

# Manual

The CADET Authors

CADET Version 3.1.4

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# 1 Simulation

This chapter gives an overview of the simulation process and the different steps involved.

CADET uses a backward-differentiation-formula (BDF) time discretization as implemented by the IDAS solver from SUNDIALS [Hin+05]. Each time step requires the solution of a nonlinear algebraic system of equations is performed by a Newton method. Since chromatographic systems can exhibit strong nonlinearity and stiff systems, the Jacobian of the equation system is always updated (i.e., an "exact" Newton method is used).

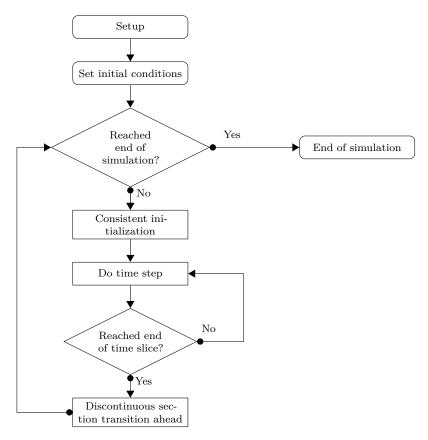


Figure 1.0.1: General time integration procedure

The general control flow of a simulation is shown in Fig. 1.0.1. Some aspects of the tasks involved are discussed below.

### 1.1 Time sections and transitions

The timespan  $[t_0, t_{\text{end}}]$  of the simulation can be divided into multiple time sections  $[t_i, t_{i+1}]$  with

$$t_0 < t_1 < \ldots < t_i < t_{i+1} < \ldots < t_{\text{end}}.$$

Time sections are used for various purposes, for example, for defining piecewise polynomials used as inlet profiles or external function, for changing operating conditions such as flow rates, or for changing the connectivity in a network of unit operations. A sequence of (one or more) time sections with smooth transitions is called a time slice (see Fig. 1.1.1).

A transition from one section to the next can either be smooth or discontinuous. On smooth transitions, the time integrator can just step over the section transition. Contrary to smooth transitions, a discontinuous transition requires some extra work in resetting the time integrator and finding consistent initial conditions for the next time slice.

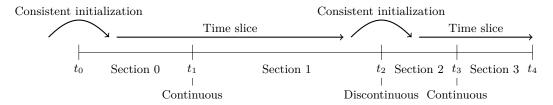


Figure 1.1.1: Time sections, time slices, and transitions between them

**Time section specification** Time sections are specified by an array (SECTION\_TIMES, see Tab. 3.5.7) which contains the  $t_i$  that denote the start- and endpoint of a time section  $[t_i, t_{i+1}]$ . This means, that the array contains n+1 values if there are n sections. The smoothness of a section transition is indicated by the array SECTION\_CONTINUITY, which contains n-1 elements if there are n sections.

# 1.2 Setup

In the setup phase, the model specification is checked and data structures are prepared for the simulation. The simulation can be run multiple times with different initial conditions, which also allows to continue a simulation. However, the model structure must not change after this point. This means, the different model and submodel types (e.g., unit operation model, binding model) as well as their discretization (i.e., number of cells) and structure (i.e., number of components, number of reactions) are fixed. On the contrary, their parameters (e.g., flow rates, porosities, dispersion coefficients) may change on different runs of a prepared simulation.

This assumption allows an accelerated simulation as there is no memory allocation performed during time integration.

#### 1.3 Consistent initialization

At the beginning of the simulation and on discontinuous section transitions, consistent initial conditions have to be computed. In CADET, the general differential-algebraic equation (DAE)

$$F(t, y, \dot{y}, p) = 0$$

consists of purely algebraic equations that do not contain  $\dot{y}$  and dynamic equations. Let  $\mathcal{I}_d$  be the index set of dynamic equations and  $\mathcal{I}_a$  the index set of algebraic equations. The general DAE can be decomposed into

$$M\dot{y}_{\mathcal{I}_d} + G_{\mathcal{I}_d}(t, y, p) = 0, \tag{1.1}$$

$$G_{\mathcal{I}_a}(t, y, p) = 0, (1.2)$$

where M is the so called "mass matrix", which essentially consist of the state time derivative Jacobian  $\partial F/\partial \dot{y}$ . Furthermore, it is assumed that the Jacobian of the algebraic equations with respect to the algebraic variables  $\partial G_{\mathcal{I}_a}/\partial y_{\mathcal{I}_a}$  is invertible. Hence, the original DAE is of differential index 1.

Given  $y_{\mathcal{I}_d}$ , consistently initialization means finding  $y_{\mathcal{I}_a}$ ,  $y_{\mathcal{I}_d}$ , and  $\dot{y}$  such that the DAE holds at the initial time point  $t_0$  and can be solved for some non-empty time span. Thus, consistent initial conditions

 $y_0$  and  $\dot{y}_0$  satisfy

$$\begin{split} 0 &= M\dot{y}_{\mathcal{I}_{d}} + G_{\mathcal{I}_{d}}\left(t_{0}, y, p\right), \\ 0 &= G_{\mathcal{I}_{a}}\left(t_{0}, y, p\right), \\ 0 &= \frac{\partial G_{\mathcal{I}_{a}}}{\partial t}\left(t_{0}, y, p\right) + \frac{\partial G_{\mathcal{I}_{a}}}{\partial y}\left(t_{0}, \dot{y}, p\right). \end{split}$$

The last equation, which determines  $\dot{y}_{\mathcal{I}_a}$ , arises from taking the total derivative with respect to time t of the second equation.

Concluding, a nonlinear algebraic equation system has to be solved in order to compute consistent initial conditions. The error in the solution of these systems is controlled by the ALGTOL setting (see Tab. 3.5.7).

# 1.4 Time stepping

Given the general differential-algebraic equation

$$F(t, y, \dot{y}, p) = 0,$$
  $y(t_0) = y_0,$   $\dot{y}(t_0) = \dot{y}_0$ 

with consistent initial values  $y_0$  and  $\dot{y}_0$ , the BDF discretization uses past time steps  $y(t_{\tau-i})$  to compute the current  $\dot{y}(t_{\tau})$  as

$$\dot{y}\left(t_{\tau}\right) = \frac{1}{\Delta t_{\tau}} \sum_{i=0}^{q_{\tau}} \alpha_{\tau,i} y\left(t_{\tau-i}\right),\,$$

where  $t_{\tau}$  denotes some time step. The IDAS time integrator adaptively changes the step size  $\Delta t_{\tau} = t_{\tau} - t_{\tau-1}$  and order  $1 \leq q_{\tau} \leq 5$  in each time step and automatically determines the corresponding coefficients  $\alpha_{\tau,i}$ . This *ansatz* is inserted into the DAE and the resulting nonlinear algebraic equation system is solved for  $y_{\tau} = y(t_{\tau})$  using a Newton method.

$$H(y_{\tau}) = F\left(t_{\tau}, y_{\tau}, \frac{1}{\Delta t_{\tau}} \sum_{i=0}^{q_{\tau}} \alpha_{\tau, i} y_{\tau-i}, p\right) = 0$$
 (1.3)

This requires (possibly many) solutions of linear equation systems involving the Jacobian of H given by

$$\frac{\mathrm{d}H}{\mathrm{d}y_{\tau}} = \frac{\partial F}{\partial y} + \frac{\alpha_{\tau,0}}{\Delta t_{\tau}} \frac{\partial F}{\partial \dot{y}}.$$

The Jacobian of H, which is the linear combination of the partial derivatives  $\partial F/\partial y$  and  $\partial F/\partial \dot{y}$ , can be calculated in two ways. The fastest way is the analytic computation, which is to be preferred if available. However, the implementation of the analytic Jacobian can be time consuming and complicated, especially when quickly testing new models. As a remedy, CADET offers to compute the complicated Jacobian  $\partial F/\partial y$  automatically by automatic differentiation (AD). The much simpler Jacobian  $\partial F/\partial \dot{y}$  has to be implemented manually.

Adaptivity The BDF order  $q_{\tau}$  and size of the time step  $t_{\tau}$  are chosen such that the error bounds are satisfied with minimal work. Equation (1.3) is solved with a Newton iteration using very few iterations (MAX\_NEWTON\_ITER from Table 3.5.7) since the starting point is assumed to be close to the solution as the time steps are small. If the solution of the Newton iteration does not pass the *convergence test*, the step size  $t_{\tau}$  is reduced and the Newton iteration is tried again. This may happen at most MAX\_CONVTEST\_FAIL times, otherwise time integration is aborted with failure. Having a solution of the Newton iteration at hand, a *local truncation error test* is performed which controls the error of the BDF method. If this test fails, step size  $t_{\tau}$  and order  $q_{\tau}$  are adapted and the process restarts with solving the nonlinear system Eq. (1.3). The error test may be failed at most MAX\_ERRTEST\_FAIL times before time integration is aborted with failure.

Error bounds for the local truncation error test are specified by an absolute tolerance (ABSTOL) and a relative tolerance (RELTOL). Note that the relative tolerance only works for non-zero values, whereas zero values are accounted for by the absolute tolerance. For example, a relative tolerance of  $10^{-4}$  and absolute tolerance of  $10^{-8}$  requests 3 significant digits (correct digits after the comma in scientific notation) and considers all numbers with magnitude smaller than  $10^{-8}$  as 0.

The size of the first time step in a time slice is given by INIT\_STEP\_SIZE from Table 3.5.7. If a simulation fails on the first time step, it might help to reduce the initial step size. It can also help to increase the error tolerances, at the cost of imprecise results over the whole course of the simulation.

It can happen that due to severe stiffness or ill-posed models a simulation may exceed its computational budget given by the maximum number of time steps (per time slice) MAX\_STEPS. In this case, the formulation of the model should be checked and, if necessary, the error tolerances increased. Note that the number of time steps is independent of possible USER\_SOLUTION\_TIMES and refers to internal steps of the time integrator.

On the other extreme, the time integrator might jump over an important change in the solution or not capture some feature of the solution because of too large time steps. This can be avoided by specifying the maximum time step size MAX\_STEP\_SIZE. However, such a situation is unlikely to occur and can also be alleviated by placing a discontinuous section transition at the time of the event.

Note that while the error in time integration is controlled and can (in theory) be made arbitrarily small, the spatial discretization error is not. Hence, it is important and necessary to test different spatial resolutions (number of cells) in order to find a sufficiently accurate (but minimal) number of grid cells. This is particularly relevant for problems with steep fronts as a low spatial resolution adds a substantial amount of numerical dispersion to the solution.

## 1.5 Parameter sensitivities

Parameter sensitivities  $s = \partial y/\partial p$  of a solution y to the DAE with respect to some parameter p are required for various tasks, for example, parameter estimation, process design, and process analysis. The CADET simulator implements the forward sensitivity approach which creates a linear companion DAE for each sensitive parameter

$$\begin{split} 0 &= \frac{\mathrm{d}}{\mathrm{d}p} F(t,y,\dot{y},p) = \frac{\partial F}{\partial y}(t,y,\dot{y},p) \frac{\partial y}{\partial p} + \frac{\partial F}{\partial \dot{y}}(t,y,\dot{y},p) \frac{\partial \dot{y}}{\partial p} + \frac{\partial F}{\partial p}(t,y,\dot{y},p) \\ &= \frac{\partial F}{\partial y}(t,y,\dot{y},p)s + \frac{\partial F}{\partial \dot{y}}(t,y,\dot{y},p)\dot{s} + \frac{\partial F}{\partial p}(t,y,\dot{y},p). \end{split}$$

These linear DAEs depend on the solution y,  $\dot{y}$  of the original DAE system. Consequently, the sensitivity systems are solved together with the original DAE system in a staggered approach [FTB97].

After the nonlinear Eq. (1.3) has been successfully solved using Newton iteration (i.e., it has passed the convergence test), each sensitivity is solved using the same Newton iteration. If direct linear solvers were used, this iteration would convergence with a single iteration as there is no nonlinearity. The Newton method for the sensitivities performs at most MAX\_NEWTON\_ITER\_SENS (see Table 3.5.7) iterations. Assuming the convergence test has passed for each sensitivity, the local truncation error test is performed for the full set of variables (i.e., original system and sensitivities). The sensitivities can be excluded from the local truncation error test by setting ERRORTEST\_SENS appropriately. Note that time integration step size is always affected by sensitivities due to possible convergence test failures.

While the Jacobians  $\partial F/\partial y$  and  $\partial F/\partial \dot{y}$  can be computed either analytically or via AD, the last term  $\partial F/\partial p$  is always computed by AD. In fact, the terms  $\partial F/\partial y$  and  $\partial F/\partial p$  can be computed by one AD-enabled evaluation of F using the vector mode with appropriate seed vectors [Püt+16].

A parameter sensitivity is specified by identifying the involved parameters, which can be more than one (see below). Parameters are identified by name and multiple indices, which may not all be used (see Tab. 3.5.6). Indices that are not used for identification (e.g., flow rate is independent of bound phase or component) are set to -1. All other indices are zero-based.

Note that the sensitivity systems need to be consistently initialized, too. However, since only linear systems are involved, no nonlinear equation system has to be solved and the procedure is much simpler computationally.

**Joint parameters** The use of AD also enables support of meta parameters or joint parameters. Consider the situation in which several parameters  $p_1, p_2, \ldots, p_n$  depend (linearly) on a single meta parameter p

$$p_i = p_i(p) = \alpha_i p$$
 for  $i = 1, \dots, n$ 

and some coefficients  $\alpha_i \in \mathbb{R}$ . Hence, the DAE residual function F becomes

$$F(t, y, \dot{y}, p_1(p), \dots, p_n(p)) = 0$$

and the partial derivative  $\partial F/\partial p$  can be computed automatically by AD:

$$\frac{\partial F}{\partial p} = \frac{\partial F}{\partial p_1} \alpha_1 + \ldots + \frac{\partial F}{\partial p_n} \alpha_n.$$

In order to use this functionality, all parameters involved have to be specified in a single sensitivity instance (param\_XXX group, see Tables 3.5.6 and 3.5.6) by using arrays for the name and indices instead of scalar values. The coefficients  $\alpha_i$  are provided in the SENS\_FACTOR vector.

A simple example for a situation, which benefits from having a single meta parameter, would be a chain of unit operations in a network in which the sensitivity with respect to the flow rate is to be computed. Instead of computing the sensitivity with respect to each single flow rate and fusing them together in a postprocessing step, a meta parameter that maps to all flow rates can be introduced.

Note that nonlinear relationships between original parameter and meta parameters are supported by updating the coefficients  $\alpha_i$  before each simulation:

$$\alpha_i = \frac{\partial p_i}{\partial p}.$$

# 2 Models

# 2.1 Network of unit operation models

Unit operation models can be composed into a network or graph, in which a node represents a unit operation and an edge denotes a connection between two unit operations. When utilized to full extent, this allows the simulation of complicated setups and processes (e.g., SMB, MCSGP). A more simple use case is the addition of plug flows and stirred tanks up- and downstream of a column in order to account for dead volume and additional dispersion from the tubing.

In a network, outlets of unit operations can be connected to any number of inlets of unit operations. Even direct cycles, where an outlet of a unit operation is connected to its own inlet, are possible. A unit operation does not have to possess both inlet and outlet, but it has to have at least one of them. Pseudo unit operations such as inlet and outlet serve as sources and sinks for the network. However, the latter is not strictly required as any terminal node (i.e., a unit operation that possesses an outlet but does not have an outgoing connection) serves as a sink.

Each connection between two unit operations (i.e., an edge in the graph) is equipped with a volumetric flow rate that determines the mass flow from source to target unit operation. Those flow rates are used to determine the weight of the different incoming feeds at a unit operation's inlet port. Some unit operations can infer their internal flow rate (e.g., interstitial velocity) from their total incoming volumetric flow rate. In general, the mass balance at a unit operation has to be closed, except for unit operations that act as source or sink in the network and variable volume units (e.g., stirred tanks).

The network of unit operations uses "connection"-variables  $c_{\text{con}}$  to connect the different unit operations with each other. The inlet port variables  $c_{\text{in},n}$  of unit operation n are attached to  $c_{\text{con},n}$  via

$$c_{\text{in},n,i} = c_{\text{con},n,i}, \qquad i = 1, \dots, N_{\text{comp},n}.$$
 (2.1)

The connection variables  $c_{\text{con},n,i}$  collect all inflows of component i into unit operation n:

$$c_{\text{con},n,i} = \frac{\sum_{m=1}^{N_{\text{units}}} \sum_{j=1}^{N_{\text{comp},m}} S_{(n,i),(m,j)} F_m c_{\text{out},m,j}}{\sum_{m=1}^{N_{\text{units}}} \hat{S}_{n,m} F_m},$$
(2.2)

where  $F_m$  denotes the volumetric flow rate of unit operation m,  $S_{(n,i),(m,j)} \in \{0,1\}$  is a connection matrix indicating whether component i of unit operation n is connected to component j of unit operation m, and  $\hat{S}_{n,m} \in \{0,1\}$  is another connection matrix indicating whether unit operation n is connected to unit operation m, that is

$$\hat{S}_{n,m} = \begin{cases} 1 & \text{if } \sum_{i=1}^{N_{\text{comp},n}} \sum_{j=1}^{N_{\text{comp},m}} S_{(n,i),(m,j)} \ge 1, \\ 0 & \text{otherwise.} \end{cases}$$

**Specification of network connections** The connections between the different unit operations in the network are specified by a table. The first two columns specify source and destination unit operation id. The next two columns give source and destination component indices. Finally, the fith column specifies the volumetric flow rate of this connection.

