Shri Mata Vaishno Devi University
School of Mathematics

B.Tech ECE/ME (Semester-III) Examination: Major

Course Title: Integral Transform and Complex Analysis
Course Code: MTL2022 Max. Marks: 50 Time: 2 hr

Note: Attempt any five questions.

1. Define continuity and differentiability of a complex variable function and show that the function $f(z) = |z|^2$, $\forall\ z$ is continuous at every point of the complex plane. Is it differentiable at any point? Discuss.

2.(*i*) Define an analytic function and show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin although Cauchy Riemann equations are satisfied.

(*ii*) Prove that the real and imaginary parts of an analytic function are both harmonic.

3.(*i*) Find an analytic function whose imaginary part is $e^x[(x^2 - y^2)\cos y - 2xy \sin y]$. Also find its conjugate function.

(*ii*) If the potential function in an electrostatic field is $\log(x^2 + y^2)$, find the flux function.

4. An alternating current after passing through a rectifier has the form

$$i = \begin{cases} I_0 \sin x, & 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi, \end{cases}$$

where I_0 is the maximum current and the period is 2π. Express i as a Fourier series in $(0, 2\pi)$.

or

Find the Fourier sine transform of e^{-x}, where $x \ge 0$ and use this result to show that

$$\int_0^\infty \frac{t \sin tx}{1+t^2} dt = \frac{\pi e^{-x}}{2}, \quad x > 0.$$

5. Solve the initial value problem $y''' + y'' = e^t + t + 1$, $y(0) = y'(0) = y''(0) = 0$ by using Laplace transform.

6. Evaluate the following Integrals by Laplace transform

(*i*) $\int_0^\infty te^{-2t} \sin t dt$

(*ii*) $L\left\{ \int_0^t \frac{e^t \sin t}{t} dt \right\}$.

Shri Mata Vaishno Devi University
Department of Mathematics

B.Tech ECE/ME (Semester-III) Examination: Minor I
Course Title: Integral Transform and Complex Analysis
Course Code: MTL2022 Max. Marks: 20 Time: 1 hr
Note: Attempt any four questions.

1. A sinusoidal voltage $E \sin \omega t$ is passed through a half wave rectifier which clips the negative portion of the wave. Developing the resulting periodic function -

$$f(t) = \begin{cases} 0, & \text{if } \frac{-T}{2} < t < 0 \\ E \sin \omega t, & \text{if } 0 < t < \frac{T}{2}, \end{cases}$$

where $T = \frac{2\pi}{\omega}$, in a Fourier series.

2. Obtain the Fourier series, in the interval $[-\frac{1}{2}, \frac{1}{2}]$ of the function f given by

$$f(x) = \begin{cases} x - [x] - \frac{1}{2}, & \text{when } x \text{ is not an integer} \\ 0, & \text{otherwise.} \end{cases}$$

Also find the period of $f(x)$.

3. Show that for all values of x in $[-\pi, \pi]$, when k is not an integer,

$$\cos kx = \frac{\sin k\pi}{\pi} \left[\frac{1}{k} + \sum_{n=1}^{\infty} \frac{(-1)^n 2k \cos nx}{k^2 - n^2} \right].$$

Deduce that $\pi \cot k\pi = \frac{1}{k} + \sum_{n=1}^{\infty} \frac{2k}{k^2 - n^2}$ and $\frac{\pi}{\sin k\pi} = \sum_{n=1}^{\infty} (-1)^n \left[\frac{1}{n+k} + \frac{1}{n+1-k} \right]$

4. If a is a real number, find the Fourier series of the function f defined by

$$f(x) = e^{ax}, \quad -\pi < x < \pi$$

$$f(x + 2\pi) = f(x), \quad x \in \mathbb{R}$$

and hence show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{a^2 + n^2} = \frac{1}{2a^2} (a\pi \operatorname{cosech} a\pi - 1)$

5. Find the Fourier sine transform of e^{-x}, $x \geq 0$ and use it to show that

$$\int_0^{\infty} \frac{t \sin tx}{1 + t^2} dt = \frac{\pi e^{-x}}{2}, \quad x > 0.$$

6. State and prove Parseval's identity. The Fourier series for x in $(0, l)$ is

$$x = \frac{l}{2} + \sum_{n=1}^{\infty} \frac{2l}{n^2 \pi^2} \left[(-1)^n - 1 \right] \cos \left(\frac{n\pi x}{l} \right).$$

Use Parseval identity to prove that

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

SHRI MATA VAISHNO DEVI UNIVERSITY, KATRA
School of Mathematics
B. Tech.III sem. (Branch Electrical) Minor /
Major Examination (Odd/Even/Summer) 2019-20

Entry No: 1 8 B E E 0 2 1 | Total Number of Pages: [1]
Date: | Total Number of Questions: [6]

Course Title: Integeral transform and complex analysis
Course Code: MTL2023

Time Allowed: 1.5 Hours | Max Marks: [30]

Instructions / NOTE

i. Attempt All Questions.

