1-4

1 Juan organizes the stamps in his collection by country and by the decade in which they were issued. The prices he paid for them at a stamp shop were: Brazil and France， 6cents each， Peru 4 cents each， and Spain 5 cents each. (Brazil and Peru are South American countries and France and Spain are in Europe.)

Number of Stamps by Decade

Country	50s	60s	70s	80s
Brazil	4	7	12	8
France	8	4	12	15
Peru	6	4	6	10
Spain	3	9	13	9

Juan's Stamp Collection

His South American stamps issued before the '70s cos t him (　　) .

A. $0.40　　B. $1.06　　C. $1.80　　D. $2.38　　E. $2.64

Answer B

Solution The number of stamps from Brazil in the '50s and '60s is $4+7=11$, and they cost 6 cents each for a total of $11 \cdot \$0.06=\$0.66$. The number of stamps from Peru in the '50s and 60s is $6+4=10$, and they cost 4 cents each for a total of $10. \$0.04=\$0.40$. In total, he paid $0.66+0.40=\$1.06$.

2 On a twenty-question test, each correct answer is worth 5 points, each unanswered question is worth 1 point and each incorrect answer is worth 0 points. Which of the following scores is NOT possible? (　　) .

A. 90　　B. 91　　C. 92　　D. 95　　E. 97

Answer E

Solution

The highest possible score is if you get every answer right, to get $5\,(20)=100$. The second highest possible score is if you get 19 questions right and leave the remaining one blank, to get a $5\,(19)+1\,(1)=96$. Therefore, no score between 96 and 100, exclusive, is possible, so 97 is not possible.

3 The six children listed below are from two families of three siblings each. Each child has blue or brown eyes and black or blond hair. Children from the same family have at least one of these characteristics in common. Which two children are Jim's siblings? ( ) .

Child	Eye Color	Hair Color
Benjamin	Blue	Black
Jim	Brown	Blonde
Nadeen	Brown	Black
Austin	Blue	Blonde
Tevyn	Blue	Black
Sue	Blue	Blonde

A. Nadeen and Austin

B. Benjamin and Sue

C. Benjamin and Austin

D. Nadeen and Tevyn

E. Austin and Sue

Answer E

Solution Jim has brown eyes and blonde hair. If you look for anybody who has brown eyes or blonde hair, you find that Nadeen, Austin, and Sue are Jim's possible siblings. However, the children have at least one common characteristics. Since Austin and Sue both have blonde hair, Nadeen is ruled out and therefore Austin and Sue are his siblings.

4 How many integers between 1000 and 2000 have all three of the numbers15 , 20 , and 25 as factors? ( ) .

A. 1 B. 2 C. 3 D. 4 E. 5

Answer C

Solution Find the least common multiple of 15, 20, 25 by turning the numbers into their prime factorization. $15 = 3 * 5$, $20 = 2^2 * 5$, $25 = 5^2$ Gather all necessary multiples 3, 2^2, 5^2 when multiplied gets 300. The multiples of $300 - 300$, 600, 900, 1200, 1500, 1800, 2100. The number of multiples between 1000 and 2000 is $\boxed{(C)3}$.

5 Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they shared, how many meals should they have ordered to have just enough food for the 12 of them? ( ) .

A. 8 B. 9 C. 10 D. 15 E. 18

Answer A

Solution Set up the proportion $\frac{12\text{meals}}{18\text{people}} = \frac{x\text{meals}}{12\text{people}}$. Solving for x gives us $x = \boxed{(A)8}$.

6 Mindy made three purchases for $\$1.98$, $\$5.04$ and $\$9.89$. What was her total, to the nearest dollar? ( ) .

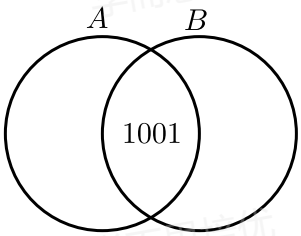
A. 10 B. 15 C. 16 D. 17 E. 18

Answer D

Solution The three prices round to $\$2$, $\$5$, and $\$10$, which has a sum of $\boxed{(D)17}$.

7 Set A and B, shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A (

) .



A. 503 B. 1006 C. 1504 D. 1507 E. 1510

Answer C

Solution Let x be the number of elements in A and B.

Since the union is the sum of all elements in A and B,

and A and B have the same number of elements then,

$2x - 1001 = 2007$

$2x = 3008$

$x = 1504$.

The answer is 1504.

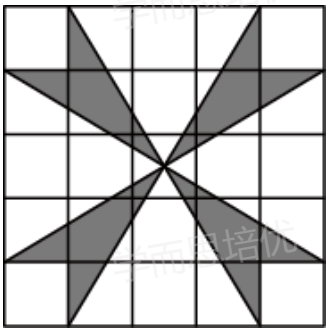
8 A mixture of 30 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of yellow tint are added to the original mixture. What is the percent of yellow tint in the new mixture? ( ) .

