Title of my Dissertation

Conner J Adlington August XX, 2024



MSc in Theoretical Physics
The University of Edinburgh 2023

Abstract This is where you summarise the contents of your dissertation. It should be at least 100 words, but not more than 250 words.

Declaration

I declare that this dissertation was composed entirely by myself.

Personal Statement

You must include a Personal Statement in your dissertation. This should describe what you did during the project, and when you did it. Give an account of problems you faced and how you attempted to overcome them. The examples below are based on personal statements from MSc and MPhys projects in previous years, with (mostly-obvious) changes to make them anonymous.

Acknowledgements

Contents

1	Intr	oducti	on	2
2	Bac	kgrour	nd Theory	4
	2.1	Magne	etism	4
		2.1.1	Magnetic Dipole	4
		2.1.2	Gyromagnetic Ratio	5
		2.1.3	Electron Magnetic Moment	6
3	То	Sort		7
	3.1	Spin		7
	3.2	Defect	Orientation	7
	3.3	Miller	Indices	7
	3.4	Lattice	e Symmetry	8
	3.5	Linear	Combination of Atomic Orbitals	8
	3.6	Quant	um Sensing	8
		3.6.1	DiVincenzo Criteria	8
		3.6.2	Crystal Defects	8
		3.6.3	Coherence	9
		3.6.4	Sensitivity	9
		3.6.5	ODMR	9
		3.6.6	Multimodal Sensors	10
	3.7	Silicon	Carbide	10
		3.7.1	Production of SiC	10
		3.7.2	Colour Defects in SiC	11
		3.7.3	Wider Scientific Context	11
4	Tas	k		12
	4.1	Brief		12
	4.2	Work		12

	4.2.1	Concepts and Nomenclature	 12
	4.2.2	System Hamiltonian	 12
5	Design		16
6	Results an	nd Analysis	17
7	Conclusion	ns	18
\mathbf{A}	Python Co	ode	19

List of Tables

List of Figures

Introduction

The field of spectroscopy studies the way atoms and molecules interact with and exchange energy with a wider physical system - specifically through electromagnetic radiation. The electric field interacts with with the electric dipole moment and the magnetic field interacts with a magnetic dipole moment. Magnetic resonance spectroscopy focusses specifically on the interaction between the ${\bf B}$ field with magnetic moments which exist in a given material. This can be broken into two distinct fields:

Nuclear Magnetic Resonance (NMR) which studies the interaction with nuclear magnetic moments.

Electron Paramagnetic Resonance (EPR) which studies the interaction with magnetic moments of electrons.

Using Planck's relationship $E = h\nu$ and $c = \lambda\nu$ we may characterise the electromagnetic radiation by its energy which is, to a constant, equivalent to the frequency or the wavelength. EPR is observed in systems where the magnetic dipole of the electron is influences by an applied, oscillating magnetic field forcing transitions between electron energy levels. In general the measurable difference in energy levels for which the transition occurs is caused by an external magnetic field via the Zeeman effect. Some systems also exhibit energy level splitting in the absence of an applied external magnetic field so called zero field splitting.

EPR is thus a tool to manipulate electron spins in solid state materials. The transition between energy levels is quantised thus the discrete amount of energy which is lost by the system is transferred into a photon or charge state which may be detected optically or electrically [1].

A particularly successful technique is Optically Detected Magnetic Resonance (ODMR) which uses an applied microwave frequency, an oscillating magnetic field with energy quanta equivalent to the transitions between Zeeman sub levels, to drive the repopulation of those Zeeman sub levels following a spin-dependent optical transition. In essence this boosts the sensitivity since the microwave driven repopulation induces a change in photoluminescence with a much higher and thus much more readily detectable energy. The techniques of ODMR are so effective that even a single electron spin may be detected this way [2].

Spintronics, a portmantau of **spin** and elec**tronics** is a technology which exploits the characteristics of spin akin to how charge is manipulated in electronics. Fundamentally, the smallest stable magnetic moment available in nature is generated by the spin of a single electron. If efficient read-out can be achieved, the sensitivity of the electron magnetic dipole cannot be matched. Careful construction of an appropriate system, or identification of a system with appropriate characteristics allows for the initialisation, manipulation and read-out of EPR from which we may infer the physical properties of the environment surrounding the system.

The properties of nitrogen-vacancy (NV) colour centres in diamond have catalysed major development in the field. The NV and ODMR have enabled the manipulation of spin states in single, atomic-sized centres at room temperature despite spin polariation being a primarily thermodynamic effect. This is possible since optical excitation of the energy levels decay faster via a spin-preserving transition, leading to an inverse population of spin sublevels in its ground state when the system is irradiated consistently for several excitation/decay cycles. Other structures with similar unique quantum properties are being identified. Specifically, Silicon carbide (SiC) is a promising candidate. A major benefit of SiC is the existence of various polytypes, which each exhibit unique spin colour centre properties. Furthermore, even within a single polytype, these centres can occupy distinct and non-equivalent lattice positions. The existence of these colour centres with similar properties but different energy quanta allows for selection of a specific defect with parameters suitable for the problem at hand.

need a reference for the thermodynamic comment

EPR can be approached by two different methods:

Continuous Wave the magnitude of the static magnetic field (B_0) is swept, while the amplitude of the driving field B_1 is constant with time.

Pulsed a time-dependent driving pulse B_1 is applied in addition to a static magnetic field B_0 .

This work looks to explore how the physical characteristics which influence the Hamiltonian, thus the energy, of the electron spin may be inferred by measuring the effects of those characteristics on EPR of the specific system. Further, it will look to explore whether the compound effect of multiple influences may be disentangled and distinguished simultaneously.

Consider writing a paragraph on ENDOR - only if relevant later in the project.

Need to write a section on using Pulsed EPR to measure relaxation timescales.

Background Theory

2.1 Magnetism

2.1.1 Magnetic Dipole

Where charge (**E**-field) has an intuitive elementary source unit of a point charge (or monopole) which may be positively or negatively charged. Conversely the elementary source unit of magnetism (**B**-field) is the magnetic dipole.

Classically, the magnetic dipole may be modelled as a closed loop that carries an electric current. Its magnetic dipole moment, $\vec{\mu}$, is defined as the vector which points out of the plane of the current loop,

$$\vec{\mu} = IS\vec{n} \tag{2.1}$$

where I is the current in the loop and S is the surface area enclosed by the loop.

