AS-DS Modeling w/ Linear Algebra

$$\begin{cases} \pi_t = 0 + \bar{v}\tilde{Y} + \bar{o} + \pi_{t-1} \\ \tilde{Y}_t = -\bar{b}\pi_t + 0 + \bar{a}_t + \bar{b}\bar{\pi} \end{cases}$$

- Write $X_t = \begin{bmatrix} \pi_t \\ Y_t \end{bmatrix}$ in terms of X_{t-1} and ε

$$\vec{x}_t = \begin{bmatrix}\pi_t \\ \tilde{y}_t\end{bmatrix} = \underbrace{\begin{array}{cc} \pi_t & \tilde{y}_t \\ \left[\begin{array}{c}0 \\ -\bar{b}\end{array}\right. & \left.\begin{array}{c}\bar{v} \\ 0\end{array}\right]\end{array}}_{\text{Some matrix } B} \underbrace{\begin{bmatrix}\pi_t \\ \tilde{y}_t\end{bmatrix}}_{\vec{x}_t} + \underbrace{\begin{bmatrix}\bar{o}_t \\ \bar{a}_t\end{bmatrix}}_{\text{Some shock, } \vec{\varepsilon}_t} + \underbrace{\begin{bmatrix}\pi_{t+1} \\ \bar{b}\bar{\pi}\end{bmatrix}}_{\text{using prev. term.}}$$

$z_t = \pi_{t-1}$

$\pi_0 = \bar{\pi}$

Write $\vec{x}_t$ as a function of π_{t-1}:

$$\vec{x}_t = B\vec{x}_t + \vec{\varepsilon}_t + \begin{bmatrix}\pi_{t-1} \\ \bar{b}\bar{\pi}\end{bmatrix}$$

$$\vec{x}_t - B\vec{x}_t = \vec{\varepsilon}_t + \begin{bmatrix}\pi_{t-1} \\ \bar{b}\bar{\pi}\end{bmatrix}$$

$$\vec{x}_t\left(I_2 - B\right) = \vec{\varepsilon}_t + \begin{bmatrix}\pi_{t-1} \\ \bar{b}\bar{\pi}\end{bmatrix}$$

$$\vec{x}_t = \left(I_2 - B\right)^{-1}\left(\vec{\varepsilon}_t + \begin{bmatrix}\pi_{t-1} \\ \bar{b}\bar{\pi}\end{bmatrix}\right)$$

$$\vec{X}_{t-1} = \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \vec{X}_{t-1} = \begin{bmatrix} \pi_{t-1} \\ 0 \end{bmatrix}$$

$$\vec{x} = (I_2 - B)^{-1} \left(\vec{\varepsilon} + \left(\begin{bmatrix} \pi_{t-1} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{b}\bar{\pi} \end{bmatrix} \right) \right)$$

$$\vec{x}_t = (I_2 - B)^{-1} \left(\vec{\varepsilon}_t + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \vec{x}_{t-1} + \begin{bmatrix} 0 \\ \tilde{b}\bar{\pi} \end{bmatrix} \right)$$

$$\vec{x}_t = \underbrace{(I_2 - B)^{-1}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\vec{x}_{t-1}}_{\text{dynamic term}} + \underbrace{(I_2 - B)^{-1}\left(\vec{\varepsilon}_t + \begin{bmatrix}0\\ \bar{b}\bar{\pi}\end{bmatrix}\right)}_{\text{some constant term (may change if } \varepsilon_t \text{ changes)}}$$

$$x_t = \begin{pmatrix}\pi_t\\ \tilde{y}_t\end{pmatrix}$$