivatives at unit operation inlet \partial e_i^l(t,0)/\partial t
                      Range: \{0,1\}
      Type: int
WRITE_SOLDOT_OUTLET Write solution time derivatives at unit operation outlet (chromatograms) \partial c_i^i(t,L)/\partial t
     Type: int
                      Range: \{0,1\}
WRITE_SOLDOT_BULK Write solution time derivatives of the bulk volume \partial c_i^l/\partial t
     Type: int
                      Range: \{0,1\}
WRITE_SOLDOT_PARTICLE Write solution time derivatives of the particle mobile phase \partial c_{i,i}^p/\partial t
     Type: int
                      Range: \{0,1\}
WRITE_SOLDOT_SOLID Write solution time derivatives of the solid phase \partial c_{j,i,m_{i,i}}^{s}/\partial t
      Type: int
                      Range: \{0,1\}
WRITE_SOLDOT_FLUX Write solution time derivatives of the bead fluxes \partial j_{f,i}/\partial t
     Type: int
                      Range: \{0,1\}
```

WRITE_SOLDOT_VOLUME Write solution time derivatives of the volume $\partial V/\partial t$

Range: $\{0,1\}$

Type: int

```
WRITE_SENS_INLET Write sensitivities at unit operation inlet \partial c_i^l(t,0)/\partial p
      Type: int
                        Range: \{0,1\}
WRITE_SENS_OUTLET Write sensitivities at unit operation outlet (chromatograms) \partial c_i^l(t,L)/\partial p
                        Range: \{0,1\}
      Type: int
WRITE_SENS_BULK Write sensitivities of the bulk volume \partial c_i^l/\partial p
                        Range: \{0,1\}
      Type: int
WRITE_SENS_PARTICLE Write sensitivities of the particle mobile phase \partial c_{j,i}^p/\partial p
      Type: int
                        Range: \{0,1\}
WRITE_SENS_SOLID Write sensitivities of the solid phase \partial c_{i,i,m_{i,i}}^{s}/\partial p
      Type: int
                        Range: \{0,1\}
WRITE_SENS_FLUX Write sensitivities of the bead fluxes \partial j_{f,i}/\partial p
      Type: int
                        Range: \{0,1\}
WRITE_SENS_VOLUME Write sensitivities of the volume \partial V/\partial p
                        Range: \{0, 1\}
      Type: int
WRITE_SENSOOT_INLET Write sensitivity time derivatives at unit operation inlet \partial^2 c_i^l(t,0)/(\partial p,\partial t)
      Type: int
                        Range: {0,1}
WRITE_SENSDOT_OUTLET Write sensitivity time derivatives at unit operation outlet (chromatograms)
      \partial^2 c_i^l(t,L)/(\partial p,\partial t)
      Type: int
                        Range: \{0,1\}
WRITE_SENSOOT_BULK Write sensitivity time derivatives of the bulk volume \partial^2 c_i^l/(\partial p, \partial t)
      Type: int
                        Range: \{0,1\}
WRITE_SENSDOT_PARTICLE Write sensitivity time derivatives of the particle mobile phase \partial^2 c_{i,i}^p/(\partial p, \partial t)
                        Range: \{0,1\}
      Type: int
WRITE_SENSOOT_SOLID Write sensitivity time derivatives of the solid phase \partial^2 c_{j,i,m_{j,i}}^s/(\partial p,\partial t)
      Type: int
                        Range: \{0,1\}
WRITE_SENSDOT_FLUX Write sensitivity time derivatives of the bead fluxes \partial^2 j_{f,i}/(\partial p, \partial t)
                        Range: \{0,1\}
      Type: int
WRITE_SENSOOT_VOLUME Write sensitivity time derivatives of the volume \partial^2 V/(\partial p, \partial t)
      Type: int
                        Range: \{0,1\}
3.5.8 Parameter sensitivities
Group /input/sensitivity
NSENS Number of sensitivities to be computed
                        Range: \geq 0
                                                                 Length: 1
```

SENS_METHOD Method used for computation of sensitivities (algorithmic differentiation)

Length: 1

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Type: int

Type: string Range: ad1

Group /input/sensitivity/param_XXX

SENS_UNIT Unit operation index

Type: int Range: ≥ 0 Length: ≥ 1