A. 25 B. 35 C. 40 D. 45 E. 50

Answer C

Solution Since 30% of the original 30 liters of paint was yellow, and 5 liters of yellow paint were added to make the new mixture, there are $9 + 5 = 14$ liters of yellow tint in the new mixture. Since only 5 liters of paint were added to the original 30, there are a total of 35 liters of paint in the new mixture. This gives 40% of yellow tint in the new mixture, which is 40.

9 What is the area of the shaded part shown in the 5×5 grid? ( ) .



A. 4 B. 6 C. 8 D. 10 E. 12

Answer B

Solution The area of the square around the pinwheel is 25. The area of the pinwheel is equal to the square - the white space. Each of the four triangles have a base of 3 units and a height of 2.5 units, and so their combined area is 15 units squared. Then the unshaded space consists of the four triangles with total area of 15, and there are four white corner squares. Therefore the area of the pinwheel is $25-(15+4)$ which is 6.

10 Mary's winning art design is shown. The smallest circle has radius 2 inches, with each successive circle's radius increasing by 2 inches. Approximately what percent of the design is black? ( ) .

A. 42 B. 44 C. 45 D. 46 E. 48

Answer A

Solution

circle#	radius	area
1	2	4π
2	4	16π
3	6	36π
4	8	64π
5	10	100π
6	12	144π

circle#	radius	area
1	2	4π
2	4	16π
3	6	36π
4	8	64π
5	10	100π
6	12	144π

The entire circle's area is 144π. The area of the black regions is $(100-64)\pi+(36-16)\pi+4\pi=60\pi$. The percentage of the design that is black is $\frac{60\pi}{144\pi}=\frac{5}{12}\approx 42$.

11 What time was it 2011 minutes after midnight on January 1, 2011? ( ) .

A. January 1 at $9:31$ PM

B. January 1 at $11:51$ PM

C. January 2 at $3:11$ AM

D. January 2 at $9:31$ AM

E. January 2 at $6:01$ PM

Answer D

Solution There are 60 minutes in an hour. $2011/60=33\text{r}31$, or 33 hours and 31 minutes. There are 24 hours in a day, so the time is 9 hours and 31 minutes after midnight on January 2, 2011.

$\Rightarrow \boxed{(\text{D})\text{January}2\text{at}9:31\text{AM}}$.

12 A shop advertises everything is "half price in today's sale." In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage off the original price? ( ) .

A. 10 B. 33 C. 40 D. 60 E. 70

Answer D

Solution Let the original price of an item be x.

First, everything is half-off, so the price is now $\frac{x}{2} = 0.5x$.

Next, the extra coupon applies 20% off on the sale price, so the price after this discount will be $100\% - 20\% = 80\%$ of what it was before. (Notice how this is not applied to the original price; if it were, the solution would be applying $50\% + 20\% = 70\%$ off the original price.)

$80\% \cdot 0.5x = \frac{4}{5} \cdot 0.5x = 0.4x$

The price of the item after all discounts have been applied is $0.4x = 40\% \cdot x$. However, we need to find the percentage off the original price, not the current percentage of the original price. We then subtract $40\%x$ from $100\%x$ (the original price of the item), to find the answer, $\boxed{(D)60}$.

Since the problem implies that the percentage off the original price will be the same for every item in the store, fakesolving is applicable here. Say we are buying an item worth 10 dollars, a convenient number to work with. First, it is clear that we'll get 50% off, which makes the price then 5 dollars. Taking 20% off of 5 dollars gives us 4 dollars. Therefore, we have saved a total of $\frac{10-4}{10} = \frac{6}{10} = \frac{60}{100} = \boxed{(D)60}\%$.

13 An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon? ( ) .

A. 4 B. 5 C. 6 D. $4\sqrt{3}$ E. $6\sqrt{3}$

Answer C

Solution Let the perimeter of the equilateral triangle be $3s$. The side length of the equilateral triangle would then be s and the sidelength of the hexagon would be $\frac{s}{2}$.

A hexagon contains six equilateral triangles. One of these triangles would be similar to the large equilateral triangle in the ratio $1:4$, since the sidelength of the small equilateral triangle is half the sidelength of the large one.Thus, the area of one of the small equilateral triangles is 1. The area of the hexagon is then $1 \times 6 = \boxed{(C)6}$.

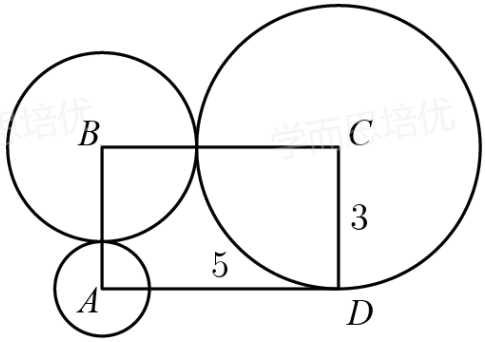
14 The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born? ( ) .

A. 1979 B. 1980 C. 1981 D. 1982 E. 1983

Answer A

Solution The seventh AMC8 would have been given in 1991. If Samantha was 12 then, that means she was born 12 years ago, so she was born in $\boxed{\text{(A)}1979}$.

15 Rectangle $ABCD$ has sides $CD = 3$ and . $DA = 5$. A circle of radius 1 is centered at A, a circle of radius 2 is centered at B, and a circle of radius 3 is centered at C. Which of the following is closest to the area of the region inside the rectangle but outside all three circles? ( ) .