With this setup it is possible to connect single components of unit operations with each other yielding a maximum in flexibility. However, the predominant case is to connect all components of the source unit operations with their respective counterparts in the destination unit. This can easily be done by setting both component indices to -1 instead of writing a separate row for each component of the connection.

Note that in case of multiple rows for one connection between two unit operations (e.g., in case of separate component connections) the flow rate of the first row of that connection is used and all following

flow rates are ignored. Consequently, there can only be one flow rate for a connection between two unit operations regardless of which components are connected.

The connection table is expected in row-major storage format (i.e., the rows are appended to one long array). See Table 3.5.1.

**Valve switches** The connectivity of the network can only change on a discontinuous section transition. Such a transition with changing connectivity is referred to as valve switch and the connectivity itself as valve configuration.

A list of valve configurations with at least one entry is required. Each valve configuration consists of a network connectivity table as described in Section 2.1 and a section index. The latter denotes the section in which the connectivity table becomes active. Hence, the one required (i.e., the first) entry must have a section index of 0 denoting the initial connectivity.

Note that the section index has to be monotonically increasing throughout the list of valve configurations. See Tables 3.5.1 and 3.5.1.

# 2.2 Unit operation models

#### 2.2.1 Inlet

A system inlet unit operation is a pseudo unit operation since there is no physical correspondence. The inlet serves as a mass source in the network of unit operations. Consequently, it only possesses an outlet port and no inlet port. Note that an inlet unit operation can provide arbitrary many components and there can be arbitrary many inlet unit operations in a network.

An inlet unit operation provides a feed in which the concentration of each component is given by a profile. The most common profile is a piecewise cubic polynomial, which can both represent discontinuous signals (e.g., pulse or step) and smooth  $C^2$  signals (cubic spline):

$$c_{i}(t) = \sum_{k=1}^{N_{\text{sect}}} \mathbb{1}_{[t_{k}, t_{k+1})}(t) \left[ a_{k,i} (t - t_{k})^{3} + b_{k,i} (t - t_{k})^{2} + d_{k,i} (t - t_{k}) + f_{k,i} \right],$$

where  $0 \le t_1 < t_2 < \ldots < t_{N_{\text{sect}}+1} \le T_{\text{sim}}$  is a decomposition of the simulation time interval  $[0, T_{\text{sim}}]$  into pieces  $[t_k, t_{k+1})$ . On each piece, the profile is given by a cubic (fourth order) polynomial shifted to the beginning  $t_k$  of the piece.

See Tables 3.5.2 and 3.5.2.

#### 2.2.2 **Outlet**

A system outlet unit operation is a pseudo unit operation since there is no physical correspondence. The outlet serves as a sink (terminal node) in the network of unit operations. Since any terminal node in the network is a sink (see Section 2.1), outlet unit operations are not strictly necessary. However, in some applications (e.g., SMB) only a certain fraction of a unit operation's output is taken out of the system and the rest is recycled. In this case, outlet unit operations are required in order to avoid unbalanced mass flow in the other unit operations.

Outlets can also be of help if the output of multiple unit operations merges together leaving the network. Instead of manually adding the streams together in a post-processing step, the unit operations can be connected to the same outlet unit.

See Table 3.5.2.

## 2.2.3 General rate model (GRM)

The general rate model is the most comprehensive model of mass transfer in column liquid chromatography, when only the axial coordinate in the column and the radial coordinate in the beads are considered [Kuč65; Gu95; Gui+06; FG04].

The main assumptions are:

- 1. The cross sections of the column are homogenous in terms of interstitial volume, fluid flow, and distribution of components. Thus, only one spatial coordinate in axial direction is needed and radial transport is neglected in the column bulk volume.
- 2. The bead radii  $r_p$  are much smaller than the column radius  $r_c$  and the column length L. Therefore, the beads can be seen as continuously distributed inside the column (i.e., at each point there is interstitial and bead volume).

Variable	Domain	Description
$\overline{i}$	$\{0,\ldots,N_{\rm comp}-1\}$	Component index
j	$\{0,\ldots,N_{\mathrm{partype}}-1\}$	Particle type index
$m_{j,i}$	$\{0,\ldots,N_{\mathrm{bnd},j,i}-1\}$	Bound state index of $i$ th component in $j$ th particle
		type
$m_{j}$	$\left\{0, \dots, \sum_{i=0}^{N_{\text{comp}}-1} N_{\text{bnd}, j, i} - 1\right\}$	Total bound state index in particle type $j$
t	$[0, T_{\mathrm{end}}]$	Time coordinate
z	[0, L]	Axial coordinate
r	$[r_{c,j}, r_{p,j}]$	Generic bead radial coordinate
$c_i^l(t,z)$	$[0, T_{\mathrm{end}}] \times [0, L]$	Interstitial concentration of the $i$ th component
$c_{i,i}^p(t,z,r)$	$[0, T_{\text{end}}] \times [0, L] \times [r_{c,j}, r_{p,j}]$	Mobile phase concentration of the $i$ th component in
J,		the $j$ th particle type
$c_{j,i,m_{j,i}}^s(t,z,r)$	$[0, T_{\mathrm{end}}] \times [0, L] \times [r_{c,j}, r_{p,j}]$	Solid phase concentration of the <i>i</i> th component's
-		$m_{j,i}$ th bound state in particles of type j
$j_{f,j,i}(t,z)$	$[0, T_{\mathrm{end}}] \times [0, L]$	Flux of the $i$ th component through stagnant film into
		the bead of type $j$

Table 2.2.1: Variables and unknowns

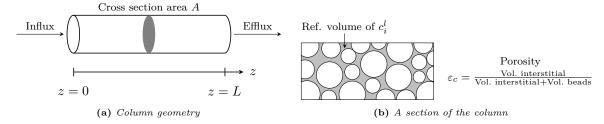


Figure 2.2.1: Column bulk model

The GRM describes transport of solute molecules through the interstitial column volume by convective flow, band broadening caused by axial dispersion, mass transfer resistance through a stagnant film around the beads, pore (and surface) diffusion in the porous beads [MWW96; SS68; Miy07], and adsorption to the inner bead surfaces.

Consider a column of length L > 0 filled with spherical beads of (possibly) multiple types with radius  $r_{p,j} \ll L$  (see Fig. 2.2.1), where j is the particle type index. The mass balance in the interstitial column volume is described by

$$\frac{\partial c_i^l}{\partial t} = -u \frac{\partial c_i^l}{\partial z} + D_{\text{ax}} \frac{\partial^2 c_i^l}{\partial z^2} - \frac{1}{\beta_c} \sum_j d_j \frac{3}{r_{p,j}} k_{f,j,i} \left[ c_i^l - c_{j,i}^p(\cdot, \cdot, r_{p,j}) \right]. \tag{2.3}$$

Here,  $c_i^l : [0, T_{\mathrm{end}}] \times [0, L] \to \mathbb{R}^{\geq 0}$  denotes the concentration in the interstitial column volume,  $c_{j,i}^p : [0, T_{\mathrm{end}}] \times [0, L] \times [r_{c,j}, r_{p,j}] \to \mathbb{R}^{\geq 0}$  the liquid phase concentration in the beads,  $k_{f,j,i}$  the film diffusion coefficient,  $D_{\mathrm{ax}}$  the dispersion coefficient, u the interstitial velocity,  $d_j$  the volume fraction of particle type j, and  $\beta_c = \varepsilon_c/(1-\varepsilon_c)$  the column phase ratio, where  $\varepsilon_c$  is the column porosity (ratio of interstitial volume to total column volume).

Danckwerts boundary conditions [Dan53] are applied to inlet and outlet of the column:

$$uc_{\text{in},i}(t) = uc_i^l(t,0) - D_{\text{ax}} \frac{\partial c_i^l}{\partial z}(t,0) \qquad \forall t > 0,$$
(2.4)

$$\frac{\partial c_i^l}{\partial z}(t, L) = 0 \qquad \forall t > 0. \tag{2.5}$$

Note that the outlet boundary condition Eq. (2.5) is also known as "do nothing" or natural outflow condition.

In the liquid phase of the porous beads (see Fig. 2.2.2) the mass balance is given by

$$\frac{\partial c_{j,i}^{p}}{\partial t} + \frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} \frac{\partial}{\partial t} \sum_{m_{j,i}} c_{j,i,m_{j,i}}^{s} = \underbrace{D_{p,j,i} \left[ \frac{\partial^{2}}{\partial r^{2}} + \frac{2}{r} \frac{\partial}{\partial r} \right] c_{j,i}^{p}}_{\text{Pore diffusion}} + \underbrace{\frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} D_{s,j,i} \left[ \frac{\partial^{2}}{\partial r^{2}} + \frac{2}{r} \frac{\partial}{\partial r} \right] \sum_{m_{j,i}} c_{j,i,m_{j,i}}^{s}}_{\text{Surface diffusion}},$$

$$(2.6)$$

where  $c_{j,i,m_{j,i}}^s$ :  $[0,T_{\mathrm{end}}] \times [0,L] \times [r_{c,j},r_{p,j}] \to \mathbb{R}^{\geq 0}$  denotes the solid phase concentration of the ith component's  $m_{j,i}$ th bound state in the beads of jth type,  $D_{p,j,i}$  the effective diffusion coefficient in the beads,  $D_{s,j,i}$  the surface diffusion coefficient,  $F_{\mathrm{acc},j,i} \in [0,1]$  the pore accessibility factor, and  $\varepsilon_{p,j}$  the particle porosity (ratio of pore volume to total bead volume). The inner bead radius  $r_{c,j} \in [0,r_{p,j})$  is assumed to be 0 by default, but can be positive in order to account for core shell particles that have an impermeable core.

The GRM is used with both quasi-stationary (Eq. (2.7)) and dynamic (Eq. (2.8)) binding models.

quasi-stationary: 
$$0 = f_{ads,j} \left( c_j^p, c_j^s \right),$$
 (2.7)

dynamic: 
$$\frac{\partial c_j^s}{\partial t} = f_{\text{ads},j} \left( c_j^p, c_j^s \right).$$
 (2.8)

Note that  $c_j^p$  and  $c_j^s$  denote the vector of all  $c_{j,i}^p$  and  $c_{j,i,m_{j,i}}^s$ , respectively.

The boundary conditions of the bead model the film diffusion and are given for all  $t \in (0, \infty)$  and  $z \in [0, L]$  by

$$k_{f,j,i} \left[ c_i^l - c_{j,i}^p(\cdot, \cdot, r_{p,j}) \right] = F_{\text{acc},j,i} \varepsilon_p D_{p,j,i} \frac{\partial c_{j,i}^p}{\partial r} (\cdot, \cdot, r_{p,j}) + (1 - \varepsilon_{p,j}) D_{s,j,i} \sum_{m_{j,i}} \frac{\partial c_{j,i,m_{j,i}}^s}{\partial r} (\cdot, \cdot, r_{p,j}), \quad (2.9)$$

$$\frac{\partial c_{j,i}^p}{\partial r}(\cdot, \cdot, r_{c,j}) = 0. \tag{2.10}$$

By default, the following initial conditions are applied for all  $z \in [0, L]$  and  $r \in [r_{c,j}, r_{p,j}]$ :

$$c_i^l(0,z)=0, \qquad \qquad c_{j,i}^p(0,z,r)=0, \qquad \qquad c_{j,i,m_{j,i}}^s(0,z,r)=0. \tag{2.11}$$

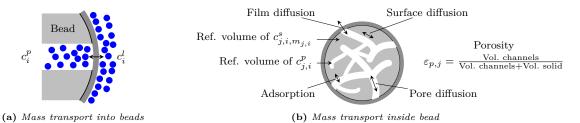


Figure 2.2.2: Column bead model

See Table 3.5.2.

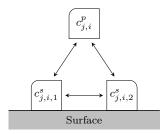


Figure 2.2.3: Binding with multiple bound states

**Multiple particle types** A particle type has its own set of mass transfer parameters  $\varepsilon_{p,j}$ ,  $D_{p,j}$ ,  $D_{s,j}$ , etc (see Eq. (2.6)) and its own binding model  $f_{\rm ads}$  (including a possibly differing number of bound states). This allows, for example, modeling of particle size distributions or potential applications with differently functionalized beads (e.g., immobilized enzymes).

The distribution of the particle types is governed by their volume fractions  $d_j$  in Eq. (2.3). The volume fractions have to sum to 1:

$$\sum_{j=0}^{N_{\text{partype}}-1} d_j = 1.$$

Size exclusion chromatography The general rate model can be used to simulate size exclusion chromatography (SEC) [Gu95]. The particle porosity  $\varepsilon_{p,j}$  on the mobile phase side of the transport equations is replaced by a component-dependent accessible porosity

$$\varepsilon_{p,j,i} = F_{\text{acc},j,i}\varepsilon_{p,j},$$
(2.12)

where the pore accessibility factor  $F_{\text{acc},j,i}$  ranges in (0,1].

Small molecules that can enter any pore have  $F_{\text{acc},j,i} = 1$ , whereas larger molecules that can enter some, but not small pores, have values  $0 < F_{\text{acc},j,i} < 1$ . The other extreme is given by molecules so large that they cannot enter any pore and, consequently,  $F_{\text{acc},j,i} = 0$ . Note that  $F_{\text{acc},j,i} = 0$  is not allowed in a simulation, which can be circumvented by setting  $k_{f,j,i} = 0$ .

By default,  $F_{\mathrm{acc},j,i}=1$  for all components i and all particle types j, which disables size exclusion chromatography. In order to simulate pure SEC, binding is disabled by setting  $N_{\mathrm{bnd},i}=0$  for all components i and applying no binding model. If adsorption is present, it is important to note that any saturation capacity (e.g.,  $q_{\mathrm{max}}$  of Langmuir-type binding models) is subject to the full pore volume fraction  $\varepsilon_{p,j}$ .

Note that multiple particle types can also be used to aid in modeling size exclusion effects, see Section 2.2.3.

**Specification of flow rate / velocity and direction** Since volumetric flow rates are specified for each network connection, the unit operation can infer its interstitial velocity via

$$u = u_{\rm int} = \frac{F_{\rm in}}{A\varepsilon_c},$$

where  $F_{\rm in}$  denotes the volumetric flow rate and A the cross section area. Note that without the bulk porosity  $\varepsilon_c$ , the superficial velocity would be obtained.

The direction of flow inside the unit operation is governed by the sign of the interstitial velocity u. A positive sign results in (standard) forward flow, whereas a negative sign reverses the flow direction. Note that in case of reversed flow, the chromatogram is returned at the unit operation's *inlet*, which may not be returned from simulation by default.

The final behavior is controlled by the interplay of cross section area and interstitial velocity:

1. If cross section area A is given and u is not, u is inferred from the volumetric flow rate.

- 2. If u is given and A is not, the volumetric flow rate is ignored and the provided interstitial velocity is used.
- 3. If both cross section area A and interstitial velocity u are given, the magnitude of the actual interstitial velocity u is inferred from the volumetric flow rate and the flow direction is given by the sign of the provided u.

