1. BASIC KNOWLEDGE

Abbreviations:

Perimeter = P Area = A or S Length = l Width = w
Height = h Circumference = C Radius = r
Side lengths of a polygon = $a, b, c, \ldots$

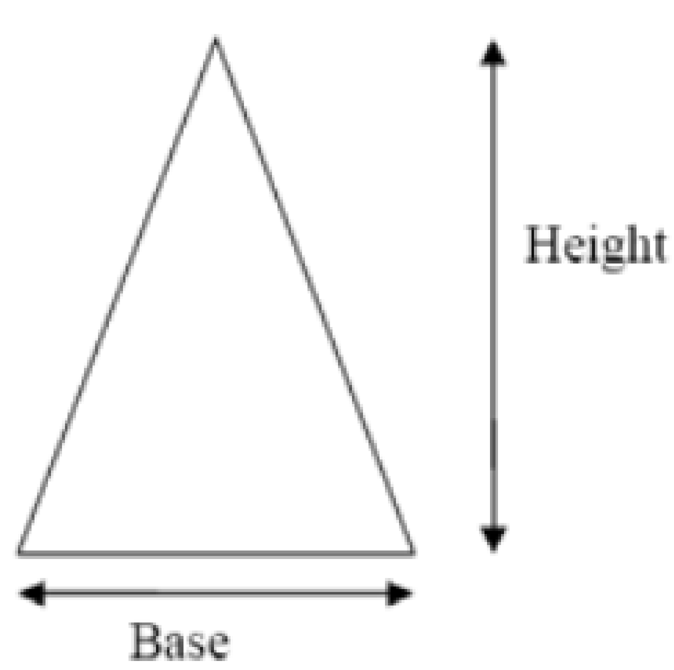
Basic Formulas for Perimeters and Areas

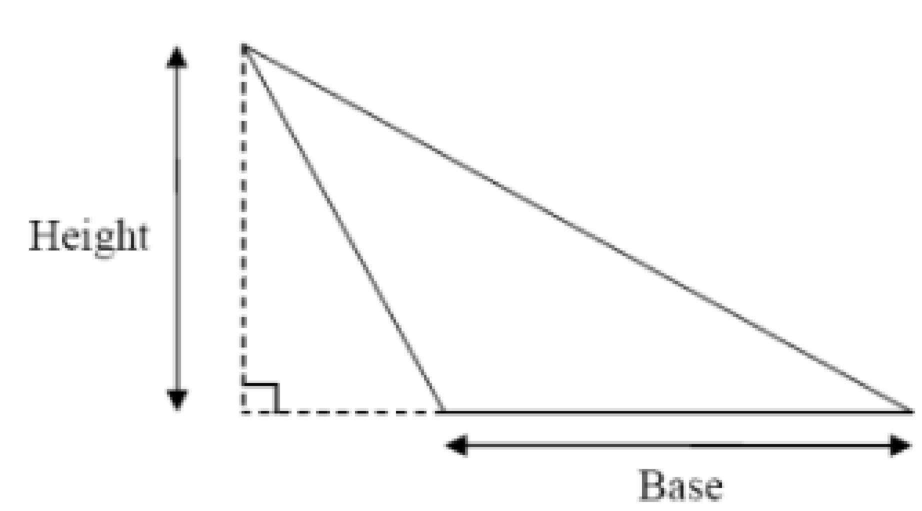
Triangle:

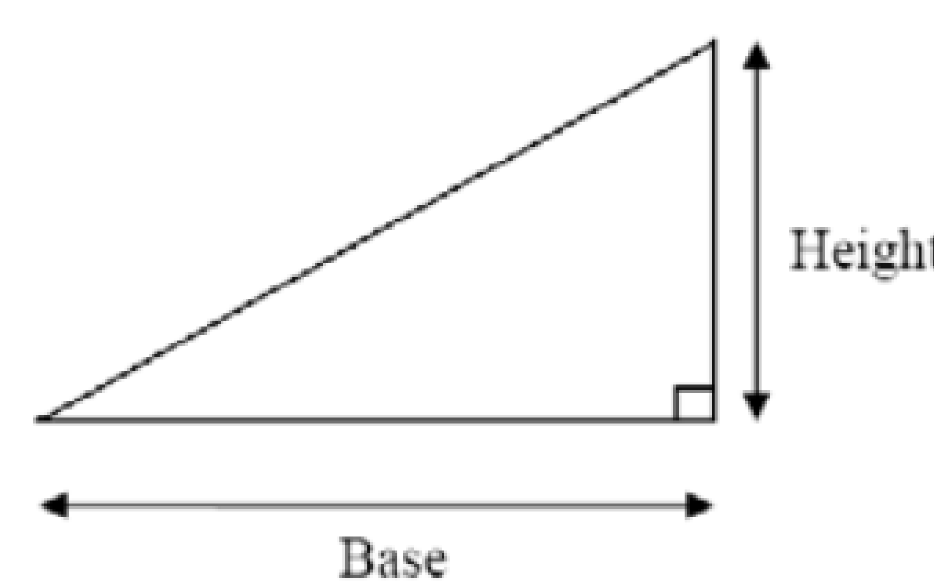
Perimeter of a triangle: $P = a + b + c$

Area of a triangle: $A = \frac{1}{2}bh_b$, h_b is the height on the side b.

For an equilateral triangle (three sides have the same length, a), the area is $A = \frac{1}{4}a^2\sqrt{3}$.





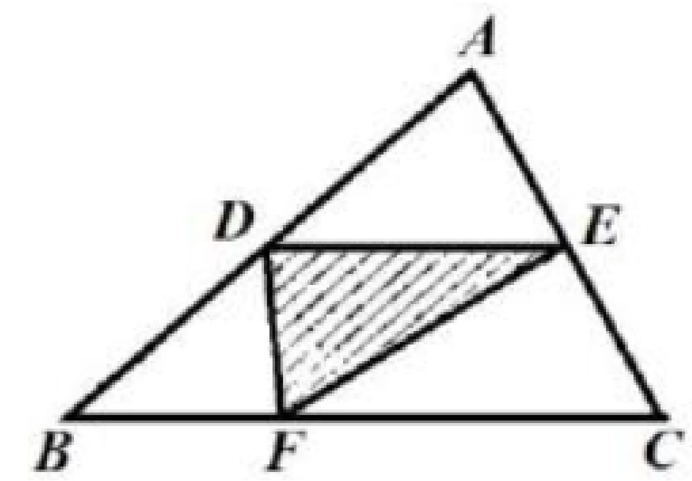


Heron Formula Area of a triangle with sides a, b, and c:

$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$ and a, b, and c are the three sides.

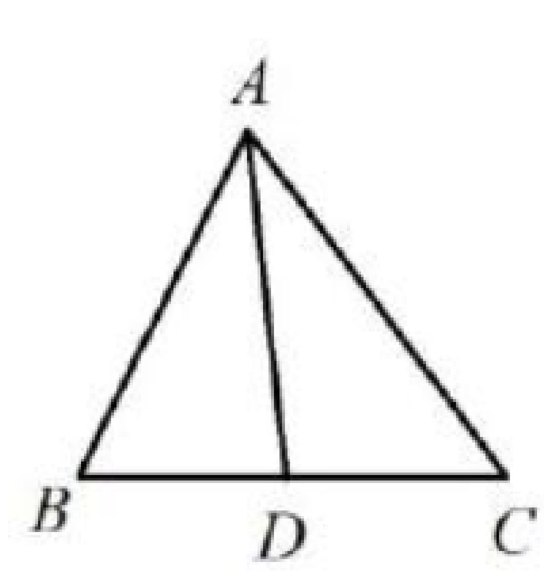
Example 1. The area of triangle ABC is 16 cm^2. D and E are midpoints of AB and AC, respectively. F is a point on BC such that $BF = 3$ cm. What is the area of triangle DEF ?

