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Regression II:

Multiple linear regression

Sessions 11-14

Programación Estadística con Python

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MASTER EN DATA ANALYTICS PARA LA EMPRESA

Goals (+ to be developed)

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- Multiple linear regression.
  - Comprehensive models
  - Gaining control over alternative explanations: *Ceteris paribus*

- Dealing with the qualitative in regression models (I)
  - Interpretation of coefficients from dichotomies

- Dealing with non linearity
  - Curvilinear modeling under linear regression

- Graphic methods in multiple linear regression:
  - Coefficent plot
  - Confidence interval plot

Multiple Regression

1. **Always DESCRIBE** the variables involved in the regression model separately. Check and validate the integrity of the data prior to any analysis.
2. **EXPLORE** bivariate relation: **Scatterplot / Pearson's r / Simple Regression**
3. **Fit your multiple linear regression model carefully. Attend to:**
   a) **Slope & intercept**
   b) **P. value**
   c) **Model fit**
   d) **Sample size**
   e) **Model Diagnostics**

Research Question

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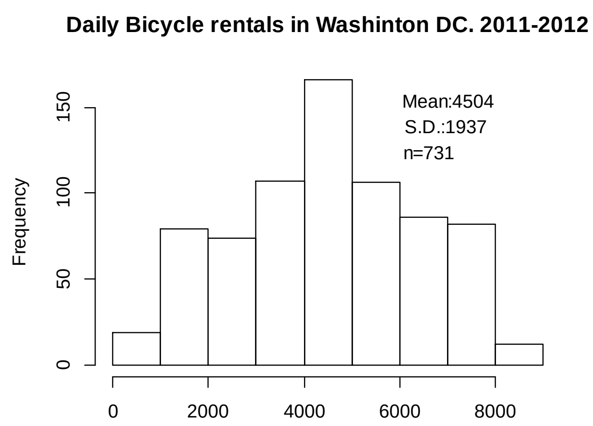
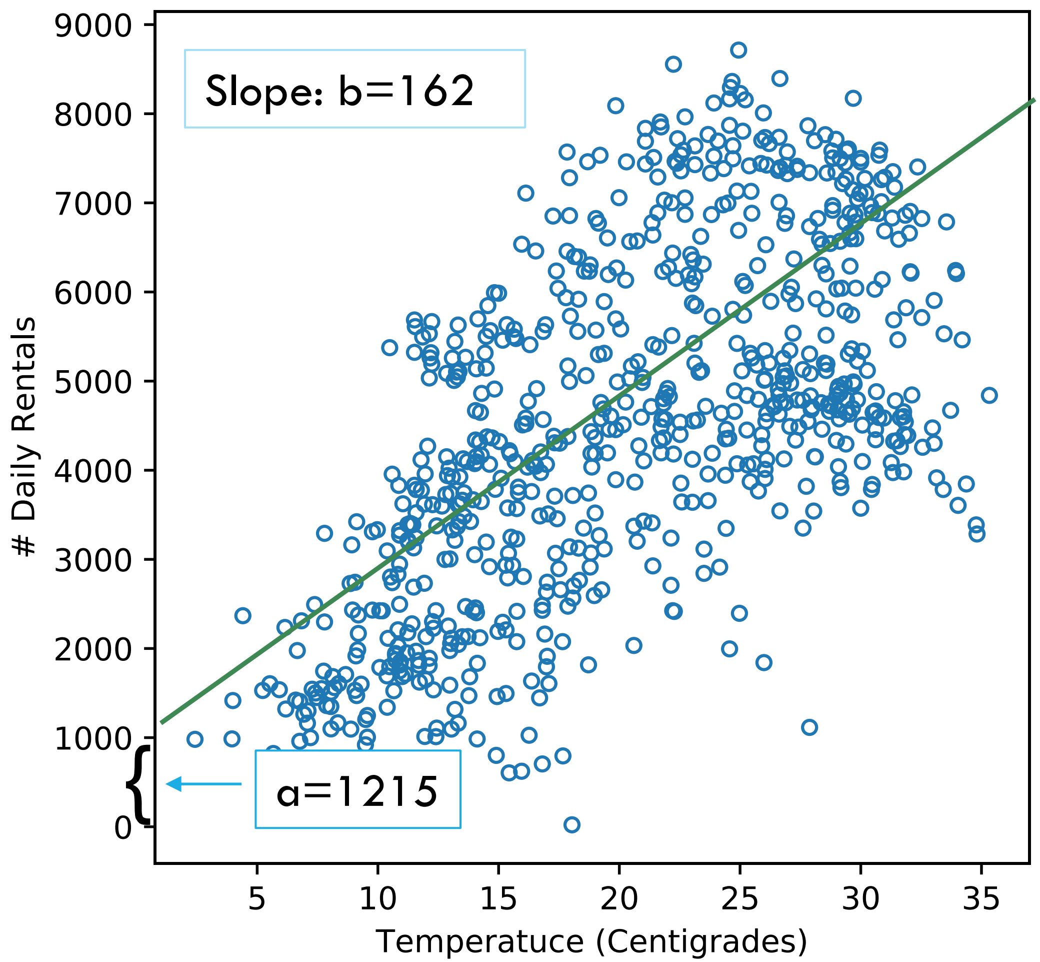
Why some days are rent *more* bikes?

