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Flywheel Simultaneous Attitude Control and Energy Storage Using a VSCMG Configuration

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Abstract

Space vehicle programs consistently endeavor to reduce satellite bus mass to increase payload capacity and/or reduce launch and fabrication costs. At the same time, performance demands on satellite systems continue to increase, creating a formidable challenge to space vehicle technology development. Flywheel-based systems providing both energy storage and attitude control functionality address both of these issues. In particular, the Flywheel Attitude Control, Energy Transmission and Storage (FACETS) system should combine all or part of the energy storage, attitude control, and power management and distribution (PMAD) subsystems into a single system, thus significantly decreasing bus mass (and volume). The control problem of mechanically-based simultaneous energy storage and attitude control is far from trivial, however, even in its simplest conceivable form. While decoupling the attitude control and energy storage may be a workable solution to the problem, research in related areas suggests it may not be the best approach. It has been shown that simultaneous momentum management and power tracking can be accomplished with four or more wheels in reaction wheel mode using the null subspace of the angular momentum dynamics of the wheels. In this way the energy storage or power tracking function does not induce attitude disturbance torques to the spacecraft. Furthermore, the null subspace was shown to be sufficient for tracking a variety of practical satellite power profiles. For some applications, however, reaction wheels produce insufficient control torque and control moment gyros (CMGs) are required. This paper extends the null subspace approach for simultaneous power tracking and attitude control, proven for flywheels in a reaction wheel mode, to an array of flywheels in a CMG configuration.

1 Introduction

Flywheel based simultaneous energy storage and attitude control holds much promise for reduced satellite bus mass and volume for future space vehicle systems. System level control for this combined functionality is far from trivial, however. For reaction wheels (RWs) which have a fixed spin axis and control momentum by changing flywheel spin rate, the attitude control and energy storage functions are completely coupled. Control moment gyros (CMGs), on the other hand, typically involve flywheels with fixed spin rates and momentum control is achieved by changing the spin axis.

Momentum control is performed using CMGs in cases where RWs cannot provide enough torque to meet mission slewing or agility requirements. CMGs can either be single or double gimbaled depending on the design application. A double gimbal CMG can arbitrarily reorient its spin axis while the single gimbal CMG spin axis is constrained to move in a plane, the gimbal axis being fixed in the spacecraft body. The single gimbal CMG does have a significant advantage over the double gimbal CMG in that the output torque exerted on the spacecraft by a single gimbaled CMG has a much higher magnitude than the gimbal motor input torque required to actuate it. This physical phenomenon is knows as Torque amplification.

Several recent results are applicable to the simultaneous energy storage and attitude control problem using RWs. Hall showed that for an underdetermined set of RWs (four or more wheels) the internal torque of the RW array can be decomposed into components corresponding to the row space and the null space of its projection onto the body frame [1]. In this way the energy storage function can be performed without exerting torque on the spacecraft, using the null space component of the internal torque. Shen and Tsiotras [2] show that practical power tracking can be achieved using Hall's result [1]. An example is given in [2] that clearly illustrates the feasibility of this approach.

While there have been no results applying CMG type devices to the problem of combined energy storage and attitude control, several results in singularity avoidance control for CMGs are applicable to the problem. Singularity avoidance entails manipulation of the CMG array to avoid configurations in which the CMGs cannot produce the desired torque (or any torque at all).

Oh and Vadali [3] develop a singularity robust (SR) gimbal acceleration steering law for singularity avoidance using fixed spin rate CMGs. Schaub, et al. [4, 5], develop singularity avoidance control laws for variable speed CMGs (VSCMGs) that allow CMG operation far away from singularity points, but drive the VSCMGs to behave more like RWs as a singularity is approached.