The magnetic dipole produces a magnetic field \vec{B} , which for points a large distance from the dipole may be calculated as:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\frac{3(\vec{\mu} \cdot \vec{r}) \cdot \vec{r}}{r^2} - \vec{\mu} \right]$$

The symmetry of the field enables us to, without any loss of generality, consider the direction of the dipole the z-axis. Then, defining x, y as usual by $r \cos \theta$ and $r \sin \theta$ respectively. We may then consider magnetic field in two separate components, parallel (B_z) and perpendicular (B_x, B_y) :

$$B_{\parallel} = \frac{\mu_0}{r^3} (3\cos^2\theta - 1), \quad B_{\perp} = \frac{3\mu_0}{r^3} \cos\theta \sin\theta.$$

Then, we may use the Pythagorean principle to determine the overall magnitude B as

$$B = \sqrt{B_{\parallel}^2 + B_{\perp}^2}.$$

Type up derivation from David Tong notes

2.1.2 Gyromagnetic Ratio

Classical Derivation

The current in equation 2.1 is proportional to the angular momentum of the charge. That is, the dipole moment is always associated with an angular momentum $\vec{G} = \vec{r} \times \vec{p}$ with \vec{r} the radius and \vec{p} the momentum.

Dividing the magnetic dipole moment by the angular momentum we find the **gyro-magnetic ratio**.

$$\gamma = \frac{\vec{\mu}}{\vec{G}}.\tag{2.2}$$

Without loss of generality we may consider the most simple case which is where the magnetic dipole moment is parallel (or anti-parallel) to the angular momentum. Then we may consider the absolute values for the dipole moment and the angular momentum:

$$\mu = IS, \quad I = \underbrace{\frac{q}{2\pi R}}_{a \text{ (charge density)}} v, \quad S = \pi R^2$$
 (2.3)

We substitute I and S to find

$$\mu = \frac{qvR}{2} \tag{2.4}$$

and further, we equate the angular momentum vector, using the model of a planar loop to

$$G = m_q v R (2.5)$$

leaving

$$\gamma = \frac{q}{2m_q}. (2.6)$$

We finally consider that we may represent the, currently unknown, charge and mass as a sum of electron charges and masses. We therefore find that the gyromagnetic ratio of the electron depends only on constants

$$\gamma = \frac{q}{2m_q} = \frac{\mathcal{N}e}{2\mathcal{N}m_e} \implies \gamma = \frac{e}{2m_e}.$$
(2.7)

[3]

Extending to Quantum Mechanics

Since the gyromagnetic ratio was calculated considering the motion of dipole in a loop, we may extend this to an electron in an orbit within the atom. The fundamental change required to extend the model to quantum mechanics is the treatment of angular momentum which should now be quantized. Thus, we replace our classical approximation of $\vec{G} = \vec{r} \times \vec{p}$ with the equation for the eigenvalues of the quantum mechanical representation of orbital angular momentum:

$$\hat{G} = \hbar \hat{J} \tag{2.8}$$

where \hat{J} is the operator of the orbital angular momentum (quantum number of orbital momentum).

919	Floatnon	Magnetic	Mamont
4.1.5	Liectron	magnenc	woment

[4]

To Sort

3.1 Spin

The magnetic moment of elementary particles is called spin.

3.2 Defect Orientation

Colour centres or defects in general are part of the crystal lattice and thus have an associated orientation and direction within the lattice. This allows the definition of a **defect axis**. For example, in diamond the NV axis is defined as the vector from the vacancy towards the Nitrogen atom when the vacancy is taken as the origin of your co-ordinate system.

In a tetragonal crystal, due to symmetry there are four possible orientations of a defect within the lattice: 111, $1\overline{11}$, $\overline{111}$ and $\overline{11}$ 1 directions.

3.3 Miller Indices

The notation for defect orientation above is known as a Miller Index, and we consider the 111 direction to be aligned with the defect axis.

This means that if we know the orientation of our crystal then we can establish the orientations of the defect axis inside. For example, using a crystal for which all surfaces belong to the {001} lattice planes, each surface normal is aligned with a Cartesian axis. Thus, by fixing the crystal in place, there remain just **four** possible angles which a defect axis can have with respect to the crystal surface.

Calculating the scalar product of any of the surface planar directions in the family of $\{001\}$ and the four possible orientations of the defect within the lattice we find $\cos \theta = \pm 0.6$. Then, considering the physical solutions (from $0, 2\pi$) gives four possible angles that the (directed) defect axis may make with the surface of the crystal: 53.13° , 306.87° , 126.9° and 233.13° (0.927, 5.355, 2.214 and 4.069 radians respectively).

3.4 Lattice Symmetry

Tetragonal lattice has the

3.5 Linear Combination of Atomic Orbitals

3.6 Quantum Sensing

Quantum sensing involves using a qubit system acting as a quantum sensor that interacts with an external variable of interest, such as a magnetic field, electric field, strain or acoustic wave, or temperature [5].

Quantum sensors have a higher sensitivity within a nanoscale or microscale sampling volume compared to a fully classical counterpart which would require higher field densities or higher volume interrogation to be effective.

sities o	r higher	volume	interroga	tion to l	oe effecti	ve.	
[6]							
[7]							
[8]							
[9]							

3.6.1 DiVincenzo Criteria

[11] [12]

[10]

3.6.2 Crystal Defects

[13]

[14]

Quantisation

Polarisation

[15]

Coherent Manipulation

[16]

[17]

[18]

- [19]
- [20]
- [21]

Efficient Readout

- [22]
- [23]
- [24]
- [25]
- [26]

3.6.3 Coherence

- [27],[28], [29] [30], [31]
- [32]

Spin Relaxation

Dephasing

Hahn Echo

[33]

Example: NV Diamond

3.6.4 Sensitivity

- [34]
- [35]
- [36]
- [37]
- [38]
- [39]

3.6.5 ODMR

- [40]
- [41]

[42]	
[43]	
[44]	
[45]	
[46]	
[47]	
[48]	
[49]	
[50]	
[51]	
[52]	
[15]	
[10]	
3.7	Silicon Carbide
0.1	Sincon Carbiac
[50]	
[53]	
[54]	
[54]	
[54] [55]	
[54][55][56][57]	
[54][55][56][57][58]	
[54][55][56][57][58][58]	
[54][55][56][57][58][58][59]	
[54][55][56][57][58][58]	
[54][55][56][57][58][58][59]	
[54][55][56][57][58][58][59]	
[54][55][56][57][58][58][59][60]	Production of SiC
[54][55][56][57][58][58][59][60]	Production of SiC
[54][55][56][57][58][58][59][60]	Production of SiC
 [54] [55] [56] [57] [58] [59] [60] 3.7.1 	Production of SiC

Multimodal Sensors

3.6.6

[63] [64]

3.7.2 Colour Defects in SiC

Electronic Structure

Charge State

Spin System

3.7.3 Wider Scientific Context

[65]

Task

4.1 Brief

I think, as a start can go through section 3.2.4 in the attached PhD thesis? In particular check in details how to diagonalise the NV centre spin S=1 Hamiltonian to get Eq. 3.31? You could also do some python simulations to plot how the spin levels (i.e. the eigenvalues of the spin Hamiltonian) change with applied magnetic field.