A. 3.5 B. 4.0 C. 4.5 D. 5.0 E. 5.5

Answer B

Solution The area in the rectangle but outside the circles is the area of the rectangle minus the area of all three of the quarter circles in the rectangle. The area of the rectangle is $3 \cdot 5 = 15$. The area of all 3 quarter circles is $\frac{\pi}{4} + \frac{\pi(2)^2}{4} + \frac{\pi(3)^2}{4} = \frac{14\pi}{4} = \frac{7\pi}{2}$. Therefore the area in the rectangle but outside the circles is $15 - \frac{7\pi}{2}$. π is approximately $\frac{22}{7}$, and substituting that in will give $15 - 11 = \boxed{\text{(B)}4.0}$.

16 Ralph went to the store and bought 12 pairs of socks for a total of $\$24$. Some of the socks he bought cost $\$1$ a pair, some of the socks he bought cost $\$3$ a pair, and some of the socks he bought cost $\$4$ a pair. If he bought at least one pair of each type, how many pairs of $\$1$ socks did Ralph buy? ( ) .

A. 4 B. 5 C. 6 D. 7 E. 8

Answer D

Solution So let there be x pairs of $\$1$ socks, y pairs of $\$3$ socks, z pairs of $\$4$ socks. We have $x+y+z=12$, $x+3y+4z=24$, and x, y, $z \geqslant 1$, Now we subtract to find $2y+3z=12$, and y, $z \geqslant 1$. It follows that y is a multiple of 3 and $2y$ is a multiple of 6, so since $0<2y<12$, we must have $2y=6$. Therefore, $y=3$, and it follows that $z=2$. Now

$x=12-y-z=12-3-2=$ $\boxed{(\mathrm{D})7}$, as desired.

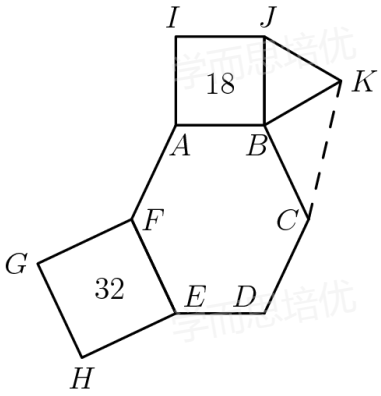
Since the total cost of the socks was $\$24$ and Ralph bought 12 pairs, the average cost of each pair of socks is $\frac{\$24}{12}=\$2$.

There are two ways to make packages of socks that average to $\$2$. You can have:

- Two $\$1$ pairs and one $\$4$ pair (package adds up to $\$6$)
- One $\$1$ pair and one $\$3$ pair (package adds up to $\$4$)

So now we need to solve $6a+4b=24$, where a is the number of $\$6$ packages and b is the number, of $\$4$ packages. We see our only solution (that has at least one of each pair of sock) is $a=2$, $b=3$, which yields the answer of $2\times 2+3\times 1=$ $\boxed{(\mathrm{D})7}$.

17 In the given figure hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE=BC$. What is the area of $\triangle KBC$? ( ) .



A. $6\sqrt{2}$ B. 9 C. 12 D. $9\sqrt{2}$ E. 32

Answer C

Solution Clearly, since $\overline{FE}$ is a side of a square with area 32, $\overline{FE} = \sqrt{32} = 4\sqrt{2}$, Now, since $\overline{FE} = \overline{BC}$, we have $\overline{BC} = 4\sqrt{2}$.

Now, $\overline{JB}$ is a side of a square with area 18, so $\overline{JB} = \sqrt{18} = 3\sqrt{2}$. Since $\triangle JBK$ is equilateral, $\overline{BK} = 3\sqrt{2}$.

Lastly, $\triangle KBC$ is a right triangle. We see that

$\angle JBA + \angle ABC + \angle CBK + \angle KBJ = 360° \rightarrow 90° + 120° + \angle CBK + 60° = 360° \rightarrow \angle CBK = 90°$

, so $\triangle KBC$ is a right triangle with legs $3\sqrt{2}$ and $4\sqrt{2}$. Now, its area is

$\frac{3\sqrt{2} \cdot 4\sqrt{2}}{2} = \frac{24}{2} = (C)12.$

18 The number N is a two-digit number.

• When N is divided by 9, the remainder is 1.

• When N is divided by 10, the remainder is 3.

What is the remainder when N is divided by 11? ( ) .

A. 0 B. 2 C. 4 D. 5 E. 7

Answer E

Solution From the second bullet point, we know that the second digit must be 3. Because there is a remainder of 1 when it is divided by 9 the multiple of 9 must end in a 2. We now look for this

one:

$9(1) = 9$

$9(2) = 18$

$9(3) = 27$

$9(4) = 36$

$9(5) = 45$

$9(6) = 54$

$9(8) = 72$

The number $72 + 1 = 73$ satisfies both conditions. We subtract the biggest multiple of 11 less than 73 to get the remainder. Thus, $73 - 11(6) = 73 - 66 = \boxed{(E)7}$.