L1 Calculation of Rational Numbers

Concept 1: Calculation of Decimals &Fractions

Math Exploration 1

1 Calculate:

(1) $2.25 \times 1.2 =$ _____ .

(2) $0.96 \times 0.15 =$ _____ .

(3) $0.693 \div 0.3 =$ _____ .

(4) $6.65 \div 3.5 =$ _____ .

Fraction Multiplication

Steps:

1. Convert all mixed fractions to improper fractions.
2. Multiply the denominators.
3. Multiply the numerators.
4. Use reduction to convert the fraction to the simplest form.

Cross Cancellation is a method to multiply fractions and quickly convert the result into the simplest form. To cross cancel, common factors present both in numerators and denominators are crossed out.

$$\frac{\cancel{3}^{1}}{\cancel{15}_{3}} \times \frac{\cancel{5}^{1}}{\cancel{6}_{2}} = \frac{1}{6}$$

Reciprocal is either of a pair of numbers whose product is one. For example, 2 and $\frac{1}{2}$.

To find the reciprocal of a mixed number, convert the mixed number to an improper fraction first.

Fraction Division

Dividing a number is the same as multiplying its reciprocal.

Math Exploration 2

2 Calculate:

$\frac{3}{8} \times \frac{5}{2} =$ _____ .

$\frac{13}{6} \times \frac{4}{39} =$ _____ .

$1\frac{1}{7} \times 1\frac{3}{4} =$ _____ .

$1\frac{2}{13} \times 2\frac{3}{5} =$ _____ .

3 Calculate:

$\frac{5}{3} \div \frac{10}{21} =$ _____ .

$3\frac{2}{3} \div 2\frac{3}{4} =$ _____ .

$4\frac{1}{8} \div \frac{9}{16} =$ _____ .

$3\frac{1}{5} \div 2\frac{2}{15} =$ _____ .

Concept 2: Calculation of Negative Numbers

1. Adding & Subtracting of Negative Numbers:

When adding two numbers with the same sign, find the sum and keep the sign.

For example, $1+2=3$, $(-1)+(-2)=-(1+2)=-3$.

When adding two numbers with different signs, subtract the number with the smaller absolute value from the one with the larger absolute value and keep the sign of the larger number.

For example, $2-1=1$, $1-2=-(2-1)=-1$.

Subtracting a number is the same as adding its opposite.

For example, $1-(-2)=1+2=3$.

Math Exploration 3

4 Calculate:

$3+(-2)=$ _____

$(-3)+2=$ _____

$(-3)+(-2)=$ _____

5 Calculate:

$3-(-2)=$ _____

$(-3)-2=$ _____

$(-3)-(-2)=$ _____

2. Multiplication & Division Negative Numbers:

Sign Rules:

(1) Even numbers of negative signs: positive.

(2) Odd numbers of negative signs: negative.

Step 1: Confirm the sign. The product or quotient of two negative numbers makes a positive number. The product or quotient of one positive number and one

negative number makes a negative number.

Step 2: Find the product of their absolute values and put it after the sign.

For example, $1 \times (-2) = -1 \times 2 = -2$, $(-1) \times (-2) = 1 \times 2 = 2$,

$2 \div (-1) = -2 \div 1 = -2$, $(-2) \div (-1) = 2 \div 1 = 2$.

Math Exploration 4

6 $-1 \times (-3) =$ _____

$-5 \times (-4) =$ _____

$12 \times (-2) =$ _____

7 $-9 \div (-3) =$ _____

$12 \div (-2) =$ _____

$-15 \div 5 =$ _____

Concept 3: Ratio & Word Problems

A **ratio** is a comparison of two or more numbers that indicates their sizes in relation to each other. It compares two quantities by division.

There are three ways to write the **ratio** of two quantities a and $b(b \neq 0)$:

$\frac{a}{b}$ a to b $a : b$

Math Exploration 5

8 $40g$ salt is dissolved in $100g$ of water. The ratio of salt to water is _____ . The ratio of salt to salty water is _____ .

