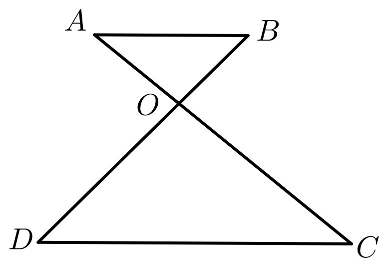
Day 7

1 As shown in the figure below, $AB//CD$, $AB=6$, $CD=14$. Find the value of $AO:OC$, $BO:OD$, and $S_{\triangle ABO}:S_{\triangle CDO}$, respectively.

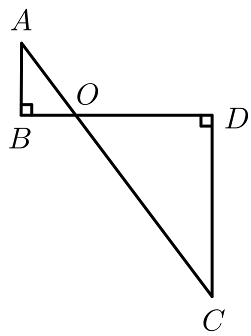


Answer $3:7;3:7;9:49$

Solution $AB//CD \Rightarrow \triangle ABO \sim \triangle CDO$

$AO:OC=BO:OD=AB:CD=6:14=3:7$, $S_{\triangle AOB}=S_{\triangle CDO}=AB^2:CD^2=9:49$.

2 As shown in the figure below, $AB\perp BD$, $CD\perp BD$, $BO=2$, and $AB=3$, $OD=6$, what is the area of $S_{\triangle COD}$?



Answer 27

Solution $\because AB \perp BD$, $CD \perp BD$;

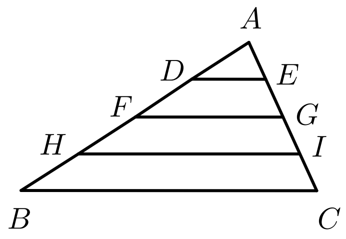
$\therefore AB // CD$;

$\therefore \triangle ABO \sim \triangle CDO$;

$\therefore AB : CD = BO : OD = 2 : 6 = 1 : 3$;

$\therefore CD = 9$, $S_{\triangle CDO} = 6 \times 9 \div 2 = 27$.

3 As shown in the figure below, in $\triangle ABC$, $AD = DF = FH = HB$, $AE = EG = GI = IC$. What fraction of the area of $\triangle ABC$ is the area of $\triangle ADE$?

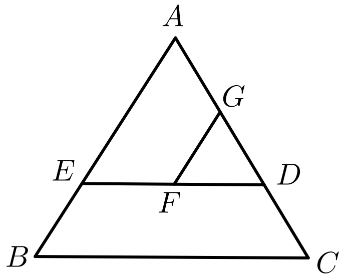


Answer $\frac{1}{16}$

Solution $AE = \frac{1}{4}AC$, $AD = \frac{1}{4}AB$;

$\frac{S_{\triangle ADE}}{S_{\triangle ABC}} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$.

4 As shown in the figure below, in $\triangle ABC$, $AB=3BE$, $AC=3CD$, G is the midpoint of AD and F is the midpoint of ED. Given that the area of quadrilateral $BCDE$ is 20 cm^2 larger than that of $\triangle DGF$, what is the area of $\triangle ABC$?



Answer 45 cm^2

Solution $\because AE=\frac{2}{3}AB$, $AD=\frac{2}{3}AC$;

$\therefore \triangle ABC \sim \triangle AED$;

$\therefore \frac{S_{BCDE}}{S_{\triangle ABC}}=1-\frac{2}{3}\times\frac{2}{3}=\frac{5}{9}$.

$\because G$, F are the midpoints of AD and ED, respectively;

$\therefore \triangle DGF \sim \triangle DAE$;

$\therefore \frac{S_{\triangle GFD}}{S_{\triangle AED}}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$;

$\therefore \frac{S_{\triangle GFD}}{S_{\triangle ABC}}=\frac{1}{4}\times\frac{4}{9}=\frac{1}{9}$;

$\therefore S_{\triangle ABC}=20\div(\frac{5}{9}-\frac{1}{9})=45$ cm^2.