- Temperature ?
- Windspeed ?
- Humidity ?
- Holiday ?
- ...



The (simple) Regression Model

Figure 9. Daily bicycle rentals, by temperature.




```
Results: Ordinary least squares
==================================
No. Observations:    731
R-squared:           0.394
----------------------------------
                  Coef.      P>|t|
----------------------------------
Intercept       1214.6421    0.0000
temp_celsius     161.9685    0.0000
----------------------------------
```

$$Y = a + b*x$$

#rentals= 1215 + 162 * temperature

The Regression Model


```
model1  = ols('cnt ~ temp_celsius', data=wbr).fit()
model1b = ols('cnt ~ windspeed_kh', data=wbr).fit()
print(model1b.summary2())
```

```
                 Results: Ordinary least squares
==================================================================
Model:              OLS              Adj. R-squared:     0.054
Dependent Variable: cnt              AIC:                13102.0108
Date:               2019-12-11 15:56 BIC:                13111.1996
No. Observations:   731              Log-Likelihood:     -6549.0
Df Model:           1                F-statistic:        42.44
Df Residuals:       729              Prob (F-statistic): 1.36e-10
R-squared:          0.055            Scale:              3.5512e+06
------------------------------------------------------------------
                Coef.   Std.Err.    t    P>|t|    [0.025    0.975]
------------------------------------------------------------------
Intercept     5621.1529 185.0624 30.3744 0.0000 5257.8341 5984.4717
windspeed_kh   -87.5062  13.4327 -6.5144 0.0000 -113.8775  -61.1348
------------------------------------------------------------------
Omnibus:              45.655       Durbin-Watson:          0.350
Prob(Omnibus):        0.000        Jarque-Bera (JB):       17.090
Skew:                 -0.026       Prob(JB):               0.000
Kurtosis:             2.253        Condition No.:          37
==================================================================
```

The Multiple Regression Model


```
model1 = ols('cnt ~ temp_celsius', data=wbr).fit()
model2 = ols('cnt ~  temp_celsius  + windspeed_kh', data=wbr).fit()
print(mode2.summary2())
```

```
 Results: Ordinary least squares
====================================================================
Model:              OLS              Adj. R-squared:     0.411
Dependent Variable: cnt              AIC:                12756.4931
Date:               2019-12-11 16:03 BIC:                12770.2763
No. Observations:   731              Log-Likelihood:     -6375.2
Df Model:           2                F-statistic:        255.6
Df Residuals:       728              Prob (F-statistic): 7.99e-85
R-squared:          0.413            Scale:              2.2106e+06
--------------------------------------------------------------------
                Coef.    Std.Err.    t     P>|t|    [0.025    0.975]
--------------------------------------------------------------------
Intercept     1991.0459 225.9615  8.8114 0.0000 1547.4319 2434.6599
temp_celsius   156.3058   7.4254 21.0500 0.0000  141.7279  170.8836
windspeed_kh   -51.8225  10.7328 -4.8284 0.0000  -72.8934  -30.7515
--------------------------------------------------------------------
Omnibus:              25.144        Durbin-Watson:           0.467
Prob(Omnibus):        0.000         Jarque-Bera (JB):        15.379
Skew:                 0.206         Prob(JB):                0.000
Kurtosis:             2.422         Condition No.:           102
====================================================================
```

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Models of increasing complexity

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```
model1 = ols('cnt ~ temp_celsius', data=wbr).fit()

model2 = ols('cnt ~  temp_celsius  + windspeed_kh ,data=wbr).fit()

model3 = ols('cnt ~ temp_celsius +  windspeed_kh + hum'
            , data=wbr).fit()

model4 = ols('cnt ~ temp_celsius +  windspeed_kh + hum + workingday'
            , data=wbr).fit()
```

The Multiple Regression Model