Q1.	Express $f(x)=\lvert x\rvert,\ -\pi\le x\le\pi$, as fourier series. Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+......=\frac{\pi^2}{8}$	5	CO1
Q2.	State and prove convolution theorem.	5	CO1
Q3.	An alternating current after passing through a rectifier has the form $i=\begin{cases} I_0 Sinx, \text{ for } 0\le x\le\pi \\ 0, \pi\le x\le 2\pi \end{cases}$ Where I_0 is the maximum current and the period is 2π. Express i as a fourier series.	5	CO1
Q4.	Find a series of cosines of multiples of x which will represent $x\sin x$ in the interval $((0,\pi)$ and show that $\frac{1}{1.3}-\frac{1}{3.5}+........\frac{\pi-2}{4}$	5	CO1
Q5.	a.Find the inverse lapace transformation of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ by using convolution theorem. b. Prove that $f\otimes g=g\otimes f$ on $[o,\infty)$	5	CO2
Q6.	State and prove first shift property and use it to evaluate the lapace transformation of $t^2e^{3t}\cos kt$	5	CO2

Course Outcomes: After the successful completion of the course, students shall be able to

CO1. Find the fourier series of the periodic functions and its applications
CO2. Find the lapace transformation of the different functions like trigonometry function, functions, exponential functions etc. and its applications.

SHRI MA… DEVI UNIVERSITY, KATRA

…Mathematics

B. Tech.III se… …lectrical) Re- Minor Exam/

Major Exami… …ven/Summer) 2019-20

Entry No: 1 8 B E E 0 2 … Total Number of Pages: [1]

Date: Total Number of Questions: [6]

Course Title: Integeral transform and complex analysis

Course Code: MTL2023

Time Allowed: 3 Hours Max Marks: [50]

Instructions / NOTE

i. Attempt Any five Questions.

Q1.	a) Define an analytic function. What is necessary condition for a function $f(x) = u(x,y) + i\,v(x,y)$ to be analytic? Hence show that the function $f(z) = 2x^2 + y + i(y^2 - x)$ is not analytic.	5	CO4
	(b) Expand $f(z) = \frac{1}{1-z}$ in a Taylor series with centre $z_0 = 2i$.	5	
Q2.	a) Define isolated singularity of a complex function with a suitable example. Find the kind of singularity of the function $\sin\left(\frac{1}{1-z}\right)$ at $z = 1$.	5	CO4
	b) Find the residue of the function $f(z) = \frac{z}{(z-3)^3(z-2)(z-1)}$ at $z = 1$.	5	
Q3.	a) An alternating current after passing through a rectifier has the form $i = \begin{cases} I_0 \sin x, & \text{for } 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi \end{cases}$ Where I_0 is the maximum current and the period is 2π. Express i as a fourier series.	5	CO2
	b) State and prove Multipication by t- property and use it to evaluate $t^n \sin kt$	5	
Q4.	a) Find a series of cosines of multiples of x which will represent $x\sin x$ in the interval ($(0, \pi)$ and show that $\frac{1}{1.3} - \frac{1}{3.5} + \ldots\ldots = \frac{\pi - 2}{4}$	5	CO1
	b) Express fourier series for $f(x) = e^{-x}$ in the interval $0 \le x \le 2\pi$.	5	
Q5.	a. State and prove convolution theorem.	5	CO2
	b. Prove that $f \otimes g = g \otimes f$ on $[0, \infty)$	5	

Q6.	Solve the following differential equations a. $(D^2+n^2)y = a\sin(nt+b), \text{ with } \; y(0)=y'(0)=0$ b. $y''+9y=\cos 2t, \text{ with } \; y(0)=1, y(\frac{\pi}{2})=-1$	5 5	CO2

<u>Course Outcomes: After the successful completion of the course, students shall be able to</u>

CO1. Find the fourier series of the periodic functions and its applications
CO2. Find the lapace transformation of the different functions like trigonometry function, functions, exponential functions etc. and its applications.
CO3. Understand, construct and write proofs.
CO4. To develop the fundamentals complex analysis and use it to solve numerical analysis.