Regression I:

Hypothesis testing & predictions
+ Graphic Methods

Session 10

Programación Estadística con Python

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MASTER EN DATA ANALYTICS PARA LA EMPRESA

Goals

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- Hypothesis testing over the relatioship of two quantitative variables by the means of regression.
  - Numeric approach (coefficients & p.values)
  - Graphic approach (Slope line)

- Measurements of Model fit
  - Numeric methods (residuals and R2)
  - Graphic methods (Scatterplot + trend line)

- Linear modeling & prediction:
  - Numeric methods (The regression function)
  - Graphic methods (Slope line + confidence interval+ rugs)

Regression (I)

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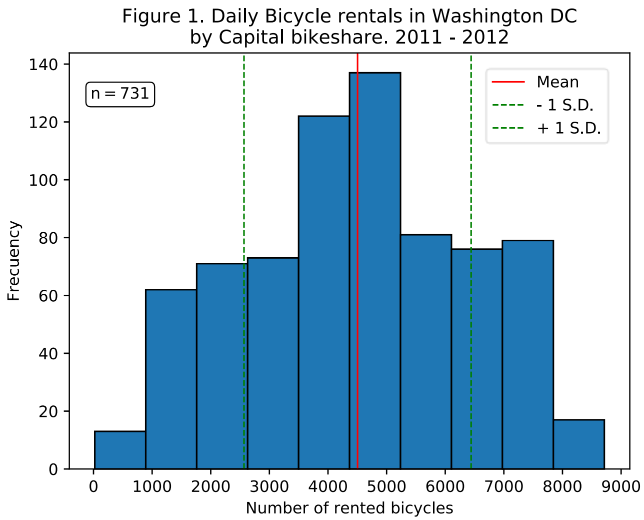
1. **Always DESCRIBE** the variables involved in the regression model separately. Check and validate the integrity of the data prior to any analysis.
2. **EXPLORE** of bivariate relation: **Scatterplot / Pearson's r**
3. **Fit your linear regression model carefully.** Pay attention to:
   a) **Slope & intercept**
   b) **P. value**
   c) **Model fit**
   d) **Sample size**
   e) **Model Diagnostics**

Research Question

Why some days are rent *more* bikes?

- Temperature ?

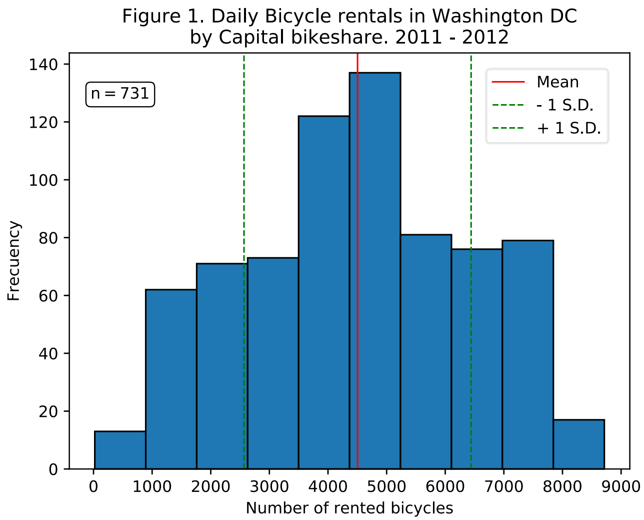


- H0.: There is no linear association (r=0) between the *number of rentals* and the *temperature*.
- H1.: There is a linear association (r≠0) between the *number of rentals* and the *temperature*.

