Numerical methods II: exercises

February 16, 2022

Exercises

Lecture 1 and 2

Exercise 1: volume of a D-sphere . Compute the volume of a sphere of radius 1 in a) D=3 and b) D=10. For $D = 3$, use the following probability distribution:

$$\pi(x_1, x_2, x_3) = 1 \quad \text{if} \quad |x_1|, |x_2|, |x_3| \leq 1 \tag{1}$$

and the following function to sample:

$$f(x_1, x_2, x_3) = 1 \quad \text{if} \quad \sum_{j=1}^{3} |x_j|^2 \leq 1 \tag{2}$$

For $D = 10$, use the following probability distribution:

$$\pi(x_1, x_2, ..., x_{10}) = 1 \quad \text{if} \quad |x_1|, |x_2|, |x_3|, ...|x_{10}| \leq 1 \tag{3}$$

and the following function to sample:

$$f(x_1, x_2, ..., x_{10}) = 1 \quad \text{if} \quad \sum_{j=1}^{10} |x_j|^2 \leq 1 \tag{4}$$

Solution: for $D = 3$, we know that the volume is $4\pi/3$, while for $D = 10$, $V = \frac{\pi^5}{120}$.

Exercise 2: error estimate as a function of sampled points . Using the same code for the evaluation of the 3D sphere, calculate, at fixed sampling of points $N \in [1000, 4000]$ the error in the evaluation of the integral, and show that it scales as $O(1/\sqrt{N})$ with a plot.

Extra to Ex. 2. Make the code general so that it can compute the volume of a d-dimensional sphere, and its error. Write a function that, assuming the error scaling is $O(N^{\alpha})$, extracts α. Perform an analysis of the extracted α for at least 3 values of N for the case of a $3D$ and $10D$ sphere, and comment on it.
You can show the scaling either as a guide to the eye (by including curves with different slopes), or with a fit.