1. (a)

$$\int_0^2 x^3 dx$$
$$= \frac{x^4}{4}\bigg|_0^2$$
$$= \frac{2^4}{4} - \frac{0^4}{4}$$
$$= 4$$

(b)

$$\int_{-1}^1 (-1 - 2y - 3y^2) dy$$
$$= -\int_{-1}^1 1 dy - 2\int_{-1}^1 y dy - 3\int_{-1}^1 y^2 dy$$
$$= -1y\bigg|_{-1}^1 - 2\frac{y^2}{2}\bigg|_{-1}^1 - 3\frac{y^3}{3}\bigg|_{-1}^1$$
$$= -[1 - (-1)] - 2\left(\frac{1^2}{2} - \frac{(-1)^2}{2}\right) - 3\left(\frac{1^3}{3} - \frac{(-1)^3}{3}\right)$$
$$= -2 - 0 + 2 = 0$$

(c)

$$
\begin{aligned}
&\int_0^{49} \sqrt{t} dt \\
&= \int_0^{49} t^{\frac{1}{2}} dt \\
&= \left.\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right|_0^{49} \\
&= \left.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^{49} \\
&= \left.\frac{2}{3} t^{\frac{3}{2}}\right|_0^{49} \\
&= \frac{2}{3}\left(49^{\frac{3}{2}} - 1\right) \\
&= \frac{2}{3}\left(7^3 - 1\right) \\
&= \frac{2 \times 342}{3} \\
&= \frac{686}{3}
\end{aligned}
$$

(d)

$$
\begin{aligned}
&\int_1^{3} \frac{2}{x^3} dx \\
&= 2\int_1^{3} x^{-3} dx \\
&= 2\left.\frac{x^{-2}}{-2}\right|_1^{3} \\
&= 2\left(\frac{3^{-2}}{-2} - \frac{1}{-2}\right) \\
&= -\frac{1}{9} + 1 \\
&= \frac{8}{9}
\end{aligned}
$$

(e)

$$
\begin{aligned}
&\int_0^{\frac{\pi}{4}} \sec^2(\theta) d\theta \\
&= \tan(\theta)\Big|_0^{\frac{\pi}{4}} \\
&= \tan\left(\frac{\pi}{4}\right) - \tan(0) \\
&= 1 - 0 = 1
\end{aligned}
$$

(f)

$$
\begin{aligned}
&\int_0^1 2^x dx \\
&= \frac{2^x}{\ln(2)}\Big|_0^1 \\
&= \left(\frac{2}{\ln(2)} - \frac{1}{\ln(2)}\right) \\
&= \frac{1}{\ln(2)}
\end{aligned}
$$

(g)

$$
\begin{aligned}
&\int \left(z^2 + 1 + \frac{1}{z^2+1}\right) dz \\
&= \int z^2 dz + \int 1 dz + \int \frac{1}{z^2+1} dz \\
&= \frac{z^3}{3} + z + \arctan(z) + C
\end{aligned}
$$

(h)

$$
\begin{aligned}
&\int (1 + \tan^2(s)) ds \\
&= \int \sec^2(s) ds \\
&= \tan(s) + C
\end{aligned}
$$

2. (a)

$$
\begin{aligned}
g(x) &= \int_1^x ln(t)dt \\
g'(x) &= f(u(x))u'(x) \\
g'(x) &= ln(x)
\end{aligned}
$$

(b)

$$
\begin{aligned}
F(x) &= \int_x^7 \sin(u^7)du \\
F(x) &= -\int_7^x \sin(u^7)du \\
F'(x) &= \sin(x^7)7x^6 \\
F'(x) &= 7x^6 \sin(x^7)
\end{aligned}
$$

(c)

$$
\begin{aligned}
h(t) &= \int_{\cos(t)}^1 \cos(x)dx \\
h(t) &= -\int_1^{\cos(t)} \cos(x)dx \\
h'(t) &= -\cos(\cos(t))(-\sin(x)) \\
h'(t) &= \cos(\cos(t))\sin(x)
\end{aligned}
$$