In this paper, the results of Oh and Vadali [3], are extended to VSCMGs, then power tracking control for integrated attitude control and energy storage is derived using this modified result. This development extends the approach of Hall [1] and Shen and Tsiotras [2] for RW arrays to VSCMGs. In particular, a null space decomposition of the internal torque is identified for VSCMGs and it is shown that this decomposition of the torque vector can be used for power tracking. We begin by defining useful mathematical preliminaries in Section 2, then present the spacecraft equations of motion in Section 3, next we derive the Lyapunov based stabilizing feedback steering law and the power tracking result for VSCMGs in Section 4 and finally, we summarize our work in Section 5 and discuss ongoing research.

2 Mathematical Preliminaries

Here we present definitions and mathematical relations that will be useful in subsequent sections. The derivative of a vector $\boldsymbol{x}$ with respect to an inertial frame of reference is defined by

$$\frac{\mathrm{d}^{\mathcal{N}}}{\mathrm{d}t}(\boldsymbol{x}) \equiv \dot{\boldsymbol{x}} \tag{1}$$

In this paper, we consider a system consisting of a rigid spacecraft with body fixed reference frame, $\mathcal{B}$, which includes an array of n rigid VSCMGs with reference frames fixed to each of the VSCMG gimbals, $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_n$. An illustration of one VSCMG is given in Figure 1 which is found in [4]. As pictured in Fig. 1, the frames attached to the n VSCMGs, $\mathcal{G}_j$, $j = 1, ..., n$, are characterized by the orthogonal set of unit vectors, $\hat{\boldsymbol{g}}_{sj} \in \mathbb{R}^3$, $\hat{\boldsymbol{g}}_{tj} \in \mathbb{R}^3$, and $\hat{\boldsymbol{g}}_{gj} \in \mathbb{R}^3$, $j = 1, ..., n$, where the subscripts s, t and g denote the spin, transverse and gimbal axes, respectively, satisfying the relation

$$\hat{\boldsymbol{g}}_g \times \hat{\boldsymbol{g}}_s = \hat{\boldsymbol{g}}_t \tag{2}$$

The matrix $\boldsymbol{L}_{\mathrm{BG}j} \in \mathbb{R}^{3\times 3}$ is a rotation matrix taking vectors from the reference frame $\mathcal{G}_j$ to frame $\mathcal{B}$ and conversely, $\boldsymbol{L}_{\mathrm{G}j\mathrm{B}} = \boldsymbol{L}_{\mathrm{BG}j}^{\mathrm{T}}$ is the rotation matrix from $\mathcal{B}$ to $\mathcal{G}_j$. The vector $\boldsymbol{\Omega} \in \mathbb{R}^n$ contains the wheel rotational speeds of the n VSCMGs, Ω_j, $j = 1, \ldots, n$, and $\boldsymbol{\gamma} \in \mathbb{R}^n$ is a vector containing the gimbal angles of the n VSCMGs, γ_j, $j = 1, \ldots, n$. The vectors $\dot{\boldsymbol{\Omega}} \in \mathbb{R}^n$, $\dot{\boldsymbol{\gamma}} \in \mathbb{R}^n$ and $\ddot{\boldsymbol{\gamma}} \in \mathbb{R}^n$ are defined similarly.

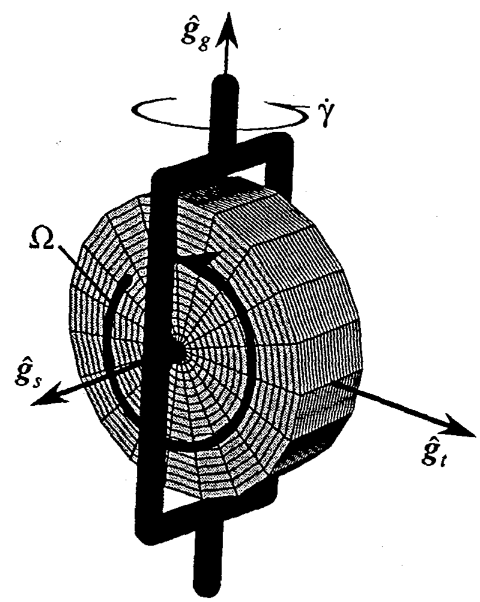

Figure 1: Variable Speed Control Moment Gyro

To simplify presentation of the results of this paper, we define the following matrices starting with $\boldsymbol{G}_s \in \mathbb{R}^{3\times n}$ whose columns are the unit vectors of the n VSCMGs in the spin axis direction, so that