## 2.2.4 Lumped rate model with pores (LRMP)

The lumped rate model with pores [Gui+06; FG04] deviates from the general rate model (see Section 2.2.3) by neglecting pore diffusion. The particle phase  $c_j^p$  is still there, but no mass transfer happens except for binding and film diffusion. Hence, the model equations are given by

$$\frac{\partial c_i^l}{\partial t} = -u \frac{\partial c_i^l}{\partial z} + D_{\text{ax}} \frac{\partial^2 c_i^l}{\partial z^2} - \frac{1}{\beta_c} \sum_i d_j \frac{3}{r_{p,j}} k_{f,j,i} \left[ c_i^l - c_{j,i}^p \right], \qquad (2.13)$$

$$\frac{\partial c_{j,i}^{p}}{\partial t} + \frac{1 - \varepsilon_{p,j}}{F_{\text{acc},j,i}\varepsilon_{p,j}} \frac{\partial}{\partial t} \sum_{m_{j,i}} c_{j,i,m_{j,i}}^{s} = \frac{3}{F_{\text{acc},j,i}\varepsilon_{p,j}r_{p,j}} k_{f,j,i} \left[ c_{i}^{l} - c_{j,i}^{p} \right]$$

$$(2.14)$$

with the same meanings of variables and parameters as in the general rate model. The equations are complemented by Danckwerts boundary conditions [Dan53]

$$uc_{\text{in},i}(t) = uc_i^l(t,0) - D_{\text{ax}} \frac{\partial c_i^l}{\partial z}(t,0) \qquad \forall t > 0,$$
  
$$\frac{\partial c_i^l}{\partial z}(t,L) = 0 \qquad \forall t > 0.$$

As for the general rate model, both quasi-stationary and dynamic binding models are supported:

quasi-stationary: 
$$0 = f_{\mathrm{ads},j} \left( c_j^p, c_j^s \right),$$
 
$$\mathrm{dynamic:} \ \frac{\partial c_j^s}{\partial t} = f_{\mathrm{ads},j} \left( c_j^p, c_j^s \right).$$

By default, the following initial conditions are applied for all  $z \in [0, L]$ :

$$c_i^l(0,z) = 0,$$
  $c_{j,i}^p(0,z) = 0,$   $c_{j,i,m_{j,i}}^s(0,z) = 0.$  (2.15)

Multiple particle types are supported (see Section 2.2.3). This model can also be used to simulate size exclusion chromatography (see Section 2.2.3). For the specification of flow rate and direction, the same holds as for the general rate model (see Section 2.2.3). See Table 3.5.2.

# 2.2.5 Lumped rate model without pores (LRM)

The lumped rate model without pores [Gui+06; FG04] deviates from the lumped rate model with pores (see Section 2.2.4) by neglecting pores completely. The particle phase  $c^p$  is removed and the porosity  $\varepsilon_t$  is taken as total porosity

$$\varepsilon_t = \varepsilon_c + (1 - \varepsilon_c) \,\varepsilon_p. \tag{2.16}$$

The phase ratio is denoted by  $\beta_t = \varepsilon_t/(1-\varepsilon_t)$  accordingly. The model equations are given by

$$\frac{\partial c_i^l}{\partial t} + \frac{1}{\beta_t} \frac{\partial}{\partial t} \sum_{m_i} c_{i,m_i}^s = -u \frac{\partial c_i^l}{\partial z} + D_{\text{ax}} \frac{\partial^2 c_i^l}{\partial z^2}, \tag{2.17}$$

where  $\beta_t = \varepsilon_t/(1-\varepsilon_t)$  denotes the (total) phase ratio. The equations are complemented by Danckwerts boundary conditions [Dan53]

$$uc_{\text{in},i}(t) = uc_i^l(t,0) - D_{\text{ax}} \frac{\partial c_i^l}{\partial z}(t,0) \qquad \forall t > 0,$$
  
$$\frac{\partial c_i^l}{\partial z}(t,L) = 0 \qquad \forall t > 0.$$

Both quasi-stationary and dynamic binding models are supported:

quasi-stationary: 
$$0 = f_{\text{ads}}\left(c^l, c^s\right)$$
, dynamic:  $\frac{\partial q}{\partial t} = f_{\text{ads}}\left(c^l, c^s\right)$ .

By default, the following initial conditions are applied for all  $z \in [0, L]$ :

$$c_i^l(0,z) = 0,$$
  $c_{i,m_i}^s(0,z) = 0.$  (2.18)

Note that by setting  $\varepsilon_t = 1$ , removing all bound states by setting  $N_{\text{bnd},i} = 0$  for all components i, and applying no binding model, a dispersive plug flow reactor (DPFR) is obtained.

For the specification of flow rate and direction, the same holds as for the general rate model (see Section 2.2.3). See Table 3.5.2.

### 2.2.6 Continuous stirred tank reactor model (CSTR)

The continuous stirred tank reactor model is a basic building block in unit operation networks and often used to model holdup volume. When combined with a binding model, it can be used to model batch uptake experiments.

Assuming that the fluid inside the tank is well-mixed and that the volume can vary, the governing equations are given by

$$\frac{\mathrm{d}}{\mathrm{d}t} \left( \left[ c_i + \frac{1 - \varepsilon}{\varepsilon} \sum_j d_j \sum_{m_{j,i}} c_{j,i,m_{j,i}}^s \right] V \right) = F_{\mathrm{in}} c_{\mathrm{in},i} - F_{\mathrm{out}} c_i,$$

which balances the mass, the binding equation

$$\begin{split} \text{quasi-stationary:} & \quad 0 = f_{\text{ads},j} \left( c, c_j^s \right), \\ \text{dynamic:} & \quad \frac{\partial c_j^s}{\partial t} = f_{\text{ads},j} \left( c, c_j^s \right), \end{split}$$

depending on whether quasi-stationary or dynamic binding is used, and the evolution of volume

$$\frac{\mathrm{d}V}{\mathrm{d}t} = F_{\mathrm{in}} - F_{\mathrm{out}} - F_{\mathrm{filter}}.$$

The porosity  $\varepsilon$  denotes the ratio of liquid phase volume to total tank volume. Thus, setting  $\varepsilon=1$ , removing all bound states by setting  $N_{\mathrm{bnd},j,i}=0$  for all components i and particle types j, and applying no binding model results in a simple tank. The additional parameter  $F_{\mathrm{filter}}$ , which denotes the flow rate of pure liquid (without any components) out of the tank, can be used to model a filtering unit.

Note that it is the user's duty to make sure that the volume of the CSTR does not fall below 0 m<sup>3</sup>. If it does, the simulation may fail to run or may produce unreasonable (e.g., unphysical) results. See Table 3.5.2.

# 2.3 Binding models

The following binding models are presented in dynamic binding mode. By replacing all occurrences of dq/dt with 0, quasi-stationary (rapid-equlibrium) binding mode is achieved. In quasi-stationary binding it is assumed that ad- and desorption take place on a much faster time scale than the other transport processes such that bead liquid phase  $c_{p,i}$  (or bulk liquid phase  $c_i$  for certain unit operation models) are always in equilibrium with the solid phase  $q_i$ .

**Equilibrium constants** For the quasi-stationary binding mode, adsorption and desorption rate are no longer separate entities. Instead, the quotient  $k_{\rm eq} = k_a/k_d$  of adsorption and desorption coefficient is the relevant parameter as shown for the linear binding model (see Section 2.3.1):

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} - k_{d,i}q_i \qquad \Rightarrow 0 = k_{a,i}c_{p,i} - k_{d,i}q_i \qquad \Leftrightarrow q_i = \frac{k_{a,i}}{k_{d,i}}c_{p,i} = k_{\mathrm{eq},i}c_{p,i}.$$

The equilibrium constant  $k_{eq,i}$  is used in CADET by setting  $k_{d,i} = 1$  and  $k_{a,i} = k_{eq,i}$ .

**Correlation of ad- and desorption rates** Note that adsorption rate  $k_{a,i}$  and desorption rate  $k_{d,i}$  are linearly correlated in both binding modes due to the form of the equilibrium constant  $k_{eq}$ :

$$k_{a,i} = k_{eq} k_{d,i}$$
.

This correlation can potentially degrade performance of some optimization algorithms. While in quasistationary binding mode this is prevented by using the technique above, a dynamic binding model has to be reparameterized in order to decouple parameters:

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} - k_{d,i}q_i = k_{d,i}\left[k_{\mathrm{eq},i}c_{p,i} - q_i\right] = k_{a,i}\left[c_{p,i} - \frac{1}{k_{\mathrm{eq},i}}q_i\right].$$

This can be achieved by a (nonlinear) parameter transform

$$F\left(k_{\mathrm{eq},i},k_{d,i}\right) = \begin{pmatrix} k_{\mathrm{eq},i}k_{d,i} \\ k_{d,i} \end{pmatrix} \text{ with Jacobian } J_F\left(k_{\mathrm{eq},i},k_{d,i}\right) = \begin{pmatrix} k_{d,i} & k_{\mathrm{eq},i} \\ 0 & 1 \end{pmatrix}.$$

**Dependence on external function** A binding model may depend on an external function or profile  $T: [0, T_{\text{end}}] \times [0, L] \to \mathbb{R}$ , where L denotes the physical length of the unit operation, or  $T: [0, T_{\text{end}}] \to \mathbb{R}$  if the unit operation model has no axial length. By using an external profile it is possible to account for effects that are not directly modeled in CADET (e.g., temperature). The dependence of each parameter is modeled by a polynomial of third degree. For example, the adsorption rate  $k_a$  is really given by

$$k_a(T) = k_{a,3}T^3 + k_{a,2}T^2 + k_{a,1}T + k_{a,0}.$$

While  $k_{a,0}$  is set by the original parameter XXX\_KA of the file format (XXX being a placeholder for the binding model), the parameters  $k_{a,3}$ ,  $k_{a,2}$ , and  $k_{a,1}$  are given by XXX\_KA\_TTT, XXX\_KA\_TT, and XXX\_KA\_T, respectively. The identifier of the externally dependent binding model is constructed from the original identifier by prepending EXT\_ (e.g., MULTI\_COMPONENT\_LANGMUIR) is changed into EXT\_MULTI\_COMPONENT\_LANGMUIR). This pattern applies to all parameters and supporting binding models (see Table 2.3.1). Note that the parameter units have to be adapted to the unit of the external profile by dividing with an appropriate power.

Each parameter of the externally dependent binding model can depend on a different external source. The 0-based indices of the external source for each parameter is given in the dataset EXTFUN. By assigning only one index to EXTFUN, all parameters use the same source. The ordering of the parameters in EXTFUN is given by the ordering in the file format specification in Section 3.5.4.

**Binding model feature matrix** A short comparison of the most prominent binding model features is given in Table 2.3.1. The implemented binding models can be divided into two main classes: Single-state and multi-state binding. While single-state models only have one bound state per component (or less),

multi-state models provide multiple (possibly different) bound states for each component, which may correspond to different binding orientations or binding site types. The models also differ in whether a mobile phase modifier (e.g., salt) is supported to modulate the binding behaviour.

Binding model	Competi- tive	Mobile phase modifier	External function	Multi-state
Linear	Х	Х	<b>√</b>	×
Multi component Langmuir	✓	X	✓	×
Multi component Anti-Langmuir	✓	×	✓	×
Steric mass action	✓	✓	✓	×
Self association	✓	✓	✓	×
Mobile phase modulator Langmuir	✓	✓	✓	×
Kumar-Langmuir	✓	✓	✓	×
Saska	X	×	✓	×
Multi component Bi-Langmuir	✓	×	✓	✓
Multi component spreading	✓	×	✓	✓
Multi-state steric mass action	✓	✓	✓	✓
Simplified multi-state steric mass action	✓	✓	×	✓
Bi steric mass action	✓	✓	✓	✓

Table 2.3.1: Supported features of the different binding models

**Reference concentrations** Some binding models use reference concentrations  $c_{\text{ref}}$  and  $q_{\text{ref}}$  of the mobile phase modulator (e.g., salt) in the particle liquid and solid phase, respectively. The reference values are mainly used for normalizing adsorption and desorption rates, but also for other parameters that appear with those concentrations. They amount to a simple parameter transformation that is exemplified at one equation of the steric mass action binding model

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i} c_{p,i} \bar{q}_0^{\nu_i} - k_{d,i} q_i c_{p,0}^{\nu_i},$$

where  $c_{p,0}$  denotes the mobile phase salt concentration and

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} (\nu_j + \sigma_j) \, q_j$$

is the number of available binding sites which is related to the number of bound salt ions. Using the parameter transformation

$$k_{a,i} = \tilde{k}_{a,i} q_{\text{ref}}^{-\nu_i},$$
  
$$k_{d,i} = \tilde{k}_{d,i} c_{\text{ref}}^{-\nu_i},$$

we obtain the modified model equation

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = \tilde{k}_{a,i} c_{p,i} \left(\frac{\bar{q}_0}{q_{\mathrm{ref}}}\right)^{\nu_i} - \tilde{k}_{d,i} q_i \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{\nu_i}.$$

This transformation serves as a (partial) nondimensionalization of the adsorption and desorption rates and, by properly choosing the reference concentrations  $c_{\text{ref}}$  and  $q_{\text{ref}}$ , may improve the optimizer performance.

Recommended choices for  $c_{\text{ref}}$  are the average or maximum inlet concentration of the mobile phase modifier  $c_0$ , and for  $q_{\text{ref}}$  the ionic capacity  $\Lambda$ . Note that setting the reference concentrations to 1.0 each results in the original binding model.

#### 2.3.1 Linear

A linear binding model, which is often employed for low concentrations or in analytic settings [Gui+06].

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} - k_{d,i}q_i \qquad i = 0,\dots, N_{\text{comp}} - 1.$$

See Table 3.5.4.

#### 2.3.2 Multi Component Langmuir

The Langmuir binding model includes a saturation term and takes into account the capacity of the resin [Lan16; Gui+06]. All components compete for the same binding sites.

$$\frac{dq_i}{dt} = k_{a,i} c_{p,i} q_{\max,i} \left( 1 - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j}{q_{\max,j}} \right) - k_{d,i} q_i \qquad i = 0, \dots, N_{\text{comp}} - 1.$$

See Table 3.5.4.

#### 2.3.3 Multi Component Anti-Langmuir

The Anti-Langmuir (or generalized Langmuir) binding model extends the Langmuir model (see Section 2.3.2). The factor  $p_j \in \{-1,1\}$  determines the shape of the isotherm. For  $p_j = 1$  (standard Langmuir) the chromatograms have sharp fronts and a dispersed tail (isotherm is concave). In case of the Anti-Langmuir  $(p_j = -1)$  it is the other way around (isotherm is convex).

$$\frac{dq_i}{dt} = k_{a,i} c_{p,i} q_{\max,i} \left( 1 - \sum_{j=0}^{N_{\text{comp}} - 1} p_j \frac{q_j}{q_{\max,j}} \right) - k_{d,i} q_i \qquad i = 0, \dots, N_{\text{comp}} - 1.$$

See Table 3.5.4.

#### 2.3.4 Steric Mass Action

The steric mass action model takes charges of the molecules into account [BC92] and is, thus, often used in ion-exchange chromatography. Each component has a characteristic charge  $\nu$  that determines the number of available binding sites  $\Lambda$  (ionic capacity) used up by a molecule. Due to the molecule's shape, some additional binding sites (steric shielding factor  $\sigma$ ) may be shielded from other molecules and are not available for binding.

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}c_{p,i} \left(\frac{\bar{q}_0}{q_{\mathrm{ref}}}\right)^{\nu_i} - k_{d,i} q_i \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{\nu_i} \qquad i = 1,\dots, N_{\mathrm{comp}} - 1,$$

where  $c_{p,0}$  and  $q_0$  denote the salt concentrations in the liquid and solid phase of the beads, respectively. The number of free binding sites

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}} - 1} (\nu_j + \sigma_j) \, q_j = q_0 - \sum_{j=1}^{N_{\text{comp}} - 1} \sigma_j q_j$$

is calculated from the number of bound counter ions  $q_0$  by taking steric shielding into account. In turn, the number of bound counter ions  $q_0$  (electro-neutrality condition) is given by

$$q_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}} - 1} \nu_j q_j,$$

which also compensates for the missing equation for  $\frac{dq_0}{dt}$ . See Table 3.5.4.

The concept of reference concentrations ( $c_{\text{ref}}$  and  $q_{\text{ref}}$ ) is explained in the respective paragraph in Section 2.3.

#### 2.3.5 Self Association

This binding model is similar to the steric mass action model (see Section 2.3.4) but is also capable of describing dimerization [Mol08; Wes+12]. The dimerization, which is the immobilization of protein at some already bound protein, is also termed "self-association". It is modeled by adding a quadratic (in  $c_{p,i}$ ) term to the adsorption part of the equation.

$$\frac{dq_i}{dt} = c_{p,i} \left(\frac{\bar{q}_0}{q_{\text{ref}}}\right)^{\nu_i} \left[k_{a,i,1} + k_{a,i,2}c_{p,i}\right] - k_{d,i} q_i \left(\frac{c_{p,0}}{c_{\text{ref}}}\right)^{\nu_i} \qquad i = 1, \dots, N_{\text{comp}} - 1,$$

$$q_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}} - 1} \nu_j q_j,$$

where the number of available binding sites is given by

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} (\nu_j + \sigma_j) q_j = q_0 - \sum_{j=1}^{N_{\text{comp}}-1} \sigma_j q_j.$$

See Table 3.5.4.

The concept of reference concentrations ( $c_{\text{ref}}$  and  $q_{\text{ref}}$ ) is explained in the respective paragraph in Section 2.3.