Describing quantitative variables


```
x=wbr['cnt']
plt.hist(x, bins=10,
edgecolor='black')
plt.xticks(np.arange(0, 10000,
step=1000))
plt.title('Figure 4. Daily Bicycle
rentals in Washington DC'
          '\n'
          'by Capital bikeshare.
2011 - 2012')
plt.ylabel('Frecuency')
plt.xlabel('Number of rented
bicycles')


props = dict(boxstyle='round',
facecolor='white', lw=0.5)
textstr = '$\mathrm{n}=%.0f$'%(n)
plt.text (-50,128, textstr ,
bbox=props)
```



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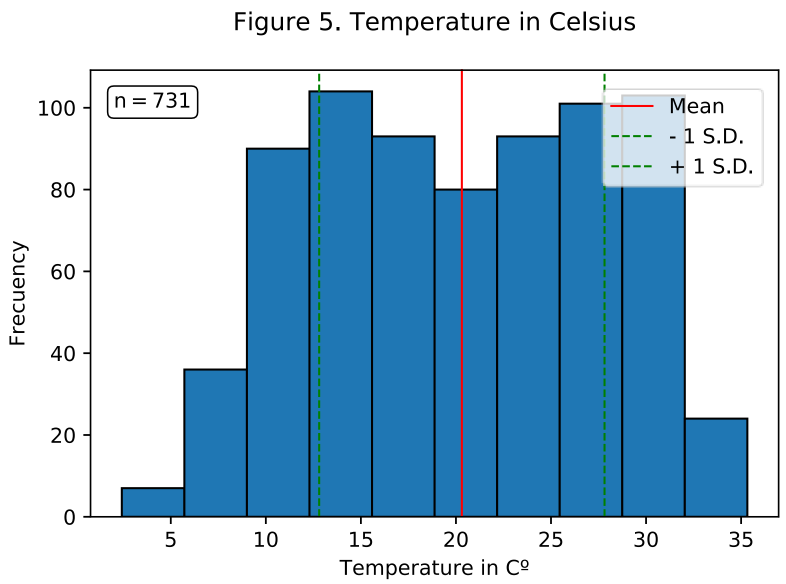
Describing quantitative variables


```
##histogram ver4
x=wbr['temp_celsius']
plt.hist(x, bins=10,
edgecolor='black')
#plt.xticks(np.arange(0, 10000,
step=1000))
plt.title('Figure 5. Temperature in
Celsius'
          '\n')
plt.ylabel('Frecuency')
plt.xlabel('Temperature in C°')
props = dict(boxstyle='round',
facecolor='white', lw=0.5)
textstr = '$\mathrm{n}=%.0f$'%(n)
plt.text (2,100, textstr ,
bbox=props)
```