$$\boldsymbol{G}_s = [\hat{\boldsymbol{g}}_{s1} \cdots \hat{\boldsymbol{g}}_{sn}] \tag{3}$$

and we define $\boldsymbol{G}_t$ and $\boldsymbol{G}_g$ similarly for the transverse and gimbal axis unit vectors, respectively. Next, we define similar matrices $\boldsymbol{G}_{tm} \in \mathbb{R}^{3\times 3n}$ and $\boldsymbol{G}_{sm} \in \mathbb{R}^{3\times 3n}$ such that

$$\boldsymbol{G}_{tm} = [\hat{\boldsymbol{g}}_{t1}\, \hat{\boldsymbol{g}}_{t1}\, \hat{\boldsymbol{g}}_{t1} \cdots \hat{\boldsymbol{g}}_{tn}\, \hat{\boldsymbol{g}}_{tn}\, \hat{\boldsymbol{g}}_{tn}] \tag{4}$$

and similarly for $\boldsymbol{G}_{sm}$. Now, we define $\boldsymbol{G}_{td} \in \mathbb{R}^{3n\times n}$ and $\boldsymbol{G}_{sd} \in \mathbb{R}^{3n\times n}$ such that

$$\boldsymbol{G}_{td} = \mathrm{diag}\{\hat{\boldsymbol{g}}_{t1}, \hat{\boldsymbol{g}}_{t2}, \cdots, \hat{\boldsymbol{g}}_{tn}\} \tag{5}$$

and similarly for $\boldsymbol{G}_{sd}$.

For convenience in representing vector cross products, we define the skew symmetric operator $\tilde{\boldsymbol{x}}$, $\boldsymbol{x} \in \mathbb{R}^3$, as

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \tag{6}$$

Also for convenience, we define the following summations

$$\begin{aligned} \boldsymbol{S}_1 &= \sum_{j=1}^{n} \boldsymbol{L}_{\mathrm{BG}j}(\boldsymbol{I}_{Wj} + \boldsymbol{I}_{Gj})\boldsymbol{L}_{\mathrm{G}j\mathrm{B}} \\ \boldsymbol{S}_2 &= \sum_{j=1}^{n} \boldsymbol{L}_{\mathrm{BG}j}(\boldsymbol{I}_{Wj} + \boldsymbol{I}_{Gj})\boldsymbol{\Gamma}_{skj} \end{aligned}$$

$$S_3 = \sum_{j=1}^{n} L_{\mathrm{BG}j} I_{Wj} (\Omega_{vj} + \Gamma_{vj})$$

$$S_4 = \sum_{j=1}^{n} L_{\mathrm{BG}j} I_{Gj} \Gamma_{vj}$$

$$S_5 = \sum_{j=1}^{n} L_{\mathrm{BG}j} \Gamma_{skj} (I_{Wj} + I_{Gj}) L_{Gj\mathrm{B}}$$

$$S_6 = \sum_{j=1}^{n} L_{\mathrm{BG}j} \Gamma_{skj} I_{Wj} (\Omega_{vj} + \Gamma_{vj})$$

$$S_7 = \sum_{j=1}^{n} L_{\mathrm{BG}j} I_{Wj} L_{Gj\mathrm{B}}$$

$$S_8 = \sum_{j=1}^{n} L_{\mathrm{BG}j} I_{Wj} \Gamma_{skj} L_{Gj\mathrm{B}}$$

$$S_9 = \sum_{j=1}^{n} (\Omega_{vj} + \Gamma_{vj})^{\mathrm{T}} I_{Wj} (\Omega_{vj} + \Gamma_{vj})$$