#### 2.3.6 Mobile Phase Modulator Langmuir

This model is a modified Langmuir model (see Section 2.3.2) which can be used to describe hydrophobic interaction chromatography [MEH89; Kar+04]. A modulator component (termed "salt",  $c_{p,0}$  and  $q_0$ ) influences ad- and desorption processes:

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = k_{a,i}e^{\gamma_i c_{p,0}} c_{p,i} \, q_{\max,i} \left( 1 - \sum_{j=1}^{N_{\text{comp}}-1} \frac{q_j}{q_{\max,j}} \right) - k_{d,i} \, c_{p,0}^{\beta_i} \, q_i \qquad i = 1, \dots, N_{\text{comp}} - 1.$$

where  $c_{p,0}$  and  $q_0$  denote the salt concentrations in the liquid and solid phase of the beads respectively. Salt is considered to be inert, therefore either

$$\frac{\mathrm{d}q_0}{\mathrm{d}t} = 0$$

is used if salt has one bound state, or salt can be used without a bound state. The parameter  $\gamma$  describes the hydrophobicity and  $\beta$  the ion-exchange characteristics. See Table 3.5.4.

#### 2.3.7 Kumar-Langmuir

This extension of the Langmuir isotherm (see Section 2.3.2) developed in [Kum+15] was used to model charge variants of monoclonal antibodies in ion-exchange chromatography. A non-binding salt component  $c_{p,0}$  is added to modulate the ad- and desorption process.

$$\frac{dq_{i}}{dt} = k_{a,i} \exp\left(\frac{k_{\text{act},i}}{T}\right) c_{p,i} q_{\text{max},i} \left(1 - \sum_{j=1}^{N_{\text{comp}}-1} \frac{q_{j}}{q_{\text{max},j}}\right) - k_{d,i} \left(c_{p,0}\right)^{\nu_{i}} q_{i} \qquad i = 1, \dots, N_{\text{comp}} - 1$$

In this model, the true adsorption rate  $k_{a,i,\text{true}}$  is governed by the Arrhenius law in order to take temperature into account

$$k_{a,i,\text{true}} = k_{a,i} \exp\left(\frac{k_{\text{act},i}}{T}\right).$$

Here,  $k_{a,i}$  is the frequency or pre-exponential factor,  $k_{\text{act},i} = E/R$  is the activation temperature (E denotes the activation energy and R the Boltzmann gas constant), and T is the temperature. The characteristic charge  $\nu$  of the protein is taken into account by the power law. See Table 3.5.4.

#### 2.3.8 Saska

In this binding model an additional quadratic term is added to the linear model [Sas+92]. The quadratic term allows to take interactions of liquid phase components into account.

$$\frac{dq_i}{dt} = H_i c_{p,i} + \sum_{j=0}^{N_{\text{comp}}-1} k_{ij} c_{p,i} c_{p,j} - q_i \qquad i = 0, \dots, N_{\text{comp}} - 1$$

See Table 3.5.4.

#### 2.3.9 Multi Component Bi-Langmuir

The multi component Bi-Langmuir model [Gui+06] adds M-1 additional types of binding sites  $q_{i,j}$  ( $0 \le j \le M-1$ ) to the Langmuir model (see Section 2.3.2) without allowing an exchange between the different sites  $q_{i,j}$  and  $q_{i,k}$  ( $k \ne j$ ). Therefore, there are no competitivity effects between the different types of binding sites and they have independent capacities.

$$\frac{\mathrm{d}q_{i,j}}{\mathrm{d}t} = k_{a,i}^{(j)} c_{p,i} q_{\max,i}^{(j)} \left( 1 - \sum_{k=0}^{N_{\text{comp}}-1} \frac{q_{k,j}}{q_{\max,k}^{(j)}} \right) - k_{d,i}^{(j)} q_{i,j} \qquad i = 0, \dots, N_{\text{comp}} - 1, \ j = 0, \dots, M - 1.$$

Note that all binding components must have exactly the same number of binding site types  $M \ge 1$ . See the Langmuir isotherm in Section 2.3.2 and Table 3.5.4.

Originally, the Bi-Langmuir model is limited to two different binding site types. Here, the model has been extended to arbitrary many binding site types.

#### 2.3.10 Multi Component Spreading

The multi component spreading model adds a second bound state  $q_{i,2}$  to the Langmuir model (see Section 2.3.2) and allows the exchange between the two bound states  $q_{i,1}$  and  $q_{i,2}$ . In the spreading model a second state of the bound molecule (e.g., a different orientation on the surface or a different folding state) is added. The exchange of molecules between the two states is allowed and, since the molecules can potentially bind in both states at the same binding site, competitivity effects are present. This is different to the Bi-Langmuir model in which another type of binding sites is added and no exchange between the different bound states is considered (see Section 2.3.9). For all components  $i = 0, \ldots, N_{\text{comp}} - 1$  the equations are given by

$$\frac{\mathrm{d}q_{i,1}}{\mathrm{d}t} = \left(k_a^A c_{p,i} - k_{12}q_{i,1}\right) q_{\max,i}^A \left(1 - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^A}{q_{\max,j}^A} - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^B}{q_{\max,j}^B}\right) - k_d^A q_{i,1} + k_{21}q_{i,2},$$

$$\frac{\mathrm{d}q_{i,2}}{\mathrm{d}t} = \left(k_a^B c_{p,i} + k_{12}q_{i,1}\right) q_{\max,i}^A \left(1 - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^A}{q_{\max,j}^A} - \sum_{j=0}^{N_{\text{comp}}-1} \frac{q_j^B}{q_{\max,j}^B}\right) - \left(k_d^B + k_{21}\right) q_{i,2}.$$

See Table 3.5.4.

#### 2.3.11 Multi-State Steric Mass Action

The multi-state steric mass action model adds  $M_i - 1$  additional bound states  $q_{i,j}$   $(j = 0, ..., M_i - 1)$  for each component i to the steric mass action model (see Section 2.3.4) and allows the exchange between the different bound states  $q_{i,0}, ..., q_{i,M-1}$  of each component. In the multi-state SMA model a variable number of states of the bound molecule (e.g., different orientations on the surface, binding strength of tentacle adsorbers) is added which are more and more strongly bound, i.e.,

$$\nu_{i,j} \le \nu_{i,j+1}$$
  $i = 1, \dots, N_{\text{comp}} - 1, \quad j = 0, \dots, M_i - 1.$ 

The exchange between the different states of each component is allowed and, since the molecules can potentially bind in all states at the same binding site, competitive effects are present.

$$\begin{split} \frac{\mathrm{d}q_{i,j}}{\mathrm{d}t} &= \ k_{a,i}^{(j)}c_{p,i} \left(\frac{\bar{q}_0}{q_{\mathrm{ref}}}\right)^{\nu_{i,j}} - k_{d,i}^{(j)} \ q_{i,j} \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{\nu_{i,j}} \\ &- \sum_{\ell=0}^{j-1} k_{j\ell}^{(i)} \ q_{i,j} \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{(\nu_{i,j}-\nu_{i,\ell})} - \sum_{\ell=j+1}^{M_i-1} k_{j\ell}^{(i)} \ q_{i,j} \left(\frac{\bar{q}_0}{q_{\mathrm{ref}}}\right)^{(\nu_{i,\ell}-\nu_{i,j})} \\ &+ \sum_{\ell=0}^{j-1} k_{\ell j}^{(i)} \ q_{i,\ell} \left(\frac{\bar{q}_0}{q_{\mathrm{ref}}}\right)^{(\nu_{i,j}-\nu_{i,\ell})} + \sum_{\ell=j+1}^{M_i-1} k_{\ell j}^{(i)} \ q_{i,\ell} \left(\frac{c_{p,0}}{c_{\mathrm{ref}}}\right)^{(\nu_{i,\ell}-\nu_{i,j})} & i = 1, \dots, N_{\mathrm{comp}} - 1, \\ j = 0, \dots, M_i - 1, \end{split}$$

where  $c_{p,0}$  and  $q_0$  denote the salt concentrations in the liquid and solid phase of the beads respectively. The number of available salt ions  $\bar{q}_0$  is given by

$$\bar{q}_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}} - 1} \sum_{\ell=0}^{M_j - 1} (\nu_{j,\ell} + \sigma_{j,\ell}) q_{j,\ell}.$$

A neutrality condition compensating for the missing equation for  $\frac{dq_0}{dt}$  is required:

$$q_0 = \Lambda - \sum_{j=1}^{N_{\text{comp}}-1} \sum_{\ell=0}^{M_j-1} \nu_{j,\ell} q_{j,\ell}.$$

See Table 3.5.4.

The concept of reference concentrations ( $c_{\text{ref}}$  and  $q_{\text{ref}}$ ) is explained in the respective paragraph in Section 2.3.

#### 2.3.12 Simplified Multi-State Steric Mass Action

The simplified multi-state steric mass action is the same as the multi-state SMA model described above (see Section 2.3.11), but with additional assumptions:

- Molecules are only exchanged between two adjacent states, that is, no transfer from state  $q_{i,1}$  to state  $q_{i,3}$  is allowed.
- Characteristic charge  $\nu_{i,j}$  and shielding factor  $\sigma_{i,j}$  only depend on the index of the state j.

Thus, the exchange parameters  $k_{j\ell}^{(i)}$ , the characteristic charge  $\nu_{i,j}$ , and the shielding  $\sigma_{i,j}$  can be parameterized with few degrees of freedom. For all  $i=1,\ldots,N_{\text{comp}}-1$  and  $j,\ell=0,\ldots,M_i-1$  let

$$k_{j\ell}^{(i)} = \begin{cases} 0, & \text{for } |j - \ell| \neq 1 \\ K_{ws}^{(i)} + j K_{ws, \text{lin}}^{(i)} - K_{ws, \text{quad}}^{(i)} j (j - M_i + 2), & \text{for } \ell = j + 1 \\ K_{sw}^{(i)} + \ell K_{sw, \text{lin}}^{(i)} - K_{sw, \text{quad}}^{(i)} \ell (\ell - M_i + 2), & \text{for } \ell = j - 1, \end{cases}$$

$$\nu_{i,j} = \nu_{\min,i} + \frac{j}{M_i - 1} \left( \nu_{\max,i} - \nu_{\min,i} \right) - \nu_{\text{quad},i} j (j - M_i + 1),$$

$$\sigma_{i,j} = \sigma_{\min,i} + \frac{j}{M_i - 1} \left( \sigma_{\max,i} - \sigma_{\min,i} \right) - \sigma_{\text{quad},i} j (j - M_i + 1).$$

Note that the characteristic charge  $\nu_{i,j}$  has to be monotonically non-decreasing in the second index j and all other rates and the steric factor  $\sigma_{i,j}$  have to be non-negative. See Table 3.5.4.

#### 2.3.13 Bi Steric Mass Action

Similar to the Bi-Langmuir model (see Section 2.3.9), the Bi-SMA model adds M-1 additional types of binding sites  $q_{i,j}$  ( $0 \le j \le M-1$ ) to the SMA model (see Section 2.3.4) without allowing an exchange between the different sites  $q_{i,j}$  and  $q_{i,k}$  ( $k \ne j$ ). Therefore, there are no competitivity effects between the two types of binding sites and they have independent capacities.

$$\frac{\mathrm{d}q_{i,j}}{\mathrm{d}t} = k_{a,i,j}c_{p,i} \left(\frac{\bar{q}_{0,j}}{q_{\mathrm{ref},j}}\right)^{\nu_{i,j}} - k_{d,i,j} \ q_{i,j} \left(\frac{c_{p,0}}{c_{\mathrm{ref},j}}\right)^{\nu_{i,j}} \qquad i = 1, \dots, N_{\mathrm{comp}} - 1, \quad j = 0, \dots, M - 1,$$

where  $c_{p,0}$  and  $q_{0,j}$  ( $0 \le j \le M-1$ ) denote the salt concentrations in the liquid and solid phases of the beads respectively. The number of available salt ions  $\bar{q}_{0,j}$  for each binding site type  $0 \le j \le M-1$  is given by

$$\bar{q}_{0,j} = \Lambda_j - \sum_{k=1}^{N_{\text{comp}}-1} (\nu_{k,j} + \sigma_{k,j}) q_{k,j}.$$

Electro-neutrality conditions compensating for the missing equations for  $\frac{dq_{0,j}}{dt}$  are required:

$$q_{0,j} = \Lambda_j - \sum_{k=1}^{N_{\text{comp}}-1} \nu_{k,j} q_{k,j}$$
  $j = 0, \dots, M-1.$ 

Note that all binding components must have exactly the same number of binding site types  $M \ge 1$ . See Table 3.5.4.

The reference concentrations  $c_{\text{ref},j}$  and  $q_{\text{ref},j}$  can be specified for each binding site type  $0 \le j \le M-1$ . The concept of reference concentrations is explained in the respective paragraph in Section 2.3.

Originally, the Bi-SMA model is limited to two different binding site types. Here, the model has been extended to arbitrary many binding site types.

# 3 CADET File Format Specifications

The CADET framework is designed to work on a file format structured into groups and datasets. This concept may be implemented by different file formats. At the moment, CADET natively supports HDF5 and XML as file formats. The choice is not limited to those two formats but can be extended as needed. In this section the general layout and structure of the file format is described.

File format versions The file format may change and evolve over time as new features are added to the simulator. This manual describes the most recent file format version that is also set as default value in /meta/FILE\_FORMAT (see Tab. 3.7). The simulator assumes that the input file uses the most recent format version and does not update old files to the current standard.

#### 3.1 Global structure

The global structure (see Fig. 3.1.1) is divided into three parts: input, output, and meta. Every valid CADET file needs an input group (see Fig. 3.1.2) which contains all relevant information for simulating a model. It does not need an output (see Fig. 3.1.4) or meta (see Fig. 3.1.1) group, since those are created when results are written. Whereas the output group is solely used as output and holds the results of the simulation, the meta group is used for input and output. Details such as file format version and simulator version are read from and written to the meta group.

If not explicitly stated otherwise, all datasets are mandatory. By convention all group names are lowercase, whereas all dataset names are uppercase. Note that this is just a description of the file format and not a detailed explanation of the meaning of the parameters. For the latter, please refer to the corresponding sections in the previous chapter.

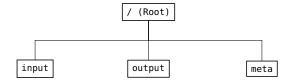


Figure 3.1.1: Structure of the groups in the root group of the file format

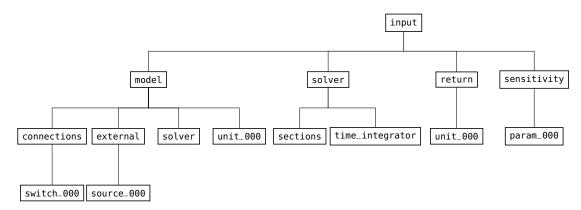


Figure 3.1.2: High-level structure of the groups in the input part of the file format

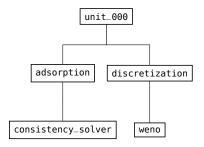


Figure 3.1.3: Structure of the groups in a column unit operation (/input/model group)

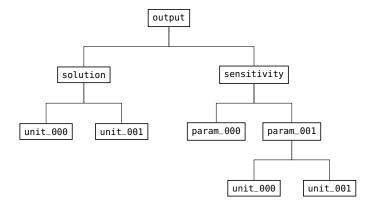


Figure 3.1.4: Structure of the groups in the output part of the file format

#### 3.2 Notation and identifiers

Reference volumes are denoted by subscripts:

 $m_{IV}^3$  Interstitial volume

 $m_{MP}^3$  Bead mobile phase volume

 $m_{SP}^3$  Bead solid phase volume

Common notation and identifiers that are used in the subsequent description are listed in Table 3.2.1.

Identifier	Meaning
NCOMP	Number of components of a unit operation
NTOTALCOMP	Total number of components in the system (sum of all unit operation components)
NPARTYPE	Number of particle types of a unit operation
$NBND_{\mathbf{i}}$	Number of bound states of component $i$ of a unit operation
NTOTALBND	Total number of bound states of a unit operation (sum of all bound states of all components)
NSTATES	Maximum of the number of bound states for each component of a unit operation
NDOF	Total number of degrees of freedom of the current unit operation model or system of unit operations
NSEC	Number of time integration sections
PARAM_VALUE	Value of a generic unspecified parameter

Table 3.2.1: Common notation and identifiers used in the file format description

# 3.3 Ordering of multi dimensional data

Some model parameters, especially in certain binding models, require multi dimensional data. Since CADET only reads one dimensional arrays, the layout of the data has to be specified (i.e., the way how the data is linearized in memory). The term "xyz-major" means that the index corresponding to xyz changes the slowest.