Once we've learned this, we can apply it to other spin defects in SiC.

4.2 Work

4.2.1 Concepts and Nomenclature

Spin-Spin Interactions

Zeeman Splitting

Hyperfine Interaction

4.2.2 System Hamiltonian

The ground state of the NV⁻ spin system in diamond is a triplet state, thus a S=1 system.

The corresponding Hamiltonian, which it seems can be generalised to an electron spin system of a defect, can be expressed as:

$$H_{\rm NV} = H_{\rm D} + H_{\rm Zeeman} + H_{\rm HF} \tag{4.1}$$

Here the labels D, Z and HF describe the electron spin-spin interactions, the Zeeman interaction with an external magnetic field and the hyperfine interaction between the nuclearable spin I and the electron spin S of the NV.

They have the following forms:

$$H_{\rm D} = DS_z^2 + E(S_x^2 + S_y^2) \tag{4.2}$$

$$H_{\rm Z} = g\mu_B \sum_{j}^{x,y,z} B_j \cdot S_j \tag{4.3}$$

$$H_{\rm HF} = \vec{S} \cdot A \cdot \vec{I}. \tag{4.4}$$

Spin-Spin Interaction

The E and D in equation 4.2 the fine structure constants of the spin system, describing the spin-spin interaction and S_i the corresponding spin operators in x,y and z-direction.

D is non-zero in system with axis of threefold (or other manifold) symmetry. The symmetry or spin quantization axis points along the connection of the nitrogen atom and vacancy forming the defect. In bulk diamond D is around 2.87 GHz at room temperature.

The definiteness, orientation and magnitude of D is thus dependent on the specific spin system being studied.

E occurs when there is a distortion of the point group symmetry, for example strain or an electrical field. In bulk diamond E is typically negligibly small but especially in NDs, E can be of the order of several MHz.

Thus, similarly, the value of E is a characteristic of the nature of the distortion and the specifics of the spin system being studied.

Zeeman Interaction

 B_j in equation 4.3 is the magnetic field along the x, y and z direction, g is the g-factor of the vacancy and μ_B the Bohr-Magneton, a constant.

It seems often the scaled parameter $g\mu_B$ is considered, for the NV⁻ system this is around 28 GHz T⁻¹, but again, will be a characteristic of the system being studied.

Hyperfine Interaction

Equation 4.4 related the nuclear spin to the electron spin via the hyperfine tensor A which has the form

$$A = \begin{pmatrix} A_{\perp} & 0 & 0 \\ 0 & A_{\perp} & 0 \\ 0 & 0 & A_{\parallel} \end{pmatrix}. \tag{4.5}$$

 A_{\parallel} and A_{\perp} are the axial and non-axial hyperfine parameters which encode two different interactions.

Fermi Contact Interaction. This interaction is calculated by

$$f_A = \frac{A_{\parallel} + 2A_{\perp}}{3}. (4.6)$$

Anisotropic Interaction. This interaction is found by considering both spins as magnetic dipoles is calculated by

$$d_A = \frac{A_{\parallel} - A_{\perp}}{3}. (4.7)$$

For the NV⁻ system in diamond specifically, using the values for A_{\parallel} and A_{\perp} we calculate that f_A is an order of magnitude stronger than d_A for both N¹⁴ and N¹⁵.

Reduced Hamiltonian

why (specifically) do we

get to neglect

this? Can we generalise?

By combining H_D and H_Z and neglecting H_{HF} we find

$$H_{\text{NV}} = DS_z^2 + E(S_x^2 + S_y^2) + g\mu_B \sum_{j}^{x,y,z} B_j \cdot S_j$$
 (4.8)

The spin operators S_j in matrix representation are

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, S_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, S_z = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{4.9}$$

Then, aligning the magnetic field (with strength B_0) along the z-axis (the quantisation axis), the reduced Hamiltonian will have the form

$$H_{\text{NV}} = \begin{pmatrix} D + B_0 & 0 & E \\ 0 & 0 & 0 \\ E & 0 & D - B_0 \end{pmatrix}, \tag{4.10}$$

with Eigenvalues

$$E_x = E_y = D \pm \sqrt{B_0^2 + E^2}, \ E_z = 0.$$
 (4.11)

The corresponding non-normalised Eigenvectors are then

$$|X\rangle = \frac{1}{E} \left(B_0 + \sqrt{B_0^2 + E^2} \right) |+1\rangle + |-1\rangle$$
 (4.12)

$$|Y\rangle = \frac{1}{E} \left(B_0 - \sqrt{B_0^2 + E^2} \right) |+1\rangle + |-1\rangle$$
 (4.13)

$$|Z\rangle = |0\rangle, \qquad (4.14)$$

with

$$|1\rangle = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \ |0\rangle = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \ |-1\rangle = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix},$$
 (4.15)

the Eigenvectors for H_{NV} with E = 0...

In the case where $E \ll B_0$ the Eigenvectors are well described by the bases $|0\rangle$ and $|\pm 1\rangle$.

For $E \gg B_0$, when transforming the spin operators S_j into the diagonalised system with Hamiltonian $H_{\rm NV}$ they read

$$\hat{S}_{x}^{\parallel} \propto \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \hat{S}_{y}^{\parallel} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \hat{S}_{z} \propto \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \tag{4.16}$$

and

$$\hat{H}_{\text{NV}} = \begin{pmatrix} D + \sqrt{B_0^2 + E^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & D - \sqrt{B_0^2 - E^2}. \end{pmatrix}$$
(4.17)

Another solution for a $\pi/2$ shifted, modulating magnetic field leads to

$$\hat{S}_{x}^{\perp} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \hat{S}_{y}^{\perp} \propto \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \hat{S}_{z} \propto \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{4.18}$$

a physical interpretation of which is that a linear modulating B-field aligned along the x-axis where strain is applied only allows transitions between the state $|X\rangle$ and $|0\rangle$, whereas fields perpendicular to the strain and the NV quantization axis only allow coupling between $|Y\rangle$ and $|0\rangle$.