SENS_NAME Name of the parameter

Type: string Range: See Length: ≥ 1

SENS_COMP Component index (-1) if parameter is independent of components

Type: int Range: ≥ -1 Length: ≥ 1

SENS_PARTYPE Particle type index (-1 if parameter is independent of particle types)

Type: int Range: ≥ -1 Length: ≥ 1

SENS_REACTION Reaction index (-1 if parameter is independent of reactions)

Type: int Range: ≥ -1 Length: ≥ 1

SENS_BOUNDPHASE Bound phase index (-1) if parameter is independent of bound phases)

Type: int Range: ≥ -1 Length: ≥ 1

SENS_SECTION Section index (-1 if parameter is independent of sections)

Type: int Range: ≥ -1 Length: ≥ 1

SENS_ABSTOL Absolute tolerance used in the computation of the sensitivities (optional). Rule of thumb: ABSTOL/PARAM_VALUE

Type: double Range: ≥ 0.0 Length: ≥ 1

SENS_FACTOR Linear factor of the combined sensitivity (optional, taken as 1.0 if left out)

Type: double Range: \mathbb{R} Length: ≥ 1

3.5.9 Solver configuration

Group /input/solver

NTHREADS Number of used threads

Type: int Range: ≥ 1 Length: 1

USER_SOLUTION_TIMES Vector with timepoints at which the solution is evaluated

Unit: s Type: double Range: ≥ 0 Length: Arbitrary

CONSISTENT_INIT_MODE Consistent initialization mode (optional, defaults to 1). Valid values are:

- 0 None
- 1 Full
- 2 Once, full
- 3 Lean
- 4 Once, lean
- $\mathbf{5}$ Full once, then lean
- 6 None once, then full
- 7 None once, then lean

Type: int Range: $\{0, \dots, 7\}$ Length: 1

CONSISTENT_INIT_MODE_SENS Consistent initialization mode for parameter sensitivities (optional, defaults to 1). Valid values are:

0 None

- 1 Full
- 2 Once, full
- 3 Lean
- 4 Once, lean
- **5** Full once, then lean
- **6** None once, then full
- 7 None once, then lean

Range: $\{0, ..., 7\}$ Type: int

Length: 1

Group /input/solver/time_integrator

ABSTOL Absolute tolerance in the solution of the original system

Type: double **Range:** > 0

Length: 1

RELTOL Relative tolerance in the solution of the original system

Type: double Range: ≥ 0

Length: 1

ALGTOL Tolerance in the solution of the nonlinear consistency equations

Type: double **Range:** > 0

Length: 1

RELTOL_SENS Relative tolerance in the solution of the sensitivity systems

Type: double Range: ≥ 0

Length: 1

INIT_STEP_SIZE Initial time integrator step size for each section or one value for all sections (0.0: IDAS default value), see IDAS guide 4.5, p. 36f.

Type: double Range: ≥ 0

Length: 1 / NSEC

MAX_STEPS Maximum number of timesteps taken by IDAS (0: IDAS default = 500), see IDAS guide Sec. 4.5

Type: int

Range: ≥ 0

Length: 1

MAX_STEP_SIZE Maximum size of timesteps taken by IDAS (optional, defaults to unlimited 0.0), see IDAS guide Sec. 4.5

Unit: s

Type: double **Range:** ≥ 0

Length: 1

ERRORTEST_SENS Determines whether (forward) sensitivities take part in local error test (optional, defaults to 1)

Type: int

Range: $\{0,1\}$

Length: 1

MAX_NEWTON_ITER Maximum number of Newton iterations in time step (optional, defaults to 3)

Type: int

Range: ≥ 0

Length: 1

MAX_ERRTEST_FAIL Maximum number of local error test failures in time step (optional, defaults to 7)

Type: int

Range: ≥ 0

Length: 1

MAX_CONVTEST_FAIL Maximum number of Newton convergence test failures (optional, defaults to 10)

Type: int

Range: ≥ 0

Length: 1

MAX_NEWTON_ITER_SENS Maximum number of Newton iterations in forward sensitivity time step (optional, defaults to 3)

Type: int

Range: ≥ 0

Length: 1

Group /input/solver/sections

NSEC Number of sections

Type: int Range: ≥ 1 Length: 1

SECTION_TIMES Simulation times at which the model changes or behaves discontinuously; including start and end times

Unit: s Type: double Range: ≥ 0 Length: NSEC + 1

SECTION_CONTINUITY Continuity indicator for each section transition: 0 (discontinuous) or 1 (continuous).