For instance, suppose a model with 2 components and 3 bound states has a "component-major" dataset. Then, the requested matrix is stored in memory such that all bound states are listed for each component (i.e., the component index changes the slowest and the bound state index the fastest):

```
comp0bnd0, comp0bnd1, comp0bnd2, comp1bnd0, comp1bnd1, comp1bnd2.
```

This linear array can be represented as a  $2 \times 3$  matrix in "row-major" storage format, or a  $3 \times 2$  matrix in "column-major" ordering.

# 3.4 Section dependent model parameters

Some model parameters (see Table 3.4.1) can be assigned different values for each section. For example, the velocity a column is operated with could differ in the load, wash, and elution phases. Section dependency is recognized by specifying the appropriate number of values for the parameters (see *Length* column in the following tables). If a parameter depends on both the component and the section, the ordering is section-major.

For instance, the Length field of the parameter VELOCITY reads "1 / NSEC" which means that it is not recognized as section dependent if only 1 value (scalar) is passed. However, if NSEC many values (vector) are present, it will be treated as section dependent.

Note that all components of component dependent datasets have to be section dependent (e.g., you cannot have a section dependency on component 2 only while the other components are not section dependent).

Dataset	Component dependent	Section dependent
COL_DISPERSION		✓
FILM_DIFFUSION	✓	✓
PAR_DIFFUSION	✓	✓
PAR_SURFDIFFUSION	✓	✓
VELOCITY		$\checkmark$

Table 3.4.1: Section dependent datasets in the unit operation models (/input/model/unit\_XXX group)

# 3.5 Input group

#### 3.5.1 System of unit operations

```
Group /input/model
```

**NUNITS** Number of unit operations in the system

Type: int Range:  $\geq 1$  Length: 1

INIT\_STATE\_Y Initial full state vector (optional, unit operation specific initial data is ignored)

Type: int Range:  $\geq 1$  Length: 1

**INIT\_STATE\_YDOT** Initial full time derivative state vector (optional, unit operation specific initial data is ignored)

Type: double Length: NDOF

INIT\_STATE\_SENSY\_XXX Number of unit operations in the system

Type: double Length: NDOF

INIT\_STATE\_SENSYDOT\_XXX Initial full state vector of the XXXth sensitivity system (optional, unit operation specific initial data is ignored)

Type: double Length: NDOF

NUNITS Initial full time derivative state vector of the XXXth sensitivity system (optional, unit operation specific initial data is ignored)

Type: double Length: NDOF

#### Group /input/model/connections

**NSWITCHES** Number of valve switches

Type: int Range:  $\geq 1$  Length: 1

#### Group /input/model/connections/switch\_XXX

**SECTION** Index of the section that activates this connection set

Type: int Range:  $\geq 0$  Length: 1

**CONNECTIONS** Matrix with list of connections in row-major storage. Columns are UnitOpID from, UnitOpID to, Component from, Component to, volumetric flow rate. If both component indices are -1, all components are connected.

Type: double Range:  $\geq -1$  Length:  $5 \cdot NCONNECTIONS$ 

#### Group /input/model/external/source\_XXX - EXTFUN\_TYPE = LINEAR\_INTERP\_DATA

**VELOCITY** Velocity of the external profile in positive column axial direction

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: 1

 ${\bf DATA}$  Function values T at the data points

Unit: [T] Type: double Range:  $\mathbb{R}$  Length: Arbitrary

**TIME** Time of the data points

Unit: s Type: double Range:  $\geq 0.0$  Length: Same as DATA

#### Group /input/model/external/source\_XXX - EXTFUN\_TYPE = PIECEWISE\_CUBIC\_POLY

**VELOCITY** Velocity of the external profile in positive column axial direction

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: 1

CONST\_COEFF Constant coefficients of piecewise cubic polynomial

Unit: [T] Type: double Range:  $\mathbb{R}$  Length: Arbitrary

LIN\_COEFF Linear coefficients of piecewise cubic polynomial

Unit:  $[T] s^{-1}$  Type: double Range:  $\mathbb{R}$  Length: Same as CONST\_COEFF

CONST\_COEFF Quadratic coefficients of piecewise cubic polynomial

Unit:  $[T] s^{-2}$  Type: double Range:  $\mathbb{R}$  Length: Same as CONST\_COEFF

**QUAD\_COEFF** Cubic coefficients of piecewise cubic polynomial

Unit:  $[T] s^{-3}$  Type: double Range:  $\mathbb{R}$  Length: Same as CONST\_COEFF

SECTION\_TIMES Simulation times at which a new piece begins (breaks of the piecewise polynomial)

Unit: s Type: double Range:  $\geq 0.0$  Length: CONST\_COEFF+1

#### Group /input/model/solver

**GS\_TYPE** Type of Gram-Schmidt orthogonalization, see IDAS guide Section 4.5.7.3, p. 41f. A value of 0 enables classical Gram-Schmidt, a value of 1 uses modified Gram-Schmidt.

Type: int Range:  $\{0,1\}$  Length: 1

**MAX\_KRYLOV** Defines the size of the Krylov subspace in the iterative linear GMRES solver (0: MAX\_KRYLOV = NDOF)

Type: int Range:  $\{0, \dots, NDOF\}$  Length: 1

MAX\_RESTARTS Maximum number of restarts in the GMRES algorithm. If lack of memory is not an issue, better use a larger Krylov space than restarts.

Type: int Range:  $\geq 0$  Length: 1

**SCHUR\_SAFETY** Schur safety factor; Influences the tradeoff between linear iterations and nonlinear error control; see IDAS guide Section 2.1 and 5.

Type: double Range:  $\geq 0$  Length: 1

### 3.5.2 Unit operation models

#### Inlet

### Group /input/model/unit\_XXX - UNIT\_TYPE = INLET

 $\ensuremath{\textbf{UNIT\_TYPE}}$  Specifies the type of unit operation model

Type: string Range: INLET Length: 1

NCOMP Number of chemical components in the chromatographic media

Type: int Range:  $\geq 1$  Length: 1

**INLET\_TYPE** Specifies the type of inlet profile

Type: string Range: PIECEWISE\_CUBIC\_POLY Length: 1

# Group /input/model/unit\_XXX/sec\_XXX

**CONST\_COEFF** Constant coefficients for inlet concentrations

Unit:  $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3}$  Type: double Range:  $\mathbb R$  Length: NCOMP

 $\textbf{LIN\_COEFF} \ \, \text{Linear coefficients for inlet concentrations}$ 

Unit:  $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3} \operatorname{s}^{-1}$  Type: double Range:  $\mathbb{R}$  Length: NCOMP

**QUAD\_COEFF** Quadratic coefficients for inlet concentrations

Unit:  $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3} \operatorname{s}^{-2}$  Type: double Range:  $\mathbb{R}$  Length: NCOMP

**CUBE\_COEFF** Cubic coefficients for inlet concentrations

Unit:  $\operatorname{mol} \operatorname{m}_{\mathrm{IV}}^{-3} \operatorname{s}^{-3}$  Type: double Range:  $\mathbb{R}$  Length: NCOMP

#### Outlet

#### Group /input/model/unit\_XXX - UNIT\_TYPE = OUTLET

UNIT\_TYPE Specifies the type of unit operation model

Type: string Range: OUTLET Length: 1

**NCOMP** Number of chemical components in the chromatographic media

Type: int Range:  $\geq 1$  Length: 1

#### General rate model

Group /input/model/unit\_XXX - UNIT\_TYPE = GENERAL\_RATE\_MODEL

UNIT\_TYPE Specifies the type of unit operation model

Type: string Range: GENERAL\_RATE\_MODEL Length: 1

**NCOMP** Number of chemical components in the chromatographic media

Type: int Range:  $\geq 1$  Length: 1

ADSORPTION\_MODEL Specifies the type of adsorption models of each particle type

Type: string Range: See Section 3.5.4 Length: NPARTYPE

INIT\_C Initial concentrations for each component in the bulk mobile phase

Unit:  $mol m_{IV}^{-3}$  Type: double Range:  $\geq 0$  Length: NCOMP

**INIT\_CP** Initial concentrations for each component in the bead liquid phase of each particle type in type-major ordering (optional, INIT\_C is used if left out)

Unit:  $mol \, m_{MP}^{-3}$  Type: double Range:  $\geq 0$  Length: NPARTYPE · NCOMP

**INIT\_Q** Initial concentrations for each bound state of each component in the bead solid phase of each particle type in type-component-major ordering

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

INIT\_STATE Full state vector for initialization (optional, INIT\_C, INIT\_CP, and INIT\_Q will be ignored;
if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range:  $\mathbb{R}$  Length: NDOF / 2NDOF

**COL\_DISPERSION** Axial dispersion coefficient

Unit:  $m_{IV}^2 s^{-1}$  Type: double Range:  $\geq 0$  Length: 1 / NSEC

**COL\_LENGTH** Column length

Unit:  $m s^{-1}$ 

Unit: m Type: double Range: > 0 Length: 1

**COL\_POROSITY** Column porosity

Unit: - Type: double Range: [0,1] Length: 1

**FILM\_DIFFUSION** Film diffusion coefficients for each component of each particle type in section-type-major ordering

Type: double Range:  $\geq 0$ 

PAR\_POROSITY Particle porosity of each particle type

Unit: - Type: double Range: [0, 1] Length: NPARTYPE

PAR\_RADIUS Particle radius of each particle type

Unit: m Type: double Range: > 0 Length: NPARTYPE

**PAR\_CORERADIUS** Particle core radius of each particle type (optional, defaults to 0 m)

Unit: m Type: double Range: [0, PAR\_RADIUS) Length: NPARTYPE

**PORE\_ACCESSIBILITY** Pore accessibility factor of each component in each particle type in type-major ordering (optional, defaults to 1)

Unit: - Type: double Range: (0,1] Length: NPARTYPE · NCOMP

**PAR\_DIFFUSION** Effective particle diffusion coefficients of each component in each particle type in section-type-major ordering

Unit:  $m_{MP}^2 s^{-1}$  Type: double Range:  $\geq 0$  Length: NPARTYPE · NCOMP / NSEC · NPARTYPE ·

Length: NPARTYPE · NCOMP / NPARTYPE · NCOMP

PAR\_SURFDIFFUSION Particle surface diffusion coefficients of each bound state of each component in each particle type in section-type-component-major ordering

Unit:  $m_{SP}^2 s^{-1}$  Type: double Range:  $\geq 0$  Length: NTOTALBND / NSEC · NTOTALBND

**VELOCITY** Interstitial velocity of the mobile phase (optional if CROSS\_SECTION\_AREA is present, see Section 2.2.3)

Type: double Range: R

Length: 1 / NSEC

**CROSS\_SECTION\_AREA** Cross section area of the column (optional if VELOCITY is present, see Section 2.2.3)

Unit:  $m^2$ 

Unit:  $m s^{-1}$ 

Type: double Range: > 0

Length: 1

PAR\_TYPE\_VOLFRAC Volume fractions of the particle types. The volume fractions can be set for all axial cells together or for each individual axial cell. In case of a spatially inhomogeneous setting, the data is expected in cell-major ordering and the SENS\_SECTION field is used for indexing the axial cell when specifying parameter sensitivities.

Unit: -

Type: double Range: [0,1]

Length: NPARTYPE / NCOL · NPARTYPE

#### Group /input/model/unit\_XXX/discretization - UNIT\_TYPE = GENERAL\_RATE\_MODEL

**NCOL** Number of axial column discretization cells

Type: int Range:  $\geq 1$ 

Length: 1

NPAR Number of particle (radial) discretization cells for each particle type

Type: int Range:  $\geq 1$  Length: NPARTYPE

NBOUND Number of bound states for each component in each particle type in type-major ordering

Type: int Range:  $\geq 0$  Length: NPARTYPE · NCOMP

**PAR\_DISC\_TYPE** Specifies the discretization scheme inside the particles for each particle type. Valid values are EQUIDISTANT\_PAR, EQUIVOLUME\_PAR, and USER\_DEFINED\_PAR.

Type: string Length: NPARTYPE

**PAR\_DISC\_VECTOR** Node coordinates for the cell boundaries (ignored if PAR\_DISC\_TYPE  $\neq$  USER\_DEFINED\_PAR). The coordinates are relative and have to include the endpoints 0 and 1. They are later linearly mapped to the true radial range  $[r_{c,j}, r_{p,j}]$ . The coordinates for each particle type are appended to one long vector in type-major ordering.

Unit: -

Type: double Range: [0,1]

Length:  $\sum_{i} (NPAR_i + 1)$ 

**USE\_ANALYTIC\_JACOBIAN** Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0)

Type: int Range:  $\{0,1\}$ 

Length: 1

**RECONSTRUCTION** Type of reconstruction method for fluxes

Type: string Range: WEN0

Length: 1

**GS\_TYPE** Type of Gram-Schmidt orthogonalization, see IDAS guide Section 4.5.7.3, p. 41f. A value of 0 enables classical Gram-Schmidt, a value of 1 uses modified Gram-Schmidt.

Type: int Range:  $\{0,1\}$ 

Length: 1

 $\label{eq:max_krylov} \textbf{MAX\_KRYLOV} \ \ \text{Defines the size of the Krylov subspace in the iterative linear GMRES solver} \ (0: \ \texttt{MAX\_KRYLOV} = \\ \textbf{NCOL} \cdot \textbf{NCOMP} \cdot \textbf{NPARTYPE})$ 

Type: int Range:  $\{0, \dots, NCOL \cdot NCOMP \cdot NPAR \text{ Like } \text{ } \text{ } 1$ 

MAX\_RESTARTS Maximum number of restarts in the GMRES algorithm. If lack of memory is not an issue, better use a larger Krylov space than restarts.

Type: int F

Range:  $\geq 0$ 

Length: 1

**SCHUR\_SAFETY** Schur safety factor; Influences the tradeoff between linear iterations and nonlinear error control; see IDAS guide Section 2.1 and 5.

Type: double Range:  $\geq 0$  Length: 1

#### Lumped rate model with pores

Group /input/model/unit\_XXX - UNIT\_TYPE = LUMPED\_RATE\_MODEL\_WITH\_PORES

UNIT\_TYPE Specifies the type of unit operation model

Type: string Range: LUMPED\_RATE\_MODEL\_WITH\_IP@Rg\$h: 1

**NCOMP** Number of chemical components in the chromatographic media

Type: int Range: > 1 Length: 1

ADSORPTION\_MODEL Specifies the type of adsorption models of each particle type

Type: string Range: See Section 3.5.4 Length: NPARTYPE

INIT\_C Initial concentrations for each component in the bulk mobile phase

Unit:  $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3}$  Type: double Range:  $\geq 0$  Length: NCOMP

INIT\_CP Initial concentrations for each component in the bead liquid phase of each particle type in type-major ordering (optional, INIT\_C is used if left out)

Unit:  $mol \, m_{\mathrm{MP}}^{-3}$  Type: double Range:  $\geq 0$  Length: NPARTYPE · NCOMP

**INIT\_Q** Initial concentrations for each bound state of each component in the bead solid phase of each particle type in type-component-major ordering

Unit:  $mol \, m_{\rm SP}^{-3}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

**INIT\_STATE** Full state vector for initialization (optional, INIT\_C, INIT\_CP, and INIT\_Q will be ignored; if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range:  $\mathbb{R}$  Length: NDOF / 2NDOF

**COL\_DISPERSION** Axial dispersion coefficient

Unit:  $m_{IV}^2 s^{-1}$  Type: double Range:  $\geq 0$  Length: 1 / NSEC

**COL\_LENGTH** Column length

Unit: m Type: double Range: > 0 Length: 1

**COL\_POROSITY** Column porosity

Unit: - Type: double Range: [0,1] Length: 1

**FILM\_DIFFUSION** Film diffusion coefficients for each component of each particle type in section-type-major ordering

Unit:  $m s^{-1}$  Type: double Range:  $\geq 0$ 

PAR\_POROSITY Particle porosity of each particle type

Unit: - Type: double Range: [0, 1] Length: NPARTYPE

PAR\_RADIUS Particle radius of each particle type

Unit: m Type: double Range: > 0 Length: NPARTYPE

**PORE\_ACCESSIBILITY** Pore accessibility factor of each component in each particle type in type-major ordering (optional, defaults to 1)

Unit: – Type: double Range: (0,1] Length: NPARTYPE · NCOMP

**VELOCITY** Interstitial velocity of the mobile phase (optional if  $CROSS\_SECTION\_AREA$  is present, see Section 2.2.3)

Unit:  $m s^{-1}$  Type: double Range:  $\mathbb{R}$  Length: 1 / NSEC

Length: NPARTYPE · NCOMP / NPARTYPE · NCOMP

CROSS\_SECTION\_AREA Cross section area of the column (optional if VELOCITY is present, see Section 2.2.3)

Unit:  $m^2$  Type: double Range: > 0

PAR\_TYPE\_VOLFRAC Volume fractions of the particle types. The volume fractions can be set for all axial cells together or for each individual axial cell. In case of a spatially inhomogeneous setting, the data is expected in cell-major ordering and the SENS\_SECTION field is used for indexing the axial cell when specifying parameter sensitivities.

