$$\int \sin(ax^2 + bx + c)dx$$
$$\int \sin((ax^2) + (bx + c))dx$$
$$\int \sin(ax^2)\cos(bx + c) + \cos(ax^2)\sin(bx + c)dx$$
$$\int \sin(ax^2)\cos(bx + c) + \int \cos(ax^2)\sin(bx + c)dx$$
$$\cos(bx + c) = \cos(bx)\cos(c) - \sin(bx)\sin(c)$$
$$\sin(bx + c) = \sin(bx)\cos(c) + \cos(bx)\sin(c)$$

$$\int \sin(ax^2)\cos(bx + c)$$
$$\int \sin(ax^2)(\cos(bx)\cos(c) - \sin(bx)\sin(c))$$
$$\int \sin(ax^2)\cos(bx)\cos(c) - \sin(ax^2)\sin(bx)\sin(c)$$
$$\int \sin(ax^2)\cos(bx)\cos(c) - \int \sin(ax^2)\sin(bx)\sin(c)$$

$$\int \cos(ax^2)\sin(bx + c)$$
$$\int \cos(ax^2)(\sin(bx)\cos(c) + \cos(bx)\sin(c))$$
$$\int \cos(ax^2)\sin(bx)\cos(c) + \cos(ax^2)\cos(bx)\sin(c)$$
$$\int \cos(ax^2)\sin(bx)\cos(c) + \int \cos(ax^2)\cos(bx)\sin(c)$$
$$\int \sin(ax^2)\cos(bx)\cos(c) - \int \sin(ax^2)\sin(bx)\sin(c)$$

$$I_1 = \int \cos(ax^2)\sin(bx)\cos(c)dx$$
$$I_2 = \int \cos(ax^2)\cos(bx)\sin(c)dx$$
$$I_3 = \int \sin(ax^2)\cos(bx)\cos(c)dx$$
$$I_4 = \int \sin(ax^2)\sin(bx)\sin(c)dx$$
$$I_1 = \cos(c)\int \cos(ax^2)\sin(bx)dx$$
$$I_2 = \sin(c)\int \cos(ax^2)\cos(bx)dx$$
$$I_3 = \cos(c)\int \sin(ax^2)\cos(bx)dx$$
$$I_4 = \sin(c)\int \sin(ax^2)\sin(bx)dx$$
$$\int \sin(ax^2 + bx + c)dx = I_1 + I_2 + I_3 - I_4$$

1 $\int I_1$

$$I_1 = \cos(c) \int \cos(ax^2)\sin(bx)dx$$
$$u = bx$$
$$du = bdx$$
$$\frac{1}{b}du = dx$$
$$\frac{1}{b}u = x$$
$$I_1 = \cos(c)\frac{1}{b}\int \cos\left(\frac{a}{b^2}u^2\right)\sin(u)du$$
$$w = \cos\left(\frac{a}{b^2}u^2\right)$$
$$dw = -\sin\left(\frac{a}{b^2}u^2\right)\left(\frac{2a}{b^2}u\right)du$$
$$dv = \sin(u)du$$
$$v = -\cos(u)$$
$$-\cos(u)\cos\left(\frac{a}{b^2}u\right) - \int \cos(u)\sin\left(\frac{a}{b^2}u\right)\left(\frac{2a}{b^2}u\right)du$$
$$-\cos(u)\cos\left(\frac{a}{b^2}u\right) - \frac{2a}{b^2}\int \cos(u)\sin\left(\frac{a}{b^2}u^2\right)udu$$
$$k = \frac{a}{b^2}u^2$$
$$dk = \frac{2a}{b^2}udu$$
$$\frac{b^2}{2a}dk = udu$$
$$\sqrt{\frac{b^2}{2a}k} = u$$