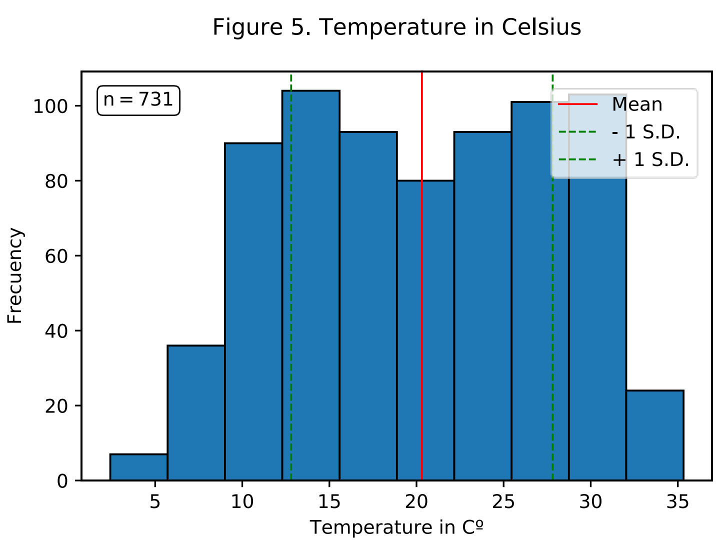


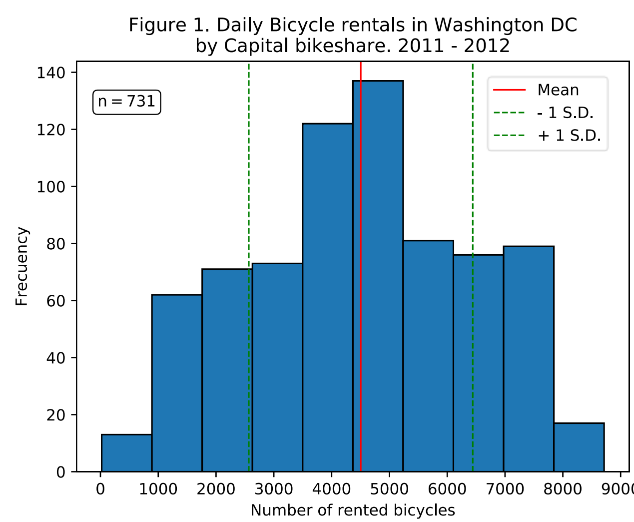
Regression

1. Describe the two variables involved in hypothesis

Temperature

Rentals





Regression

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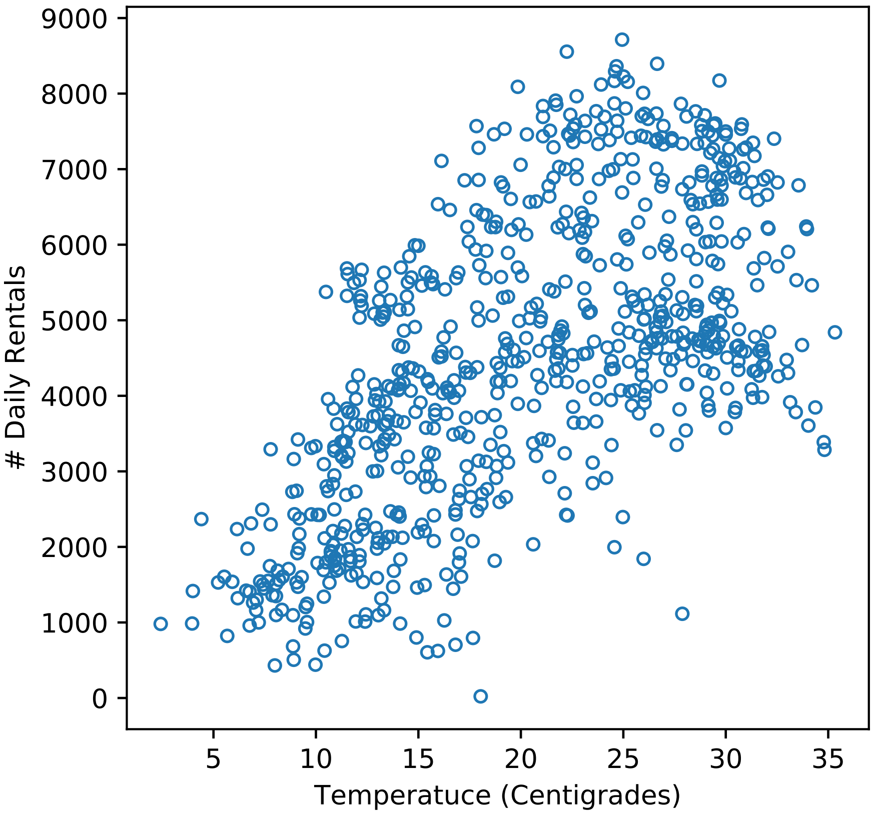
2. Scatterplot

```
x=wbr.temp_Celsius
y=wbr.cnt
plt.scatter (x,y)
```

Figure 9. Daily bicycle rentals, by temperature.



