# Expanding on the Nyquist Sampling Theorem for DSP

# Sampling

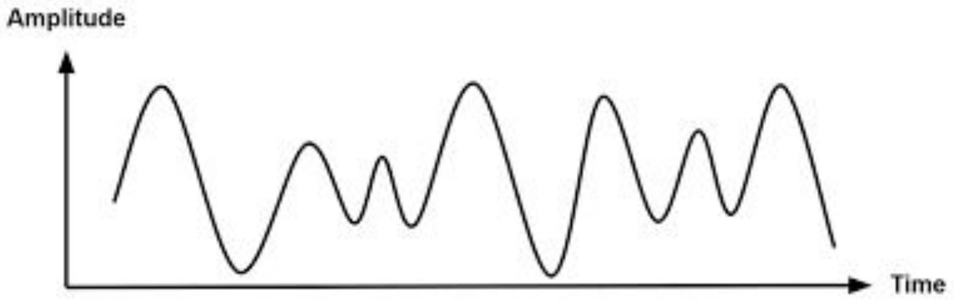
In the real world, signals are analog meaning that they are continuous and can take on any value. Computers operate in the digital world and can only take on signals with discrete values.

This leads us to the main issue we face when sampling signals from the real world with computers. How can our computers accurately process our signals if they can’t take on all of their values?

The Nyquist Sampling Theorem is a solution posed by DSP Theory to solve this issue.

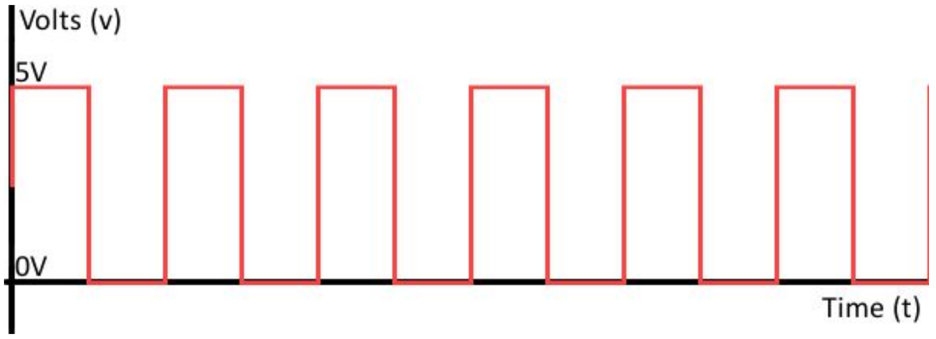
# Analog vs. Digital Signals - A Closer Look

## Analog Signal




Can take on any value, leading to it's continuous nature

## Digital Signal




Can only take on two discrete values (zero and five, which are common in digital electronics applications).

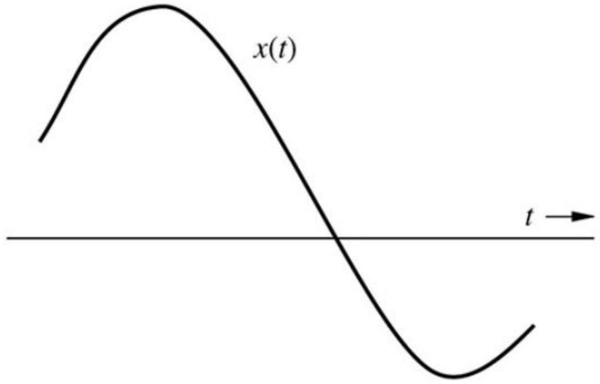
# The Nyquist Sampling Theorem

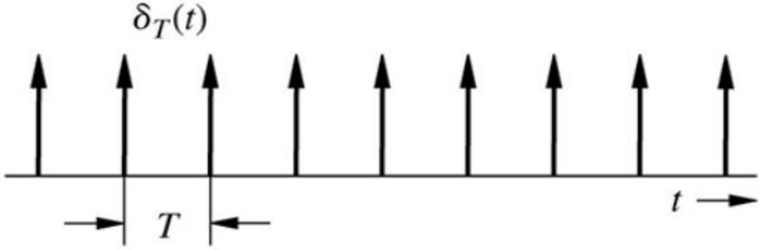
It's not obvious how we can sample an analog signal in the digital world without losing any information. Many would think that you would have to have a sampling rate (the rate at which you take in the information) of infinity in order to capture all of the info needed. This is not the case, however. According to the Nyquist Sampling Theorem, the sampling frequency only has to be twice the signal bandwidth B (in hertz).

