1.

2.

$$det\left(\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} - \lambda I\right) = 0$$

$$det\left(\begin{bmatrix}-\lambda & 1\\ 1 & -\lambda\end{bmatrix}\right) = 0$$

$$\lambda^2 - 1 = 0$$

$$(\lambda_1, \lambda_2) = (1, -1)$$

Para $\lambda_1 = 1$

$$\left(\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} - 1I\right) x_1 = 0$$

$$\left(\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\right) x_1 = 0$$

$$\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$

$$\begin{bmatrix}-a+b\\ a-b\end{bmatrix} = 0$$

$$\begin{cases}-a+b=0 \implies a=b\\ a-b=0 \implies a=b\end{cases}$$

$$x_1 = k\begin{bmatrix}1\\ 1\end{bmatrix}$$

Para $\lambda_2 = -1$

$$\left(\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} + 1I\right) x_2 = 0$$

$$\left(\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right) x_2 = 0$$

$$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}c\\ d\end{bmatrix} = 0$$

$$\begin{bmatrix}c+d\\ c+d\end{bmatrix} = 0$$

$$\begin{cases}c+d=0 \implies c=-d\\ c+d=0 \implies c=-d\end{cases}$$

$$x_2 = k\begin{bmatrix}-1\\ 1\end{bmatrix}$$

3.

$$det\left(\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} - \lambda I\right) = 0$$
$$det\left(\begin{bmatrix}-1-\lambda & 0\\ 0 & 1-\lambda\end{bmatrix}\right) = 0$$
$$(-1-\lambda)(1-\lambda) = 0$$
$$(\lambda_1, \lambda_2) = (1, -1)$$

Para $\lambda_1 = 1$

$$\begin{bmatrix}-1-1 & 0\\ 0 & 1-1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$\begin{bmatrix}-2 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$\begin{bmatrix}-2a+0b\\ 0\end{bmatrix} = 0$$
$$\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}0\\ k\end{bmatrix}$$

Para $\lambda_2 = -1$

$$\begin{bmatrix}-1+1 & 0\\ 0 & 1+1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$\begin{bmatrix}0 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$\begin{bmatrix}0\\ 2b\end{bmatrix} = 0$$
$$\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}k\\ 0\end{bmatrix}$$

4.

$$det\left(\begin{bmatrix}-2 & 2\\ 0 & 2\end{bmatrix} - \lambda I\right) = 0$$
$$det\left(\begin{bmatrix}-2-\lambda & 2\\ 0 & 2-\lambda\end{bmatrix}\right) = 0$$
$$(\lambda_1, \lambda_2) = (2, -2)$$

Para $\lambda_1 = 2$
$$\begin{bmatrix}-2-2 & 2\\ 0 & 2-2\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$\begin{bmatrix}-4 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$-4a + 2b = 0 \implies 2a = b$$
$$k\begin{bmatrix}1\\ 2\end{bmatrix}$$

Para $\lambda_2 = -2$
$$\begin{bmatrix}-2+2 & 2\\ 0 & 2+2\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$\begin{bmatrix}0 & 2\\ 0 & 4\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = 0$$
$$\begin{bmatrix}2a\\ 4b\end{bmatrix} = 0$$
$$a = b = 0$$

5.

$$det(AA^{-1}) = 0$$
$$det(I) = 0$$