Regression

3. Pearson's r

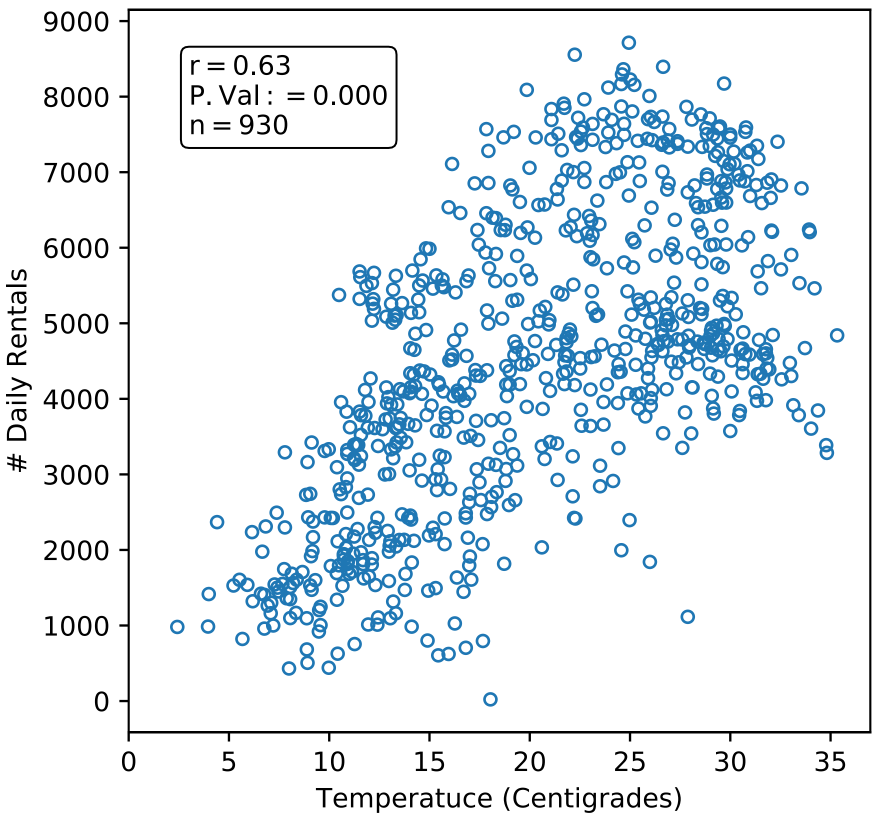
```
from scipy.stats.stats import pearsonr
res = pearsonr(x, y)
print (res)

[1] (0.627494009033349195, 2.810622397590l415e-81)
```

This is
Perason's r

This is
The P.Value

Scatterplot + Pearson's r + test

Figure 9. Daily bicycle rentals, by temperature.



The Regression Model

$Y= a + bx$

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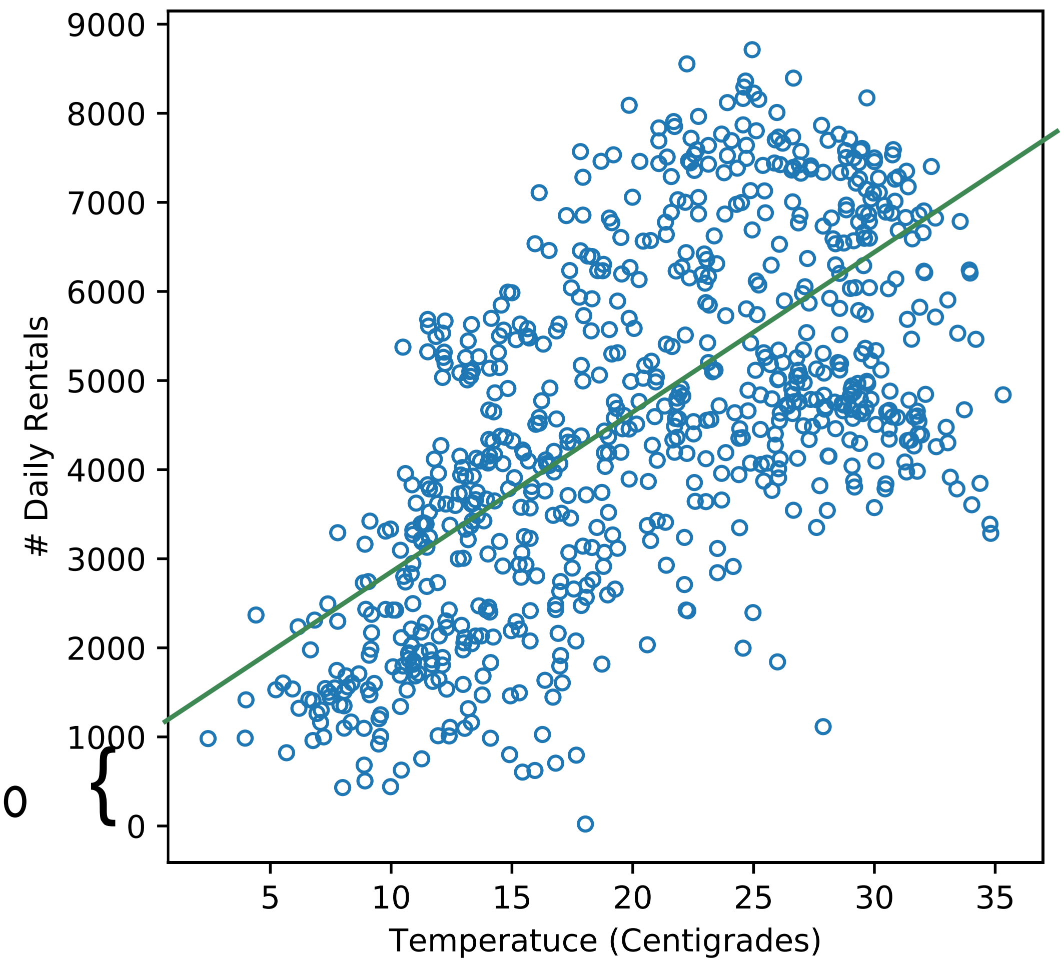
Figure 9. Daily bicycle rentals, by temperature.