For an arbitrary external magnetic field, H_{NV} can be expressed using spherical coordinates:

$$H_{\text{NV}} = \begin{pmatrix} D + B_0 \cdot \cos \theta & \frac{B_0}{\sqrt{2}} \cdot e^{-i \cdot \varphi} \cdot \sin \theta & E \\ \frac{B_0}{\sqrt{2}} \cdot e^{i \cdot \varphi} \cdot \sin \theta & 0 & \frac{B_0}{\sqrt{2}} e^{-i \cdot \varphi} \cdot \sin \theta \\ E & \frac{B_0}{\sqrt{2}} \cdot e^{i \cdot \varphi} \cdot \sin \theta & D - B_0 \cdot \cos \theta \end{pmatrix}$$
(4.19)

Here, we transformed the magnitude of the arbitrary magnetic field into spherical coordinates as

$$B_x = B_0 \cos \varphi \sin \theta \tag{4.20}$$

$$B_{\nu} = B_0 \sin \varphi \sin \theta \tag{4.21}$$

$$B_z = B_0 \cos \theta \tag{4.22}$$

with θ the azimuthal and φ the polar angle. Then using equations 4.8 and 4.9 we compute 4.19.

It immediately follows from the characteristic equation that Eigenvalues λ satisfy

$$0 = \lambda^{3} - 2 \cdot \lambda^{2} \cdot D + \frac{D \cdot B_{0}^{2}}{2} + \lambda (D^{2} - E^{2} - B_{0}^{2}) - \frac{1}{2} B_{0}^{2} \underbrace{\left(D \cdot \cos(2\theta) - 2 \cdot E \cos(2\varphi) \cdot \sin(\theta)^{2}\right)}_{\Delta_{\varphi\theta}}$$

$$(4.23)$$

Design

Results and Analysis

Conclusions

Appendix A

Python Code

```
'main.py'
import matplotlib.pyplot as plt
from sympy import Matrix
from numpy import cos, sin, sqrt, exp, linspace, pi, absolute, arccos
# DEFAULT VALUES
B_{array} = linspace(0, 0.01, 20)
B = 0.05
E = 0
D = 2.87 * 10**9
theta = 0 \# Radians
phi = 0 \# Radians
def makeMatrix(B=B, E=E, D=D, theta=theta, phi=phi) -> Matrix:
    M = Matrix(
              [D + B * cos(theta), (B / sqrt(2)) *
               \exp(-1j * phi) * \sin(theta), E],
                   (B / sqrt(2)) * exp(1j * phi) * sin(theta),
                   (B / \operatorname{sqrt}(2)) * \exp(-1j * \operatorname{phi}) * \sin(\operatorname{theta}),
              [E, (B / sqrt(2)) * exp(1j * phi) *
               \sin(\text{theta}), D - (B * \cos(\text{theta}))],
    \# M = Matrix(
                [D \ / \ 3 \ + \ B \ * \ cos(theta), \ (B \ / \ sqrt(2)) \ * \ sin(theta), \ 0],
    #
                     (B / sqrt(2)) * sin(theta),
```

```
-2 * D / 3,
                    (B / sqrt(2)) * sin(theta),
    #
    #
               [0, (B / sqrt(2)) * sin(theta), D / 3 - (B * cos(theta))]
    #
    #
    # )
    return M
def evals (B=B, E=E, D=D, theta=theta, phi=phi) -> list:
    M = makeMatrix(B=B, E=E, D=D, theta=theta, phi=phi)
    return absolute (list (M. eigenvals (). keys ()))
def addToPlot(
    label, colour, opacity=0.4, B_array=B_array, E=E, D=D, theta=theta,
):
    y_array = []
    y_array.append([])
    y_array.append([])
    \# y_array.append([])
    x_array = []
    \# for B in B_array:
           es = evals(B=B, E=E, D=D, theta=theta, phi=phi)
    #
           es = [num \ for \ num \ in \ es \ if \ abs(num) >= 10**7]
    #
    #
           x_{-}array.append(B)
    #
    #
           for count, e in enumerate(es):
    #
               y_array / count / .append(abs(e))
    #
               print (count)
    #
    #
           y_array [2]. append(D)
    for B in B_array:
         es = sorted(evals(B=B, E=E, D=D, theta=theta, phi=phi))
         if len(es) == 2:
             es.append(es[1])
        \# es = [num \ for \ num \ in \ es \ if \ abs(num) >= 10**7]
         x_array.append(B)
         y_{array}[0]. append (abs(es[2] - es[0]))
         y_{array}[1]. append (abs(es[1] - es[0]))
        # for count, e in enumerate(es):
        \# y_array[count].append(abs(e))
    plt.plot\left(x\_array\;,\;\;y\_array\left[0\right]\;,\;\;label=label\;,\;\;color=colour\;,\;\;alpha=opac\;
    plt.plot(x_array, y_array[1], label=label, color=colour, alpha=opac
    \# plt.plot(x_array, y_array[2])
```

```
plt.xlabel("$B_0$")
plt.ylabel("Eigenvalues_of_$H_{NV}$")
def millerAngle(miller_1, miller_2):
    return arccos (
                 ( miller_1 [0] * miller_2 [0] )
                 + (miller_1[1] * miller_2[1])
                 + (miller_1 [2] * miller_2 [2])
             / sqrt(
                 (miller_1[0] ** 2 + miller_1[1] ** 2 + miller_1[2] ** 2)
                 * (miller_2 [0] ** 2 + miller_2 [1] ** 2 + miller_2 [2] ** 2)
    )[0]
applied_B_miller = (0, 0, 1)
defect_axis = (1, 1, 1)
defect_2 = (-1, 1, 1)
defect_3 = (1, -1, 1)
defect_4 = (1, 1, -1)
miller Angle (applied_B_miller, defect_axis)
addToPlot(
    \label{label} \verb"label" Defect_1.$(111) $",
    colour="blue",
    opacity = 0.3,
    E=0,
    theta=millerAngle(applied_B_miller, defect_axis),
)
addToPlot(
    label="Defect_2_$(\ \ overline \{1\}_11)$",
    colour="purple",
    opacity = 0.3,
    E=0,
    theta=millerAngle(applied_B_miller, defect_2),
)
```

```
addToPlot (
    label="Defect_3_$(1_\) overline {1}_1)$",
    colour="green",
    opacity = 0.3,
    E=0,
    theta=millerAngle(applied_B_miller, defect_3),
)
addToPlot(
    label=" Defect_4_$(11\setminus overline\{1\})$",
    colour="red",
    opacity = 0.3,
    E=0,
    theta=millerAngle(applied_B_miller, defect_4),
## Generic Results
\# addToPlot(label="$E=0, \land theta=0$", colour="black", opacity=1, E=0
# addToPlot(
       label="$E=0,\\text{theta}=\\pi/8$", colour="blue", opacity=0.5, E=0
#
\# addToPlot(label="$E= 0.005, \land theta = 0$",
#
             colour = "orange", opacity = 0.5, E = 0.005)
\# addToPlot(label="$E= 0.010, \land theta = 0$",
             colour = "Purple", opacity = 0.5, E = 0.010
#
# addToPlot(
       label="$E=0,\\text{theta}=\\pi/3$", colour="green", opacity=0.5, E-
#
# )
#
# Diamond Results with B aligned to suface of cubic crystal
\# addToPlot(label="$E=0, \land theta=0.927$",
             colour = "blue", opacity = 0.8, E=0, theta = 0.927)
\# \ addToPlot(label="$E=0, \ \ \ theta=2.214$",
             colour = "blue", opacity = 0.8, E=0, theta = 2.214)
#
#
# Diamond results with B aligned to [210] (to generate 4 different angle
\# addToPlot(label="$E=0, \land theta=0.68$",
             colour = "blue", opacity = 0.5, E=0, theta = 0.68)
#
#
\# addToPlot(label="$E=0, \land theta = 1.83$",
#
             colour = "blue", opacity = 0.5, theta = 1.83)
\# addToPlot(label="$E=0, \land theta = 1.31$",
#
             colour = "blue", opacity = 0.5, E=0, theta = 1.31)
#
\# addToPlot(label="Base", colour="black", opacity=1, E=0, theta=0.927)
\# addToPlot(label="Base", colour="black", opacity=1, E=0, theta=5.355)
```

```
 \# \ add To Plot (label="Base", \ colour="black", \ opacity=1, \ E=0, \ theta=2.214) \\ \# \ add To Plot (label="Base", \ colour="black", \ opacity=1, \ E=0, \ theta=4.069) \\ \# \ add To Plot (label="Base", \ colour="black", \ opacity=1, \ E=0, \ theta=pi \ / \ 4) \\ \# \ add To Plot (label="Base \ Lower \ D", \ colour="black", \ \# \ opacity=1, \ D=2.6 * 10**9, \ theta=0) \\ \# \ add To Plot (label="Base, \ E=50$", \ colour="green", \ E=5 * 10**6, \ theta=0) \\ \# \ add To Plot (label="Base, \ E=50$", \ colour="green", \ E=5 * 10**6, \ theta=pi * 0. \\ \# \ add To Plot (label="Base, \ E=500$", \ colour="blue", \ E=9 * 10**6, \ theta=0) \\ \# \ handles, \ labels=plt.gca().get_legend_handles_labels() \\ by_label=dict(zip(labels, handles)) \\ plt.legend(by_label.values(), \ by_label.keys()) \\ plt.title("$B_0$_aligned_with_" + "".join(map(str, applied_B_miller))) \\ plt.xlim(left=0) \\ plt.show()
```

