Introduction to Modern Astrophysics

Notes on BOB: the Big Orange Book

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Preface

Background

I discovered computational physics far too late. My first exposure was through my astrodynamics coursework, completed in the final year of my aerospace engineering M.S. program. I explored the introductory material — Kepler's Laws, conic sections, restricted two-body propagation, etc. — with a new-to-me programming language, Julia. With the mechanics of Julia under my belt, I continued on to more complicated first-year-graduate concepts, including Halo orbit solvers and manifold computations within the circular restricted three-body problem. Using computation to find periodic orbits, and free-energy paths through the solar system is still an exhilerating idea to me; through approximately 4000 lines of code, I could learn the behavior of places across our solar system. These simple simulations allowed me to feel more connected to the universe I inhabit.

After (recently) coming to understand my love for learning about the universe through computation, I could not wait to learn more. I ordered *Introduction to Modern Astrophysics* by Caroll and Ostlie, and started working through each chapter. Therefore: this text!

Methodology

As I read each chapter, I am using this space to summarize the information I learned. This will be incredibly difficult to do on my own, in spare time. At the time of writing, August 13th 2023, I truly have no idea if I will succeed in finishing this book; the odds certainly point to **no**. Still, the attempt will help me to further understand my place within the space industry. Do I contribute to science, or exploration?

As I write, I will reference equation and figure numbers from the Big Orange Book using phrases such as *in the text*. All other references are internal to this document. As a rule of thumb, all click-able links are internal links!

Acknowledgements

Finally, I'm so grateful to Quarto, Jupyter, Julia, and other free software tools that enable quick and clear technical communication.

Ad astra!

1 The Celestial Sphere

Chapter 1 walks through the history Copernican Revolution in astronomy — our worldview's transition from geocentric to heliocentric. The following approximate calculations are presented: sidereal and synodic periods, the Equitorial Coordinate System, perturbations to the Equitorial Coorrdinates caused by a Earth's precession and the observed object's tangential velocity in the sky, and proper time.

1.1 Retrograde Motion

From the point of view of observers on Earth, Mars appears to change its direction of motion in the night sky. This effect — retrograde motion — was a great motivator for the early field of astronomy. The appearance of retrograde motion is caused by the planets' relative positions changing.

The synodic period S, and the sidereal period P are related to the discussion on retrograde motion. Both assume **circular orbits** about the Sun, with constant velocities. The synodic period relates to the oscillation of the relative position between the Earth, and the target planet. The sidereal period relates to the duration of Earth's orientation with respect to background stars. The two periods are related by equation (1.1) in the text, where $P_{\oplus}$ is the sidereal period of Earth: 365.256308 days [1].

$$S = \begin{cases} 1/P - 1/P_{\oplus} & \text{(inferior)} \\ 1/P_{\oplus} - 1/P & \text{(superior)} \end{cases} \tag{1.1}$$

1.2 Altitude/Azimuth

After I was given an Orion ST-80 telescope in 2022, my father (a phycisist in his own right) introduced me to altitude/azimuth coordinates. We could use these coordinates to track objects in the night sky by orienting the telescope's two degrees of freedom along the horizon, and vertically "up" from the horizon to the point on the *celestial sphere* directly overhead the observer: the **zenith**. The celestial sphere coincides with two points: the point being *observed*, and the **zenith**. The altitude coordinate h is the angle from the horizontal to the observed object. The azimuth coordinate A is the angle from north along the observer-zenith axis (clockwise). This coordinate system *says nothing* about Earth's rotation about its axis (diurnal motion), or Earth's motion about the Sun (annual motion).

1.3 Equitorial Coordinates

The **Equitorial Coordinate System** also uses three coordinates, which "are based on the latitude-longitude system of Earth but does not participate in the planet's rotation" [1]. The angle of declination — δ — corresponds to latitude. The right ascension — α — is a kind of *longitude* angle. The angle of right ascension is measured from the **vernal equinox** Υ, counter-clockwise about the *celestial polar axis* or meridian: the axis from celestial south pole to celestial north pole. Neither δ nor α are affected by the Earth's annual motion.

