Compsci 166 Homework 3

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Question 1.

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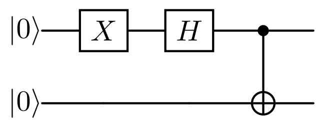
1) Design a two quibit circuit that starts in the state $|00\rangle$ and ends with $|\Psi^-\rangle$.



2) Write down $|0\rangle$ and $|1\rangle$ as a weighted sum of $|\pi/6\rangle$ and $|4\pi/6\rangle$ in the above basis.

$$|0\rangle = \frac{\sqrt{3}}{2}|\pi/6\rangle - \frac{1}{2}|4\pi/6\rangle$$
$$|1\rangle = \frac{1}{2}|\pi/6\rangle + \frac{\sqrt{3}}{2}|4\pi/6\rangle$$

3) If Alice measures her quibit in the standard basis, what are the probabilities of each outcome, and the state of the two quibits after the measurement?

$$|0\rangle : 1/2 \text{ Probability and state collapses to } |01\rangle$$
$$|1\rangle : 1/2 \text{ Probability and state collapses to } |10\rangle$$

4) If Alice instead chooses to measure in the $\{|\pi/6\rangle, |4\pi/6\rangle\}$ basis, what are the porbailities of each outcome, and the state of the two quibits after the measurement?

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}((\frac{\sqrt{3}}{2}|\pi/6\rangle - \frac{1}{2}|4\pi/6\rangle) \otimes |1\rangle - (\frac{1}{2}|\pi/6\rangle + \frac{\sqrt{3}}{2}|4\pi/6\rangle) \otimes |0\rangle)$$

Alice measures:

$$|\pi/6\rangle \text{ w/ probability } 1/2\text{, collapses to } \tfrac{\sqrt{3}}{2}|\pi/6\rangle|1\rangle - \tfrac{1}{2}|\pi/6\rangle|0\rangle$$
$$|4\pi/6\rangle \text{ w/ probability } 1/2\text{, collapses to } \tfrac{-1}{2}|4\pi/6\rangle|1\rangle - \tfrac{\sqrt{3}}{2}|4\pi/6\rangle|0\rangle$$

5) Verbally describe what happens to the second quibit when the first quibit of a $|\Psi\rangle$ state gets measures.

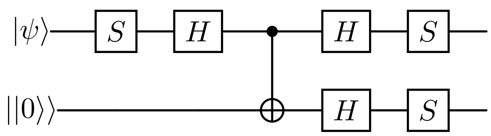
Answer:
The second quibit collapses into a state of $R_{-\theta}|0\rangle$ for $\theta =$ the state of the first quibit.

Question 2.

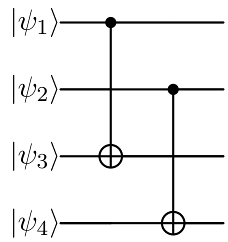
1) Determine whether a state $|\psi\rangle$ that we know to be one of the two states is clonable or not. Briefly justify your answer.

Answer:
Yes. Since $|i\rangle$ and $|-i\rangle$ form an orthonormal basis, there must exist a U that can clone a quibit that we know to be in one of the two above states.

2) Design a quantum circuit that clones a state if we know it is in the above basis.



3) Design a 4 quibit quantum circuit that clones 2 quibit standard basis states.



4) Design a 4 quibit quantum circuit that clones the Bell basis states.