Midterm - FA24

YOU DO NOT NEED TO SIMPLIFY YOUR ANSWERS

YOU MUST SHOW WORK ON ALL PROBLEMS

VERSION A

Name:

EID:

YOUR SIGNATURE:

(1) (20 points) You do not need to simplify your answers.

(a) In how many ways can 6 oaks, 4 pines, and 2 maples be arranged along a property line if one does not distinguish between trees of the same kind?

(b) How many different ways can 3 cats: Archie, Leo, and Stein, and 4 dogs: Bella, Stella, Amelia, and Lucy sit in a row if the cats and dogs must alternate?

(c) How many ways can I distribute 20 identical pennies to my 5 children?

(d) If you roll five 6-sided dice, how many ways can you roll 3 of the same and two of the same? (For example: 1,1,1,2,2)

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(2) (12 points) Suppose that X is a random variable having possible values 0 and 1. Suppose that $P(X = 0) = 4P(X = 1)$.

(a) Find $E(X)$

(b) What is $P(X = 1)$?

(c) Find $Var(X)$.

(3) (6 points) For a discrete random variable, X, show that $E[aX + b] = aE[X] + b$

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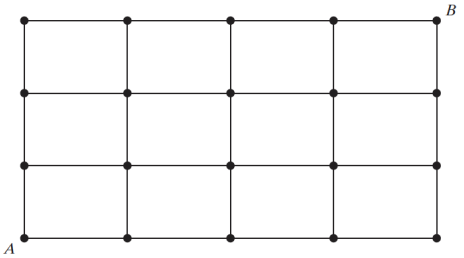
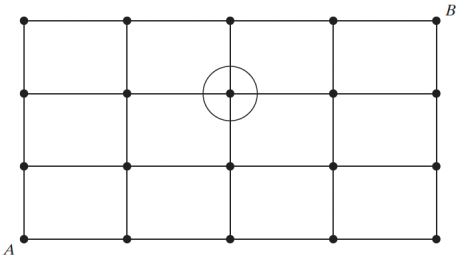
(4) (12 points)

(a) 

Consider the grid of points shown here. Suppose that, starting at the point labeled A, you can go one step up or one step to the right at each move. This procedure is continued until the point labeled B is reached. How many different paths from A to B are possible?

(b) 

How many different paths are there from A to B that go through the point circled in the following lattice?

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(5) (10 points) Parents like to hide presents. With probability 0.6, the present was hidden by mom; with probability 0.4, it was hidden by dad. When mom hides the present, she hides it upstairs 70 percent of the time and downstairs 30 percent of the time. Dad is equally likely to hide it upstairs or downstairs.

(a) What is the probability that the present is upstairs?

(b) Given that it is downstairs, what is the probability it was hidden by Dad?

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(6) (28 points) A pair of fair dice is tossed. Let X represent the number of dice showing an even number, and let Y represent the number of dice showing either a 1 or a 6.

$$X = \{0, 1, 2\} \quad Y = \{0, 1, 2\}$$

(a) (10 points) Complete a Joint Distribution table, including the marginals.

(b) (6 points) Find $E(X)$

(c) (6 points) Find $P(X = 2|Y = 2)$

(d) (6 points) Find $E[X|X \text{ is odd}]$.

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Multiple Choice. Circle the correct answer for the following multiple-choice questions. You do not need to justify your answer. (12 points)

(a) A multiple-choice exam has 12 questions each with 4 answer choices. Each question has only one correct answer. In how many ways can one choose an answer to each question, so that the student gets them all wrong?

(i) 2^{12}
(ii) 12^4
(iii) 4^{12}
(iv) 12^{12}

(b) Which of the following is false? (Circle all that apply)

(i) $19! = 19 \cdot 18 \cdot 17 \cdot 16!$
(ii) $\dfrac{12!}{3!} = 4!$
(iii) $3! * 0! = 0!$
(iv) $6! + 3! = 9!$
(v) $\dfrac{9!}{7!2!} = 36$

(c) If A and B are independent events, with $P(A) = \dfrac{1}{5}$, $P(B) = \dfrac{2}{5}$, then $P(A \cap B)$ is

(i) $\emptyset$
(ii) $\dfrac{3}{5}$
(iii) $\dfrac{2}{25}$
(iv) It cannot be determined from the given information.

(d) How many ways can I select 3 out of 10 people to join my committee?

(i) $10 * 9 * 8 * 7 * 6 * 5 * 4$
(ii) 10 P 3
(iii) 3!
(iv) 10 C 3