Unit: - Type: double Range: [0,1]

Length: NPARTYPE / NCOL · NPARTYPE

Length: 1

# Group /input/model/unit\_XXX/discretization - UNIT\_TYPE = LUMPED\_RATE\_MODEL\_WITH\_PORES

NCOL Number of axial column discretization cells

Type: int Range:  $\geq 1$  Length: 1

NBOUND Number of bound states for each component in each particle type in type-major ordering

Type: int Range:  $\geq 0$  Length: NPARTYPE · NCOMP

**USE\_ANALYTIC\_JACOBIAN** Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0)

Type: int Range:  $\{0,1\}$  Length: 1

 $\mbox{\bf RECONSTRUCTION}$  Type of reconstruction method for fluxes

Type: string Range: WEN0 Length: 1

**GS\_TYPE** Type of Gram-Schmidt orthogonalization, see IDAS guide Section 4.5.7.3, p. 41f. A value of 0 enables classical Gram-Schmidt, a value of 1 uses modified Gram-Schmidt.

Type: int Range:  $\{0,1\}$  Length: 1

 $\label{eq:max_krylov} \textbf{MAX\_KRYLOV} \ \ \textbf{Defines the size of the Krylov subspace in the iterative linear GMRES solver (0: \texttt{MAX\_KRYLOV} = \texttt{NCOL} \cdot \texttt{NCOMP} \cdot \texttt{NPARTYPE})$ 

Type: int Range:  $\{0, \dots, NCOL \cdot NCOMP \cdot NPART$   $\{0, \dots, NCOL \cdot NCOMP \cdot NPART \}$ 

MAX\_RESTARTS Maximum number of restarts in the GMRES algorithm. If lack of memory is not an issue, better use a larger Krylov space than restarts.

Type: int Range:  $\geq 0$  Length: 1

**SCHUR\_SAFETY** Schur safety factor; Influences the tradeoff between linear iterations and nonlinear error control; see IDAS guide Section 2.1 and 5.

Type: double Range:  $\geq 0$  Length: 1

#### Lumped rate model without pores

#### Group /input/model/unit\_XXX - UNIT\_TYPE = LUMPED\_RATE\_MODEL\_WITHOUT\_PORES

UNIT\_TYPE Specifies the type of unit operation model

Type: string Range: LUMPED\_RATE\_MODEL\_WITHOUTINESS

NCOMP Number of chemical components in the chromatographic media

Type: int Range:  $\geq 1$  Length: 1

ADSORPTION\_MODEL Specifies the type of adsorption model

Type: string Range: See Section 3.5.4 Length: 1

INIT\_C Initial concentrations for each component in the bulk mobile phase

Unit:  $\operatorname{mol} \operatorname{m}_{\operatorname{IV}}^{-3}$  Type: double Range:  $\geq 0$  Length: NCOMP

**INIT\_Q** Initial concentrations for each bound state of each component in the bead solid phase in component-major ordering

Unit:  $mol m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

**INIT\_STATE** Full state vector for initialization (optional, INIT\_C and INIT\_Q will be ignored; if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range:  $\mathbb{R}$  Length: NDOF / 2NDOF

**COL\_DISPERSION** Axial dispersion coefficient

Unit:  $m_{IV}^2 s^{-1}$  Type: double Range:  $\geq 0$  Length: 1 / NSEC

**COL\_LENGTH** Column length

Unit: m Type: double Range: > 0 Length: 1

TOTAL\_POROSITY Total porosity

Unit: - Type: double Range: [0,1] Length: 1

**VELOCITY** Interstitial velocity of the mobile phase (optional if  $CROSS\_SECTION\_AREA$  is present, see Section 2.2.3)

Unit:  $m s^{-1}$  Type: double Range:  $\mathbb{R}$  Length: 1 / NSEC

CROSS\_SECTION\_AREA Cross section area of the column (optional if VELOCITY is present, see Section 2.2.3)

Unit:  $m^2$  Type: double Range: > 0 Length: 1

#### Group /input/model/unit\_XXX/discretization - UNIT\_TYPE = LUMPED\_RATE\_MODEL\_WITHOUT\_PORES

**NCOL** Number of axial column discretization cells

Type: int Range:  $\geq 1$  Length: 1

NBOUND Number of bound states for each component

Type: int Range: > 0 Length: NCOMP

**USE\_ANALYTIC\_JACOBIAN** Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0)

Type: int Range:  $\{0,1\}$  Length: 1

**RECONSTRUCTION** Type of reconstruction method for fluxes

Type: string Range: WEN0 Length: 1

#### Continuous stirred tank reactor model

Group /input/model/unit\_XXX - UNIT\_TYPE = CSTR

**UNIT\_TYPE** Specifies the type of unit operation model

Type: string Range: CSTR Length: 1

 ${\tt NCOMP}$  Number of chemical components in the chromatographic media

Type: int Range:  $\geq 1$  Length: 1

NBOUND Number of bound states for each component in each particle type in type-major ordering (optional, defaults to all 0)

Type: int Range:  $\geq 0$  Length: NPARTYPE · NCOMP

**USE\_ANALYTIC\_JACOBIAN** Determines whether analytically computed Jacobian matrix (faster) is used (value is 1) instead of Jacobians generated by algorithmic differentiation (slower, value is 0)

Type: int Range:  $\{0,1\}$  Length: 1

ADSORPTION\_MODEL Specifies the type of adsorption models of each particle type (optional, defaults to NONE)

Type: string Range: See Section 3.5.4 Length: NPARTYPE

INIT\_C Initial concentrations for each component in the mobile phase

Unit:  $mol m_{vv}^{-3}$  Type: double Range:  $\geq 0$  Length: NCOMP

INIT\_VOLUME Initial tank volume

Unit:  $m^3$  Type: double Range:  $\geq 0$  Length: 1

**INIT\_Q** Initial concentrations for each bound state of each component in each particle type's solid phase in type-component-major ordering (optional, defaults to all 0)

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

INIT\_STATE Full state vector for initialization (optional, INIT\_C, INIT\_Q, and INIT\_VOLUME will be ignored; if length is 2NDOF, then the second half is used for time derivatives)

Unit: various Type: double Range:  $\mathbb{R}$  Length: NDOF / 2NDOF

**POROSITY** Porosity  $\varepsilon$  (defaults to 1)

Unit: - Type: double Range: [0,1] Length: 1

**FLOWRATE\_FILTER** Flow rate of pure liquid without components (optional, defaults to  $0 \, \mathrm{m}^3 \, \mathrm{s}^{-1}$ )

Unit:  $m^3 s^{-1}$  Type: double Range:  $\geq 0$  Length: 1 / NSEC

PAR\_TYPE\_VOLFRAC Volume fractions of the particle types

Unit: - Type: double Range: [0, 1] Length: NPARTYPE

#### 3.5.3 Flux reconstruction methods

Group /input/model/unit\_XXX - WENO parameters

**BOUNDARY\_MODEL** Boundary model type: 0 = lower WENO order (stable),  $1 = \text{zero weights (unstable for small } D_{\text{ax}}$ ),  $2 = \text{zero weights for p} \neq 0$  (stable?), 3 = large ghost points

**Type:** int **Range:**  $\{0, 1, 2, 3\}$  **Length:** 1

WENO\_EPS WENO  $\varepsilon$ 

Type: double Range:  $\geq 0$  Length: 1

WENO\_ORDER WENO Order: 1 = standard upwind scheme (order 1), 2 (order 3), 3 (order 5); also called WENO <math>k

Type: int Range:  $\{1, 2, 3\}$  Length: 1

#### 3.5.4 Adsorption models

Externally dependent binding models Some binding models have a variant that can use external sources as specified in Section 3.5.1 (also see Section 2.3 and Table 2.3.1 on which binding models support this feature). For the sake of brevity, only the standard variant of those binding models is specified below. In order to obtain the format for the externally dependent variant, first replace the binding model name XXX by EXT\_XXX. Each parameter p (except for reference concentrations XXX\_REFC0 and XXX\_REFQ) depends on a (possibly distinct) external source in a polynomial way:

$$p(T) = p_{TTT}T^3 + p_{TT}T^2 + p_{T}T + p.$$

Thus, a parameter XXX\_YYY of the standard binding model variant is replaced by the four parameters EXT\_XXX\_YYY, EXT\_XXX\_YYY\_T, EXT\_XXX\_YYY\_TT, and EXT\_XXX\_YYY\_TTT. Since each parameter can depend on a different external source, the dataset EXTFUN (not listed in the standard variants below) should

contain a vector of 0-based integer indices of the external source of each parameter. The ordering of the parameters in EXTFUN is given by the ordering in the standard variant. However, if only one index is passed in EXTFUN, this external source is used for all parameters.

**Non-binding components** For binding models that do not support multiple bound states, many parameters can vary per component and their length is taken as NCOMP. However, these models still support non-binding components. In this case, the entries in their parameters that correspond to non-binding components are simply ignored.

Multiple particle types The group that contains the parameters of a binding model in unit operation with index XXX reads /input/model/unit\_XXX/adsorption. This is valid for models with a single particle type. If a model has multiple particle types, it may have a different binding model in each type. The parameters are then placed in the group /input/model/unit\_XXX/adsorption\_YYY instead, where YYY denotes the index of the particle type.

Note that, in any case, /input/model/unit\_XXX/adsorption\_000 contains the parameters of the first (and possibly sole) particle type. This group also takes precedence over a possibly existing /input/model/unit\_XXX/adsorption group.

#### Group /input/model/unit\_XXX/adsorption/consistency\_solver — Nonlinear consistency solver parameters

**SOLVER\_NAME** Name of the solver. Available solvers are LEVMAR, ATRN\_RES, ATRN\_ERR, and COMPOSITE.

Type: string Length: 1

**INIT\_DAMPING** Initial damping factor (default is 0.01)

Type: double Range:  $\geq 0$  Length: 1

MIN\_DAMPING Minimal damping factor (default is 0.0001; ignored by LEVMAR)

Type: double Range:  $\geq 0$  Length: 1

SUBSOLVERS Vector with names of solvers for the composite solver (only required for composite solver).

See SOLVER\_NAME for available solvers.

Type: string Length: > 1

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = LINEAR

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

 ${f LIN\_KA}$  Adsorption rate constants for each component

Unit:  $m_{MP}^3 m_{SP}^{-3} s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

 ${ t LIN\_KD}$  Desorption rate constants for each component

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = MULTI\_COMPONENT\_LANGMUIR

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

MCL\_KA Adsorption rate constants

Type: double Range:  $\geq 0$  Length: NCOMP

MCL\_KD Desorption rate constants

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

MCL\_QMAX Maximum adsorption capacities

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range: > 0 Length: NCOMP

Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = MULTI\_COMPONENT\_ANTILANGMUIR

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

MCAL\_KA Adsorption rate constants

Unit:  $m_{MP}^3 \mod^{-1} s^-$ Type: double Range:  $\geq 0$  Length: NCOMP

MCAL\_KD Desorption rate constants

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

MCAL\_QMAX Maximum adsorption capacities

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range: > 0 Length: NCOMP

MCAL\_ANTILANGMUIR Anti-Langmuir coefficients (optional)

Unit:  $mol m_{SP}^{-3}$  Type: double Range:  $\{-1,1\}$  Length: NCOMP

Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = MOBILE\_PHASE\_MODULATOR

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

MPM\_KA Adsorption rate constants

Unit:  $m_{MP}^3 \mod^{-1} s$ -Type: double Range:  $\geq 0$  Length: NCOMP

 $\ensuremath{\mathsf{MPM\_KD}}$  Desorption rate constants

Unit:  $m_{MP}^{3\beta} \mod^{-\beta} s$  Type: double Range:  $\geq 0$  Length: NCOMP

 $MPM\_QMAX$  Maximum adsorption capacities

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: NCOMP

MPM\_BETA Parameters describing the ion-exchange characteristics (IEX)

Unit:  $mol \, m_{\rm SP}^{-3}$  Type: double Range:  $\geq 0$  Length: NCOMP

MPM\_GAMMA Parameters describing the hydrophobicity (HIC)

Unit:  $m_{MP}^3 \mod^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = STERIC\_MASS\_ACTION

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

SMA\_KA Adsorption rate constants

Unit:  $m_{MP}^3 m_{SP}^{-3} s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

 $\textbf{SMA\_KD} \ \ \mathrm{Desorption} \ \ \mathrm{rate} \ \ \mathrm{constants}$ 

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

SMA\_NU Characteristic charges of the protein; The number of sites  $\nu$  that the protein interacts with on

the resin surface

Type: double Range:  $\geq 0$  Length: NCOMP

SMA\_SIGMA Steric factors of the protein; The number of sites  $\sigma$  on the surface that are shielded by the protein and prevented from exchange with the salt counterions in solution

Type: double Range:  $\geq 0$  Length: NCOMP

SMA\_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites available on the resin surface

Unit:  $mol m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: 1

**SMA\_REFCO** Reference liquid phase concentration (optional, defaults to 1.0)

Unit:  $mol m_{MP}^{-3}$  Type: double Range: > 0 Length: 1

**SMA\_REFQ** Reference solid phase concentration (optional, defaults to 1.0)

Unit:  $mol m_{SP}^{-3}$  Type: double Range: > 0 Length: 1

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = SELF\_ASSOCIATION

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

 $\textbf{SAI\_KA1} \ \, \textbf{Adsorption} \ \, \textbf{rate} \ \, \textbf{constants}$ 

Unit:  $m_{MP}^3 m_{SP}^{-3} s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

SAI\_KA2 Adsorption rate constants

Unit:  $m_{MP}^6 m_{SP}^{-6} s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

**SAI\_KD** Desorption rate constants

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

**SAI\_NU** Characteristic charges  $\nu$  of the protein

Type: double Range:  $\geq 0$  Length: NCOMP

SAI\_SIGMA Steric factors  $\sigma$  of the protein

Type: double Range: > 0 Length: NCOMP

SAI\_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites

available on the resin surface

Unit:  $mol m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: 1

**SAI\_REFCO** Reference liquid phase concentration (optional, defaults to 1.0)

Unit:  $mol m_{MP}^{-3}$  Type: double Range: > 0 Length: 1

**SAI\_REFQ** Reference solid phase concentration (optional, defaults to 1.0)

Unit:  $mol m_{SP}^{-3}$  Type: double Range: > 0 Length: 1

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = SASKA

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

SASKA\_H Henry coefficient

Unit:  $m_{MP}^3 m_{SP}^{-3} s^{-1}$  Type: double Range:  $\mathbb{R}$  Length: NCOMP

SASKA\_K Quadratic factors

Unit:  $m_{MP}^6 m_{SP}^{-3} \mod T$ ype: double Range:  $\mathbb{R}$  Length: NCOMP<sup>2</sup>

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = MULTI\_COMPONENT\_BILANGMUIR

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length:

MCBL\_KA Adsorption rate constants in state-major ordering

Unit:  $m_{MP}^3 \mod^{-1} s^-$ Type: double Range:  $\geq 0$  Length: NSTATES · NCOMP

MCBL\_KD Desorption rate constants in state-major ordering

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NSTATES · NCOMP

MCBL\_QMAX Maximum adsorption capacities in state-major ordering

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range: > 0.0 Length: NSTATES · NCOMP

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = KUMAR\_MULTI\_COMPONENT\_LANGMUIR

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

KMCL\_KA Adsorption pre-exponential factors

Unit:  $m_{MP}^3 \mod^{-1} s$  Type: double Range:  $\geq 0$  Length: NCOMP

 $KMCL\_KD$  Desorption rate

Unit:  $m_{MP}^{3\nu_i} \mod^{-\nu_i} s$  Type: double Range:  $\geq 0$  Length: NCOMP

 $\textbf{KMCL\_KACT} \ \, \text{Activation temperatures}$ 

Unit: K Type: double Range:  $\geq 0$  Length: NCOMP

KMCL\_QMAX Maximum adsorption capacities

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range: > 0 Length: NCOMP

KMCL\_NU Salt exponents / characteristic charges

Type: double Range: > 0 Length: NCOMP

 $\begin{tabular}{ll} KMCL\_TEMP & Temperature \\ \end{tabular}$ 

Unit: K Type: double Range:  $\geq 0$  Length: 1

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = MULTI\_COMPONENT\_SPREADING