9 There are 12 boys in the class. The ratio of boys to girls is $1:2$. How many students are there in the class?

Concept 4: Exponents

a^b means a is multiplied by itself b times: $a^b = a \cdot a \cdot \ldots \cdot a$

$1^n = 1$

Three Rules for Calculation of Exponent:

Product Rule: $a^m \cdot a^n = a^{m+n}$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

Power Rule: $(a^m)^n = a^{m \cdot n}$

Math Exploration 6

10 Calculate:

(1) $4^3 =$ _____

(2) $(0.2)^3 =$ _____

(3) $(\frac{2}{3})^3 =$ _____

11 Calculate:

$2^3 \times 2^2 =$ _____ .

$2^3 \div 2^2 =$ _____ .

$(2^3)^2 =$ _____ .

Concept 5: Arithmetic Mixed Operations

The Order of Operations: PEMDAS

PEMDAS stands for **P- Parentheses, E- Exponents, M- Multiplication, D-Division, A- Addition, and S- Subtraction**

Steps:

1. Operations in brackets should be carried out first.
2. Next, solve the exponents in the expression.
3. Move from left to right and carry out multiplication or division, whichever comes first.
4. Move from left to right and carry out addition or subtraction, whichever comes first.

Math Exploration 7

12 Calculate:

$5-(-7)+2=$ _____

$(-4)+4-(-4)=$ _____

$(-3)-(-3)+(-3)=$ _____

13 $-2\times(-3)\times(-4)=$_____

$-48\div(-4)\div(-3)=$_____

$6\times10\div(-5)=$_____

14 Calculate:

(1) $-\frac{6}{5}\div3\frac{3}{4}\times(-0.25)=$_____ .

(2) $\frac{8}{3}\div(-4\frac{1}{2})\times\frac{9}{16}=$_____ .

(3) $-2\frac{1}{6}\times(-\frac{3}{13})\div(-\frac{1}{4})=$_____ .

(4) $-11\frac{4}{7}\times5\frac{4}{9}\times0.1=$_____ .

15 Calculate:

(1) $\frac{3}{11}\div0.2\div\frac{5}{22}=$_____

(2) $\frac{4}{5}\times\frac{1}{7}\div\frac{2}{5}-\frac{4}{7}=$_____

(3) $\frac{13}{14}\div\left(\frac{6}{7}+\frac{13}{14}\right)=$_____

(4) $16\times(1\frac{1}{8}-2.125)=$_____

Homework

16 Calculate:

(1) $75 \times 1.2 =$ _____ .

(2) $16.8 \times 3.2 =$ _____ .

(3) $7.83 \div 9 =$ _____

(4) $51.3 \div 0.27 =$ _____

17 Calculate:

$7 + (-11) =$ _____

$-12 - (-1) =$ _____

$-3 \times 7 =$ _____

$-24 \div (-6) =$ _____

18 Calculate:

$-7 \times 6 \div (-3) =$ _____

$(-12) \div (-2) \times 5 =$ _____

19 $2\frac{4}{5} \times \frac{10}{3} =$ _____ .

$-2\frac{5}{8} \times (-3\frac{3}{7}) =$ _____ .

20 Calculate:

$\frac{4}{7} \div \frac{8}{7} =$ _____ .

$-5\frac{1}{9} \div 3\frac{5}{6} =$ _____ .

21 Calculate:

$3^3 \times 3^2 =$ _____ .

$3^3 \div 3^2 =$ _____ .

$(3^3)^2 =$ _____ .

22 A bakery made 33 mufins, 50 crossiants, and 13 bagels.

(1) The ratio of the number of mufins to the number of crossiants is _____ .

(2) The ratio of the number of mufins to the number of bagels is _____ .

(3) The ratio of the number of bagels to the number of crossiants is _____ .

23 There were 20 people on a bus. The ratio of the number of adults to the number of children was $1:4$. There were _____ adults.

24 Calculate:

(1) $(5.6+2.8) \div \frac{3}{10} =$ _____

(2) $\frac{3}{7} \times 0.625 \div \frac{3}{8} =$ _____

25 Calculate:

(1) $\frac{3}{5} \div (2 - \frac{1}{4}) =$ _____

(2) $\frac{2}{3} \div \frac{8}{9} \times (0.75 - 1) =$ _____