$$S_{10} = \sum_{j=1}^{n} L_{\mathrm{BG}j} I_{Wi} (\dot{\Omega}_{vj} + \dot{\Gamma}_{vj})$$

where

$$\Omega_{vj} = \begin{bmatrix} \Omega_j \\ 0 \\ 0 \end{bmatrix}, \quad \Gamma_{vj} = \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma}_j \end{bmatrix} \tag{7}$$

and $\Gamma_{skj} = \tilde{\Gamma}_{vj}$.

It will also be convenient to define several matrices involving the inertia properties of the VSCMG . The inertia values of the VSCMGs are decomposed into the contributions of the wheel and the gimbal structure using the scalar variables I_{Wsj}, I_{Wtj}, I_{Wnj}, I_{Gsj}, I_{Gtj} and I_{Ggj}, $j = 1, \ldots, n$, where the subscripts W and G denote the wheel and gimbal structure contributions, respectively. It should be noted that we assume the VSCMGs are perfectly balanced and aligned so that the unit vectors h_j, $j = 1, \ldots, n$, represent principle directions for the VSCMG frames. Now, we define matrices $I_{Gg} \in \mathbb{R}^{n\times n}$, $I_{Gs} \in \mathbb{R}^{n\times n}$, $I_{Wg} \in \mathbb{R}^{n\times n}$ and $I_{Ws} \in \mathbb{R}^{n\times n}$ such that

$$I_{Gg} = \mathrm{diag}\{I_{Gg1}, I_{Gg2}, \ldots, I_{Ggn}\} \tag{8}$$

and similarly for I_{Gs}, I_{Gt}, I_{Ws}, I_{Ws} and I_{Wt}. Next, we define matrices $I_{Gsm} \in \mathbb{R}^{3n\times 3n}$, $I_{Gtm} \in \mathbb{R}^{3n\times 3n}$, $I_{Wsm} \in \mathbb{R}^{3n\times 3n}$ and $I_{Wtm} \in \mathbb{R}^{3n\times 3n}$ such that

$$I_{Gsm} = \mathrm{blockdiag}\{I_{Gs1} \cdot \mathbf{I}_3, I_{Gs2} \cdot \mathbf{I}_3, \ldots, I_{Gsn} \cdot \mathbf{I}_3\} \tag{9}$$

and similarly for I_{Gtm}, I_{Wsm} and I_{Wtm}, where $\mathbf{I}_3$ is the 3×3 identity matrix. At times it is convenient to combine the inertia contributions of the wheel and gimbal frame, so we define $J_s \in \mathbb{R}^{n\times n}$, $J_t \in \mathbb{R}^{n\times n}$, $J_g \in \mathbb{R}^{n\times n}$, $J_{sm} \in \mathbb{R}^{3n\times 3n}$ and $J_{tm} \in \mathbb{R}^{3n\times 3n}$ such that

$$
\begin{aligned}
J_s &= I_{Gs} + I_{Ws} \\
J_t &= I_{Gt} + I_{Wt} \\
J_g &= I_{Gg} + I_{Wg} \\
J_{sm} &= I_{Gsm} + I_{Wsm} \\
J_{tm} &= I_{Gtm} + I_{Wtm}
\end{aligned}
$$

We also define matrices $\Omega_d \in \mathbb{R}^{n\times n}$ and $\omega_d \in \mathbb{R}^{3n\times n}$ such that

$$
\begin{aligned}
\Omega_d &= \mathrm{diag}\{\Omega_1, \Omega_2, \ldots, \Omega_n\} \\
\omega_d &= \mathrm{blockdiag}\{\omega, \omega, \ldots, \omega\}
\end{aligned}
$$

where the spacecraft body angular velocity vector ω is repeated n times in the definition of ω_d.