(A) 12 (B) 9 (C) 8 (D) 4 (E) 3



Example 2. What is the area of a right triangle whose legs measure 10 cm and 18 cm respectively?
(A) 180 cm^2 (B) 100 cm^2 (C) 90 cm^2 (D) 60 cm^2 (E) 30 cm^2

Example 3. If $BD = DC$ and the area of the triangle ABD is 8 cm^2, find the area of triangle ABC.
(A) 16 cm^2 (B) 8 cm^2 (C) 14 cm^2 (D) 32 cm^2 (E) 4 cm^2



Example 4. The area of a triangle is 24 square units, and its base is 6 units. How many units are in the length of the height ?

(A) 24 (B) 12 (C) 10 (D) 8 (E) 6

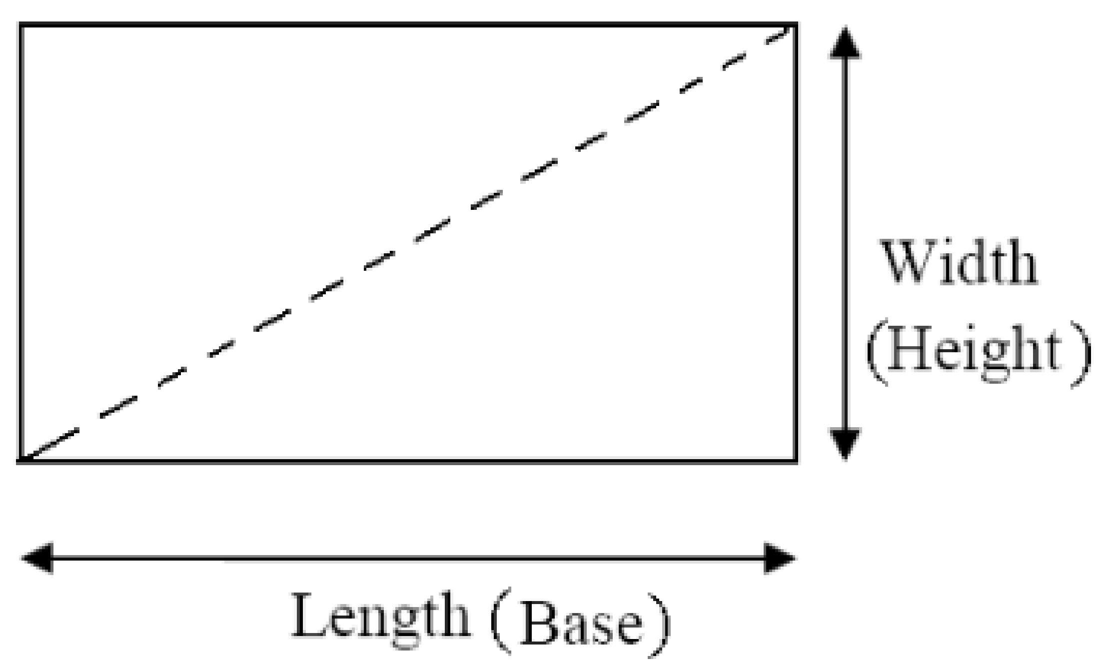
Example 5. Find the area of the triangle with the side lengths of 21, 28, and 35.

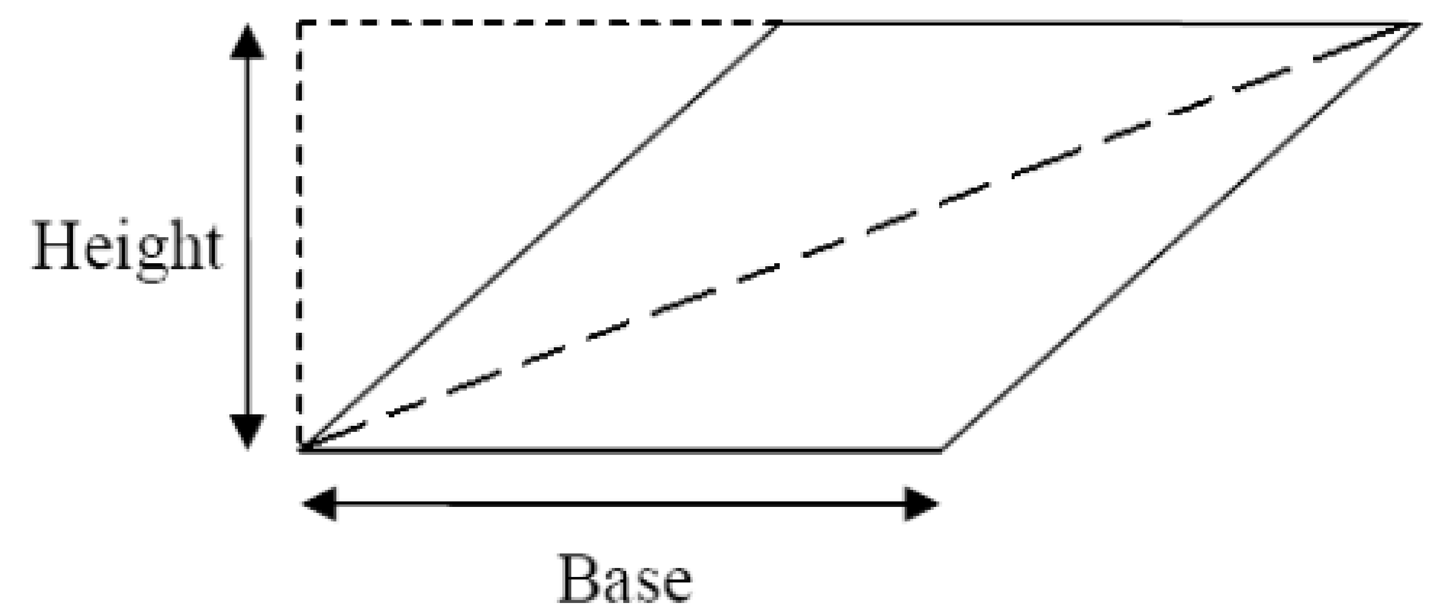
(A) 84 (B) 42 (C) 144 (D) 288 (E) 294

Rectangle and Parallelogram:

Perimeter: $P = 2(L + W)$ Area: $A = L \times W$

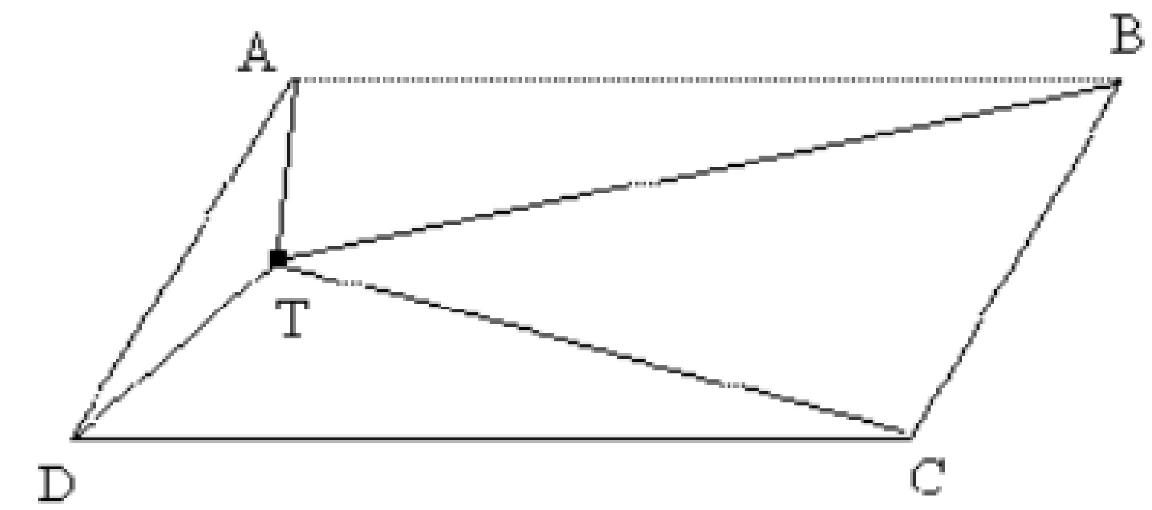
This can be thought as the area of two triangles added together.





In the parallelogram $ABCD$, T is any interior point, we have always:

$$S_{\Delta TAB} + S_{\Delta TCD} = \frac{1}{2} \times S_{ABCD}$$



Area of A Rectangle with Four Cut Areas

The rectangle is divided into four rectangles with areas as shown.

a	b
d	c

The following relationship is true: $a \times c = b \times d$

☆**Example 6.** (AMC 12) A large rectangle is partitioned into four rectangles by two segments parallel to its sides. The areas of three of the resulting rectangles are shown. What is the area of the fourth rectangle?

6	14
x	35

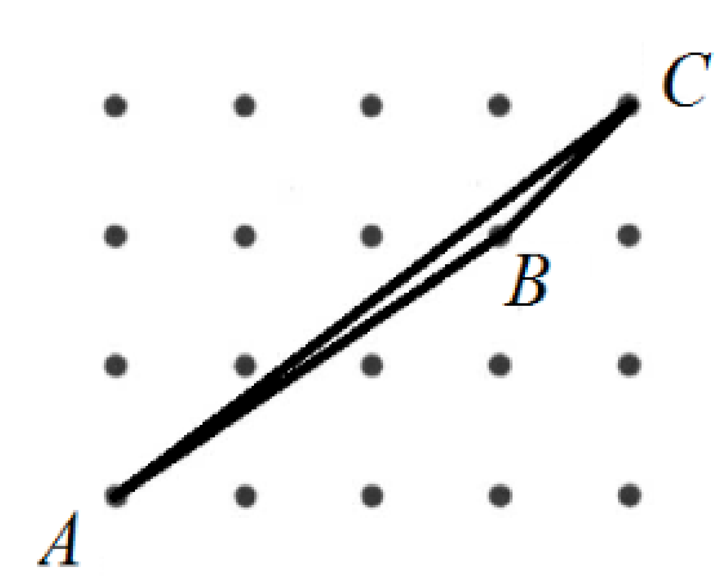
(A) 10 (B) 15 (C) 20 (D) 21 (E) 25

The Pick's Law (Finding the area of the region bounded by grids)

For unit rectangular grid:

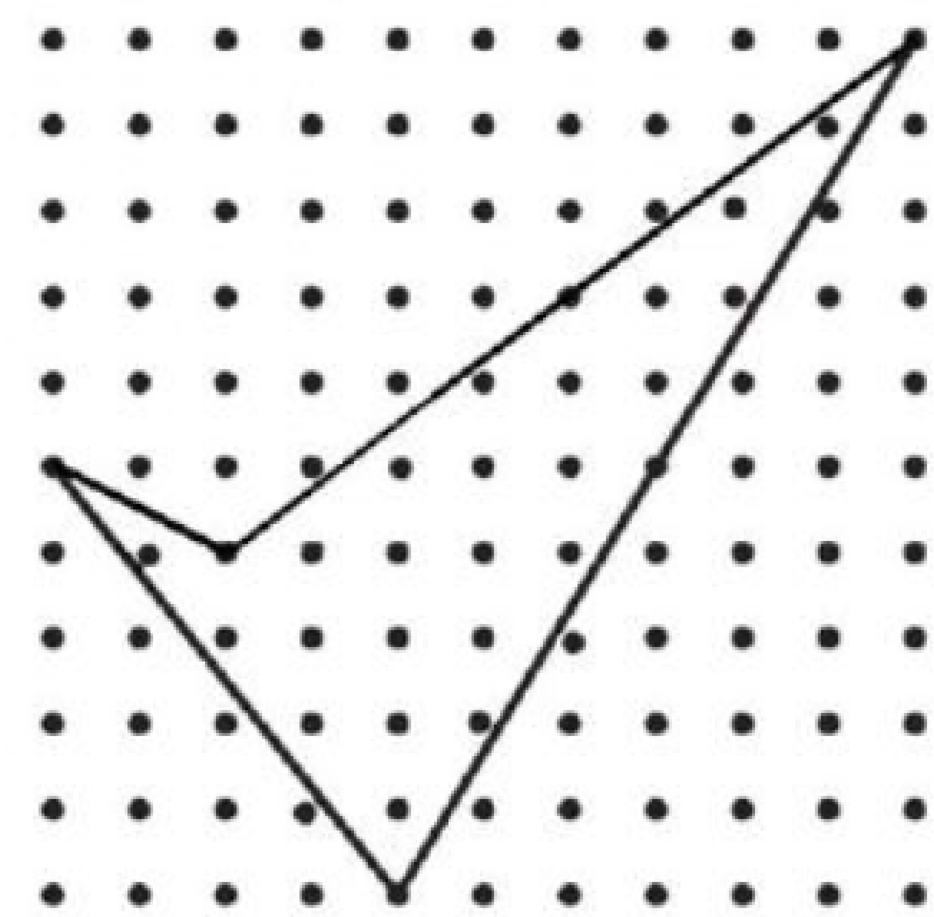
For unit triangular grid: $Area = B + 2I - 2$
B: Number of boundary points. I: Number of interior point.

☆**Example 7.** (AMC 8) The horizontal and vertical distances between adjacent points equal 1 unit. The area of triangle ABC is
(A) 1/4 (B) 1/2 (C) 3/4 (D) 1 (E) 5/4



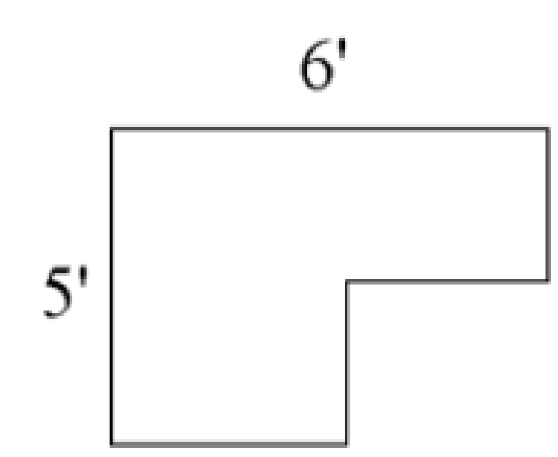
☆**Example 8.** What is the area enclosed by the geoboard quadrilateral below?