The third parameter brings information about Earth's annual motion into the Equitorial Coordinate System: the **local sidereal time**. The hour-angle H of the vernal equinox is "equivalent to" local sidereal time; the hour-angle is the angle between the object and the observer's *meridian*, "measured in the direction of the object's" motion around the celestial sphere" [1]. Every word in that last quote is important. We are encoding information about the Earth's annual motion by relating the angle of the object about Earth's meridian from the position of the vernal equinox, and that angle's *direction* is defined *by the object's motion.* Yuck!

1.4 Precession

Earth's rotational-axis's wobble, or **precession**, causes position of the vernal equinox Υ to change, and therefore causes the right ascension and declination angles to change. The hour-angle of the vernal equinox, H, is **unchanged**. An epoch, commonly J2000, is used to set the *origin* local sidereal time. The drift of δ and α due to precession can be approximated by equations (1.2) and (1.3) in the text, where t is the current date specified in fractions of a year [1].

$$\Delta\alpha = M + N \sin\alpha \tan\alpha \tag{1.2}$$

$$\Delta\delta = N \cos\alpha \tag{1.3}$$

$$\begin{aligned} M &= 1.2812323^\circ T + 0.0003879^\circ T^2 + 0.0000101 T^3 \\ N &= 0.5567530^\circ T + 0.0001185^\circ T^2 + 0.0000116 T^3 \end{aligned} \tag{1.4}$$

$$T = (t - 2000)/100 \tag{1.5}$$

1.5 Tangential Velocity

The motion of objects in space too causes the declination and right ascension angles to *drift*. With a star's radial distance to from the observer r, and the star's **transverse velocity** defined as v_θ, the star's proper motion μ can be found using the text's equation (1.5) [1]. Using spherical trigonometry, Caroll and Ostlie derive equation (1.8) in the text, which relates the change in angular distance traveled with the corresponding change in declination and right ascension [1].

$$\mu = \frac{d\theta}{dt} = \frac{v_\theta}{r} \tag{1.6}$$

$$(\Delta\theta)^2 = (\Delta\alpha \cos\delta)^2 + (\Delta\delta)^2 \tag{1.7}$$

2 Celestial Mechanics

Chapter 2 walks through the historical contributions of Kepler's Laws, Galileo's scientific confirmation of heliocentrism, and Newton's contributions to physics which successfully connected Galileo (and others') astronomical observations with *gravity*. Caroll and Ostlie introduce work and energy, and derive Kepler's Laws before proving the *virial theorem*: that the total energy of a gravitationally bound systems is equivalent to "one-half of the time-averaged potential energy" @bob.

2.1 Kepler's Laws

The following laws — Kepler's Laws — are quoted directly from Caroll and Ostlie [1]. Each are derived later in Chapter 2.

1. A planet orbits the Sun in an ellipse, with the Sun at one focus[1] of the ellipse.
2. A line connecting a planet to the Sun sweeps out equal areas in equal time intervols.
3. The Harmonic Law: $P^2 = a^3$.

Kepler's definitions place the paths of all orbits on *conic sections*: cross-sections of a cone. While circular, elliptical, parabolic, and hyperbolic paths are all conic sections, ach of the planets Kepler identified in the solar system have elliptical orbits. For this reason, a substantial section of Chapter 2 reviews the geometry of ellipses.

An ellipse has two focal points, with semimajor axis a defined as one-half the length of the diameter — the line which passes through both focal points — of the ellipse. The central body — in our case, the Sun — is located at one of the two focal points; that point is *principle focus* [1]. The distance from the principle focus to an orbiting object is referred to in Equation 2.1 as r, and the distance from the other focus is referred to as r'. The semiminor axis b is perpendicular to, and bisects, the semimajor axis. The eccentricity e is the distance between both focal points divided by the major axis $2a$ [1]. The label *perihelion* on the ellipse, and on the major axis, which is closes to the principle focus; the opposite point along the major axis

[1] As Caroll and Ostlie explain later in Chapter 2, objects in our solar system *actually* orbit the solar system barycenter, which is located just outside the surface of the sun.

is referred to as *aphelion*. The suffix *helion* refers to our Sun. More general terms are *periapsis* and *apoapsis*.