3 System Model

In this section we present the spacecraft system model extending the results of Oh and Vadali [3] to the case of variable speed CMGs (VSCMGs).

3.1 Dynamics

The equations of motion are derived using the well-known Euler equations [6]

$$\dot{H} = L \tag{10}$$

where H is the total angular momentum of the spacecraft and VSCMG array, given by

$$H = H_B + H_G + H_W \tag{11}$$

and L is the sum of all external torques on the spacecraft. From Eqs. (10) and (11), the system dynamic equations of motion may be expressed as

$$I_{\mathrm{T}}\dot{\omega} = -\tilde{\omega} I_{\mathrm{T}} \omega - B\ddot{\gamma} - B_3\dot{\Omega} - D\dot{\gamma} - B_4\Omega + \tau_e \tag{12}$$

where I_{sc} is the spacecraft inertia matrix, ω is the angular velocity vector of the spacecraft body in $\mathcal{N}$ expressed in $\mathcal{B}$, τ_e is the vector of external torques and I_{T} is the *total* inertia matrix given by

$$I_{\mathrm{T}} = I_{sc} + S_1 \tag{13}$$

The coefficient matrices of Eq. 12 are defined as

$$
\begin{aligned}
B &= B_1 + B_2 && (14) \\
D &= D_1 + D_2 + D_3 && (15)
\end{aligned}
$$

where

$$
\begin{aligned}
B_1 &= G_g I_{Gg} && (16) \\
B_2 &= G_g I_{Wg} && (17) \\
B_3 &= G_s I_{Ws} && (18) \\
B_4 &= \tilde{\omega} G_s I_{Ws} && (19) \\
D_1 &= G_t I_{Ws} \Omega_{\mathrm{d}} \\
&\quad + (G_t G_s I_{Ws} - G_{sm} G_t I_{Wtm}) \omega_d && (20) \\
D_2 &= \tilde{\omega} G_g J_g && (21) \\
D_3 &= [(G_t G_s I_{Gsm} - G_{sm} G_t I_{Gtm}) \\
&\quad -(G_{tm} G_s J_{tm} - G_{sm} G_t J_{sm})]\omega_d && (22)
\end{aligned}
$$

3.2 Kinematics

The spacecraft kinematics follow the development in [2], using the so-called 'Modified Rodrigues Parameters' (MRPs). The MRPs are defined in terms of the Euler principle vector, $\boldsymbol{e}$, and angle ϕ, by

$$\boldsymbol{\sigma} = \boldsymbol{e} \tan\left(\frac{\phi}{4}\right) \tag{23}$$

where the subscript B used in [2] to denote the body frame has been dropped. The MRPs may also be expressed in terms of the components of the quaternion vector $\boldsymbol{\beta}$ according to [4]

$$\sigma_i = \frac{\beta_i}{1+\beta_0}, \quad i = 1,2,3 \tag{24}$$

Using this formulation, the differential equation of the spacecraft kinematics is given by [2]

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{G}(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{25}$$

where

$$\boldsymbol{G}(\boldsymbol{\sigma}) = \frac{1}{2}\left(\mathbf{I}_3 + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\sigma}^{\mathrm{T}} - \frac{1+\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\sigma}}{2}\,\mathbf{I}_3\right) \tag{26}$$

Equations 12 and 26 together form the system equations of motion.

4 Simultaneous Energy Storage and Attitude Control

In this section we develop the control laws for simultaneous energy storage and attitude control using VSCMGs. We begin by developing a Lyapunov based steering control as developed in [3], then we formulate the power tracking control in a somewhat analagous manner to what was done for RWs in [1, 2]. In fact the power tracking result in terms of VSCMGs can be shown to specialize to the case of reaction wheels by setting $\dot{\boldsymbol{\Omega}} = \dot{\boldsymbol{\gamma}} = 0$.