(A) 16 (B) 18 (C) 20 (D) 25 (E) 21



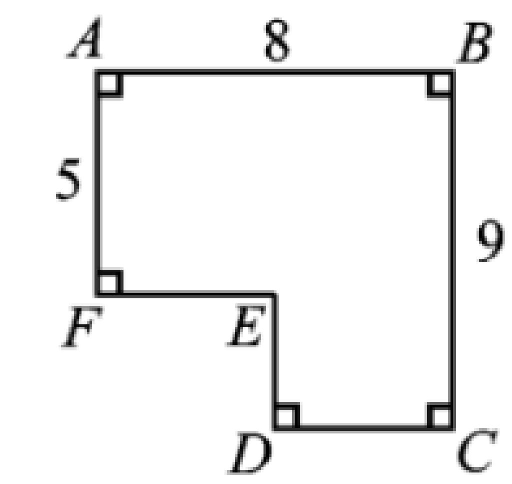
Example 9. A rectangle is cut from the corner of a larger rectangle as shown. How many feet are in the perimeter of the shape?

(A) 24 (B) 22 (C) 20 (D) 18 (E) 10



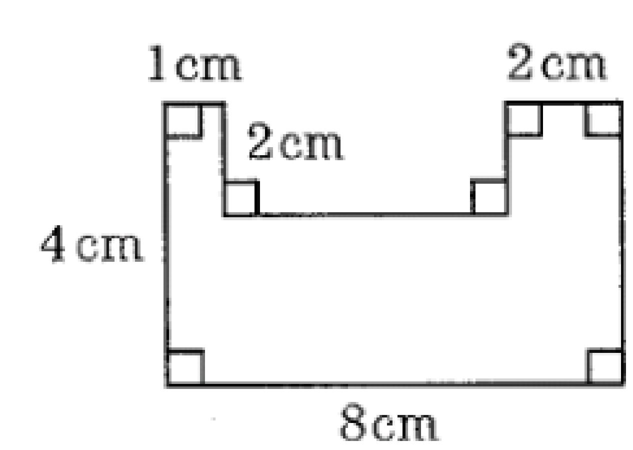
☆**Example 10.** (AMC 8) The area of polygon $ABCDEF$ is 52 with $AB = 8, BC = 9$ and $FA = 5$. What is $DE + EF$?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11



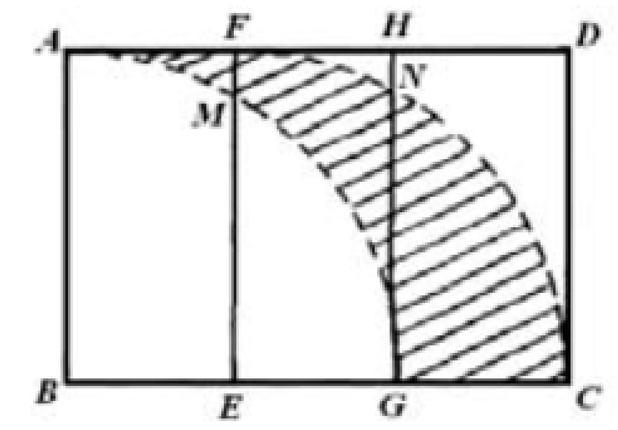
Example 11. What is the minimum number of 1cm square tiles that would cover this figure?

(A) 48 (B) 22 (C) 36 (D) 100 (E) 60



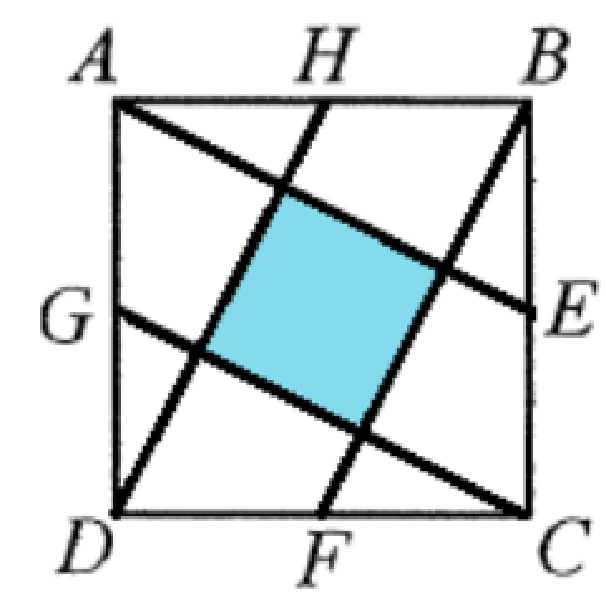
Example 12. $ABCD$ is a 4×6 rectangle formed by three 4×2 small rectangles. B and E are the centers of the arcs AG and FC, respectively. Find the shaded area.

(A) 6 (B) 9 (C) 8 (D) 10 (E) 12



Example 13. Square $ABCD$ has midpoints E, F, G, and H. $AB = 15$ centimeters. Find the area of the shaded interior square in square centimeters.

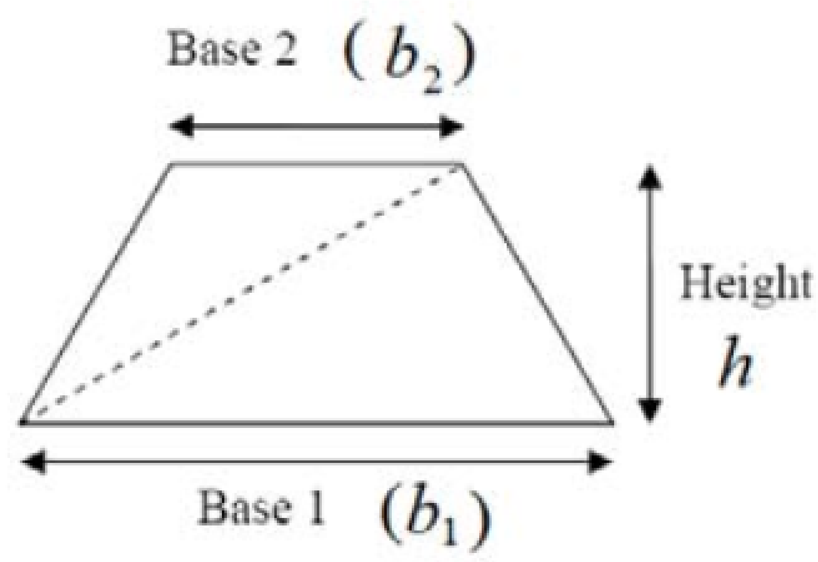
(A) 46 (B) 49 (C) 45 (D) 56 (E) 55



Trapezoid:

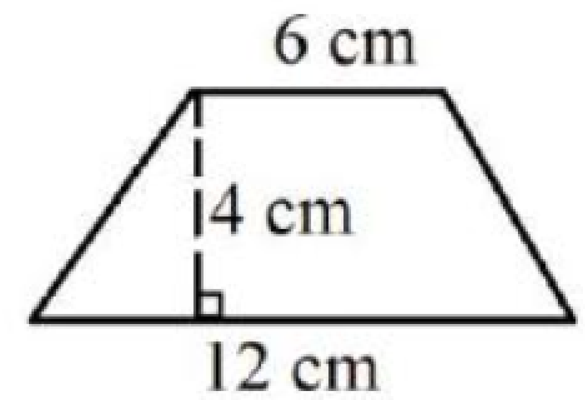
Area of a Trapezoid: $A = \frac{(b_1 + b_2)}{2}h$

