Fundamentals of Robotics

A review of fundamental concepts in robotics, from coordinate frames to manipulator dynamics and control.

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Preface

The following topics will ultimately be explored in this note-set.

Review of Coordinate Transformations

- Rotations Matrices
- Passive, active rotations
- 4×4 coordinate transformation matrix

Kinematic Descriptions

- Going from robot diagram to modified DH parameters
- Assigning coordinate frames

Forward Kinematics

- Use your kinematic description to map joint angles to Cartesian positions & orientations

Translational & Rotational Jacobians

- Differentiate your forward kinematic solutions with respect to joint angles to find how **joint velocities** map to **Cartesian velocities** and **anglular velocities**

Inverse Kinematics

- Find joint angles to match some Cartesian *pose*
- Generally relies on the Jacobian having full rank

Singularities

- Find when the Jacobian does not have full rank

Dynamics

- Apply forces through joint angles
- Incorporate mass properties, and how they interact with Cartesian velocities & angular rates
- Incorporating moments of inertia, other dynamical elements

Control

- PID Control

Applications

- ROS: *Robot Operating System*
- MoveIt: motion planning framework within ROS
- Orocos KDL: Open Source Kinematics & Dynamics Solvers in C++

1 Reference Frames

Reference frames exist at some point in space, at some orientation in space, relative to another reference frame. There are two ways to describe multi-axis rotations: fixed axes, and euler (rotating) axes.

1.1 Why Rotations

We use rotations to answer two different questions...

1. Given P, what are the coordinates of **that same point** relative to F_j?
    - Answer looks like: ${}^{j}(P_0) = R_j P$
2. Given P, what are the coordinates of a point **in** F_2 that are rotated about some axis?
    - Answer looks like: ${}^{2}(P) = R_i P$

How can we reconcile this?

- $R_j \equiv R_i^T$

1.2 Rotation Matrices

Relating reference frames requires many (somewhat arbitrary) definitions. We may choose to relate frames by rotating axes, or fixed axes. We may choose to change the basis of a three-valued vector through rotation, or we may choose to rotate a three-valued vector in its original reference frame. For these reasons, you may see two different definitions of rotation matrices about principle axes. Both definitions are the transpose — and due to orthonormality, the inverse — of one another.

The definitions below are the principle rotation matrices we will use moving forward. The following matrices rotate a three-valued vector by an angle of θ about each of the three basis vectors; see Equations 2.77, 2.78, and 2.79 in Craig's Introduction to Robotics [1].

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{1.1}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{1.2}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.3}$$

1.3 Euler Axis Rotations

There are many ways to describe rotations about multiple axes. One way is to apply a sequence of rotations about principle axes, applying each rotation to the preceding *intermediate* frame. This technique is known as *euler* sequence rotations.

You might say: to get frame B, rotate frame A by θ_1 degrees about frame A's X axis, then rotate **that intermediate frame** θ_2 degrees about the intermediate frame's Y axis, etc.

Euler ZYX: $R_z(\theta_3)R_y(\theta_2)R_x(\theta_1)$

1.4 Fixed Axis Rotations

Fixed XYZ: $R_x(\theta_1)R_y(\theta_2)R_z(\theta_3)$

1.5 Translations & Rotations (Transformations)

2 Jacobians

Jacobians map joint velocities to translational and angular (Cartesian) velocities.

$$\dot{p} = J_T \dot{q} \tag{2.1}$$

$$\dot{R} = J_\Omega \dot{q} \tag{2.2}$$

References

[1] Craig, J. J. *Introduction to Robotics*. Pearson Educacion, 2006.