CSE330

Assignment - 04, 05

Assignment-04

1. Consider a function **f(x) =** $x^3 + x^2 - 4x - 4$.

(a) (5 marks) State the exact roots of f(x) and construct two different fixed point functions g(x) such that f(x) = 0.

(b) (5 marks) Compute the convergence rate of each fixed point function g(x) obtained in the previous part, and state which root it is converging to or diverging.

2. Consider the following function: **f(x) =** $xe^x - 1$.
(a) (5 marks) Find solution of f(x) = 0 up to 5 iterations using Newton's method starting with x_0 = 1.5. Keep up to four significant figures.

(b) (5 marks) Consider the fixed point function, **g(x) =** $\frac{2x+1}{\sqrt{x+1}}$. Show that to be super-linearly convergent, the root must satisfy **x* =** $\frac{-3}{2}$.

3. (a) (5 marks) Consider a cubic function, **f(x) = 2x³ − 2x − 5**. Compute a **superlinearly convergent fixed point function g(x)** for the given function f(x) using **Newton's method.**

Assignment – 05

1. A linear system is described by the following equations:

$$x_1 + 6x_2 + 2x_3 = 10$$
$$3x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 5x_2 + 2x_3 = 9.$$

Based on these equations, answer the questions below.
(a) [3 marks] From the given linear equations, identify the matrices A, x and b such that the linear system can be expressed as a matrix equation.

(b) [4 marks] Construct the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ from this system.

(c) [3 marks] Compute the unit lower triangular matrix L.

(d) [5 marks] Now find the solution of the linear system using the LU decomposition method. Use the unit lower triangular matrix found in the previous question.

2. A function is given by **f(x) =** $\mathbf{e^{0.5x} + \sin x}$ which is to be integrated on the interval [0, 2].

a. (2 marks) Evaluate the **exact integral I(f)**.
b. (3 marks) Compute the numerical integral by using the **Newton-Cotes formula with n = 1.**
c. (5 marks) Evaluate the numerical integral $\mathbf{C_{1,4}}$ by using the **Composite Newton-Cotes** formula and also find the percentage relative error.

01. $f(x) = x^3 + x^2 - 4x - 4$

(a) $f(x) = 0$

$\Rightarrow x^3 + x^2 - 4x - 4 = 0 \qquad \Rightarrow x = 2, -1, -2$

$$\Rightarrow g_1(x) = \frac{x^3 + x^2 - 4}{4}$$

$$\Rightarrow g_2(x) = \sqrt{-x^3 + 4x + 4}$$

(b) $\lambda = |g_1'(x)|$

$$= \left|\frac{3x^2 + 2x}{4}\right|$$

$\xrightarrow{2}$ $\lambda = 4$ (divergent)

$\xrightarrow{-1}$ $\lambda = 1/4$ (linear convergent)

$\xrightarrow{-2}$ $\lambda = 2$ (divergent)

$\lambda = |g_2'(x)|$

$$= \left|(-x^3 + 4x + 4)^{-1/2} \cdot (-3x^2 + 4)\right|$$

$\xrightarrow{2}$ 4 (divergent)

$\xrightarrow{-1}$ 1 (linear convergen

$\xrightarrow{-2}$ 4 (divergent)

Q2. (a) $f(x) = xe^{x} - 1$

$f'(x) = x \cdot e^{x} + e^{x}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{-1}{1}$

$= 1$

$x_2 = 1 - \frac{1.718}{5.436} = 0.684$

iteration	x_k	x_{k+1}	$\lvert f(x_{k+1})\rvert <$ error
1	0	1	1.71 $\not<$ 0.001
2	1.718	0.684	0.355
3	0.684	0.577	0.029
4	0.577	0.5672	0.0002 < 0.001
?? 5	0.5672	0.5671	0.00001 < 0.001

$f'(x_k) = 3.337, 2.81, 2.76,$

$$x_1 + 6x_2 + 2x_3 = 10$$
$$3x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 5x_2 + 2x_3 = 9.$$

Based on these equations, answer the questions below.

(a) [3 marks] From the given linear equations, identify the matrices A, x and b such that the linear system can be expressed as a matrix equation.

(a)

$$A = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

(b)

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$$A^{(2)} = F^{(1)} \times A^{(1)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix}$$

$$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix}$$

(d)

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{bmatrix}$$

(e)

$$[L] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$$a_1 = 10$$

$$3a_1 + a_2 = 6 \qquad \Rightarrow a_2 = -24$$

$$4a_1 + \frac{19}{16} a_2 + a_3 = 9 \qquad \Rightarrow a_3 = -\frac{5}{2}$$

$$[U] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & 5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -24 \\ -\frac{5}{2} \end{bmatrix}$$

$$\Rightarrow -\frac{1}{16} x_3 = -\frac{5}{2}$$

$$\Rightarrow x_3 = 40$$

$$-16x_2 + 5x_3 = -24$$

$$\Rightarrow x_2 = -11$$

$$x + 6x_2 + 2x_3 = 10$$

$$\Rightarrow x_1 = -4$$

2. A function is given by **f(x) = e$^{0.5x}$ + sin x** which is to be integrated on the interval [0, 2].

a. (2 marks) Evaluate the **exact integral I(f)**.
b. (3 marks) Compute the numerical integral by using the **Newton-Cotes formula with n = 1.**
c. (5 marks) Evaluate the numerical integral **$C_{1,4}$** by using the **Composite Newton-Cotes** formula and also find the percentage relative error.

(a) $$\int_0^2 f(x) = \int_0^2 e^{0.5x} + \sin x \, dx$$

$$= \left[\frac{1}{0.5} e^{0.5x} - \cos x\right]_0^2$$

$$= 4.8526$$

(b) $$I_1 = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$= \frac{2-0}{2}\left[f(0) + f(2)\right]$$

$$= 4.62$$

(c) $$h = \frac{b-a}{m} = \frac{2-0}{4} = \frac{1}{2}$$

$$C_{1,4} = \frac{h}{2}\left[f(0) + 2f\left(\tfrac{1}{2}\right) + 2f(1) + 2f\left(\tfrac{3}{2}\right) + f(2)\right]$$

$$= 4.835$$

$$\therefore \text{Error} = \left|\frac{4.8526 - 4.835}{4.8526}\right| \times 100\%$$

$$= 0.36\%$$