This can be thought as the area of two triangles added together



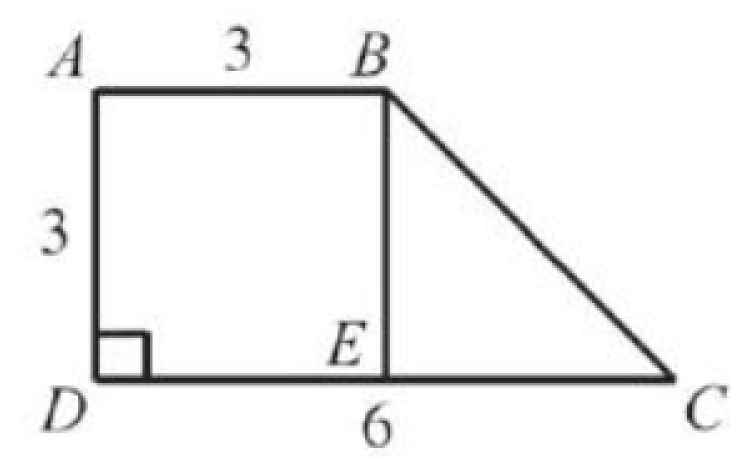
Example 14. What is the number of square centimeters in the area of the trapezoid shown?

(A) 36 (B) 38 (C) 40 (D) 48 (E) 56



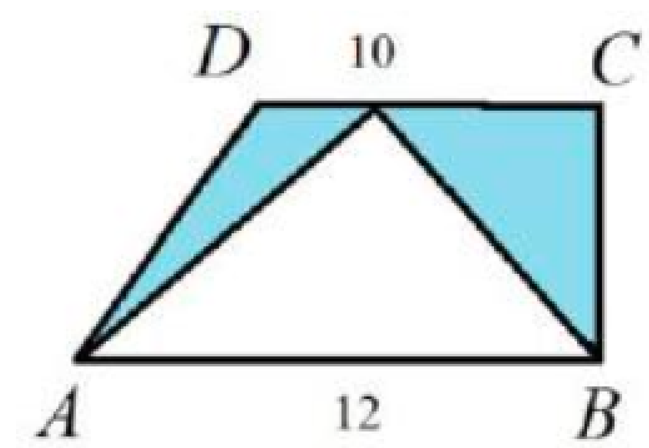
☆**Example 15.** (AMC 8) In trapezoid $ABCD$, AD is perpendicular to DC, $AD = AB = 3$, and $DC = 6$. In addition, E is on DC, and BE is parallel to AD. Find the area of ΔBEC.

(A) 3 (B) 4.5 (C) 6 (D) 9 (E) 18



Example 16. Find the area of the shaded region within the trapezoid if $AB = 12$, $BC = 8$, $CD = 10$, and $\overline{AB}$ is perpendicular to $\overline{BC}$.

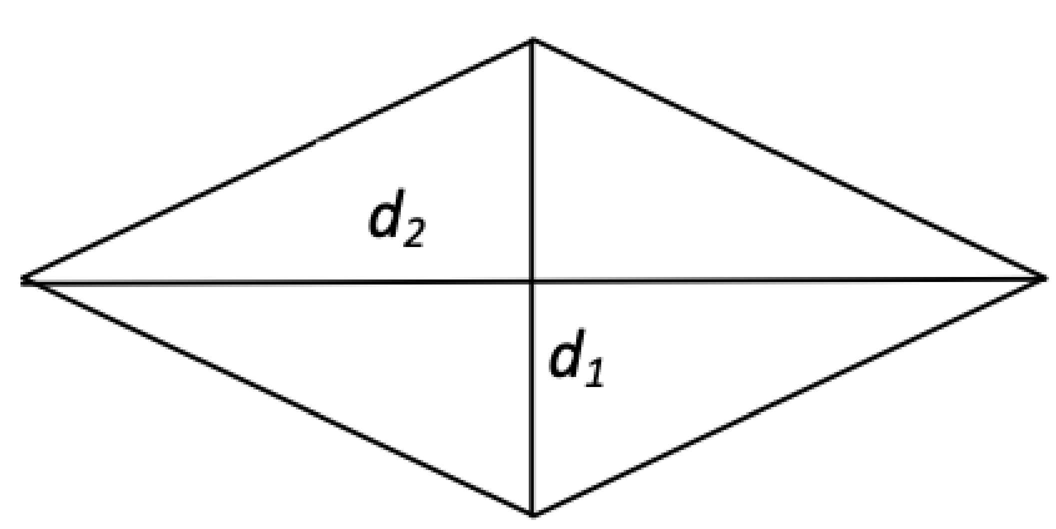
(A) 88 (B) 40 (C) 60 (D) 44 (E) 20



Rhombus:

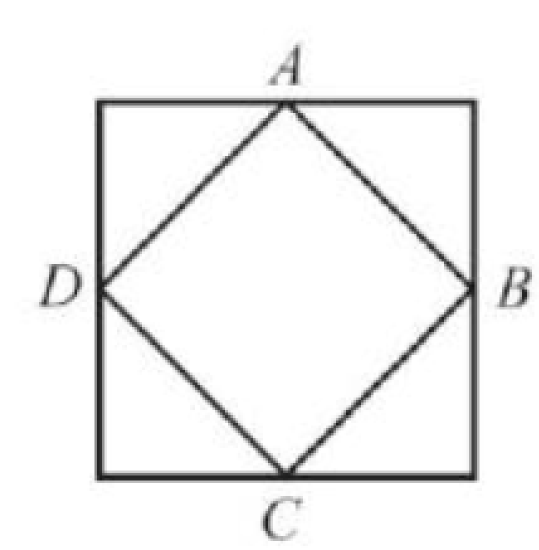
Perimeter of a rhombus: $P = 4a$ (a is the side length).

Area of a rhombus: $A =$ (d_1 and d_2 are diagonals).