```
model4 = ols('cnt ~ temp_celsius +  windspeed_kh + hum + workingday'
             , data=wbr).fit()


print(model4.summary2())
```

```
====================================================================
Model:              OLS              Adj. R-squared:     0.459
Dependent Variable: cnt              AIC:                12696.4930
Date:               2019-12-11 16:20 BIC:                12719.4650
No. Observations:   731              Log-Likelihood:     -6343.2
Df Model:           4                F-statistic:        155.7
Df Residuals:       726              Prob (F-statistic): 3.61e-96
R-squared:          0.462            Scale:              2.0309e+06
--------------------------------------------------------------------
                Coef.   Std.Err.    t     P>|t|    [0.025    0.975]
--------------------------------------------------------------------
Intercept      4009.3688 344.5244 11.6374 0.0000 3332.9858 4685.7517
temp_celsius    161.2124   7.1558 22.5289 0.0000  147.1639  175.2609
windspeed_kh    -71.6672  10.5792 -6.7743 0.0000  -92.4367  -50.8976
hum             -31.0683   3.8398 -8.0911 0.0000  -38.6067  -23.5299
workingday      125.8049 113.5505  1.1079 0.2683  -97.1217  348.7315
--------------------------------------------------------------------
Omnibus:               10.037       Durbin-Watson:          0.404
Prob(Omnibus):         0.007        Jarque-Bera (JB):       7.868
Skew:                  0.160        Prob(JB):               0.020
Kurtosis:              2.604        Condition No.:          449
====================================================================
```

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Models of increasing complexity

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```
model1 = ols('cnt ~ temp_celsius', data=wbr).fit()

model2 = ols('cnt ~  temp_celsius  + windspeed_kh ,data=wbr).fit()

model3 = ols('cnt ~ temp_celsius +  windspeed_kh + hum'
             , data=wbr).fit()

model4 = ols('cnt ~ temp_celsius +  windspeed_kh + hum + workingday'
            , data=wbr).fit()
#Stargazer
#!pip install stargazer
from stargazer.stargazer import Stargazer

Stargazer([model1,model2,model3,model4]).render_html()
```

Tip: Visit https://pypi.org/project/stargazer/ for stargazer functionalities
Visit https://github.com/mwburke/stargazer/blob/master/examples.ipynb for use examples

Tip: Stargezer will output HTML code. 1) You can render it into a nice (and editable) table in:
https://htmledit.squarefree.com/
Or 2) You can save the code in a plain text document with .html extensión and read it in word

Model reporting with Stargazer

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Table 1. Models of number of daily bicycle rentals in Washington D.C.

	Model 1	Model 2	Model 3	Model 4
Temperature Cº	162.0***	156.3***	161.6***	161.2***
	(7.4)	(7.4)	(7.1)	(7.2)
Windspeed_k/h		-51.8***	-71.7***	-71.7***
		(10.7)	(10.6)	(10.6)
Humidity			-31.0***	-31.1***
			(3.8)	(3.8)
Workingday (0/1)				125.8
				(113.6)
Intercept	1214.6***	1991.0***	4084.4***	4009.4***
	(161.2)	(226.0)	(337.9)	(344.5)
Observations	731	731	731	731
R^2	0.4	0.4	0.5	0.5
Note:				*p<0.1; **p<0.05; ***p<0.01

Models of increasing complexity

	Dependent Variable: Number of bicycle rentals in Washington				
	Model 1	Model 2	Model 3	Model 4	Model 6
Temperature in Cº	161.969***	156.306***	161.598***	161.212***	646.078***
	(7.444)	(7.425)	(7.148)	(7.156)	(38.263)
Temperature in Cº squared					-12.022***
					(0.935)
Windspeed (Km/h)		-51.822***	-71.745***	-71.667***	-85.550***
		(10.733)	(10.581)	(10.579)	(9.614)
Humidity (in %)			-31.001***	-31.068***	-42.666***
			(3.840)	(3.840)	(3.583)
Working day (0:No, 1:Yes)				125.805	85.370
				(113.551)	(102.588)
Constant	1,214.642***	1,991.046***	4,084.363***	4,009.369***	730.179*
	(161.164)	(225.962)	(337.862)	(344.524)	(402.305)
Observations	731	731	731	731	731
R^2	0.394	0.413	0.461	0.462	0.562
Note:					*p<0.1 **p<0.05 ***p<0.01