(d)

$$
\begin{aligned}
S(x) &= \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt \\
u &= \frac{\pi t^2}{2} \\
du &= \frac{\pi}{2}\frac{t^3}{3} dx \\
du &= \frac{\pi t^2}{2}\frac{t}{3} dx \\
du &= \frac{ut}{3} dt \\
dt &= \frac{du}{ut} \\
S(x) &= \int_0^x \frac{\sin(u)}{ut} du \\
S(x) &= \frac{1}{t}\int_0^x \frac{\sin(u)}{u} du
\end{aligned}
$$

3.

4.

5. (a)

$$
\begin{aligned}
&\int x\sin(x^2)dx \\
&u = x^2 \implies du = 2xdx \implies \frac{du}{2} = xdx \\
&\frac{1}{2}\int \sin(u)du \\
&\frac{1}{2}\cos(u) + C \\
&-\frac{1}{2}\cos(x^2) + C
\end{aligned}
$$

(b)

$$
\begin{aligned}
&\int u^2 e^{u^3} du \\
&x = u^3 \\
&dx = 3u^2 du \\
&\frac{dx}{3} = u^2 du \\
&\frac{1}{3}\int e^x dx \\
&\frac{1}{3}e^x + C \\
&\frac{1}{3}e^{u^3} + C
\end{aligned}
$$

6.

$$
\begin{aligned}
&\int_0^1 \left(3x^2 - g'(x)\right) dx \\
&= x^3\Big|_0^1 - \int_0^1 g'(x)dx \\
&= x^3\Big|_0^1 - g(x)\Big|_0^1 \\
&= 1 - (3-1) = 1 - 2 = -1
\end{aligned}
$$

7.

$$
\begin{aligned}
&\int_1^8 f(x)dx = 6 \\
&\int_1^2 x^2 f(x^3)dx \\
&u = x^3 \\
&du = 3x^2 dx \\
&\frac{1}{3}du = x^2 dx \\
&(1,2) \implies (1,8) \\
&\frac{1}{3}\int_1^8 f(u)du = \frac{6}{3} = 2
\end{aligned}
$$

8. (a)

$$\int \cos^3(x)dx$$

$$\int \left(1-\sin^2(x)\right)\cos(x)dx$$

$$u = \sin(x)$$

$$du = \cos(x)dx$$

$$\int (1-u^2)du$$

$$u - \frac{u^3}{3} + C$$

$$\sin(x) - \frac{\sin^3(x)}{3} + C$$

(b)

$$\int \tan^4(x)dx$$

$$\int \frac{\sin^4(x)}{\cos^4(x)}dx$$

$$\int \frac{(1-\cos^2(x))^2}{\cos^4(x)}dx$$

9. (a)

$$y^2 = 2x + 4$$
$$y = x + 2$$
$$(x+2)^2 = 2x + 4$$
$$x^2 + 4x + 4 = 2x + 4$$
$$x^2 + 2x = 0 \implies (x_1, x_2) = (0, -2)$$

$$V = \pi \int_{-2}^{0} [(x+2)^2 - 2x - 4] dx$$
$$V = \pi \int_{-2}^{0} (x^2 + 2x) dx$$
$$\frac{x^3}{3}\Big|_{-2}^{0} + x^2\Big|_{-2}^{0}$$
$$\frac{(-2)^3}{3} + (-2)^2$$
$$-\frac{8}{3} + 4$$
$$V = \frac{4}{3}\pi$$

(b)

$$x = \frac{\sqrt{2y}}{y^2 + 1}$$
$$x = 0$$
$$y = 1$$

$$\frac{\sqrt{2y}}{y^2+1} = 0$$
$$\sqrt{2y} = 0 \implies y = 0$$

10. (a)