Bibliography

- [1] A. Carrington and A.D. McLachlan. *Introduction to Magnetic Resonance with Applications to Chemistry and Chemical Physics*. A Harper international edition. Harper & Row, 1967.
- [2] J. Köhler, J. A. J. M. Disselhorst, M. C. J. M. Donckers, E. J. J. Groenen, J. Schmidt, and W. E. Moerner. Magnetic resonance of a single molecular spin. *Nature*, 363(6426):242–244, May 1993.
- [3] D.A. Bromley and W. Greiner. *Quantum Mechanics: An Introduction*. Physics and Astronomy. Springer Berlin Heidelberg, 2000.
- [4] X. Fan, T. G. Myers, B. A. D. Sukra, and G. Gabrielse. Measurement of the electron magnetic moment. *Phys. Rev. Lett.*, 130:071801, Feb 2023.
- [5] S Castelletto, C T-K Lew, Wu-Xi Lin, and Jin-Shi Xu. Quantum systems in silicon carbide for sensing applications. *Reports on Progress in Physics*, 87(1):014501, dec 2023.
- [6] Gary Wolfowicz, F. Joseph Heremans, Christopher P. Anderson, Shun Kanai, Hosung Seo, Adam Gali, Giulia Galli, and David D. Awschalom. Quantum guidelines for solid-state spin defects. *Nature Reviews Materials*, 6(10):906–925, April 2021.
- [7] Yiu Yung Pang Wai Kuen Leung Nan Zhao Kin On Ho, Yang Shen and Sen Yang. Diamond quantum sensors: from physics to applications on condensed matter research. *Functional Diamond*, 1(1):160–173, 2021.
- [8] Corey J. Cochrane, Jordana Blacksberg, Mark A. Anders, and Patrick M. Lenahan. Vectorized magnetometer for space applications using electrical readout of atomic scale defects in silicon carbide. *Scientific Reports*, 6(1), November 2016.
- [9] Tianyu Xie, Zhiyuan Zhao, Xi Kong, Wenchao Ma, Mengqi Wang, Xiangyu Ye, Pei Yu, Zhiping Yang, Shaoyi Xu, Pengfei Wang, Ya Wang, Fazhan Shi, and Jiangfeng Du. Beating the standard quantum limit under ambient conditions with solid-state spins. *Science Advances*, 7(32), August 2021.
- [10] David P. DiVincenzo. Quantum computation. Science, 270(5234):255–261, October 1995.
- [11] C. L. Degen, F. Reinhard, and P. Cappellaro. Quantum sensing. *Rev. Mod. Phys.*, 89:035002, Jul 2017.