$a \approx 1200$

$b \approx 800/5 \approx 160$

$a \approx 1200$

The Regression Model


```python
# Regression
from statsmodels.formula.api import ols

model1 = ols('cnt ~ temp_celsius', data=wbr).fit()
model1.summary2()
```

```
                 Results: Ordinary least squares
==================================================================
Model:              OLS              Adj. R-squared:     0.393
Dependent Variable: cnt              AIC:                12777.5357
Date:               2019-12-11 12:23 BIC:                12786.7245
No. Observations:   731              Log-Likelihood:     -6386.8
Df Model:           1                F-statistic:        473.5
Df Residuals:       729              Prob (F-statistic): 2.81e-81
R-squared:          0.394            Scale:              2.2783e+06
------------------------------------------------------------------
                 Coef.   Std.Err.    t    P>|t|   [0.025    0.975]
------------------------------------------------------------------
Intercept      1214.6421 161.1635  7.5367 0.0000 898.2421 1531.0421
temp_celsius    161.9685   7.4436 21.7594 0.0000 147.3551  176.5820
------------------------------------------------------------------
Omnibus:               20.477        Durbin-Watson:          0.468
Prob(Omnibus):         0.000         Jarque-Bera (JB):       12.566
Skew:                  0.167         Prob(JB):               0.002
Kurtosis:              2.452         Condition No.:          63
==================================================================
```

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The Regression Model

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The Regression Model


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R-squared:           0.394            Scale:               2.2783e+06
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                 Coef.    Std.Err.     t     P>|t|   [0.025    0.975]
--------------------------------------------------------------------
Intercept      1214.6421  161.1635  7.5367  0.0000  898.2421 1531.0421
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--------------------------------------------------------------------
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```