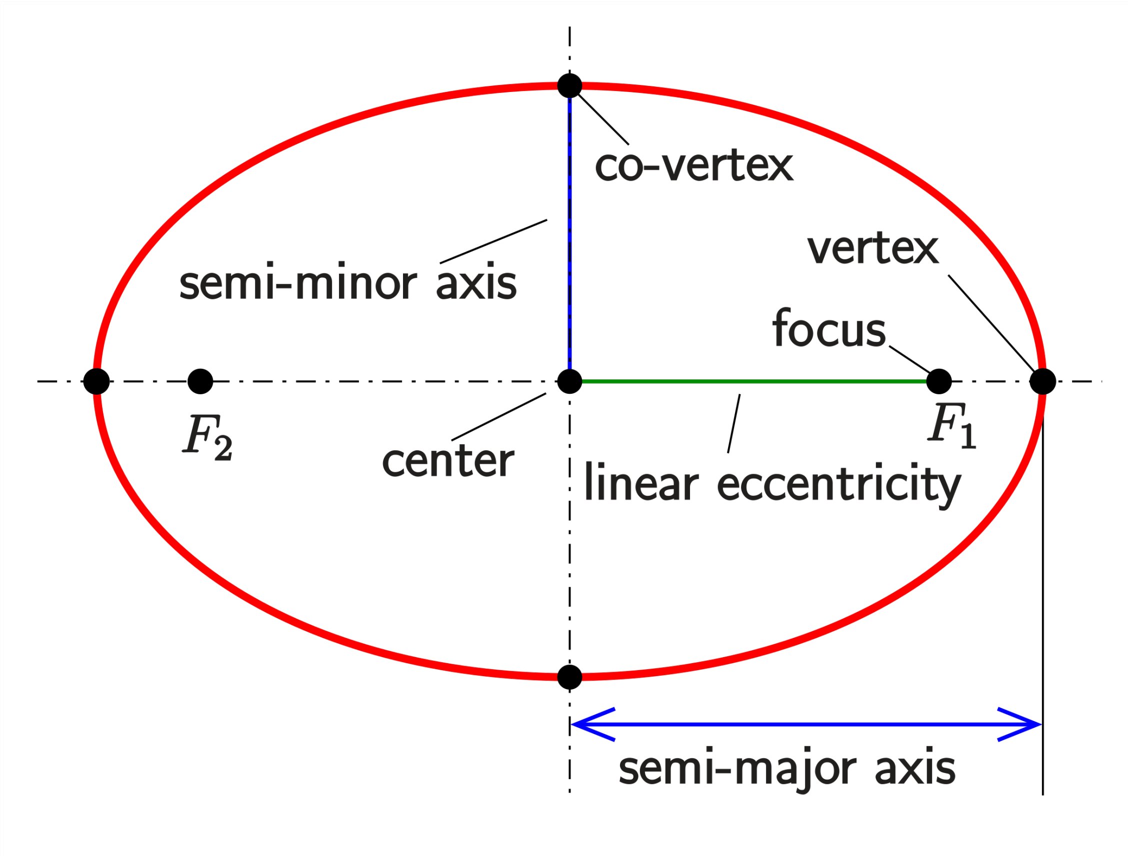

Figure 2.1: The Geometry of an Ellipse (Wikipedia)

The following equations hold for ellipses. All are pulled from Chapter 2 of the text [1].

$$r + r' = 2a \tag{2.1}$$

$$b^2 = a^2(1 - e^2) \tag{2.2}$$

Polar coordinates are convenient when describing conic orbits. The angle θ is the *true anamoly*, and represents the orbiting object's angular position past perihelion. The distance from the principle focus r is related to true anamoly; the precise relationship relies on the *type* of conic orbit. For elliptical orbits, Equation 2.3 relates r and θ; note that circular orbits are simply a special case where eccentricity e is zero. Equation 2.4 relates r and θ for parabolic orbits, with p defined as the distance from the *single* parabolic focus to perihelion. Finally, for hyperbolic orbits, equation Equation 2.5 relates r and θ. An object on a parabolic orbit will be at rest when *infinitely far* from the central gravitational body. A hyperbolic orbit would keep the object's speed positive while it travels away from the central gravitational body. An object with a velocity *equal to* to the central body's escape velocity v_{esc} would follow a parabolic orbit. An object follows a hyperbolic orbit if its velocity *exceeds* the central body's excape velocity.