This may seem crazy that you only need such a small amount of samples to cover the entire continuous signal, but the proof is really quite simple (in terms of DSP proofs).

# A Brief Explanation of the Nyquist Sampling Theorem

To prove the theorem we must consider a signal, called x(t) per convention, whose bandwidth is limited to B hertz. To sample x(t) at the sampling frequency, we must multiply x(t) by a unit impulse train (the Dirac Delta function), with each unit impulse repeating with a period of 1/(sampling frequency).



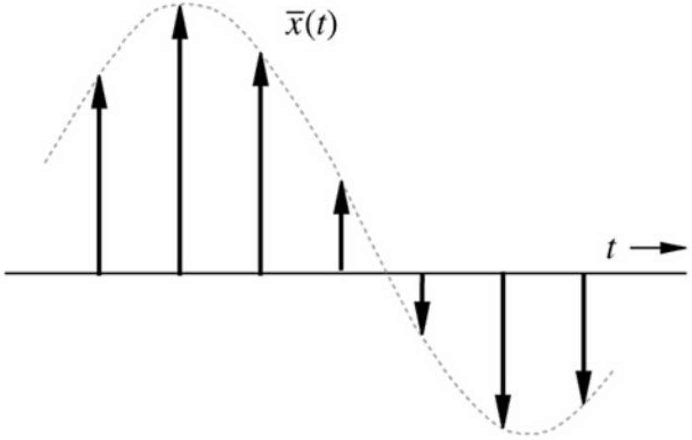

The “train” of unit impulses

# A Brief Explanation - Continued

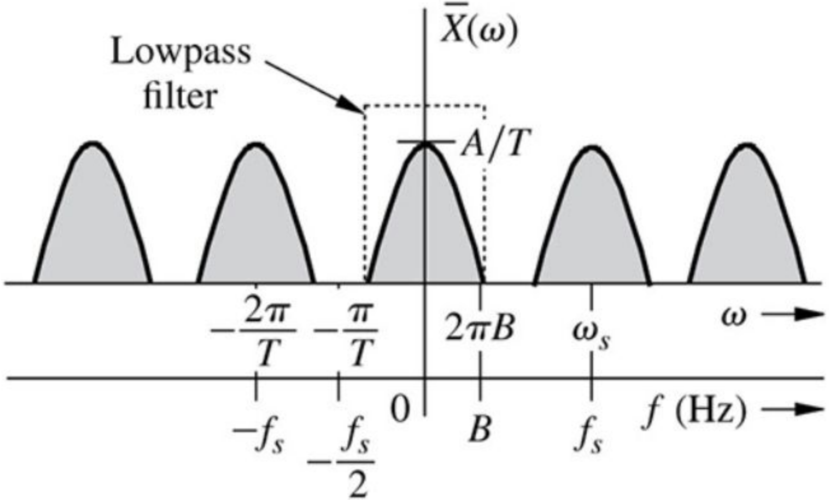
The impulse train is periodic, we can express it with a Trigonometric Fourier Series. We then take the Fourier Transform of this series times x(t) which we call “x-bar”(t). We can capitalize on the fact that we can use the frequency shifting property when we have a sinusoid (Fourier Series expresses a periodic function as a sum of sinusoids) with Fourier Transforms. The frequency shifting property creates an infinite amount of copies in the spectra domain of our signal.

Then, we use the process of demodulation to recover the signal from the spectra domain (since we modulated it with frequency shifting). The use of demodulation is the reason why the Nyquist Theorem is the way it is. Recovering with demodulation requires that the sampling frequency is greater than twice the bandwidth so that we can properly use a low pass filter and capture our original signal.

# A Brief Explanation - Continued




“x-bar”(t), the signal we get in the time domain when we multiply by the unit impulse




The effect modulation has in the spectra domain. Note that in the spectra domain we have the magnitude of the signal. The vertical axis represents how much of a certain frequency you have. The low pass filter captures the original signal.

# Connection to the Audio Sampler Project

In the audio sampler project, we must use the Nyquist Sampling Theorem because we are sampling an analog audio signal. The theorem allows us to choose a proper sampling rate so that we don’t lose any of our signal.

The average person’s hearing goes up to 20 kilohertz. This is the bandwidth of our signal. To fulfill the requirements of the Nyquist Sampling Theorem, we multiply this frequency by two to get the base sampling rate.

However, due to constraints of the low pass filters in our ADC converters, we must add a bit of “buffer frequency” to this value. 4.1 kilohertz is the standard in digital signal processing for audio which is why our final sampling rate is 44.1 kilohertz.