The Regression Model

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```

```
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------------------------------------------------------------------
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Intercept      1214.6421 161.1635  7.5367  0.0000 898.2421 1531.0421
temp_celsius    161.9685   7.4436 21.7594  0.0000 147.3551  176.5820
------------------------------------------------------------------
==================================================================
```
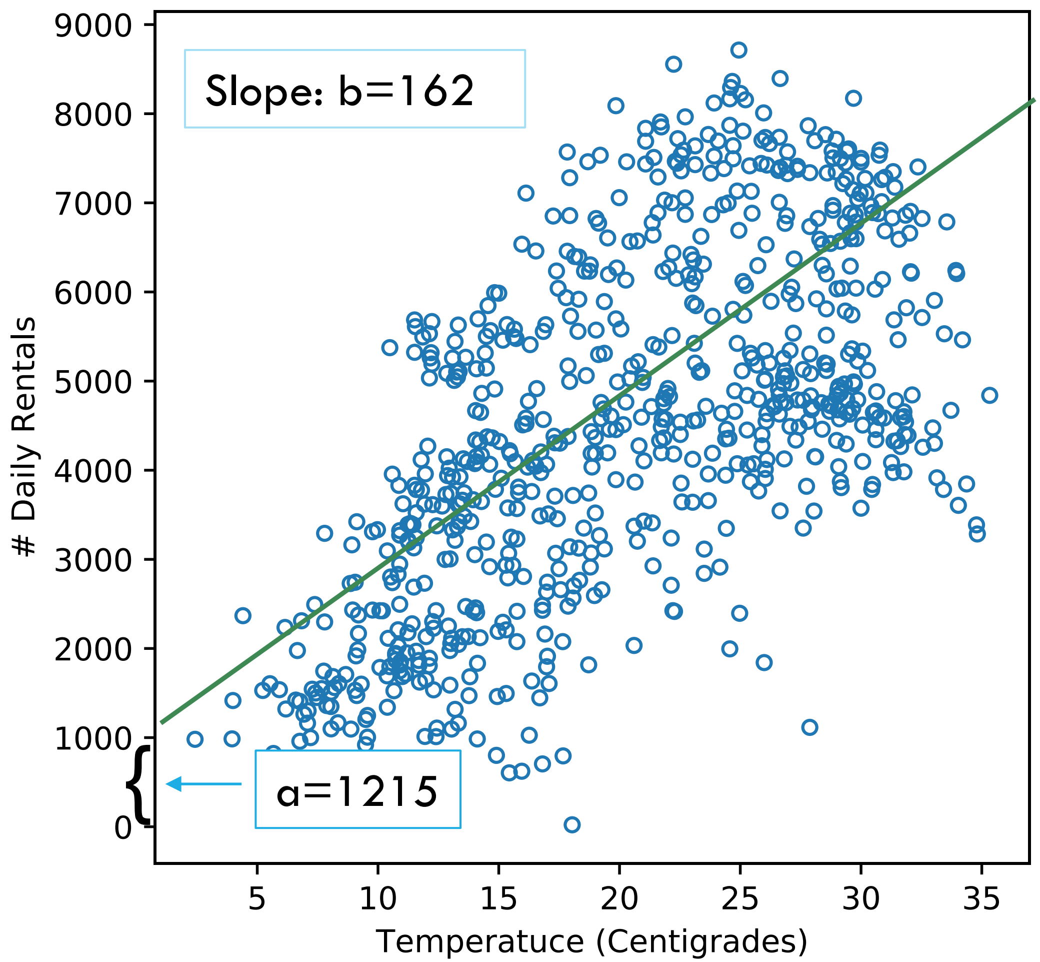

The Regression Model

Figure 9. Daily bicycle rentals, by temperature.

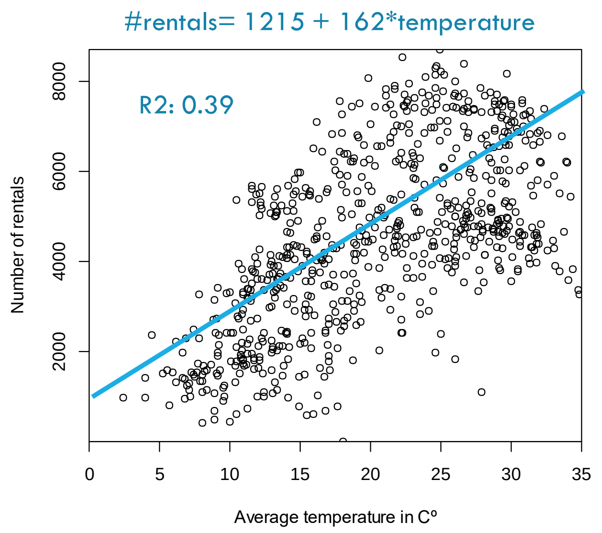



```
Results: Ordinary least squares
===================================
No. Observations:    731
R-squared:               0.394
-----------------------------------
                  Coef.       P>|t|
-----------------------------------
Intercept      1214.6421    0.0000
temp_celsius    161.9685    0.0000
-----------------------------------
```

Y = a + b*x

#rentals= 1215 + 162 * temperature

Conclussion



Conclussion:

As P. Value < 0.000

We can reject H0 with a confidence higer tan 99.9

Using (**only**) temperature as predictors, we can anticipate as much as **40 % of the variability in bike rentals!!!!**

❌ H0.: There is no linear association between the *number of rentals* and the *temperature*. (the slope of the regression line=0).