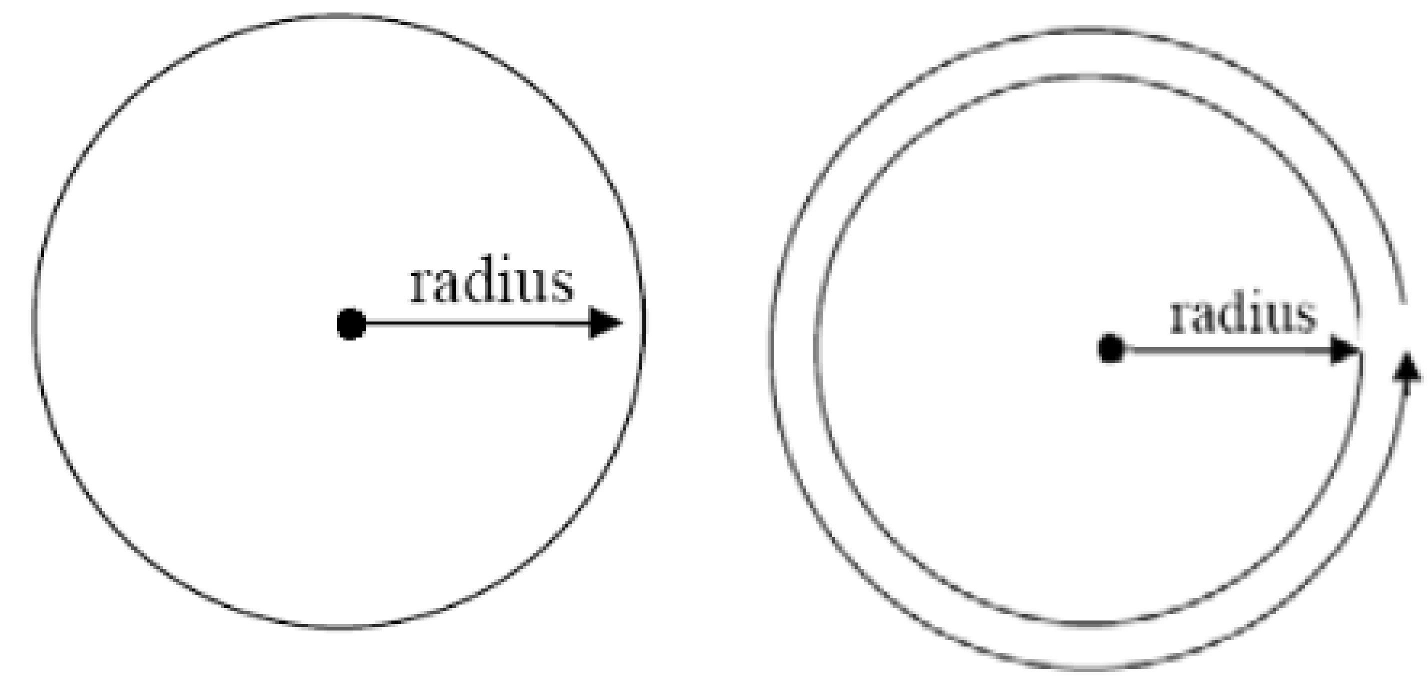


☆**Example 17.** (AMC 8) Points A, B, C and D are midpoints of the sides of the larger square. If the larger square has area 60, what is the area of the smaller square?

(A) 15 (B) 20 (C) 24 (D) 30 (E) 40



Circle:

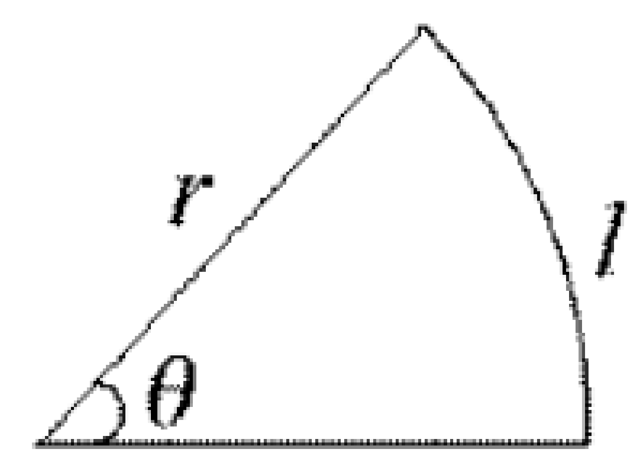


Circumference (perimeter) $C = 2\pi r$

Area of a circle: $A = \pi r^2 = \frac{\pi}{4} d^2$

d is the diameter of the circle. $d = 2r$

Sector:



Given a sector of a circle where l is the length of the arc and A is the area of the sector:

$l = 2\pi r \times \frac{\theta}{360}$. $A = \pi r^2 \times \frac{\theta}{360}$.

Example 18. What is the area of a circle whose radius measures 4 cm?

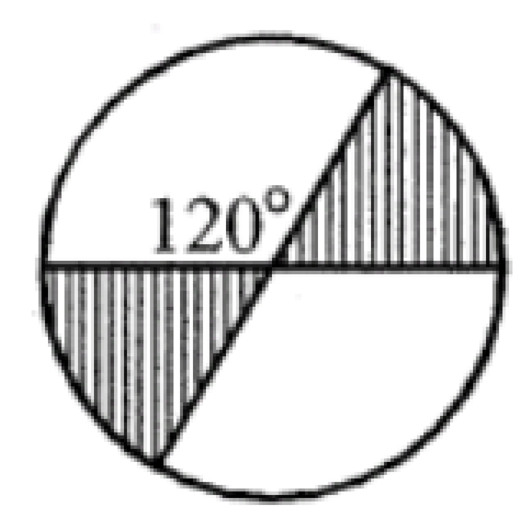
(A) 16π (B) 18π (C) 20π (D) 64π (E) 49π

Example 19. What is the radius of a circle whose area is 64π cm^2?

(A) 64 (B) 32 (C) 16 (D) 8 (E) 128

Example 20. Find in terms of π the number of square inches in the area of the shaded region formed by the intersecting diameters of a circle with radius 6.

(A) 16π (B) 12π (C) 20π (D) 64π (E) 36π

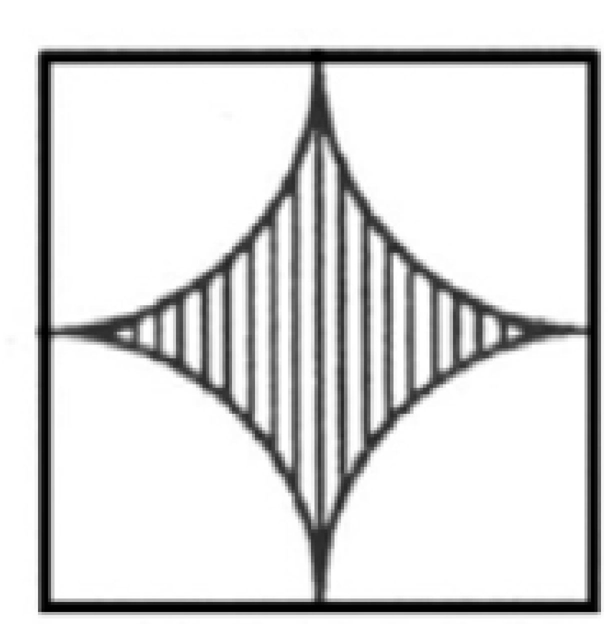


☆**Example 21.** Four circles of radius 3 are drawn with the centers at the vertices of a square. The regions inside the square are shown. Find the area of the shaded region.