Basic diagnostics for simple LM

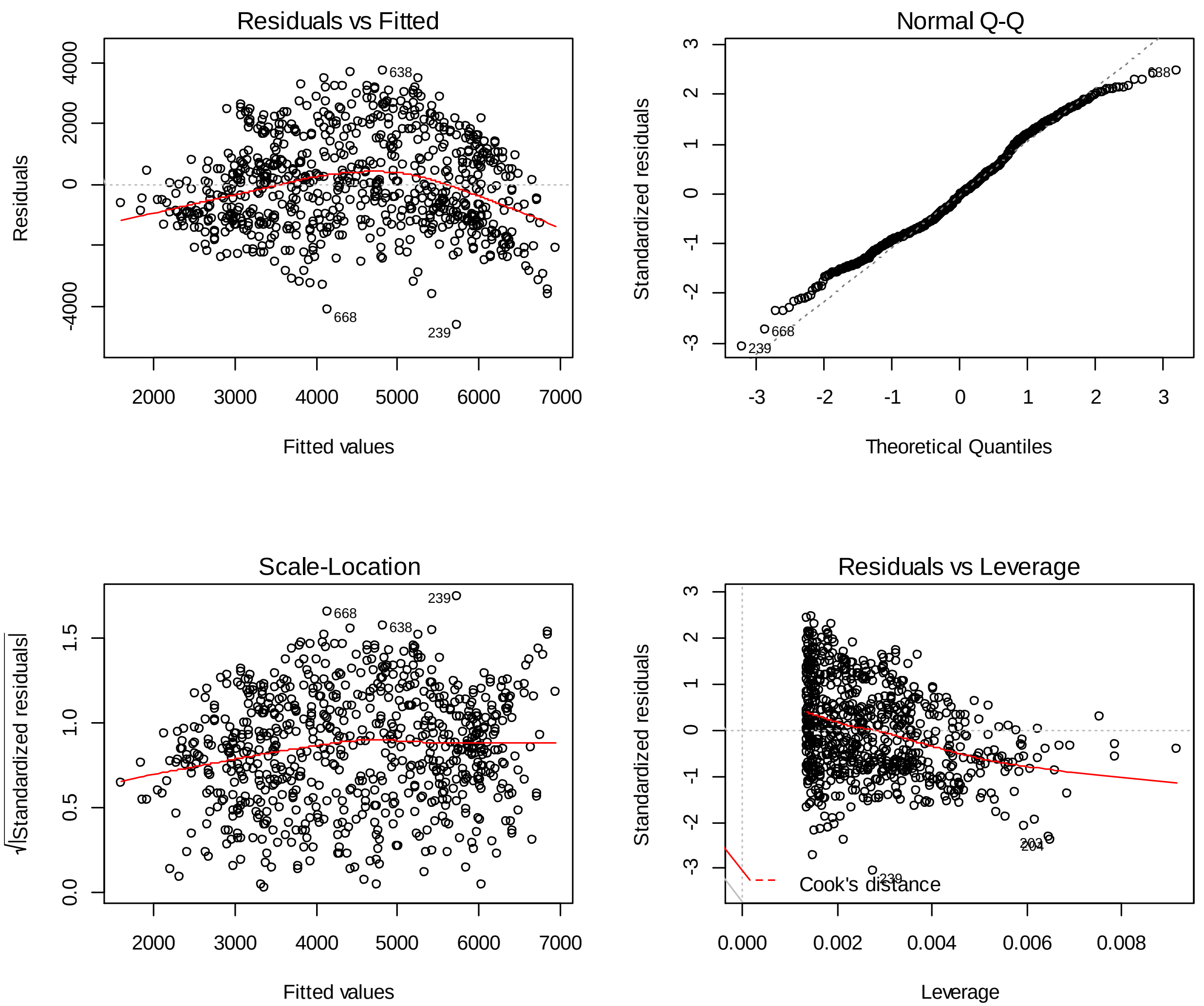
- Linear relation:
  - Check Residuals vs. Fitted (or predicted) plot
- Normality of residuals:
  - Check Normal Q-Q plot
- Homocedasticity:
  - Check Scale-Location plot

- No influential observations
  - Check the Residuals vs. Leverage plot

Basic diagnostics for simple LM

Code available at:

https://robert-alvarez.github.io/2018-06-04-diagnostic_plots/



Regression I. Summing UP

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Questions?

Thank you !

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