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

MCSPR\_KA Adsorption rate constants in state-major ordering

Unit:  $m_{MP}^3 \mod^{-1} s^-$ Type: double Range:  $\geq 0$  Length: NTOTALBND

 ${\tt MCSPR\_KD}$  Desorption rate constants in state-major ordering

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

MCSPR\_QMAX Maximum adsorption capacities in state-major ordering

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range: > 0 Length: NTOTALBND

 $MCSPR\_K12$  Exchange rates from the first to the second bound state

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

MCSPR\_K21 Exchange rates from the second to the first bound state

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = MULTISTATE\_STERIC\_MASS\_ACTION

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

MSSMA\_KA Adsorption rate constants of the components to the different bound states in component-major ordering

Unit:  $m_{MP}^3 m_{SP}^{-3} s^{-1}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

MSSMA\_KD Desorption rate constants of the components in the different bound states in component-major ordering

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

MSSMA\_NU Characteristic charges of the components in the different bound states in component-major ordering

Type: double Range:  $\geq 0$  Length: NTOTALBND

MSSMA\_SIGMA Steric factors of the components in the different bound states in component-major ordering

Type: double Range: > 0

Length: NTOTALBND

 $\textbf{MSSMA\_RATES} \ \ \text{Conversion rates between different bound states in component-row-major ordering}$ 

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length:  $\sum_{i=0}^{\mathsf{NCOMP}-1} \mathsf{NBND}_i^2$ 

MSSMA\_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites available on the resin surface

Unit:  $mol m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: 1

MSSMA\_REFCO Reference liquid phase concentration (optional, defaults to 1.0)

Unit:  $mol m_{MP}^{-3}$  Type: double Range: > 0 Length: 1

MSSMA\_REFQ Reference solid phase concentration (optional, defaults to 1.0)

Unit:  $mol m_{SP}^{-3}$  Type: double Range: > 0 Length: 1

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = SIMPLE\_MULTISTATE\_STERIC\_MASS\_ACTION

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary

Type: int Range:  $\{0,1\}$  Length: 1

SMSSMA\_LAMBDA Stationary phase capacity (monovalent salt counterions); The total number of binding sites available on the resin surface

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range:  $\geq 0$  Length: 1

SMSSMA\_KA Adsorption rate constants of the components to the different bound states in component-major ordering

Unit:  $m_{\mathrm{MP}}^{3} m_{\mathrm{SP}}^{-3} \, \mathrm{s}^{-1} \, \mathrm{Type}$ : double Range:  $\geq 0$  Length: NTOTALBND

 $\textbf{SMSSMA\_KD} \ \, \text{Desorption rate constants of the components to the different bound states in component-major ordering}$ 

Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NTOTALBND

SMSSMA\_NU\_MIN Characteristic charges of the components in the first (weakest) bound state Unit:  $s^{-1}$  Type: double Range:  $\geq 0$  Length: NCOMP

SMSSMA\_NU\_MAX Characteristic charges of the components in the last (strongest) bound state

Type: double Range:  $\geq 0$  Length: NCOMP

SMSSMA\_NU\_QUAD Quadratic modifiers of the characteristic charges of the different components depending on the index of the bound state Type: double Range:  $\mathbb{R}$ Length: NCOMP SMSSMA\_SIGMA\_MIN Steric factors of the components in the first (weakest) bound state Type: double Range:  $\geq 0$ Length: NCOMP SMSSMA\_SIGMA\_MAX Steric factors of the components in the last (strongest) bound state Length: NCOMP Type: double Range:  $\geq 0$ SMSSMA\_SIGMA\_QUAD Quadratic modifiers of steric factors of the different components depending on the index of the bound state Type: double Range:  $\mathbb{R}$ Length: NCOMP SMSSMA\_KWS Exchange rates from a weakly bound state to the next stronger bound state Type: double Range:  $\geq 0$ Length: NCOMP SMSSMA\_KWS\_LIN Linear exchange rate coefficients from a weakly bound state to the next stronger bound state Unit:  $s^{-1}$ Type: double Range:  $\mathbb{R}$ Length: NCOMP SMSSMA\_KWS\_QUAD Quadratic exchange rate coefficients from a weakly bound state to the next stronger bound state Unit:  $s^{-1}$ Type: double Range:  $\mathbb{R}$ Length: NCOMP SMSSMA\_KSW Exchange rates from a strongly bound state to the next weaker bound state Unit:  $s^{-1}$ Type: double Range:  $\geq 0$ Length: NCOMP SMSSMA\_KSW\_LIN Linear exchange rate coefficients from a strongly bound state to the next weaker bound state Unit:  $s^{-1}$ Type: double Range:  $\mathbb{R}$ Length: NCOMP SMSSMA\_KSW\_QUAD Quadratic exchange rate coefficients from a strongly bound state to the next weaker bound state Unit:  $s^{-1}$ Type: double Range:  $\mathbb{R}$ Length: NCOMP SMSSMA\_REFCO Reference liquid phase concentration (optional, defaults to 1.0) Unit:  $mol m_{MP}^{-3}$ Type: double Range: > 0Length: 1 **SMSSMA\_REFQ** Reference solid phase concentration (optional, defaults to 1.0) Unit:  $mol m_{SP}^{-3}$ **Type:** double **Range:** > 0Length: 1

#### Group /input/model/unit\_XXX/adsorption - ADSORPTION\_MODEL = BI\_STERIC\_MASS\_ACTION

**IS\_KINETIC** Selects kinetic or quasi-stationary adsorption mode: 1 = kinetic, 0 = quasi-stationary**Type:** int **Range:**  $\{0,1\}$  **Length:** 1

BISMA\_KA Adsorption rate constants in state-major ordering

Unit:  $m_{MP}^3 m_{SP}^{-3} s^{-1}$  Type: double Range:  $\geq 0.0$  Length: NSTATES · NCOMP

 $\textbf{BISMA\_KD} \ \ \text{Desorption rate constants in state-major ordering}$ 

Unit:  $s^{-1}$  Type: double Range:  $\geq 0.0$  Length: NSTATES · NCOMP

**BISMA\_NU** Characteristic charges  $\nu_{i,j}$  of the *i*th protein with respect to the *j*th binding site type in state-major ordering

Unit:  $s^{-1}$  Type: double Range:  $\geq 0.0$  Length: NSTATES · NCOMP

**BISMA\_SIGMA** Steric factors  $\sigma_{i,j}$  of the *i*th protein with respect to the *j*th binding site type in state-major ordering

Unit:  $s^{-1}$  Type: double Range:  $\geq 0.0$  Length: NSTATES · NCOMP

**BISMA\_LAMBDA** Stationary phase capacity (monovalent salt counterions) of the different binding site types  $\lambda_i$ 

Unit:  $mol \, m_{SP}^{-3}$  Type: double Range:  $\geq 0.0$  Length: NSTATES

**BISMA\_REFCO** Reference liquid phase concentration for each binding site type or one value for all types (optional, defaults to 1.0)

Unit:  $mol m_{MP}^{-3}$  Type: double Range: > 0 Length:  $\{1, NSTATES\}$ 

**BISMA\_REFQ** Reference solid phase concentration for each binding site type or one value for all types (optional, defaults to 1.0)

Unit:  $mol m_{SP}^{-3}$  Type: double Range: > 0 Length:  $\{1, NSTATES\}$ 

#### 3.5.5 Return data

#### Group /input/return

WRITE\_SOLUTION\_TIMES Write times at which a solution was produced (optional, defaults to 1)

Type: int Range:  $\{0,1\}$ 

WRITE\_SOLUTION\_LAST Write full solution state vector at last time point (optional, defaults to 0)

Type: int Range:  $\{0,1\}$ 

WRITE\_SENS\_LAST Write full sensitivity state vectors at last time point (optional, defaults to 0)

Type: int Range:  $\{0,1\}$ 

SPLIT\_COMPONENTS\_DATA Determines whether a joint dataset (matrix) for all components is created or if each component is put in a separate dataset (XXX\_COMP\_000, XXX\_COMP\_001, etc.) (optional, defaults to 1)

Type: int Range:  $\{0,1\}$ 

#### Group /input/return/unit\_XXX

WRITE\_SOLUTION\_INLET Write solutions at unit operation inlet  $c_i^l(t,0)$ 

Type: int Range:  $\{0,1\}$ 

**WRITE\_SOLUTION\_OUTLET** Write solutions at unit operation outlet (chromatograms)  $c_i^l(t,L)$ 

Type: int Range:  $\{0,1\}$ 

WRITE\_SOLUTION\_BULK Write solutions of the bulk volume  $c_i^l$ 

Type: int Range:  $\{0,1\}$ 

WRITE\_SOLUTION\_PARTICLE Write solutions of the particle mobile phase  $c_{i,i}^p$ 

Type: int Range:  $\{0,1\}$ 

WRITE\_SOLUTION\_SOLID Write solutions of the solid phase  $c_{j,i,m_{j,i}}^s$ 

Type: int Range:  $\{0,1\}$ 

WRITE\_SOLUTION\_FLUX Write solutions of the bead fluxes  $j_{f,i}$ 

Type: int Range:  $\{0,1\}$ 

WRITE\_SOLUTION\_VOLUME Write solutions of the volume V

Type: int Range:  $\{0,1\}$ 

```
WRITE_SOLDOT_INLET Write solution time derivatives at unit operation inlet \partial c_i^l(t,0)/\partial t
      Type: int
                        Range: \{0,1\}
WRITE_SOLDOT_OUTLET Write solution time derivatives at unit operation outlet (chromatograms) \partial c_i^l(t,L)/\partial t
                        Range: \{0,1\}
      Type: int
WRITE_SOLDOT_BULK Write solution time derivatives of the bulk volume \partial c_i^l/\partial t
                        Range: \{0,1\}
      Type: int
WRITE_SOLDOT_PARTICLE Write solution time derivatives of the particle mobile phase \partial c_{i,i}^{p}/\partial t
      Type: int
                        Range: \{0,1\}
WRITE_SOLDOT_SOLID Write solution time derivatives of the solid phase \partial c_{i,i,m_{i,i}}^s/\partial t
                        Range: \{0,1\}
      Type: int
WRITE_SOLDOT_FLUX Write solution time derivatives of the bead fluxes \partial j_{f,i}/\partial t
      Type: int
                        Range: \{0,1\}
WRITE_SOLDOT_VOLUME Write solution time derivatives of the volume \partial V/\partial t
                        Range: \{0,1\}
      Type: int
WRITE_SENS_INLET Write sensitivities at unit operation inlet \partial c_i^l(t,0)/\partial p
      Type: int
                        Range: {0,1}
WRITE_SENS_OUTLET Write sensitivities at unit operation outlet (chromatograms) \partial c_i^l(t,L)/\partial p
                        Range: \{0,1\}
      Type: int
WRITE_SENS_BULK Write sensitivities of the bulk volume \partial c_i^l/\partial p
                        Range: \{0,1\}
      Type: int
WRITE_SENS_PARTICLE Write sensitivities of the particle mobile phase \partial c_{i,i}^p/\partial p
                        Range: \{0,1\}
      Type: int
WRITE_SENS_SOLID Write sensitivities of the solid phase \partial c_{j,i,m_{i,i}}^s/\partial p
                        Range: \{0,1\}
      Type: int
WRITE_SENS_FLUX Write sensitivities of the bead fluxes \partial j_{f,i}/\partial p
      Type: int
                        Range: \{0,1\}
WRITE_SENS_VOLUME Write sensitivities of the volume \partial V/\partial p
      Type: int
                        Range: \{0,1\}
WRITE_SENSOOT_INLET Write sensitivity time derivatives at unit operation inlet \partial^2 c_i^l(t,0)/(\partial p,\partial t)
      Type: int
                        Range: \{0,1\}
WRITE_SENSDOT_OUTLET Write sensitivity time derivatives at unit operation outlet (chromatograms)
      \partial^2 c_i^l(t,L)/(\partial p,\partial t)
      Type: int
                        Range: \{0,1\}
WRITE_SENSDOT_BULK Write sensitivity time derivatives of the bulk volume \partial^2 c_i^l/(\partial p, \partial t)
      Type: int
                        Range: \{0,1\}
WRITE_SENSOOT_PARTICLE Write sensitivity time derivatives of the particle mobile phase \partial^2 c_{i,i}^p/(\partial p,\partial t)
                        Range: \{0,1\}
      Type: int
WRITE_SENSOOT_SOLID Write sensitivity time derivatives of the solid phase \partial^2 c_{i,i,m_{i,i}}^s/(\partial p,\partial t)
      Type: int
                        Range: \{0,1\}
```

**WRITE\_SENSOOT\_FLUX** Write sensitivity time derivatives of the bead fluxes  $\partial^2 j_{f,i}/(\partial p, \partial t)$ 

Type: int **Range:**  $\{0,1\}$ 

**WRITE\_SENSDOT\_VOLUME** Write sensitivity time derivatives of the volume  $\partial^2 V/(\partial p, \partial t)$ 

Type: int **Range:**  $\{0,1\}$ 

#### 3.5.6 Parameter sensitivities

#### Group /input/sensitivity

**NSENS** Number of sensitivities to be computed

Type: int Range:  $\geq 0$ Length: 1

SENS\_METHOD Method used for computation of sensitivities (algorithmic differentiation)

Type: string Range: ad1 Length: 1

#### Group /input/sensitivity/param\_XXX

**SENS\_UNIT** Unit operation index

Range:  $\geq 0$ Length:  $\geq 1$ Type: int

**SENS\_NAME** Name of the parameter

Type: string Range: See **Length:**  $\geq 1$ 

**SENS\_COMP** Component index (-1) if parameter is independent of components

Type: int Range:  $\geq -1$ Length:  $\geq 1$ 

**SENS\_REACTION** Reaction index (-1 if parameter is independent of reactions)

Range:  $\geq -1$ **Length:**  $\geq 1$ Type: int

**SENS\_BOUNDPHASE** Bound phase index (-1) if parameter is independent of bound phases)

Type: int Range:  $\geq -1$ Length:  $\geq 1$ 

**SENS\_SECTION** Section index (-1) if parameter is independent of sections

Range:  $\geq -1$ Type: int Length:  $\geq 1$ 

**SENS\_ABSTOL** Absolute tolerance used in the computation of the sensitivities (optional). Rule of thumb:

ABSTOL / PARAM\_VALUE

Type: double Range:  $\geq 0.0$ Length:  $\geq 1$ 

**SENS\_FACTOR** Linear factor of the combined sensitivity (optional, taken as 1.0 if left out)

Type: double Range:  $\mathbb{R}$ Length:  $\geq 1$ 

#### 3.5.7 Solver configuration

#### Group /input/solver

NTHREADS Number of used threads

Range:  $\geq 1$ Length: 1 Type: int

USER\_SOLUTION\_TIMES Vector with timepoints at which the solution is evaluated

Unit: s Type: double Range:  $\geq 0$ Length: Arbitrary

**CONSISTENT\_INIT\_MODE** Consistent initialization mode (optional, defaults to 1). Valid values are 0 (none),

1 (full), 2 (once, full), 3 (lean), 4 (once, lean), 5 (full once, then lean), 6 (none once, then full), and 7 (none once, then lean).

