Compsci 166 Homework 4

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Question 1.

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1) What is your guess on the winning probability for this strategy?

Answer:
The strategy seems to result in a winning probability of 1/2.

2) Conisder the case where $x = 0$ and $y = 1$. Let's suppose that Bob measures first for our analysis. What are the possible states after Bob's measurement?

Answer:
If both players receive a 1, Bob measures $|0\rangle$ with probability 1/2 and $|1\rangle$ with probability 1/2.

3) For each case, Alice measures the same quibit as Bob with probability 1.

4) The probability that Alice and Bob win using this strategy is 1.

5) Bob measures $|+\rangle$ and $|-\rangle$ with probability 1/2 for each case.

6) For each case, Alice measures $|0\rangle$ and $|1\rangle$ with probability 1/2.

7) The probability that Alice and Bob win using this strategy is 1/2 as they need to output the same bit.

8) If all four input pairs occur uniformly, the probability that Alice and Bob win the game is 1/2.

Question 2.

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1) A classical strategy that allows for a win rate of 3/4 would be to fix one individual to output a 1 and the others to output 0. This results in an odd number of 1's being output which leads to them winning 3/4ths of the time.

2) If all individuals are given 1, after the first individual measures their quibit, it collapses to the following positions.

$$|0\rangle : \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|1\rangle : \frac{1}{\sqrt{2}}(-|01\rangle - |10\rangle)$$

In both of these cases, the total number of 1's is even, meaning that they win with probability 1.

3) Consider the case where Alice gets a 1. The quibit collapses to the following:

$$
\begin{aligned}
|0\rangle :& \frac{1}{\sqrt{2}}(|{+}{+}\rangle - |{-}{-}\rangle) \\
& \frac{1}{\sqrt{2}}(\frac{1}{2}(|00\rangle + |10\rangle + |01\rangle + |11\rangle) - \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)) \\
& \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle) \\
|1\rangle :& \frac{1}{\sqrt{2}}(|{+}{-}\rangle + |{-}{+}\rangle) \\
& \frac{1}{\sqrt{2}}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) + \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle + |01\rangle - |11\rangle) \\
& \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)
\end{aligned}
$$

For both cases, since the total number of 1's is odd, they win with probability 1.

Question 3.

1) A great strategy would be to give both bits to the frogs and output whatever they tell you to.

2) $X \cdot Y = 3 \bmod 2 = 1$

3) If both players send the other their n bits, they can each calculate $X \cdot Y$ locally and output the answer.

4) A strategy where both players send one bit to the other and can deduce the correct inner product is as follows. They send the other individual a 1 if their frog output an odd amount of 1's, else they send a 0. Now, if there are an even amount of 1's, they know that the inner product is 0. Else, they know that there are an odd number.

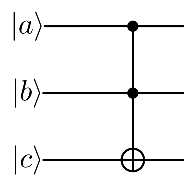
6) A strategy do derive the result of a function is similar. The cardinality of the number of 1's and 0's can be used to determine the cardinality of the result. Therefore, the mod 2 of that result can be deduced.

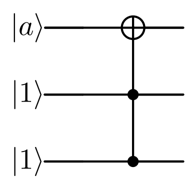
Question 4.

1)

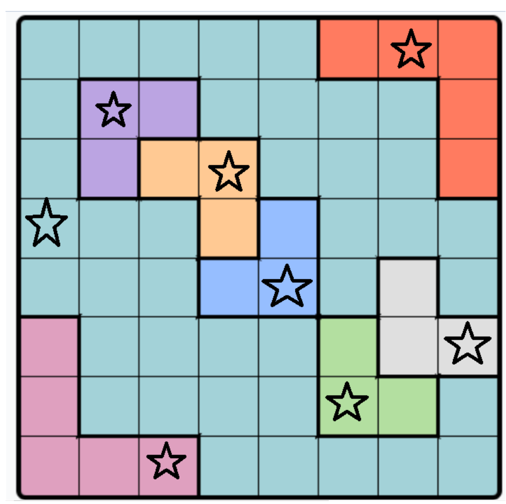
Input	Output
000	000
001	001
010	010
011	011
100	100
101	101
110	111
111	110

2)





3)



4) Queens is in NP since it can be checked in polynomial time by making sure all rows/columns have 1 queen.

5) Collatz is in NP as we assume that any sequence either repeats or reaches 1 in polynomial time. Therefore, running the algorithm will give the answer to whether or not it it or is not in the language.