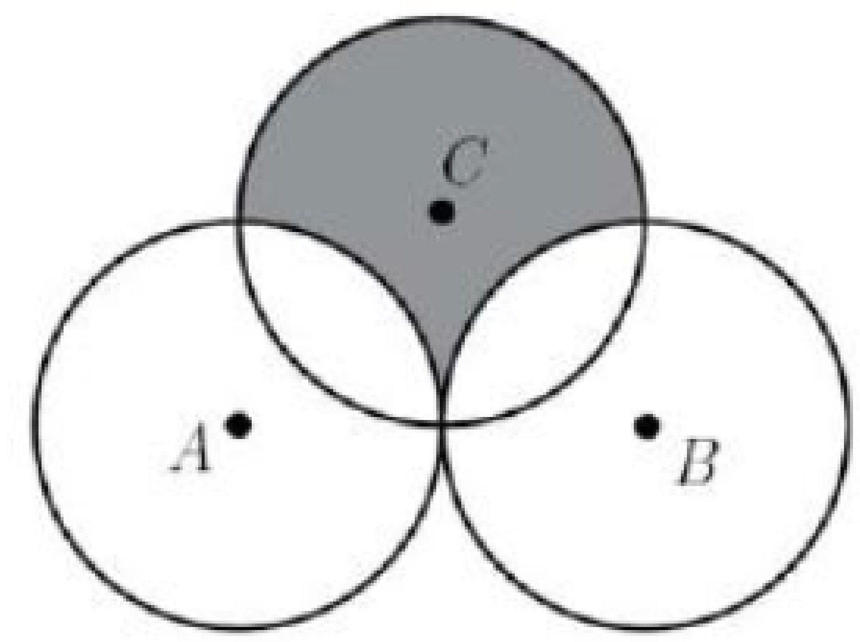
(A) $36-24\pi$ (B) $36-12\pi$

(C) $36-9\pi$ (D) $81-12\pi$

(E) $81-9\pi$

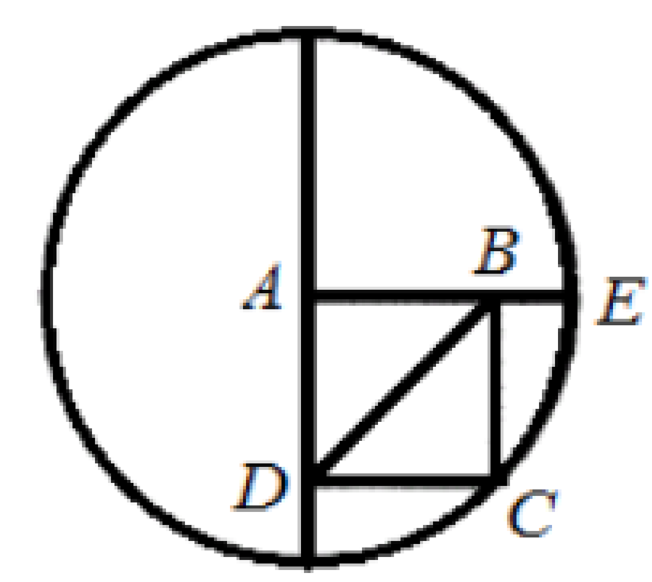


☆**Example 22.** (AMC 10) Circles A, B, and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of AB. What is the area inside circle C but outside circle A and circle B ?



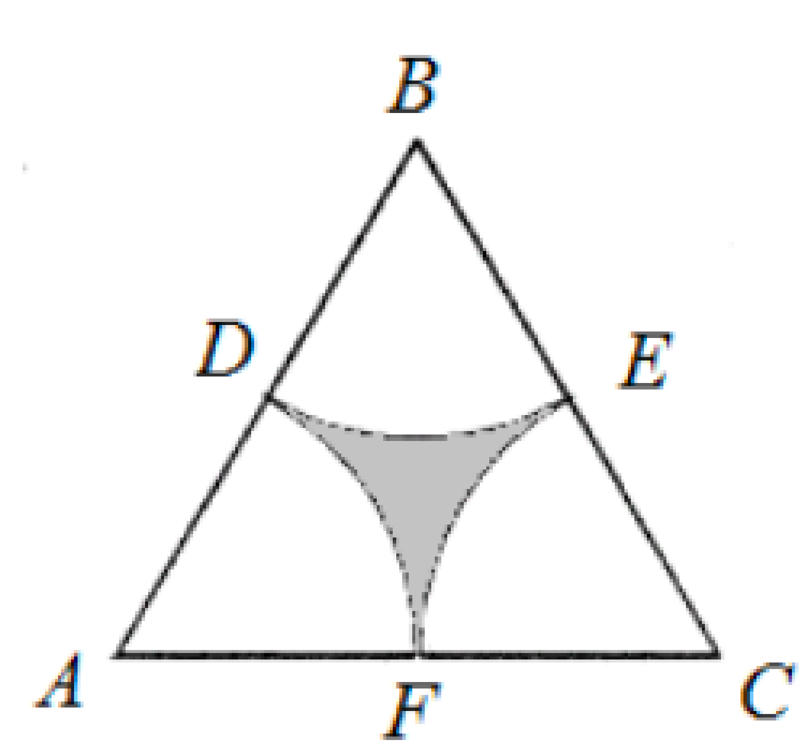
(A) $3-\frac{\pi}{2}$ (B) (C) 2 (D) $\frac{3\pi}{4}$ (E) $1+\frac{\pi}{2}$

Example 23. $ABCD$ is a square with vertex A at the center of the circle. AE = 10 in. What is the number of square inches in the area of ΔBCD?



(A) 100 (B) 50 (C) 25 (D) 8π (E) 12π

Example 24. The length of a side of equilateral triangle ABC is 12. D, E, and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively. A, B, and C are the centers of the circles that contain arcs DF, DE, and FE, respectively. What is the area of the shaded region?

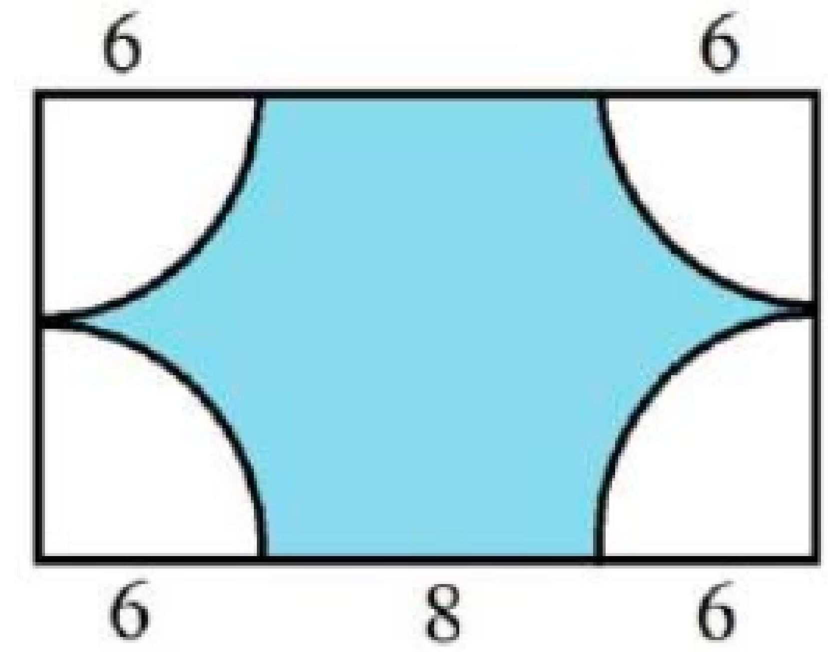


(A) $72\sqrt{3}-18\pi$ (B) $36\sqrt{3}-9\pi$

(C) $72\sqrt{3}-9\pi$ (D) $36\sqrt{2}-18\pi$

(E)

Example 25. In the rectangle shown, the radius of each quarter circle is 6. What is the area of the shaded region?



(A) $120 - 36\pi$

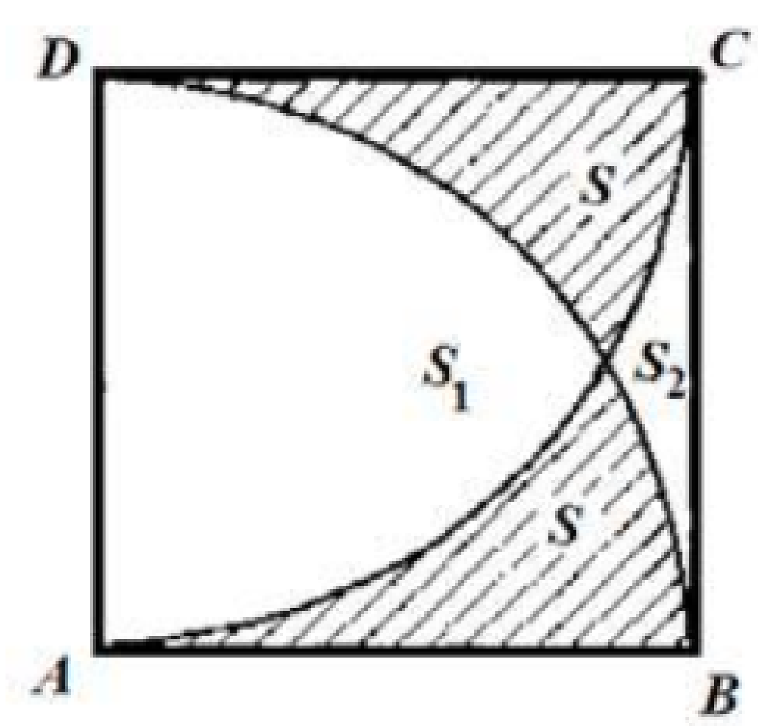
(B) 36π

(C) $200 - 36\pi$

(D) $120 - 18\pi$

(E) $240 - 36\pi$

Example 26. In square *ABCD*, $AB = 1$. *BD* and *AC* are arcs of radius 1. Two shaded areas are the same. Find the difference of the unshaded areas.

