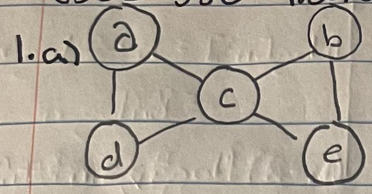
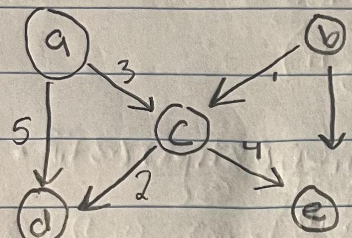
CSCE 350 : Data structure & Algorithm Caelen W.

1.a) 

$$\begin{array}{c|ccccc|l} & a & b & c & d & e & \\ \hline a & 0 & 0 & 1 & 1 & 0 & a \to c \to d \\ b & 0 & 0 & 1 & 0 & 1 & b \to c \to e \\ c & 1 & 1 & 0 & 1 & 1 & c \to a \to b \to d \to e \\ d & 1 & 0 & 1 & 0 & 0 & d \to a \to c \\ e & 0 & 1 & 1 & 0 & 0 & e \to b \to c \end{array}$$



$$\begin{array}{c|ccccc|l} & a & b & c & d & e & \\ \hline a & \infty & \infty & 3 & 5 & \infty & a \to (c,3) \to (d,5) \\ b & \infty & \infty & 1 & \infty & 7 & b \to (c,1) \to (e,7) \\ c & \infty & \infty & \infty & 2 & 4 & c \to (d,2) \to (e,4) \\ d & \infty & \infty & \infty & \infty & \infty & d \\ e & \infty & \infty & \infty & \infty & \infty & e \end{array}$$

b) Figure-1 : yes (a-c-d-a), (e-c-b-e), (a-c-b-e-c-d-

Figure-2: No

2. $(n^2+n)^{10}$ $(n^2)^{20} = n^{20}$

$$\frac{f(n)}{g(n)} = \lim_{n \to \infty} \left(\frac{(n^2+n)}{n^{20}}\right)^{20}$$

$$= \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{10} = 1$$

$$= 1$$

$$= \boxed{(n^2+n)^{10} \in \Theta(n^{20})}$$

2. $n \log[n^2] + (n-2)^2 \log \frac{n}{2}$

$n\log[n^2] = n \cdot 2\log n$

$= 2n\log n$

$(n-2)^2 \log n/_2 = n^2 - 4n + 4$

$\lim_{n\to\infty} \frac{t(n)}{g(n)} = \lim_{n\to\infty} n\log[n^2] + (n-2)^2 \log \frac{n}{2}$

$= \lim_{n\to\infty} 2n\log n + n^2 - 4n + 4$

$= \lim_{n\to\infty} 2n\log n + n^2\log n + 4\log n - (n^2 - 4n + 4)\log 2$

$= \lim_{n\to\infty} n^2\log n - 2n\log n + 4\log n - (n^2 \log 2 - 4n\log 2 + 4\log 2)$

$= n^2 \log n - n^2 \log 2 + 2n\log 2 + 4\log n - 4\log 2$

$n\log[n^2] + (n-2)^2 \log n/_2 \in \Theta(n^2 \log n)$

3. $\sum_{i=0}^{n-1} (i^2 - 2)^2 = i^4 - 4i^2 + 4$ $\sum_{i=0}^{n-1} i^4 = \frac{n(n-1)(2n-1)(3n^2+3n-1)}{30}$

$= \frac{n(n-1)(2n-1)(3n^2+3n-1)}{30} - 4\left(\frac{n(n-1)(2n-1)}{6}\right) + 4n$

$= n(n-1)\left(\frac{2n-1}{30}\right) - (2n(n-1)\left(\frac{2n-1}{30}\right) + (4n \times 30/30)$

$= n(n-1)\left(\frac{2n-1}{30}\right) - 2n(n-1)\left(\frac{2n-1}{30}\right) + 4n$

$= g(n) \in O(n^3$

$\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j) = \sum_{i=0}^{n-1} n\left(\frac{n-1}{2}\right) = 1/2 \sum_{i=0} (n(n-1)) = (1/2)(n-n-1)\left(\frac{n}{3}-2\right))$

$= n(n-1)\frac{n-2}{6}) = n^3$

$\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j) \in O(n^3)$

4. -The algorithm will compute the sum of cube integers starting from 1 and goes to 'n'.

b- basic operation: Addition

c- the basic operation will execute for n-times

d- Big-Oh Notation / O: As 'n' grows the amount of operation grows with 'n'.