- [12] Scott E. Crawford, Roman A. Shugayev, Hari P. Paudel, Ping Lu, Madhava Syamlal, Paul R. Ohodnicki, Benjamin Chorpening, Randall Gentry, and Yuhua Duan. Quantum sensing for energy applications: Review and perspective. *Advanced Quantum Technologies*, 4(8), June 2021.
- [13] H. Kraus, V. A. Soltamov, F. Fuchs, D. Simin, A. Sperlich, P. G. Baranov, G. V. Astakhov, and V. Dyakonov. Magnetic field and temperature sensing with atomic-scale spin defects in silicon carbide. *Scientific Reports*, 4(1), July 2014.
- [14] Shun Kanai, F. Joseph Heremans, Hosung Seo, Gary Wolfowicz, Christopher P. Anderson, Sean E. Sullivan, Mykyta Onizhuk, Giulia Galli, David D. Awschalom, and Hideo Ohno. Generalized scaling of spin qubit coherence in over 12, 000 host materials. Proceedings of the National Academy of Sciences, 119(15), April 2022.
- [15] P. V. Klimov, A. L. Falk, B. B. Buckley, and D. D. Awschalom. Electrically driven spin resonance in silicon carbide color centers. *Phys. Rev. Lett.*, 112:087601, Feb 2014.
- [16] Matthias Widmann, Sang-Yun Lee, Torsten Rendler, Nguyen Tien Son, Helmut Fedder, Seoyoung Paik, Li-Ping Yang, Nan Zhao, Sen Yang, Ian Booker, Andrej Denisenko, Mohammad Jamali, S. Ali Momenzadeh, Ilja Gerhardt, Takeshi Ohshima, Adam Gali, Erik Janzén, and J¨ org Wrachtrup. Coherent control of single spins in silicon carbide at room temperature. *Nature Materials*, 14(2):164–168, December 2014.
- [17] A. Csóré, I. G. Ivanov, N. T. Son, and A. Gali. Fluorescence spectrum and charge state control of divacancy qubits via illumination at elevated temperatures in 4h silicon carbide. *Phys. Rev. B*, 105:165108, Apr 2022.
- [18] Fei-Fei Yan, Ai-Lun Yi, Jun-Feng Wang, Qiang Li, Pei Yu, Jia-Xiang Zhang, Adam Gali, Ya Wang, Jin-Shi Xu, Xin Ou, Chuan-Feng Li, and Guang-Can Guo. Room-temperature coherent control of implanted defect spins in silicon carbide. npj Quantum Information, 6(1), May 2020.
- [19] William F. Koehl, Bob B. Buckley, F. Joseph Heremans, Greg Calusine, and David D. Awschalom. Room temperature coherent control of defect spin qubits in silicon carbide. *Nature*, 479(7371):84–87, November 2011.
- [20] Zhao Mu, Soroush Abbasi Zargaleh, Hans Jürgen von Bardeleben, Johannes E. Fröch, Milad Nonahal, Hongbing Cai, Xinge Yang, Jianqun Yang, Xingji Li, Igor Aharonovich, and Weibo Gao. Coherent manipulation with resonant excitation and single emitter creation of nitrogen vacancy centers in 4h silicon carbide. *Nano Letters*, 20(8):6142–6147, July 2020.
- [21] Jun-Feng Wang, Fei-Fei Yan, Qiang Li, Zheng-Hao Liu, He Liu, Guo-Ping Guo, Li-Ping Guo, Xiong Zhou, Jin-Ming Cui, Jian Wang, Zong-Quan Zhou, Xiao-Ye Xu, Jin-Shi Xu, Chuan-Feng Li, and Guang-Can Guo. Coherent control of nitrogen-vacancy center spins in silicon carbide at room temperature. *Phys. Rev. Lett.*, 124:223601, Jun 2020.

- [22] Erik R. Eisenach, John F. Barry, Michael F. O'Keeffe, Jennifer M. Schloss, Matthew H. Steinecker, Dirk R. Englund, and Danielle A. Braje. Cavity-enhanced microwave readout of a solid-state spin sensor. *Nature Communications*, 12(1), March 2021.
- [23] Christopher P Anderson, Elena O Glen, Cyrus Zeledon, Alexandre Bourassa, Yu Jin, Yizhi Zhu, Christian Vorwerk, Alexander L Crook, Hiroshi Abe, Jawad Ul-Hassan, Takeshi Ohshima, Nguyen T Son, Giulia Galli, and David D Awschalom. Five-second coherence of a single spin with single-shot readout in silicon carbide. Sci. Adv., 8(5):eabm5912, February 2022.
- [24] Matthias Niethammer, Matthias Widmann, Torsten Rendler, Naoya Morioka, Yu-Chen Chen, Rainer Stöhr, Jawad Ul Hassan, Shinobu Onoda, Takeshi Ohshima, Sang-Yun Lee, Amlan Mukherjee, Junichi Isoya, Nguyen Tien Son, and Jörg Wrachtrup. Coherent electrical readout of defect spins in silicon carbide by photo-ionization at ambient conditions. *Nature Communications*, 10(1), December 2019.
- [25] Andrea Morello, Jarryd J. Pla, Floris A. Zwanenburg, Kok W. Chan, Kuan Y. Tan, Hans Huebl, Mikko Möttönen, Christopher D. Nugroho, Changyi Yang, Jessica A. van Donkelaar, Andrew D. C. Alves, David N. Jamieson, Christopher C. Escott, Lloyd C. L. Hollenberg, Robert G. Clark, and Andrew S. Dzurak. Single-shot readout of an electron spin in silicon. *Nature*, 467(7316):687–691, September 2010.
- [26] Yu-Wei Liao, Qiang Li, Mu Yang, Zheng-Hao Liu, Fei-Fei Yan, Jun-Feng Wang, Ji-Yang Zhou, Wu-Xi Lin, Yi-Dan Tang, Jin-Shi Xu, Chuan-Feng Li, and Guang-Can Guo. Deep-learning-enhanced single-spin readout in silicon carbide at room temperature. *Phys. Rev. Appl.*, 17:034046, Mar 2022.
- [27] David J. Christle, Abram L. Falk, Paolo Andrich, Paul V. Klimov, Jawad Ul Hassan, Nguyen T. Son, Erik Janzén, Takeshi Ohshima, and David D. Awschalom. Isolated electron spins in silicon carbide with millisecond coherence times. *Nature Materials*, 14(2):160–163, December 2014.
- [28] V. A. Soltamov, C. Kasper, A. V. Poshakinskiy, A. N. Anisimov, E. N. Mokhov, A. Sperlich, S. A. Tarasenko, P. G. Baranov, G. V. Astakhov, and V. Dyakonov. Excitation and coherent control of spin qudit modes in silicon carbide at room temperature. *Nature Communications*, 10(1), April 2019.
- [29] Carmem M Gilardoni, Tom Bosma, Danny van Hien, Freddie Hendriks, Björn Magnusson, Alexandre Ellison, Ivan G Ivanov, N T Son, and Caspar H van der Wal. Spin-relaxation times exceeding seconds for color centers with strong spin-orbit coupling in sic. New Journal of Physics, 22(10):103051, October 2020.
- [30] Oscar Bulancea-Lindvall, Nguyen T. Son, Igor A. Abrikosov, and Viktor Ivády. Dipolar spin relaxation of divacancy qubits in silicon carbide. *npj Computational Materials*, 7(1), December 2021.
- [31] T. Astner, P. Koller, C. M. Gilardoni, J. Hendriks, N. T. Son, I. G. Ivanov, J. U. Hassan, C. H. van der Wal, and M. Trupke. Vanadium in silicon carbide: Telecom-ready spin centres with long relaxation lifetimes and hyperfine-resolved optical transitions, 2022.