✓ H1.:There is a linear association between the *number of rentals* and the *temperature*. (the slope of the regression line ≠ 0).

The Regression Model


```
model1  = ols('cnt ~ temp_celsius', data=wbr).fit()
model1b = ols('cnt ~ windspeed_kh', data=wbr).fit()
print(model1b.summary2())
```

```
                  Results: Ordinary least squares
==================================================================
Model:              OLS              Adj. R-squared:     0.054
Dependent Variable: cnt              AIC:                13102.0108
Date:               2019-12-11 15:56 BIC:                13111.1996
No. Observations:   731              Log-Likelihood:     -6549.0
Df Model:           1                F-statistic:        42.44
Df Residuals:       729              Prob (F-statistic): 1.36e-10
R-squared:          0.055            Scale:              3.5512e+06
------------------------------------------------------------------
                Coef.   Std.Err.    t    P>|t|    [0.025    0.975]
------------------------------------------------------------------
Intercept     5621.1529 185.0624 30.3744 0.0000 5257.8341 5984.4717
windspeed_kh   -87.5062  13.4327 -6.5144 0.0000 -113.8775  -61.1348
------------------------------------------------------------------
Omnibus:               45.655        Durbin-Watson:           0.350
Prob(Omnibus):         0.000         Jarque-Bera (JB):        17.090
Skew:                  -0.026        Prob(JB):                0.000
Kurtosis:              2.253         Condition No.:           37
==================================================================
```

The Multiple Regression Model


```
model1 = ols('cnt ~ temp_celsius', data=wbr).fit()
model2 = ols('cnt ~  temp_celsius  + windspeed_kh', data=wbr).fit()
print(mode2.summary2())
```

```
 Results: Ordinary least squares
====================================================================
Model:              OLS              Adj. R-squared:     0.411
Dependent Variable: cnt              AIC:                12756.4931
Date:               2019-12-11 16:03 BIC:                12770.2763
No. Observations:   731              Log-Likelihood:     -6375.2
Df Model:           2                F-statistic:        255.6
Df Residuals:       728              Prob (F-statistic): 7.99e-85
R-squared:          0.413            Scale:              2.2106e+06
--------------------------------------------------------------------
                Coef.    Std.Err.    t     P>|t|    [0.025    0.975]
--------------------------------------------------------------------
Intercept     1991.0459 225.9615  8.8114 0.0000 1547.4319 2434.6599
temp_celsius   156.3058   7.4254 21.0500 0.0000  141.7279  170.8836
windspeed_kh   -51.8225  10.7328 -4.8284 0.0000  -72.8934  -30.7515
--------------------------------------------------------------------
Omnibus:              25.144       Durbin-Watson:          0.467
Prob(Omnibus):        0.000        Jarque-Bera (JB):       15.379
Skew:                 0.206        Prob(JB):               0.000
Kurtosis:             2.422        Condition No.:          102
====================================================================
```

Models of increasing complexity


```
model1 = ols('cnt ~ temp_celsius', data=wbr).fit()

model2 = ols('cnt ~  temp_celsius  + windspeed_kh ,data=wbr).fit()

model3 = ols('cnt ~ temp_celsius +  windspeed_kh + hum'
            , data=wbr).fit()

model4 = ols('cnt ~ temp_celsius +  windspeed_kh + hum + workingday'
            , data=wbr).fit()
```