$$r = \tfrac{a(1-e^2)}{1+e\cos\theta} \quad (0 \le e < 1) \tag{2.3}$$

$$r = \tfrac{2p}{1+\cos\theta} \quad (e = 1) \tag{2.4}$$

$$r = \tfrac{a(e^2-1)}{1+e\cos\theta} \quad (e > 1) \tag{2.5}$$

$$v_{\text{esc}} = \sqrt{2GM/r} \tag{2.6}$$

2.2 Newton's Laws

Each of Newton's Laws below are directly quoted from Caroll and Ostlie [1].

1. *The Law of Inertia.* An object at rest will remain at rest and an object in motion will remain in motion in a straight line at a constant speed unless acted upon by an external force.
2. The *net* force (the sum of all forces) acting on an object is proportional to the object's mass and its resultant acceleration.
3. For every action there is an equal and opposite reaction[2]

These laws, which are taught in high school physics classes, revolutionized physics. Newton's Law of Universal Gravitation, shown in Equation 2.7, succesfully linked the motions of our solar system's bodies to gravitation. Caroll and Ostlie also prove that the gravitational force due to a spherical *shell*[3] is equivalent to the gravitational force due to the shell's *point of center of mass* [1].

$$F = G\frac{Mm}{r^2} \tag{2.7}$$

[2] This law is clear when considering forces as *vectors*. "Mathematically, the third law can be represented as... $F_{12} = -F_{21}$" [1].

[3] Qualitatively, the word *shell* feels unimportant to me. The proof presented by Caroll and Ostlie is specific to a spherical shell, but I suspect that proof can be extended to *all spheres*; after integrating over cross-sectional *rings* on each shell, integrate the force due to each infinitesimally thin *shell*. I should try to work this out!

2.3 Work and Energy

The expression for gravitational potential energy U is shown in Equation 2.8, and gravitational kinetic energy K is shown in Equation 2.9.

$$U = -G\frac{Mm}{r} \tag{2.8}$$

$$K = \frac{1}{2}mv^2 \tag{2.9}$$

2.4 Gravitational Constraints

The center of mass of a gravitationally bound system is **constant**. This produces a useful simplification in the two-body case: the dynamics can be simplified to a reduced mass μ with position $\mathbb{r}$ orbiting the center of mass of the original system with total mass M. The equations below are pulled directly from equations 2.22, 2.23, and 2.24 in the text [1].

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2} \tag{2.10}$$

$$r_1 = -\frac{\mu}{m_1}\mathbf{r}$$
$$r_2 = -\frac{\mu}{m_2}\mathbf{r}$$

In this reduced case of the binary system, the total angular momentum L can be found using Equation 2.11.

$$\mathbf{L} = \mu\mathbf{r} \times \mathbf{v} = \mathbf{r} \times \mathbf{p} \tag{2.11}$$

The derivations of Kepler's Laws in Chapter 2 culminate in more general forms of the laws' previous mathematical expressions. The following equations, pulled from the text, are valid for binary orbits [1]. Equation 2.14 and Equation 2.15 represents the velocity and orbital period of the reduced mass μ, respectively.

$$\frac{dA}{dt} = \frac{1}{2}\frac{L}{\mu} \tag{2.12}$$

$$E = -G\frac{M\mu}{2a} = -G\frac{m_1 m_2}{2a} \tag{2.13}$$

$$v^2 = G\left(m_1 + m_2\right)\left(\frac{2}{r} - \frac{1}{a}\right) \tag{2.14}$$

$$P^2 = \frac{r\pi^2}{G\left(m_1 + m_2\right)} \tag{2.15}$$

2.5 The Virial Theorem

For another day!

References

[1] Carroll, B. W., and Ostlie, D. A. *An Introduction to Modern Astrophysics.* Cambridge University Press, 2017.