4.1 Lyapunov Feedback Steering Law

As in [3], we define a positive definite Lyapunov function

$$\begin{aligned} V &= k(\boldsymbol{\sigma}-\boldsymbol{\sigma}_f)^{\mathrm{T}}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_f) \\ &\quad + \tfrac{1}{2}(\boldsymbol{\omega}-\boldsymbol{\omega}_f)^{\mathrm{T}}\boldsymbol{I}_{\mathrm{T}}(\boldsymbol{\omega}-\boldsymbol{\omega}_f) \end{aligned} \tag{27}$$

where $\boldsymbol{\sigma}_f$ and $\boldsymbol{\omega}_f$ are the desired final values. As shown in [3], the derivative of the Lyapunov function can be expressed as

$$\dot{V} = -(\boldsymbol{\omega}-\boldsymbol{\omega}_f)^{\mathrm{T}}\left[k\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{\sigma})\boldsymbol{\sigma}_f + \boldsymbol{I}_{\mathrm{T}}\dot{\boldsymbol{\omega}}_f - \boldsymbol{I}_{\mathrm{T}}\dot{\boldsymbol{\omega}}\right] \tag{28}$$

It is evident then that $\dot{V}$ can be made non-positive if we set

$$k\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{\sigma})\boldsymbol{\sigma}_f + \boldsymbol{I}_{\mathrm{T}}\dot{\boldsymbol{\omega}}_f - \boldsymbol{I}_{\mathrm{T}}\dot{\boldsymbol{\omega}} = K(\boldsymbol{\omega}-\boldsymbol{\omega}_d) \tag{29}$$

where K is a positive definite gain matrix. Combining Eqs. (12), (28) and (29) yields the stability condition

$$\begin{aligned} \boldsymbol{B}\ddot{\boldsymbol{\gamma}} + \boldsymbol{B}_3\dot{\boldsymbol{\Omega}} + \boldsymbol{B}_4\boldsymbol{\Omega} + \boldsymbol{D}\dot{\boldsymbol{\gamma}} &= K(\boldsymbol{\omega}-\boldsymbol{\omega}_f) \\ &\quad - k\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{\sigma})\boldsymbol{\sigma}_f - \boldsymbol{I}_{\mathrm{T}}\dot{\boldsymbol{\omega}}_f + \tilde{\boldsymbol{\omega}}\boldsymbol{I}_{\mathrm{T}}\boldsymbol{\omega} + \boldsymbol{\tau}_e \end{aligned} \tag{30}$$

Now, if we define the so-called torque required vector $\boldsymbol{\tau}_r \in \mathbb{R}^3$ as

$$\boldsymbol{\tau}_r \equiv K(\boldsymbol{\omega}-\boldsymbol{\omega}_f) - k\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{\sigma})\boldsymbol{\sigma}_f - \boldsymbol{I}_{\mathrm{T}}\dot{\boldsymbol{\omega}}_f + \tilde{\boldsymbol{\omega}}\boldsymbol{I}_{\mathrm{T}}\boldsymbol{\omega} + \boldsymbol{\tau}_e \tag{31}$$

then Eq. (30) can be written

$$\boldsymbol{B}\ddot{\boldsymbol{\gamma}} + \boldsymbol{B}_3\dot{\boldsymbol{\Omega}} + \boldsymbol{B}_4\boldsymbol{\Omega} + \boldsymbol{D}\dot{\boldsymbol{\gamma}} = \boldsymbol{\tau}_r \tag{32}$$

which expresses the torque required for stability in terms of the physical parameters (or states) of the system.

As shown by Oh and Vadali [3] if we choose

$$\ddot{\boldsymbol{\gamma}} = K_\delta\left[\boldsymbol{D}^{\mathrm{T}}(\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}})^{-1}\boldsymbol{\tau}_r - \dot{\boldsymbol{\gamma}}\right] \tag{33}$$

and desired gimbal accelerations are assumed small, we can keep $\dot{\boldsymbol{\gamma}}$ close to that required by $\boldsymbol{\tau}_r$ so that we take full advantage of the CMG torque amplification effect.

