# Nakayama's Lemma

math center

This note tries to cover some basic commutative algebra. So *all rings are commutative.*

Indeed, Nakayama's lemma is used to reduce finite modules to linear algebra. Let's start with the linear algebra.

**Theorem 0.1** (Hamilton-Cayley)**.** Let $R$ be a ring and let $A \in \mathrm{Mat}_n(R)$. Then $\chi_A(A) = 0$.

*Proof.* There are of course a ton of ways to prove this. The most standard proof is

$$\chi_A(t) := \det(t - A) \implies \chi_A(A) = \det(0) = 0.$$

In more details, let $Y = R[t]$ be a formal polynomial ring, and $V := R^{\oplus n}$ has a $Y$-module structure given by $t \rightsquigarrow A$. Consider the base change $V_Y = Y^{\oplus n}$ of $V$ and $t - A \in \mathrm{End}(V_Y)$. $\square$

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## The End

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