

$$\overline{\overline{1}}$$

a) $x^3 + x^2 - 8x - 8 = 0$

$$\Rightarrow x^2(x+1) - 8(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 8) = 0$$

$$x = -1, \sqrt{8}, -\sqrt{8}$$

$$g_1(x) = \frac{x^3 + x^2 - 8}{8}$$

$$g_2(x) = \frac{x^3 + x^2 + x - 8}{9}$$

(Any other valid manipulation is also allowed)

$$x^3 + x^2 - 8x - 8 = 0$$

$$\Rightarrow x^3 + 3x^2 - 2x^2 - 8x - 8 = 0$$

$$\Rightarrow x^2 = \frac{x^3 + 3x^2 - 8x - 8}{2}$$

$$\Rightarrow x = \sqrt{\frac{x^3 + 3x^2 - 8x - 8}{2}}$$

so,

$$g_3(x) = \quad \text{..}$$

$$\frac{b}{\overline{}}$$

$$g_1(x) = \frac{x^3 + x^2 - 8}{8}$$

$$g_1'(x) = \frac{3x^2 + 2x}{8}$$

$$\lambda = \left\{ g_1'(\alpha) \right\}$$

$$= \left\{ \frac{3x^2 + 2x}{8} \right\}$$

$$\sqrt{x = -1} \quad \left| \begin{array}{l} \sqrt{8} \\ -\sqrt{8} \end{array} \right.$$

0.23

3.71

2.29

Linear

Div

Div

Conv

$$g_1(n) = \frac{n^3 + n^2 + n - 8}{9}$$

$$g_2'(n) = \frac{3n^2 + 2n + 1}{9}$$

$$\lambda = |g_2'(n)|$$

$$= \left| \frac{3n^2 + 2n + 1}{9} \right|$$

$\sqrt{8}$ $-\sqrt{8}$
 $n = -2$ 2.15

0.22

Linear
conv

3.42

Div

Div

$$g_3(n) = \sqrt{\frac{n^3 + 3n^2 - 8n - 8}{2}}$$

$$g_3'(n) = \frac{1}{\sqrt{2}} \frac{3n^2 + 6n - 8}{\sqrt{n^3 + 3n^2 - 8n - 8}}$$

$$\lambda = \left| \frac{1}{\sqrt{2}} \frac{3n^2 + 6n - 8}{\sqrt{n^3 + 3n^2 - 8n - 8}} \right|$$

$$n \rightarrow \infty$$

5.5

Div

5.83

Div

0.17

Linear
Conv

2

b) $g(x) = \frac{2x+5}{\sqrt{x+3}}$

$$\Rightarrow g'(x) = \frac{2\sqrt{x+3} - (2x+5) \frac{1}{2\sqrt{x+3}}}{x+3}$$

$$= \frac{(x+3) \cdot 4 - (2x+5)}{x+3}$$

$$= \frac{2x+7}{2(x+3)^{\frac{3}{2}}}$$

As α is given

$$-\frac{7}{2},$$

$$\frac{-\frac{7}{2} \cdot 2 + 2}{2(\alpha + 3)^{\frac{3}{2}}}$$

$$= 0$$

As $\lambda = 0$, so

it is super linear
convergent

\approx

a) $f'(x) = 2x \ln x + x + e^{-x}$

$x_0 = 0.23$

$f(x_0) = -0.87228$

$f'(x_0) = 0.34848$

Iteration 1 :

$$x_1 = 0.23 - \frac{-0.87228}{0.34848}$$
$$= 2.7331$$

Relative Error,

$$| 2.7331 - 0.23 |$$

$$= 2.5031$$

$$f(x_1) = 7.4454$$

$$f'(x_1) = 8.2940$$

Iter 1

$$x_2 = 2.7331 -$$

$$\frac{7.4454}{8.2940}$$

$$= 1.8354$$

$$f(x_2) = 1.8862$$

$$f'(x_1) = 4.2241$$

R.E.

$$\left| 1.8354 - 2.7331 \right|$$

$$= 0.8927$$

Iteration

$$x_3 = 1.8354$$

$$\begin{array}{r} 1.8862 \\ - 4.2241 \\ \hline \end{array}$$

$$= 1.3889$$

$$f(x_3) = 0.38434$$

$$f'(x_3) = 2.5508$$

R.E.,

$$| 1.3889 - 1.8354 |$$

$$= 0.4465$$

Iterq:

$$\pi_1 = 1.3889 -$$

$$\frac{0.38434}{2.5908}$$

$$= 1.2382$$

$$f(u) = 0.037663$$

$$f'(u) = 2.0572$$

R.E.,

$$\left| \frac{1.2382 - 1.3889}{1} \right|$$

$$= 0.1507$$

Iteration:

$$x_5 = 1.2382 -$$

$$\overline{0.037663}$$

$$= 1.2199$$

$$f(x_5) = 0.00053942$$

$$f'(x_5) = 2.0001$$

$$R.E. =$$

$$\left| 1.2199 - 1.2382 \right| \\ = 0.0183$$

Iter 6 :

$$x_6 = \frac{1.2199 - 0.60053942}{2.0001}$$

$$= 1.2196$$

$$f(x_6) = 0.000060471$$

$$f'(x_6) = 1.9992$$

R.E.

$$\begin{aligned} & |1.2196 - 1.2199| \\ & = 3 \times 10^{-4} \end{aligned}$$