4.2 Power Tracking

The kinetic energy stored in the VSCMGs is given by

$$T_w = \tfrac{1}{2}\boldsymbol{\Omega}_c^{\mathrm{T}}\boldsymbol{I}_{Ws}\boldsymbol{\Omega}_c \tag{34}$$

where $\boldsymbol{\Omega}_c = \boldsymbol{\Omega} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{\omega}$ and $\boldsymbol{A} = [\boldsymbol{L}_{\mathrm{BG1}}\, \boldsymbol{L}_{\mathrm{BG2}} \cdots \boldsymbol{L}_{\mathrm{BG}n}]$. Then the power or rate of change of kinetic energy is

$$\frac{\mathrm{d}T_w}{\mathrm{d}t} = P_w = \boldsymbol{\Omega}_c^{\mathrm{T}}\boldsymbol{I}_{Ws}\dot{\boldsymbol{\Omega}}_c \tag{35}$$

Using Eqs. (32) and (35), and defining $\boldsymbol{\chi} \in \mathbb{R}^{2n}$ as $\boldsymbol{\chi} \equiv [\boldsymbol{\omega}^{\mathrm{T}}\ \dot{\boldsymbol{\gamma}}^{\mathrm{T}}]^{\mathrm{T}}$, with some algebraic manipulation we can write

$$\begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{bmatrix}\dot{\boldsymbol{\chi}} = \begin{bmatrix} \boldsymbol{F}_p \\ \boldsymbol{P}_f \end{bmatrix} \tag{36}$$

where

$$\begin{aligned} \boldsymbol{C}_{11} &= \boldsymbol{G}_s\boldsymbol{I}_{Ws} && (37)\\ \boldsymbol{C}_{12} &= \boldsymbol{G}_g\boldsymbol{J}_g && (38)\\ \boldsymbol{C}_{21} &= (\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{G}_s + \boldsymbol{\Omega})\boldsymbol{I}_{Ws} && (39)\\ \boldsymbol{C}_{22} &= (\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{G}_g + \dot{\boldsymbol{\gamma}})\boldsymbol{I}_{Wg} && (40)\\ F_p &= \boldsymbol{\tau}_r + \tilde{\boldsymbol{\omega}}\boldsymbol{I}_{\mathrm{T}}\boldsymbol{\omega} - \tilde{\boldsymbol{\omega}}\boldsymbol{I}_{sc}\boldsymbol{\omega} - \boldsymbol{I}_{sc}\dot{\boldsymbol{\omega}} + F && (41)\\ P_f &= P_r - \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{S}_7\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{S}_8\boldsymbol{\omega} \\ &\quad - \boldsymbol{S}_6^{\mathrm{T}}\boldsymbol{\omega} - \boldsymbol{S}_3^{\mathrm{T}}\dot{\boldsymbol{\omega}} && (42) \end{aligned}$$

and where

$$\begin{aligned} F &= -\boldsymbol{S}_5\boldsymbol{\omega} - \boldsymbol{S}_6 - \tilde{\boldsymbol{\omega}}(\boldsymbol{S}_3 + \boldsymbol{S}_4 + \boldsymbol{S}_1\boldsymbol{\omega}) \\ &\quad - \boldsymbol{S}_1\dot{\boldsymbol{\omega}} + \boldsymbol{S}_2\boldsymbol{\omega} && (43)\\ P_r &= \boldsymbol{S}_9 + \boldsymbol{\omega}^{\mathrm{T}}(\boldsymbol{S}_{10} - \boldsymbol{S}_8\boldsymbol{\omega} + \boldsymbol{S}_7\dot{\boldsymbol{\omega}}) \\ &\quad + \boldsymbol{S}_3^{\mathrm{T}}\dot{\boldsymbol{\omega}} + \boldsymbol{S}_6^{\mathrm{T}}\boldsymbol{\omega} && (44) \end{aligned}$$