1.1 $\int V_1$

$$V_1 = \int \frac{\sin(ax^2 + bx)}{2} dx$$
$$V_1 = \frac{1}{2} \int \sin(ax^2 + bx) dx$$
$$\sin(ax^2 + bx) = \sin(ax^2)\cos(bx) + \cos(ax^2)\sin(bx)$$
$$V_1 = \frac{1}{2} \int \sin(ax^2)\cos(bx) + \cos(ax^2)\sin(bx) dx$$
$$V_1 = \frac{1}{2} \int \sin(ax^2)\cos(bx) dx + \int \cos(ax^2)\sin(bx) dx$$
$$V_1 = \frac{1}{2} \left(\frac{I_3}{\cos(c)} + \frac{I_1}{\cos(c)} \right)$$
$$V_1 = \frac{I_3 + I_1}{2\cos(c)}$$

1.2 $\int V_2$

$$V_2 = \int \frac{\sin(ax^2 - bx)}{2} dx$$
$$V_2 = \frac{1}{2} \int \sin(ax^2 - bx) dx$$
$$\sin(ax^2 - bx) = \sin(ax^2)\cos(bx) - \cos(ax^2)\sin(bx)$$
$$V_2 = \frac{1}{2} \int \sin(ax^2)\cos(bx) - \cos(ax^2)\sin(bx) dx$$
$$V_2 = \frac{1}{2} \left(\int \sin(ax^2)\cos(bx) dx - \int \cos(ax^2)\sin(bx) dx \right)$$
$$V_2 = \frac{1}{2} \left(\frac{I_3}{\cos(c)} - \frac{I_1}{\cos(c)} \right)$$
$$V_2 = \frac{I_3 - I_1}{2\cos(c)}$$

$$I_1 = \cos(c)(V_1 + V_2)$$
$$I_1 = \cos(c) \left(\frac{I_3 + I_1}{2\cos(c)} + \frac{I_3 - I_1}{2\cos(c)} \right)$$
$$I_1 = I_3$$

2 $\int I_2$

$$I_2 = \sin(c) \int \cos(ax^2)\cos(bx)dx$$

$$\cos(ax^2)\cos(bx) = \frac{\cos(ax^2+bx)}{2} + \frac{\cos(ax^2-bx)}{2}$$

$$I_2 = \sin(c) \int \frac{\cos(ax^2+bx)}{2} + \frac{\cos(ax^2-bx)}{2} dx$$

$$V_1 = \int \frac{\cos(ax^2+bx)}{2} dx$$

$$V_2 = \int \frac{\cos(ax^2-bx)}{2} dx$$

2.1 $\int V_1$

$$V_1 = \int \frac{\cos(ax^2+bx)}{2} dx$$

$$V_1 = \frac{1}{2} \int \cos(ax^2+bx)dx$$

$$\cos(ax^2+bx) = \cos(ax^2)\cos(bx) - \sin(ax^2)\sin(bx)$$

$$V_1 = \frac{1}{2} \int \cos(ax^2)\cos(bx) - \sin(ax^2)\sin(bx)dx$$

$$V_1 = \frac{1}{2} \left(\int \cos(ax^2)\cos(bx)dx - \int \sin(ax^2)\sin(bx)dx \right)$$

$$V_1 = \frac{1}{2} \left(\frac{I_2}{\sin(c)} - \frac{I_4}{\sin(c)} \right)$$

$$V_1 = \frac{I_2 - I_4}{2\sin(c)}$$

2.2 $\int V_2$

$$V_2 = \int \frac{\cos(ax^2 - bx)}{2} dx$$
$$V_2 = \frac{1}{2} \int \cos(ax^2 - bx) dx$$
$$\cos(ax^2 - bx) = \cos(ax^2)\cos(bx) + \sin(ax^2)\sin(bx)$$
$$V_2 = \frac{1}{2} \int \cos(ax^2)\cos(bx) + \sin(ax^2)\sin(bx) dx$$

$$I_2 = \sin(c)(V_1 + V_2)$$
$$I_2 = \sin(c)\left(\frac{I_2 - I_4}{2\sin(c)} + \frac{I_2 + I_4}{2\sin(c)}\right)$$

3 $\int I_3$

4 $\int I_4$