Type: int **Range:**  $\{0, ..., 7\}$ Length: 1 CONSISTENT\_INIT\_MODE\_SENS Consistent initialization mode for parameter sensitivities (optional, defaults to 1). Valid values are 0 (none), 1 (full), 2 (once, full), 3 (lean), 4 (once, lean), 5 (full once, then lean), 6 (none once, then full), and 7 (none once, then lean). **Range:**  $\{0, ..., 7\}$ Type: int Length: 1 Group /input/solver/time\_integrator **ABSTOL** Absolute tolerance in the solution of the original system **Type:** double **Range:** > 0Length: 1 **RELTOL** Relative tolerance in the solution of the original system Type: double Range:  $\geq 0$ Length: 1 ALGTOL Tolerance in the solution of the nonlinear consistency equations Type: double Range: > 0Length: 1 **RELTOL\_SENS** Relative tolerance in the solution of the sensitivity systems Type: double Range:  $\geq 0$ Length: 1 INIT\_STEP\_SIZE Initial time integrator step size for each section or one value for all sections (0.0: IDAS default value), see IDAS guide 4.5, p. 36f. **Type:** double **Range:**  $\geq 0$ Length: 1 / NSEC MAX\_STEPS Maximum number of timesteps taken by IDAS (0: IDAS default = 500), see IDAS guide Sec. 4.5 Range:  $\geq 0$ Length: 1 Type: int MAX\_STEP\_SIZE Maximum size of timesteps taken by IDAS (optional, defaults to unlimited 0.0), see IDAS guide Sec. 4.5 Unit: s Type: double Range:  $\geq 0$ Length: 1 ERRORTEST\_SENS Determines whether (forward) sensitivities take part in local error test (optional, defaults to 1) Length: 1 Type: int **Range:**  $\{0,1\}$ MAX\_NEWTON\_ITER Maximum number of Newton iterations in time step (optional, defaults to 3) Range:  $\geq 0$ Length: 1 Type: int MAX\_ERRTEST\_FAIL Maximum number of local error test failures in time step (optional, defaults to 7) Type: int Range: > 0Length: 1 MAX\_CONVTEST\_FAIL Maximum number of Newton convergence test failures (optional, defaults to 10) Type: int Range: > 0Length: 1 MAX\_NEWTON\_ITER\_SENS Maximum number of Newton iterations in forward sensitivity time step (optional, defaults to 3) Type: int Range:  $\geq 0$ Length: 1

#### Group /input/solver/sections

**NSEC** Number of sections

Type: int Range:  $\geq 1$  Length: 1

**SECTION\_TIMES** Simulation times at which the model changes or behaves discontinuously; including start and end times

Unit: s Type: double Range:  $\geq 0$  Length: NSEC + 1

**SECTION\_CONTINUITY** Continuity indicator for each section transition: 0 (discontinuous) or 1 (continuous).

Type: int Range:  $\{0,1\}$  Length: NSEC -1

### 3.6 Output group

#### **Group /output/solution**

 $\textbf{LAST\_STATE} \ \ \text{Full state vector at the last time point of the time integrator if $WRITE\_SOLUTION\_LAST$ in $$/input/return is enabled $$$ 

Type: double

LAST\_STATE\_YDOT Full time derivative state vector at the last time point of the time integrator if WRITE\_SOLUTION\_LAST in /input/return is enabled

Type: double

LAST\_STATE\_SENSY\_XXX Full state vector of the XXXth sensitivity system at the last time point of the time integrator if WRITE\_SENS\_LAST in /input/return is enabled

Type: double

LAST\_STATE\_SENSYDOT\_XXX Full time derivative state vector of the XXXth sensitivity system at the last time point of the time integrator if WRITE\_SENS\_LAST in /input/return is enabled

Type: double

#### **Group /output/solution**

**SOLUTION\_TIMES** Time points at which the solution is written if WRITE\_SOLUTION\_TIMES in /input/return is enabled

Unit: s Type: double

#### Group /output/solution/unit\_XXX

SOLUTION\_BULK Interstitial solution as  $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$  tensor in row-major storage

Unit:  $mol m_{IV}^{-3}$  Type: double

**SOLUTION\_PARTICLE** Mobile phase solution inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol m_{MP}^{-3}$  Type: double

SOLUTION\_PARTICLE\_PARTYPE\_XXX Mobile phase solution inside the particles of type XXX as  $n_{\mathrm{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol m_{MP}^{-3}$  Type: double

**SOLUTION\_SOLID** Solid phase solution inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol m_{MP}^{-3}$  Type: double

**SOLUTION\_SOLID\_PARTYPE\_XXX** Solid phase solution inside the particles of type XXX as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol m_{SP}^{-3}$  Type: double

**SOLUTION\_FLUX** Flux solution as  $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$  tensor in row-major storage

Unit:  $mol m^{-2} s^{-1}$  Type: double

 $\textbf{SOLUTION\_VOLUME} \ \ \mathrm{Volume} \ \ \mathrm{solution}$ 

Unit: m<sup>3</sup> Type: double

**SOLUTION\_OUTLET** Matrix of solutions at the unit operation outlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $mol m_{IV}^{-3}$  Type: double

**SOLUTION\_INLET** Matrix of solutions at the unit operation inlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $mol m_{IV}^{-3}$  Type: double

**SOLUTION\_OUTLET\_COMP\_XXX** Component XXX of the solution at the unit operation outlet with. Only present if SPLIT\_COMPONENTS\_DATA is enabled.

Unit:  $mol m_{IV}^{-3}$  Type: double

**SOLUTION\_INLET\_COMP\_XXX** Component XXX of the solution at the unit operation inlet with. Only present if SPLIT\_COMPONENTS\_DATA is enabled.

Unit:  $mol m_{IV}^{-3}$  Type: double

**SOLDOT\_BULK** Interstitial solution time derivative as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage Unit: mol m<sub>IV</sub><sup>-3</sup> s<sup>-1</sup> Type: double

**SOLDOT\_PARTICLE** Mobile phase solution time derivative inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol m_{MP}^{-3} s^{-1}$  Type: double

SOLDOT\_PARTICLE\_PARTYPE\_XXX Mobile phase solution time derivative inside the particles of type XXX as  $n_{\mathrm{Time}} \times \mathsf{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol m_{MP}^{-3} s^{-1}$  Type: double

**SOLDOT\_SOLID** Solid phase solution time derivative inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol m_{MP}^{-3} s^{-1}$  Type: double

SOLDOT\_SOLID\_PARTYPE\_XXX Solid phase solution time derivative inside the particles of type XXX as  $n_{\mathrm{Time}} \times \mathtt{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol m_{SP}^{-3} s^{-1}$  Type: double

 ${\tt SOLDOT\_FLUX} \ \ {\rm Flux} \ \ {\rm solution} \ \ {\rm time} \ \ {\rm derivative} \ \ {\rm as} \ \ n_{\rm Time} \times {\tt UNITOPORDERING} \ \ {\rm tensor} \ \ {\rm in} \ \ {\rm row-major} \ \ {\rm storage}$ 

Unit:  $mol m^{-2} s^{-2}$  Type: double

**SOLDOT\_VOLUME** Volume solution time derivative

Unit:  $m^3 s^{-1}$  Type: double

**SOLDOT\_OUTLET** Matrix of solution time derivatives at the unit operation outlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $mol m_{IV}^{-3} s^{-1}$  Type: double

**SOLDOT\_INLET** Matrix of solution time derivatives at the unit operation inlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $mol m_{IV}^{-3} s^{-1}$  Type: double

 ${\tt SOLDOT\_OUTLET\_COMP\_XXX} \ \, {\tt Component} \ \, {\tt XXX} \ \, {\tt of the solution time derivative at the unit operation outlet} \\ \, {\tt with.} \ \, {\tt Only present if SPLIT\_COMPONENTS\_DATA is enabled.}$ 

Unit:  $mol m_{IV}^{-3} s^{-1}$  Type: double

**SOLDOT\_INLET\_COMP\_XXX** Component XXX of the solution time derivative at the unit operation inlet with. Only present if SPLIT\_COMPONENTS\_DATA is enabled.

Unit:  $mol m_{IV}^{-3} s^{-1}$  Type: double

#### Group /output/sensitivity/param\_XXX/unit\_YYY

 $\textbf{SENS\_BULK} \ \ \text{Interstitial sensitivity as} \ \ n_{\text{Time}} \times \textbf{UNITOPORDERING} \ \text{tensor} \ \text{in row-major storage}$ 

Unit:  $mol m_{IV}^{-3}$  [Parafflype: double

**SENS\_PARTICLE** Mobile phase sensitivity inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol m_{MP}^{-3}$  [ParaType: double

SENS\_PARTICLE\_PARTYPE\_XXX Mobile phase sensitivity inside the particles of type XXX as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol \, m_{MP}^{-3}$  [ParaType: double

**SENS\_SOLID** Solid phase sensitivity inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol m_{MP}^{-3}$  [ParaTrype: double

SENS\_SOLID\_PARTYPE\_XXX Solid phase sensitivity inside the particles of type XXX as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol m_{SP}^{-3}$  [Parafflype: double

SENS\_FLUX Flux sensitivity as  $n_{\mathrm{Time}} imes \mathtt{UNITOPORDERING}$  tensor in row-major storage

Unit: mol m<sup>-2</sup> s<sup>-1</sup> [Payane]: double

SENS\_VOLUME Volume sensitivity

Unit:  $m^3$  [Param]<sup>-1</sup> Type: double

**SENS\_OUTLET** Matrix of sensitivities at the unit operation outlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $\operatorname{mol} \operatorname{m}_{\mathrm{IV}}^{-3}$  [Paraffiype: double

**SENS\_INLET** Matrix of sensitivities at the unit operation inlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $mol m_{IV}^{-3}$  [Parafflype: double

**SENS\_OUTLET\_COMP\_XXX** Component XXX of the sensitivity at the unit operation outlet with. Only present if SPLIT\_COMPONENTS\_DATA is enabled.

Unit:  $mol m_{IV}^{-3}$  [Paraffi] ple: double

**SENS\_INLET\_COMP\_XXX** Component XXX of the sensitivity at the unit operation inlet with. Only present if SPLIT\_COMPONENTS\_DATA is enabled.

Unit: mol m<sub>IV</sub><sup>-3</sup> [Parafflype: double

**SENSDOT\_BULK** Interstitial sensitivity time derivative as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage

Unit:  $mol m_{IV}^{-3} s^{-1}$  [PLyape]: double

**SENSDOT\_PARTICLE** Mobile phase sensitivity time derivative inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol m_{MP}^{-3} s^{-1}$  [Phype] double

SENSDOT\_PARTICLE\_PARTYPE\_XXX Mobile phase sensitivity time derivative inside the particles of type XXX as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol \, m_{MP}^{-3} \, s^{-1}$  [Phypne] double

**SENSDOT\_SOLID** Solid phase sensitivity time derivative inside the particles as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if just one particle type is defined.

Unit:  $mol \, m_{MP}^{-3} \, s^{-1} \, [Playane] \, double$ 

**SENSDOT\_SOLID\_PARTYPE\_XXX** Solid phase sensitivity time derivative inside the particles of type XXX as  $n_{\text{Time}} \times \text{UNITOPORDERING}$  tensor in row-major storage. Only present if more than one particle type is defined.

Unit:  $mol \, m_{SP}^{-3} \, s^{-1}$  [PLyapae]: double

SENSDOT\_FLUX Flux sensitivity time derivative as  $n_{\rm Time} \times {\tt UNITOPORDERING}$  tensor in row-major storage Unit: mol m<sup>-2</sup> s<sup>-2</sup> [PAlyape: double

SENSDOT\_VOLUME Volume sensitivity time derivative

Unit: s<sup>-3</sup> m [Param] **Type:** double

**SENSDOT\_OUTLET** Matrix of sensitivity time derivatives at the unit operation outlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $mol m_{IV}^{-3} s^{-1}$  [PALyapue]: double

**SENSDOT\_INLET** Matrix of sensitivity time derivatives at the unit operation inlet with components as columns in row-major storage. Only present if SPLIT\_COMPONENTS\_DATA is disabled.

Unit:  $mol m_{IV}^{-3} s^{-1}$  [Phyapoe: double

**SENSDOT\_OUTLET\_COMP\_XXX** Component XXX of the sensitivity time derivative at the unit operation outlet with. Only present if SPLIT\_COMPONENTS\_DATA is enabled.

Unit:  $mol m_{IV}^{-3} s^{-1}$  [PLyapae]: double

**SENSDOT\_INLET\_COMP\_XXX** Component XXX of the sensitivity time derivative at the unit operation inlet with. Only present if SPLIT\_COMPONENTS\_DATA is enabled.

Unit:  $\operatorname{mol} \operatorname{m}_{\mathrm{IV}}^{-3} \operatorname{s}^{-1}$  [PLyapae]: double

## 3.7 Meta group

Group /meta

**FILE\_FORMAT** Version of the file format (defaults to 030102 = 3.1.2 if omitted) with two digits per part (Major.Minor.Patch)

In/out: In Type: int

CADET\_VERSION Version of the executed CADET simulator

In/out: Out Type: string

CADET\_COMMIT Git commit SHA1 from which the CADET simulator was built

In/out: Out Type: string

CADET\_BRANCH Git branch from which the CADET simulator was built

In/out: Out Type: string

TIME\_SIM Time that the time integration took (excluding any preparations and postprocessing)

Unit: s In/out: Out Type: double

# **Bibliography**

- [BC92] Clayton A. Brooks and Steven M. Cramer. "Steric mass-action ion exchange: Displacement profiles and induced salt gradients". In: *AIChE Journal* 38.12 (Dec. 1992), pp. 1969–1978. DOI: 10.1002/aic.690381212.
- [Dan53] P.V. Danckwerts. "Continuous flow systems: Distribution of residence times". In: Chemical Engineering Science 2.1 (Feb. 1953), pp. 1–13. DOI: 10.1016/0009-2509(53)80001-1.
- [FG04] A. Felinger and G. Guiochon. "Comparison of the Kinetic Models of Linear Chromatography". In: *Chromatographia* 60.S1 (May 2004), pp. 175–180. DOI: 10.1365/s10337-004-0288-7.
- [FTB97] William F. Feehery, John E. Tolsma, and Paul I. Barton. "Efficient sensitivity analysis of large-scale differential-algebraic systems". In: *Applied Numerical Mathematics* 25.1 (Oct. 1997), pp. 41–54. DOI: 10.1016/S0168-9274(97)00050-0.
- [Gu95] Tingyue Gu. Mathematical Modeling and Scale-up of Liquid Chromatography. Berlin, Heidelberg: Springer Berlin Heidelberg, 1995. DOI: 10.1007/978-3-642-79541-1.
- [Gui+06] Georges Guiochon et al. Fundamentals of Preparative and Nonlinear Chromatography. 2nd. Amsterdam: Elsevier Academic Press, 2006, p. 990.
- [Hin+05] Alan C. Hindmarsh et al. "SUNDIALS: Suite of nonlinear and differential/algebraic equation solvers". In: *ACM Transactions on Mathematical Software* 31.3 (Sept. 2005), pp. 363–396. DOI: 10.1145/1089014.1089020.
- [Kar+04] David Karlsson et al. "Model-based optimization of a preparative ion-exchange step for antibody purification". In: *Journal of Chromatography A* 1055.1-2 (Nov. 2004), pp. 29–39. DOI: 10.1016/j.chroma.2004.08.151.
- [Kuč65] Eugene Kučera. "Contribution to the theory of chromatography". In: *Journal of Chromatography* 19 (Jan. 1965), pp. 237–248. DOI: 10.1016/S0021-9673(01)99457-9.
- [Kum+15] Vijesh Kumar et al. "Mechanistic modeling of ion-exchange process chromatography of charge variants of monoclonal antibody products". In: *Journal of Chromatography A* 1426 (Dec. 2015), pp. 140–153. DOI: 10.1016/j.chroma.2015.11.062.
- [Lan16] Irving Langmuir. "The Constitution and Fundamental Properties of Solids and Liquids. Part
   I. Solids". In: Journal of the American Chemical Society 38.11 (Nov. 1916), pp. 2221–2295.
   DOI: 10.1021/ja02268a002.
- [MEH89] Wayne R. Melander, Ziad El Rassi, and Csaba Horváth. "Interplay of hydrophobic and electrostatic interactions in biopolymer chromatography". In: *Journal of Chromatography* A 469 (Jan. 1989), pp. 3–27. DOI: 10.1016/S0021-9673(01)96437-4.
- [Miy07] Kanji Miyabe. "Surface diffusion in reversed-phase liquid chromatography using silica gel stationary phases of different C1 and C18 ligand densities." In: *Journal of Chromatography* A 1167.2 (Oct. 2007), pp. 161–70. DOI: 10.1016/j.chroma.2007.08.045.
- [Mol08] Jørgen M. Mollerup. "A Review of the Thermodynamics of Protein Association to Ligands, Protein Adsorption, and Adsorption Isotherms". In: Chemical Engineering & Technology 31.6 (June 2008), pp. 864–874. DOI: 10.1002/ceat.200800082.
- [MWW96] Z. Ma, R. D. Whitley, and N.-H. L. Wang. "Pore and surface diffusion in multicomponent adsorption and liquid chromatography systems". In: AIChE Journal 42.5 (May 1996), pp. 1244–1262. DOI: 10.1002/aic.690420507.
- [Püt+16] Andreas Püttmann et al. "Utilizing algorithmic differentiation to efficiently compute chromatograms and parameter sensitivities". In: *Chemical Engineering Science* 139 (Jan. 2016), pp. 152–162. DOI: 10.1016/j.ces.2015.08.050.

- [Sas+92] M. Saska et al. "Continuous Separation of Sugarcane Molasses with a Simulated Moving-Bed Adsorber. Adsorption Equilibria, Kinetics, and Application". In: Separation Science and Technology 27.13 (Oct. 1992), pp. 1711–1732. DOI: 10.1080/01496399208019442.
- [SS68] Petr Schneider and J. M. Smith. "Chromatographic study of surface diffusion". In: AIChE Journal 14.6 (Nov. 1968), pp. 886–895. DOI: 10.1002/aic.690140613.
- [Wes+12] K. Westerberg et al. "Model-Based Process Challenge of an Industrial Ion-Exchange Chromatography Step". In: *Chemical Engineering & Technology* 35.1 (Jan. 2012), pp. 183–190. DOI: 10.1002/ceat.201000560.