- [32] Hosung Seo, Abram L Falk, Paul V Klimov, Kevin C Miao, Giulia Galli, and David D Awschalom. Quantum decoherence dynamics of divacancy spins in silicon carbide. *Nat. Commun.*, 7(1):12935, September 2016.
- [33] Yuzhou Wu, Fedor Jelezko, Martin B Plenio, and Tanja Weil. Diamond quantum devices in biology. *Angewandte Chemie International Edition*, 55(23):6586–6598, April 2016.
- [34] John F. Barry, Jennifer M. Schloss, Erik Bauch, Matthew J. Turner, Connor A. Hart, Linh M. Pham, and Ronald L. Walsworth. Sensitivity optimization for nv-diamond magnetometry. Rev. Mod. Phys., 92:015004, Mar 2020.
- [35] D. Simin, F. Fuchs, H. Kraus, A. Sperlich, P. G. Baranov, G. V. Astakhov, and V. Dyakonov. High-precision angle-resolved magnetometry with uniaxial quantum centers in silicon carbide. *Phys. Rev. Appl.*, 4:014009, Jul 2015.
- [36] John B. S. Abraham, Cameron Gutgsell, Dalibor Todorovski, Scott Sperling, Jacob E. Epstein, Brian S. Tien-Street, Timothy M. Sweeney, Jeremiah J. Wathen, Elizabeth A. Pogue, Peter G. Brereton, Tyrel M. McQueen, Wesley Frey, B. D. Clader, and Robert Osiander. Nanotesla magnetometry with the silicon vacancy in silicon carbide. *Phys. Rev. Appl.*, 15:064022, Jun 2021.
- [37] Ilja Fescenko, Andrey Jarmola, Igor Savukov, Pauli Kehayias, Janis Smits, Joshua Damron, Nathaniel Ristoff, Nazanin Mosavian, and Victor M. Acosta. Diamond magnetometer enhanced by ferrite flux concentrators. *Phys. Rev. Res.*, 2:023394, Jun 2020.
- [38] Stefania Castelletto, Abdul Salam Al Atem, Faraz Ahmed Inam, Hans Jürgen von Bardeleben, Sophie Hameau, Ahmed Fahad Almutairi, Gérard Guillot, Shin-ichiro Sato, Alberto Boretti, and Jean Marie Bluet. Deterministic placement of ultrabright near-infrared color centers in arrays of silicon carbide micropillars. *Beilstein Journal of Nanotechnology*, 10:2383–2395, December 2019.
- [39] Ji-Yang Zhou, Qiang Li, Zhi-He Hao, Wu-Xi Lin, Zhen-Xuan He, Rui-Jian Liang, Liping Guo, Hao Li, Lixing You, Jian-Shun Tang, Jin-Shi Xu, Chuan-Feng Li, and Guang-Can Guo. Plasmonic-enhanced bright single spin defects in silicon carbide membranes. Nano Letters, 23(10):4334–4343, May 2023.
- [40] Jun-Feng Wang, Jin-Ming Cui, Fei-Fei Yan, Qiang Li, Ze-Di Cheng, Zheng-Hao Liu, Zhi-Hai Lin, Jin-Shi Xu, Chuan-Feng Li, and Guang-Can Guo. Optimization of power broadening in optically detected magnetic resonance of defect spins in silicon carbide. *Phys. Rev. B*, 101:064102, Feb 2020.
- [41] Gary Wolfowicz, Christopher P. Anderson, Andrew L. Yeats, Samuel J. Whiteley, Jens Niklas, Oleg G. Poluektov, F. Joseph Heremans, and David D. Awschalom. Optical charge state control of spin defects in 4h-sic. *Nature Communications*, 8(1), November 2017.
- [42] A. N. Anisimov, D. Simin, V. A. Soltamov, S. P. Lebedev, P. G. Baranov, G. V. Astakhov, and V. Dyakonov. Optical thermometry based on level anticrossing in silicon carbide. *Scientific Reports*, 6(1), September 2016.

- [43] A. V. Poshakinskiy and G. V. Astakhov. Optically detected spin-mechanical resonance in silicon carbide membranes. *Phys. Rev. B*, 100:094104, Sep 2019.
- [44] Lin Liu, Jun-Feng Wang, Xiao-Di Liu, Hai-An Xu, Jin-Ming Cui, Qiang Li, Ji-Yang Zhou, Wu-Xi Lin, Zhen-Xuan He, Wan Xu, Yu Wei, Zheng-Hao Liu, Pu Wang, Zhi-He Hao, Jun-Feng Ding, Hai-Ou Li, Wen Liu, Hao Li, Lixing You, Jin-Shi Xu, Eugene Gregoryanz, Chuan-Feng Li, and Guang-Can Guo. Coherent control and magnetic detection of divacancy spins in silicon carbide at high pressures. *Nano Letters*, 22(24):9943–9950, December 2022.
- [45] Yu Zhou, Junfeng Wang, Xiaoming Zhang, Ke Li, Jianming Cai, and Weibo Gao. Self-protected thermometry with infrared photons and defect spins in silicon carbide. *Phys. Rev. Appl.*, 8:044015, Oct 2017.
- [46] G. Wolfowicz, S. J. Whiteley, and D. D. Awschalom. Electrometry by optical charge conversion of deep defects in 4h-sic. *Proceedings of the National Academy of Sciences*, 115(31):7879–7883, July 2018.
- [47] O. O. Soykal, Pratibha Dev, and Sophia E. Economou. Silicon vacancy center in 4h-sic: Electronic structure and spin-photon interfaces. *Phys. Rev. B*, 93:081207, Feb 2016.
- [48] S. A. Tarasenko, A. V. Poshakinskiy, D. Simin, V. A. Soltamov, E. N. Mokhov, P. G. Baranov, V. Dyakonov, and G. V. Astakhov. Spin and optical properties of silicon vacancies in silicon carbide a review. *physica status solidi* (b), 255(1), September 2017.
- [49] Fei-Fei Yan, Jun-Feng Wang, Qiang Li, Ze-Di Cheng, Jin-Ming Cui, Wen-Zheng Liu, Jin-Shi Xu, Chuan-Feng Li, and Guang-Can Guo. Coherent control of defect spins in silicon carbide above 550 k. *Phys. Rev. Appl.*, 10:044042, Oct 2018.
- [50] Qin-Yue Luo, Shuang Zhao, Qi-Cheng Hu, Wei-Ke Quan, Zi-Qi Zhu, Jia-Jun Li, and Jun-Feng Wang. High-sensitivity silicon carbide divacancy-based temperature sensing. *Nanoscale*, 15:8432–8436, 2023.
- [51] Junfeng Wang, Fupan Feng, Jian Zhang, Jihong Chen, Zhongcheng Zheng, Liping Guo, Wenlong Zhang, Xuerui Song, Guoping Guo, Lele Fan, Chongwen Zou, Liren Lou, Wei Zhu, and Guanzhong Wang. High-sensitivity temperature sensing using an implanted single nitrogen-vacancy center array in diamond. *Phys. Rev. B*, 91:155404, Apr 2015.
- [52] Wei-Ke Quan, Lin Liu, Qin-Yue Luo, Xiao-Di Liu, and Jun-Feng Wang. Fiber-coupled silicon carbide divacancy magnetometer and thermometer. *Opt. Express*, 31(10):15592–15598, May 2023.
- [53] C. R. Eddy and D. K. Gaskill. Silicon carbide as a platform for power electronics. *Science*, 324(5933):1398–1400, June 2009.
- [54] J.B. Casady and R.W. Johnson. Status of silicon carbide (sic) as a wide-bandgap semiconductor for high-temperature applications: A review. *Solid-State Electronics*, 39(10):1409–1422, 1996.

