1.

$$\lim_{n\to+\infty}\left[\frac{1}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\dots\sqrt{\frac{n}{n}}\right)\right]$$

2.

$$f(x)=\int_{\pi}^{x}\sin\left(\sqrt{s^2+4}\right)ds$$
$$u=\sqrt{s^2+4}$$
$$u^2=s^2+4$$
$$2udu=2sds$$
$$udu=sds$$
$$ds=\frac{u}{s}du$$
$$f(x)=\frac{1}{s}\int_{\sqrt{\pi^2+4}}^{\sqrt{x^2+4}}\frac{\sin(u)}{u}du$$

3. (a)

$$\frac{dy}{dx}$$
$$e^{2x+3y}=x^2-\ln(xy^3)$$
$$\frac{d}{dx}\left(e^{2x+3y}\right)=\frac{d}{dx}\left(x^2-\ln(xy^3)\right)$$
$$e^{2x+3y}\left(2+3\frac{dy}{dx}\right)=2x-\frac{1}{xy^3}\left(y^3+3xy^2\frac{dy}{dx}\right)$$
$$2e^{2x+3y}+3\left(e^{2x+3y}\right)\frac{dy}{dx}=2x-\frac{1}{x}+\frac{3}{y}\frac{dy}{dx}$$
$$3\left(e^{2x+3y}\right)\frac{dy}{dx}-\frac{3}{y}\frac{dy}{dx}=2x-\frac{1}{x}-2e^{2x+3y}$$
$$\frac{dy}{dx}\left(3(e^{2x+3y})-\frac{3}{y}\right)=2x-\frac{1}{x}-2e^{2x+3y}$$

4.

$$f(x) = x^2$$
$$g(x) = -x^2 + 8$$
$$x^2 = -x^2 + 8 \implies x = \pm 2 \implies -2 < x < 2$$
$$y = x^2 \implies x = \sqrt{y} = f(y)$$
$$y = -x^2 + 8 \implies x = \sqrt{8 - y} = g(y)$$
$$V = \pi \int_{-2}^{2} [f(y)^2 - g(y)^2] dy$$
$$V = \pi \int_{-2}^{2} (y - 8 + y) dy$$
$$V = \pi \int_{-2}^{2} (2y - 8) dy$$
$$V = y^2 \Big|_{-2}^{2} - 8y \Big|_{-2}^{2}$$
$$V = 2^2 - (-2)^2 - 8(2 - (-2))$$
$$V = -32\pi$$

$$V = \pi \int_{-2}^{2} (8 - y - y) dy$$
$$V = \pi \int_{-2}^{2} (8 - 2y) dy$$
$$V = 32\pi$$

5.

$$\int_{-1}^{1} \arctan(x)$$

$$\int udv = uv - \int vdu$$

$$u = \arctan(x)$$

$$du = \frac{1}{x^2+1}dx$$

$$dv = dx$$

$$v = x$$

$$x\arctan(x) - \int_{-1}^{1} x\frac{1}{x^2+1}dx$$

$$u = x^2 + 1$$

$$du = 2xdx$$

$$\frac{du}{2} = xdx$$

$$\int_{-1}^{1} \frac{1}{u}du = \ln|u| = \ln|1+x^2|$$

$$\int_{-1}^{1} x\frac{1}{x^2+1}dx = \frac{\ln|1+x^2|}{2}$$

$$\left(x\arctan(x) - \frac{\ln|1+x^2|}{2}\right)\Bigg|_{-1}^{1}$$

$$\arctan(-1) - \frac{\ln(2)}{2} - \arctan(1) + \frac{\ln(2)}{2}$$

$$\frac{\pi}{4} - \frac{\pi}{4}$$

$$\int_{-1}^{1} \arctan(x) = 0$$

6.

$$x(x^2-1)=0 \implies (x_1,x_2,x_3)=(-1,0,1)$$
$$A=\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}(x^3-x)dx$$
$$\int(x^3-x)dx=\frac{x^4}{4}-\frac{x^2}{2}$$

$$\left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Bigg|_{-1}^{0}$$
$$=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}$$

$$\left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Bigg|_{0}^{1}$$
$$0-\left(-\frac{1}{4}\right)=\frac{1}{4}$$
$$A=-\frac{1}{4}+\frac{1}{4}=0$$

$$V=\pi\int_{-1}^{0}(x^3-x)^2dx+\pi\int_{0}^{1}(x^3-x)^2dx$$

$$\int(x^3-x)^2dx$$
$$\int x^2(x^2-1)^2dx$$
$$\int x^2(x^4-2x^2+1)dx$$
$$\int(x^6-2x^4+x^2)dx$$
$$\frac{x^7}{7}-\frac{2x^5}{5}+\frac{x^3}{3}\Bigg|_{-1}^{0}$$
$$-\frac{1}{7}+\frac{2}{5}-\frac{1}{3}$$

$$\frac{x^7}{7}-\frac{2x^5}{5}+\frac{x^3}{3}\Bigg|_{0}^{1}$$
$$\frac{1}{7}-\frac{2}{5}+\frac{1}{3}$$

$$V=-\frac{\pi}{7}+\frac{2\pi}{5}+\frac{\pi}{3}+\frac{\pi}{7}-\frac{2\pi}{5}+\frac{\pi}{3}$$
$$V=0$$

7.

$$\frac{dC}{dt} = 5 + 10te^{-t^2}$$
$$dC = 5 + 10te^{-t^2}dt$$
$$C = \int 5 + 10te^{-t^2}dt$$
$$C = 5t + 10\int te^{-t^2}dt$$
$$u = -t^2$$
$$du = -2tdt \implies -\frac{1}{2}du = tdt$$
$$\int te^{-t^2}dt = -\frac{1}{2}\int e^u du$$
$$= -\frac{1}{2}e^u = -\frac{1}{2}e^{-t^2}$$
$$C = 5t + 10\left(-\frac{1}{2}e^{-t^2}\right)$$
$$C = 5t - 5e^{-t^2}$$