1.

$$A = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

$$B = \begin{bmatrix} cos(\omega) & -sin(\omega) \\ sin(\omega) & cos(\omega) \end{bmatrix}$$

(a)

$$AB = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} cos(\omega) & -sin(\omega) \\ sin(\omega) & cos(\omega) \end{bmatrix}$$

$$AB = \begin{bmatrix} cos(\theta)cos(\omega) - sin(\theta)sin(\omega) & -cos(\theta)sin(\omega) - sin(\theta)cos(\omega) \\ sin(\theta)cos(\omega) + cos(\theta)sin(\omega) & -sin(\theta)sin(\theta) + cos(\theta)cos(\omega) \end{bmatrix}$$

$$AB = \begin{bmatrix} cos(\theta+\omega) & -sin(\theta+\omega) \\ sin(\theta+\omega) & cos(\theta+\omega) \end{bmatrix}$$

(b)

$$AB = \begin{bmatrix} cos(\theta)cos(\omega) - sin(\theta)sin(\omega) & -cos(\theta)sin(\omega) - sin(\theta)cos(\omega) \\ sin(\theta)cos(\omega) + cos(\theta)sin(\omega) & -sin(\theta)sin(\theta) + cos(\theta)cos(\omega) \end{bmatrix}$$

$$BA = \begin{bmatrix} cos(\omega) & -sin(\omega) \\ sin(\omega) & cos(\omega) \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

$$BA = \begin{bmatrix} cos(\omega)cos(\theta) - sin(\omega)sin(\theta) & -cos(\omega)sin(\theta) - sin(\omega)cos(\theta) \\ sin(\omega)cos(\theta) + cos(\omega)sin(\theta) & -sin(\omega)sin(\theta) + cos(\omega)cos(\theta) \end{bmatrix}$$

$$BA = \begin{bmatrix} cos(\omega+\theta) & -sin(\omega+\theta) \\ sin(\omega+\theta) & cos(\omega+\theta) \end{bmatrix} = \begin{bmatrix} cos(\theta+\omega) & -sin(\theta+\omega) \\ sin(\theta+\omega) & cos(\theta+\omega) \end{bmatrix} = AB$$

(c) Sim. Pois uma matriz de rotação é inversível e seu valor é igual a da sua transposta

$$A^{-1} = A^T = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix}$$

2.

3.

4.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$B = \begin{bmatrix} x & y \\ w & z \end{bmatrix}$$

(a)

$$det(A^T) = det(A)$$
$$det\left(\begin{bmatrix} a & c \\ b & d \end{bmatrix}\right) = det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$
$$ad - bc = ad - bc \iff det(A^T) = det(A)$$

(b)

$$det(AB) = det(A)det(B)$$
$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x & y \\ w & z \end{bmatrix}\right)$$
$$det\left(\begin{bmatrix} ax + bw & ay + bz \\ cx + dw & cy + dz \end{bmatrix}\right)$$
$$det(AB) = (ax + bw)(cy + dz) - (ay + bz)(cx + dw)$$
$$det(AB) = axcy + bwcy - aycx - bzcx + axdz + bwdz - aydw - bzdw$$
$$det(AB) = bwcy - bzcx + axdz - aydw$$
$$det(A) = ad - bc$$
$$det(B) = xz - yw$$
$$det(A)det(B) = (ad - bc)(xz - yw)$$
$$det(A)det(B) = adxz - adyw - bcxz + bcyw = det(AB)$$

(c)

$$det(\lambda A) = \lambda^2 det(A)$$
$$det\left(\begin{bmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{bmatrix}\right)$$
$$\lambda^2 ad - \lambda^2 bc = \lambda^2(ad - bc) = \lambda^2 det(A)$$