- [55] Qiang Li, Jun-Feng Wang, Fei-Fei Yan, Ji-Yang Zhou, Han-Feng Wang, He Liu, Li-Ping Guo, Xiong Zhou, Adam Gali, Zheng-Hao Liu, Zu-Qing Wang, Kai Sun, Guo-Ping Guo, Jian-Shun Tang, Hao Li, Li-Xing You, Jin-Shi Xu, Chuan-Feng Li, and Guang-Can Guo. Room-temperature coherent manipulation of single-spin qubits in silicon carbide with a high readout contrast. *National Science Review*, 9(5):nwab122, 07 2021.
- [56] Nguyen T. Son and Ivan G. Ivanov. Charge state control of the silicon vacancy and divacancy in silicon carbide. *Journal of Applied Physics*, 129(21), June 2021.
- [57] Matthias Niethammer, Matthias Widmann, Sang-Yun Lee, Pontus Stenberg, Olof Kordina, Takeshi Ohshima, Nguyen Tien Son, Erik Janzén, and Jörg Wrachtrup. Vector magnetometry using silicon vacancies in 4h-sic under ambient conditions. *Phys. Rev. Appl.*, 6:034001, Sep 2016.
- [58] Zhengzhi Jiang, Hongbing Cai, Robert Cernansky, Xiaogang Liu, and Weibo Gao. Quantum sensing of radio-frequency signal with nv centers in sic. *Science Advances*, 9(20), May 2023.
- [59] Stefania Castelletto, Alberto Peruzzo, Cristian Bonato, Brett C. Johnson, Marina Radulaski, Haiyan Ou, Florian Kaiser, and Joerg Wrachtrup. Silicon carbide photonics bridging quantum technology. ACS Photonics, 9(5):1434–1457, April 2022.
- [60] Stefania Castelletto and Alberto Boretti. Silicon carbide color centers for quantum applications. *JPhys Photonics*, 2(2):022001, April 2020.
- [61] F. Fuchs, B. Stender, M. Trupke, D. Simin, J. Pflaum, V. Dyakonov, and G. V. Astakhov. Engineering near-infrared single-photon emitters with optically active spins in ultrapure silicon carbide. *Nature Communications*, 6(1), July 2015.
- [62] Takeshi Ohshima, Takahiro Satoh, Hannes Kraus, Georgy V Astakhov, Vladimir Dyakonov, and Pavel G Baranov. Creation of silicon vacancy in silicon carbide by proton beam writing toward quantum sensing applications. *Journal of Physics D: Applied Physics*, 51(33):333002, July 2018.
- [63] Jun-Feng Wang, Qiang Li, Fei-Fei Yan, He Liu, Guo-Ping Guo, Wei-Ping Zhang, Xiong Zhou, Li-Ping Guo, Zhi-Hai Lin, Jin-Ming Cui, Xiao-Ye Xu, Jin-Shi Xu, Chuan-Feng Li, and Guang-Can Guo. On-demand generation of single silicon vacancy defects in silicon carbide. ACS Photonics, 6(7):1736–1743, May 2019.
- [64] F. Sardi, T. Kornher, M. Widmann, R. Kolesov, F. Schiller, T. Reindl, M. Hagel, and J. Wrachtrup. Scalable production of solid-immersion lenses for quantum emitters in silicon carbide. *Applied Physics Letters*, 117(2), July 2020.
- [65] H. Kraus, V. A. Soltamov, D. Riedel, S. Väth, F. Fuchs, A. Sperlich, P. G. Baranov, V. Dyakonov, and G. V. Astakhov. Room-temperature quantum microwave emitters based on spin defects in silicon carbide. *Nature Physics*, 10(2):157–162, December 2013.

- [66] Edlyn V. Levine, Matthew J. Turner, Pauli Kehayias, Connor A. Hart, Nicholas Langellier, Raisa Trubko, David R. Glenn, Roger R. Fu, and Ronald L. Walsworth. Principles and techniques of the quantum diamond microscope. *Nanophotonics*, 8(11):1945–1973, 2019.
- [67] Abram L. Falk, Paul V. Klimov, Bob B. Buckley, Viktor Ivády, Igor A. Abrikosov, Greg Calusine, William F. Koehl, Ádám Gali, and David D. Awschalom. Electrically and mechanically tunable electron spins in silicon carbide color centers. *Phys. Rev. Lett.*, 112:187601, May 2014.
- [68] Gopalakrishnan Balasubramanian, Philipp Neumann, Daniel Twitchen, Matthew Markham, Roman Kolesov, Norikazu Mizuochi, Junichi Isoya, Jocelyn Achard, Johannes Beck, Julia Tissler, Vincent Jacques, Philip R. Hemmer, Fedor Jelezko, and Jörg Wrachtrup. Ultralong spin coherence time in isotopically engineered diamond. *Nature Materials*, 8(5):383–387, April 2009.
- [69] Johann-Martin Spaeth, Jürgen R. Niklas, and Ralph H. Bartram. Structural Analysis of Point Defects in Solids. Springer Berlin Heidelberg, 1992.
- [70] Johann-Martin Spaeth and Harald Overhof. Point Defects in Semiconductors and Insulators: Determination of Atomic and Electronic Structure from Paramagnetic Hyperfine Interactions. Springer Berlin Heidelberg, 2003.
- [71] John A. Weil and James R. Bolton. Electron Paramagnetic Resonance: Elementary Theory and Practical Applications. Wiley, May 2006.
- [72] J. Wrachtrup, C. von Borczyskowski, J. Bernard, M. Orrit, and R. Brown. Optical detection of magnetic resonance in a single molecule. *Nature*, 363(6426):244–245, May 1993.