Type: int Range: $\{0,1\}$ Length: NSEC -1

3.6 Output group

Group /output/solution

LAST_STATE Full state vector at the last time point of the time integrator if WRITE_SOLUTION_LAST in /input/return is enabled

Type: double

LAST_STATE_YDOT Full time derivative state vector at the last time point of the time integrator if WRITE_SOLUTION_LAST in /input/return is enabled

Type: double

LAST_STATE_SENSY_XXX Full state vector of the XXXth sensitivity system at the last time point of the time integrator if WRITE_SENS_LAST in /input/return is enabled

Type: double

 $\textbf{LAST_STATE_SENSYDOT_XXX} \ \, \text{Full time derivative state vector of the XXXth sensitivity system at the last time point of the time integrator if WRITE_SENS_LAST in /input/return is enabled$

Type: double

Group /output/solution

 $\textbf{SOLUTION_TIMES} \ \, \text{Time points at which the solution is written if WRITE_SOLUTION_TIMES in /input/return is enabled}$

Unit: s Type: double

Group /output/solution/unit_XXX

SOLUTION_BULK Interstitial solution as $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$ tensor in row-major storage

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_PARTICLE Mobile phase solution inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $mol m_{MP}^{-3}$ Type: double

SOLUTION_PARTICLE_PARTYPE_XXX Mobile phase solution inside the particles of type XXX as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined.

Unit: $mol m_{MP}^{-3}$ Type: double

SOLUTION_SOLID Solid phase solution inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $mol m_{MP}^{-3}$ Type: double

SOLUTION_SOLID_PARTYPE_XXX Solid phase solution inside the particles of type XXX as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined.

Unit: $mol m_{SP}^{-3}$ Type: double

SOLUTION_FLUX Flux solution as $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$ tensor in row-major storage

Unit: $mol m^{-2} s^{-1}$ Type: double

SOLUTION_VOLUME Volume solution

Unit: m³ Type: double

SOLUTION_OUTLET Tensor of solutions at the unit operation outlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_INLET Tensor of solutions at the unit operation inlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_OUTLET_COMP_XXX Component XXX of the solution at all outlet ports of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_INLET_COMP_XXX Component XXX of the solution at all inlet ports of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_OUTLET_PORT_XXX All components at outlet port XXX of the solution of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_INLET_PORT_XXX All components at inlet port XXX of the solution of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_OUTLET_PORT_XXX_COMP_YYY Component YYY at outlet port XXX of the solution of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple outlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

Unit: $mol m_{IV}^{-3}$ Type: double

SOLUTION_INLET_PORT_XXX_COMP_YYY Component YYY at inlet port XXX of the solution of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple inlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

Unit: $mol m_{IV}^{-3}$ Type: double

- **SOLDOT_BULK** Interstitial solution time derivative as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage Unit: mol m_{IV}⁻³ s⁻¹ Type: double
- SOLDOT_PARTICLE Mobile phase solution time derivative inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $mol m_{MP}^{-3} s^{-1}$ Type: double

SOLDOT_PARTICLE_PARTYPE_XXX Mobile phase solution time derivative inside the particles of type XXX as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined.

Unit: $mol m_{MP}^{-3} s^{-1}$ Type: double

SOLDOT_SOLID Solid phase solution time derivative inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $mol m_{MP}^{-3} s^{-1}$ Type: double

SOLDOT_SOLID_PARTYPE_XXX Solid phase solution time derivative inside the particles of type XXX as $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined.

