# Fundamentals of Robotics

**A review of fundamental concepts in robotics, from coordinate frames to manipulator dynamics and control.**

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# Table of contents

# Preface

This is still being written! The following topics will eventually be explored in this note-set.

## Review of Coordinate Transformations

- Rotations Matrices
- Passive, active rotations
- $4 \times 4$ coordinate transformation matrix

## Kinematic Descriptions

- Going from robot diagram to modified DH parameters
- Assigning coordinate frames

## Forward Kinematics

- Use your kinematic description to map joint angles to Cartesian positions & orientations

## Translational & Rotational Jacobians

- Differentiate your forward kinematic solutions with respect to joint angles to find how **joint velocities** map to **Cartesian velocities** and **anglular velocities**

## Inverse Kinematics

- Find joint angles to match some Cartesian *pose*
- Generally relies on the Jacobian having full rank

## Singularities

- Find when the Jacobian does not have full rank

## Dynamics

- Apply forces through joint angles
- Incorporate mass properties, and how they interact with Cartesian velocities & angular rates
- Incorporating moments of inertia, other dynamical elements

## Control

- PID Control

## Applications

- ROS: *Robot Operating System*
- MoveIt: motion planning framework within ROS
- Orocos KDL: Open Source Kinematics & Dynamics Solvers in C++

# 1 Reference Frames

Reference frames exist at some point in space, at some orientation in space, relative to another reference frame. There are two ways to describe multi-axis rotations: fixed axes, and euler (rotating) axes.

## 1.1 Why Rotations

We use rotations to answer two different questions...

1. Given $P$, what are the coordinates of **that same point** relative to $F_j$?

   - Answer looks like: $^j(P_0) = R_j P$

2. Given $P$, what are the coordinates of a point **in** $F_2$ that are rotated about some axis?

   - Answer looks like: $^2(P) = R_i P$

How can we reconcile this?

- $R_j \equiv R_i^T$

## 1.2 Rotation Matrices

Relating reference frames requires many (somewhat arbitrary) definitions. We may choose to relate frames by rotating axes, or fixed axes. We may choose to change the basis of a three-valued vector through rotation, or we may choose to rotate a three-valued vector in its original reference frame. For these reasons, you may see two different definitions of rotation matrices about principle axes. Both definitions are the transpose — and due to orthonormality, the inverse — of one another.

The definitions below are the principle rotation matrices we will use moving forward. The following matrices rotate a three-valued vector by an angle of $\theta$ about each of the three basis vectors; see Equations 2.77, 2.78, and 2.79 in Craig's Introduction to Robotics [1].

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{1.1}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{1.2}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.3}$$

## 1.3 Euler Axis Rotations

There are many ways to describe rotations about multiple axes. One way is to apply a sequence of rotations about principle axes, applying each rotation to the preceding *intermediate* frame. This technique is known as *euler* sequence rotations.

You might say: to get frame $B$, rotate frame $A$ by $\theta_1$ degrees about frame $A$'s $X$ axis, then rotate **that intermediate frame** $\theta_2$ degrees about the intermediate frame's $Y$ axis, etc.

Euler ZYX: $R_z(\theta_3)R_y(\theta_2)R_x(\theta_1)$

## 1.4 Fixed Axis Rotations

Fixed XYZ: $R_x(\theta_1)R_y(\theta_2)R_z(\theta_3)$

## 1.5 Translations & Rotations (Transformations)

You can combine translations and rotations to create *transformations*.