The Multiple Regression Model


```
model4 = ols('cnt ~ temp_celsius +  windspeed_kh + hum + workingday'
          , data=wbr).fit()

print(model4.summary2())
```

```
====================================================================
Model:              OLS              Adj. R-squared:     0.459
Dependent Variable: cnt              AIC:                12696.4930
Date:               2019-12-11 16:20 BIC:                12719.4650
No. Observations:   731              Log-Likelihood:     -6343.2
Df Model:           4                F-statistic:        155.7
Df Residuals:       726              Prob (F-statistic): 3.61e-96
R-squared:          0.462            Scale:              2.0309e+06
--------------------------------------------------------------------
                Coef.   Std.Err.    t    P>|t|    [0.025    0.975]
--------------------------------------------------------------------
Intercept     4009.3688 344.5244 11.6374 0.0000 3332.9858 4685.7517
temp_celsius   161.2124   7.1558 22.5289 0.0000  147.1639  175.2609
windspeed_kh   -71.6672  10.5792 -6.7743 0.0000  -92.4367  -50.8976
hum            -31.0683   3.8398 -8.0911 0.0000  -38.6067  -23.5299
workingday     125.8049 113.5505  1.1079 0.2683  -97.1217  348.7315
--------------------------------------------------------------------
Omnibus:              10.037       Durbin-Watson:          0.404
Prob(Omnibus):        0.007        Jarque-Bera (JB):       7.868
Skew:                 0.160        Prob(JB):               0.020
Kurtosis:             2.604        Condition No.:          449
====================================================================
```

Models of increasing complexity

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```
model1 = ols('cnt ~ temp_celsius', data=wbr).fit()

model2 = ols('cnt ~  temp_celsius  + windspeed_kh ,data=wbr).fit()

model3 = ols('cnt ~ temp_celsius +  windspeed_kh + hum'
              , data=wbr).fit()

model4 = ols('cnt ~ temp_celsius +  windspeed_kh + hum + workingday'
            , data=wbr).fit()

#Stargazer
#!pip install stargazer
from stargazer.stargazer import Stargazer

Stargazer([model1,model2,model3,model4]).render_html()
```

Tip: Visit https://pypi.org/project/stargazer/ for stargazer functionalities
Visit https://github.com/mwburke/stargazer/blob/master/examples.ipynb for use examples

Tip: Stargezer will output HTML code. 1) You can render it into a nice (and editable) table in:
https://htmledit.squarefree.com/
Or 2) You can save the code in a plain text document with .html extensión and read it in word

Model reporting with Stargazer

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Table 1. Models of number of daily bicycle rentals in Washington D.C.

	Model 1	Model 2	Model 3	Model 4
Temperature Cº	162.0***	156.3***	161.6***	161.2***
	(7.4)	(7.4)	(7.1)	(7.2)
Windspeed_k/h		-51.8***	-71.7***	-71.7***
		(10.7)	(10.6)	(10.6)
Humidity			-31.0***	-31.1***
			(3.8)	(3.8)
Workingday (0/1)				125.8
				(113.6)
Intercept	1214.6***	1991.0***	4084.4***	4009.4***
	(161.2)	(226.0)	(337.9)	(344.5)
Observations	731	731	731	731
R^2	0.4	0.4	0.5	0.5
Note:				*p<0.1; **p<0.05; ***p<0.01

Models of increasing complexity

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	Dependent Variable: Number of bicycle rentals in Washington				
	Model 1	Model 2	Model 3	Model 4	Model 6
Temperature in Cº	161.969***	156.306***	161.598***	161.212***	646.078***
	(7.444)	(7.425)	(7.148)	(7.156)	(38.263)
Temperature in Cº squared					-12.022***
					(0.935)
Windspeed (Km/h)		-51.822***	-71.745***	-71.667***	-85.550***
		(10.733)	(10.581)	(10.579)	(9.614)
Humidity (in %)			-31.001***	-31.068***	-42.666***
			(3.840)	(3.840)	(3.583)
Working day (0:No, 1:Yes)				125.805	85.370
				(113.551)	(102.588)
Constant	1,214.642***	1,991.046***	4,084.363***	4,009.369***	730.179*
	(161.164)	(225.962)	(337.862)	(344.524)	(402.305)
Observations	731	731	731	731	731
R^2	0.394	0.413	0.461	0.462	0.562
Note:					*p<0.1 **p<0.05 ***p<0.01

Regression I. Summing UP

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1. **Always DESCRIBE** the variables involved in the regression model separately. Check and validate the integrity of the data prior to any analysis.
2. **EXPLORE** of bivariate relation: **Scatterplot / Pearson's r**
3. **Fit your linear regression model carefully.** Pay attention to:
   a) **Slope & intercept**
   b) **P. value**
   c) **Model fit**
   d) **Sample size**

Questions?

Thank you !

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