Unit: $mol m_{SP}^{-3} s^{-1}$ Type: double

SOLDOT_FLUX Flux solution time derivative as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage Unit: mol m⁻² s⁻² Type: double

 ${f SOLDOT_VOLUME}$ Volume solution time derivative

Unit: $m^3 s^{-1}$ Type: double

SOLDOT_OUTLET Tensor of solution time derivatives at the unit operation outlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

SOLDOT_INLET Tensor of solution time derivatives at the unit operation inlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

SOLDOT_OUTLET_COMP_XXX Component XXX of the solution time derivative at all outlet ports of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

SOLDOT_INLET_COMP_XXX Component XXX of the solution time derivative at all inlet ports of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

SOLDOT_OUTLET_PORT_XXX All components at outlet port XXX of the solution time derivative of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

SOLDOT_INLET_PORT_XXX All components at inlet port XXX of the solution time derivative of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

SOLDOT_OUTLET_PORT_XXX_COMP_YYY Component YYY at outlet port XXX of the solution time derivative of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple outlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

SOLDOT_INLET_PORT_XXX_COMP_YYY Component YYY at inlet port XXX of the solution time derivative of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple inlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ Type: double

Group /output/sensitivity/param_XXX/unit_YYY

SENS_BULK Interstitial sensitivity as $n_{\mathrm{Time}} imes \mathtt{UNITOPORDERING}$ tensor in row-major storage

Unit: mol m_{IV} [Paraffiype: double

SENS_PARTICLE Mobile phase sensitivity inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{MP}}^{-3}$ [Para**Ty** $\overline{\mathbf{p}}$ **e**: double

SENS_PARTICLE_PARTYPE_XXX Mobile phase sensitivity inside the particles of type XXX as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined.

Unit: $mol m_{MP}^{-3}$ [Para**Try** $\bar{\mathbf{p}}$ **e**: double

SENS_SOLID Solid phase sensitivity inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{MP}}^{-3} [\operatorname{Para} \mathbf{T}_{\mathbf{N}} \mathbf{\bar{p}}^{1} \mathbf{e} : \operatorname{double}]$

SENS_SOLID_PARTYPE_XXX Solid phase sensitivity inside the particles of type XXX as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined.

Unit: mol m_{SP} [Paraffype: double

SENS_FLUX Flux sensitivity as $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$ tensor in row-major storage

Unit: $mol m^{-2} s^{-1}$ [Paryappe]: double

SENS_VOLUME Volume sensitivity

Unit: $m^3 [Param]^{-1}$ Type: double

SENS_OUTLET Tensor of sensitivities at the unit operation outlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: mol m_{IV} [Paraffiype: double

SENS_INLET Tensor of sensitivities at the unit operation inlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3}$ [Parafflype: double

SENS_OUTLET_COMP_XXX Component XXX of the sensitivity at all outlet ports of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3}$ [Parafflype: double

SENS_INLET_COMP_XXX Component XXX of the sensitivity at all inlet ports of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3}$ [Parafflype: double

SENS_OUTLET_PORT_XXX All components at outlet port XXX of the sensitivity of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3}$ [Parafilype: double

SENS_INLET_PORT_XXX All components at inlet port XXX of the sensitivity of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3}$ [Parafflype: double

SENS_OUTLET_PORT_XXX_COMP_YYY Component YYY at outlet port XXX of the sensitivity of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple outlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

Unit: $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3} [\operatorname{Paraff}]_{\overline{\mathbf{v}}} \overline{\mathbf{p}} \mathbf{e}$: double

SENS_INLET_PORT_XXX_COMP_YYY Component YYY at inlet port XXX of the sensitivity of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple inlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

Unit: mol m_{IV} [Parafflype: double

SENSDOT_BULK Interstitial sensitivity time derivative as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage

Unit: $mol m_{IV}^{-3} s^{-1}$ [Payene]: double

SENSDOT_PARTICLE Mobile phase sensitivity time derivative inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $mol m_{MP}^{-3} s^{-1}$ [Phype] double

SENSDOT_PARTICLE_PARTYPE_XXX Mobile phase sensitivity time derivative inside the particles of type XXX as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined.