Defining $\boldsymbol{C}_1 \equiv [\boldsymbol{C}_{11}\ \boldsymbol{C}_{12}]$ and $\boldsymbol{C}_2 \equiv [\boldsymbol{C}_{21}\ \boldsymbol{C}_{22}]$, the equation for $\boldsymbol{F}_p$ from (36) can be written

$$\boldsymbol{C}_1\dot{\boldsymbol{\chi}} = F_p \tag{45}$$

The general solution to Eq. (45) is given by

$$\dot{\boldsymbol{\chi}} = C_1^{\dagger}P_f + \dot{\boldsymbol{\chi}}_n \tag{46}$$

where the symbol $\dagger$ denotes the suitable generalized inverse, and $\boldsymbol{C}_1\dot{\boldsymbol{\chi}}_n = 0$ ($\dot{\boldsymbol{\chi}}$ is in the null space of $\boldsymbol{C}_1$, $N(\boldsymbol{C}_1)$). Now we can substitute Eq. (46) into the equation for $\boldsymbol{P}_f$ from (36) so that

$$\boldsymbol{C}_2\dot{\boldsymbol{\chi}} = \boldsymbol{C}_2(\boldsymbol{C}_1^{\dagger}\boldsymbol{F}_p + \dot{\boldsymbol{\chi}}_n) = \boldsymbol{P}_f \tag{47}$$

so

$$\boldsymbol{C}_2\dot{\boldsymbol{\chi}}_n = \boldsymbol{P}_m \tag{48}$$

where $\boldsymbol{P}_m = \boldsymbol{P}_f - \boldsymbol{C}_2\boldsymbol{C}_1^{\dagger}F_p$. Now since $\dot{\boldsymbol{\chi}}_n \in N(\boldsymbol{C}_1)$, we can find a vector $\boldsymbol{\nu}$ such that

$$\dot{\boldsymbol{\chi}}_n = \boldsymbol{P}_N\boldsymbol{\nu} \tag{49}$$

where $\boldsymbol{P}_N = \mathbf{I}_n - \boldsymbol{C}_1^{\mathrm{T}}(\boldsymbol{C}_1\boldsymbol{C}_1^{\mathrm{T}})^{-1}\boldsymbol{C}_1$ is an orthogonal projection onto $N(\boldsymbol{C}_1)$. Now, from Eqs. (48) and (49), and making use of the fact that $\boldsymbol{P}_N$ is a projection matrix, we see that we can choose $\dot{\boldsymbol{\chi}}_n$ such that

$$\dot{\boldsymbol{\chi}}_n = \boldsymbol{P}_N\boldsymbol{C}_2^{\mathrm{T}}(\boldsymbol{C}_2\boldsymbol{P}_N\boldsymbol{C}_2^{\mathrm{T}})^{-1}\boldsymbol{P}_m \tag{50}$$

which completes the solution $\dot{\boldsymbol{\chi}}$ of Eq. (46) for combined attitude stabilization and power tracking.

5 Conclusions

In this paper we have modified previous results for CMG singularity avoidance as well as making use of VSCMG control law development, to develop a methodology for combined attitude control and energy storage using VSCMGs. The main result is a general power tracking approach for VSCMG control making use of the null space of the control variable solution for required attitude torque. The methodology has yet to be verified in simulation, however it is based on a method developed for RW arrays (RWAs) that has been proven by simulation.

On-going work includes addressing the singularity avoidance issue, both in terms of attitude torque singularities as well as power tracking singularities. The solution presented in this paper will be modified to compensate for such singularities. The results presented in this paper as well as the modified result for singularity avoidance will then be verified by simulation. Also, the power tracking result presented here will be used to develop a voltage regulation control law for practical implementation in a spacecraft attitude control simulator. Finally, the culmination of this work will be a ground demonstration of flywheel hardware on a 3 degree-of-freedom spacecraft simulator to validate combined attitude control and energy storage functionality.

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