Unit: $mol \, m_{\mathrm{MP}}^{-3} \, s^{-1} \, [PFype]$ double

SENSDOT_SOLID Solid phase sensitivity time derivative inside the particles as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if just one particle type is defined.

Unit: $mol m_{MP}^{-3} s^{-1}$ [PRype] double

SENSDOT_SOLID_PARTYPE_XXX Solid phase sensitivity time derivative inside the particles of type XXX as $n_{\text{Time}} \times \text{UNITOPORDERING}$ tensor in row-major storage. Only present if more than one particle type is defined

Unit: $mol m_{SP}^{-3} s^{-1}$ [Pareque]: double

SENSDOT_FLUX Flux sensitivity time derivative as $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$ tensor in row-major storage

Unit: $mol m^{-2} s^{-2}$ [PLyapue]: double

SENSDOT_VOLUME Volume sensitivity time derivative

Unit: s^{-3} m [Param] Type: double

SENSDOT_OUTLET Tensor of sensitivity time derivatives at the unit operation outlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ [PLyapse]: double

SENSDOT_INLET Tensor of sensitivity time derivatives at the unit operation inlet with components as columns in time-port-major storage. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are both disabled. If the unit operation only has a single port, the port-dimension is removed if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ [PLyapue]: double

SENSDOT_OUTLET_COMP_XXX Component XXX of the sensitivity time derivative at all outlet ports of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ [PLyappe]: double

SENSDOT_INLET_COMP_XXX Component XXX of the sensitivity time derivative at all inlet ports of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is enabled and SPLIT_PORTS_DATA is disabled. If the unit operation only has a single port, a vector (1D array) is returned instead of a matrix if SINGLE_AS_MULTI_PORT is disabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ [PLyappe]: double

SENSDOT_OUTLET_PORT_XXX All components at outlet port XXX of the sensitivity time derivative of the unit operation as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ [PLyape]: double

SENSDOT_INLET_PORT_XXX All components at inlet port XXX of the sensitivity time derivative of the unit operation inlet as matrix in time-major storage. Only present if SPLIT_COMPONENTS_DATA is disabled and SPLIT_PORTS_DATA is enabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ [Phyapae]: double

SENSDOT_OUTLET_PORT_XXX_COMP_YYY Component YYY at outlet port XXX of the sensitivity time derivative of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple outlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

 $\mathbf{Unit:} \; \mathrm{mol} \, \mathrm{m}_{\mathrm{IV}}^{-3} \, \mathrm{s}^{-1} \, [P\textbf{Tryappe}] \text{-double}$

SENSDOT_INLET_PORT_XXX_COMP_YYY Component YYY at inlet port XXX of the sensitivity time derivative of the unit operation. Only present if SPLIT_COMPONENTS_DATA and SPLIT_PORTS_DATA are enabled, and the unit operation has multiple inlet ports. If the unit operation only has a single port, the field is created if SINGLE_AS_MULTI_PORT is enabled.

Unit: $mol m_{IV}^{-3} s^{-1}$ [Phyapue]: double

Group /output/coordinates/unit_XXX

AXIAL_COORDINATES Axial coordinates of the bulk discretization nodes

Unit: m Type: double Length: NCOL

RADIAL_COORDINATES Radial coordinates of the bulk discretization nodes (only for 2D unit operations)

Unit: m Type: double Length: NRAD

PARTICLE_COORDINATES_XXX Coordinates of the particle discretization nodes in particles of type XXX

Unit: m Type: double Length: NPAR

3.7 Meta group

Group /meta

FILE_FORMAT Version of the file format (defaults to 040000 = 4.0.0 if omitted) with two digits per part

(Major.Minor.Patch)

In/out: In Type: int

 $\textbf{CADET_VERSION} \ \ \text{Version of the executed CADET simulator}$

In/out: Out Type: string

CADET_COMMIT Git commit SHA1 from which the CADET simulator was built

In/out: Out Type: string

CADET_BRANCH Git branch from which the CADET simulator was built

In/out: Out Type: string

TIME_SIM Time that the time integration took (excluding any preparations and postprocessing)

Unit: s In/out: Out Type: double

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