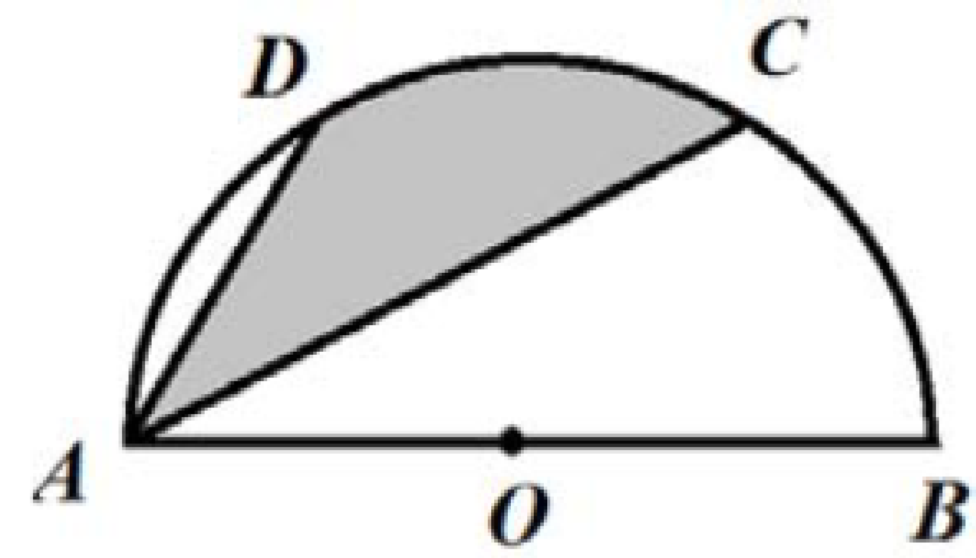


(A) $\frac{\pi}{2} - 1$. (B) $1 - \frac{\pi}{4}$. (C) $\frac{\pi}{3} - 1$.

(D) $1 - \frac{\pi}{6}$. (E) $2 - \frac{\pi}{2}$.

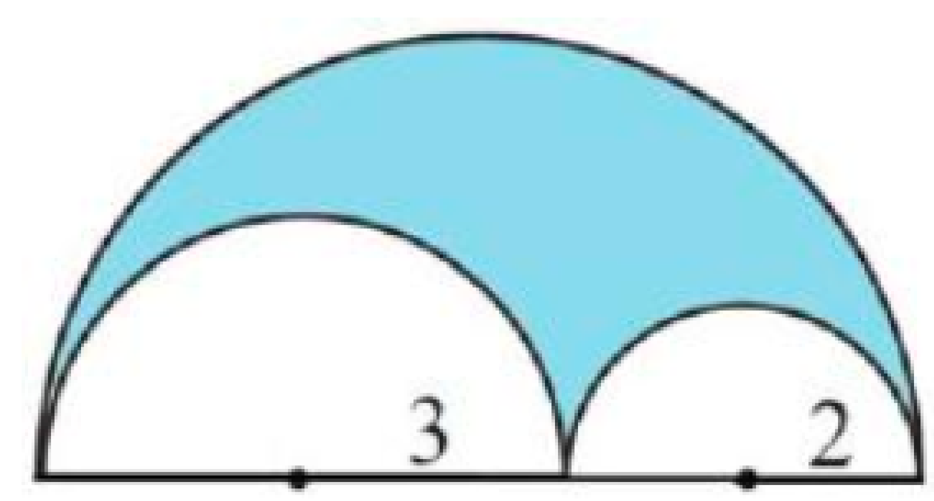
Example 27. D and C trisect the arc of the half circle as shown in the figure. Find the shaded area if the area of the half circle is 9π.

(A) 3π (B) 4π (C) 5π (D) 6π (E) 2π



☆**Example 28.** Semi-circles of radius 2 and 3 are externally tangent and are circumscribed by a third semi-circle, as shown in the figure. Find the area of the shaded region.

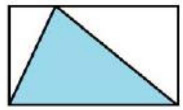
(A) 3π (B) 4π (C) 6π (D) 9π (E) 12π



PROBLEMS

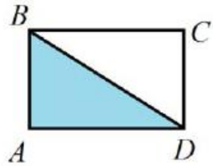
Problem 1. In the rectangle shown, the ratio of width to length is 1: 4. What percent of the rectangle is shaded?

(A) 80 (B) 20 (C) 50 (D) 44 (E) 30



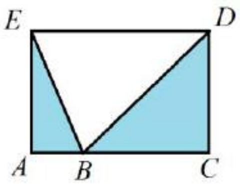
Problem 2. If the area of rectangle *ABCD* is 24, find the area of ΔABD.

(A) 20 (B) 12 (C) 10 (D) 8 (E) 6



Problem 3. In rectangle *ACDE*, *B* lies on $\overline{AC}$, $DC = 4$ cm, and $DE = 8$ cm. Find the area of the shaded region.

(A) 16 cm^2 (B) 32 cm^2 (C) 64 cm^2 (D) 8 cm^2 (E) 10 cm^2



Problem 4. If the perimeter of an equilateral triangle is 60, what is the area of the triangle?

(A) $200\sqrt{3}$ (B) $100\sqrt{3}$ (C) 300 (D) 400 (E) $50\sqrt{3}$

Problem 5. The sides of a triangle are 5, 12, and 13. What is the number of square units in the area of the triangle?

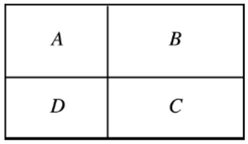
(A) 78 (B) 30 (C) 121 (D) 156 (E) 312

Problem 6. What is the number of square centimeters in the area of a triangle whose sides measure 8 cm, 15 cm, and 17 cm?

(A) 120 (B) 60 (C) 255 (D) 68 (E) 34

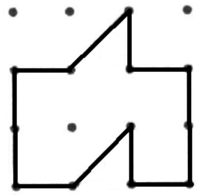
Problem 7. In the figure shown, the lengths and widths of rectangles A, B, C, and D are whole numbers. The areas of rectangles A, B, and C are $35, 45$, and 36, respectively. What is the area of the entire figure?

(A) 144 (B) 121 (C) 100 (D) 162 (E) 28



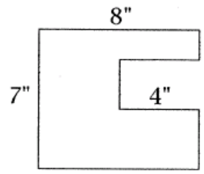
☆**Problem 8.** (AMC 8) Dots are spaced one unit apart, horizontally and vertically. The number of square units enclosed by the polygon is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9



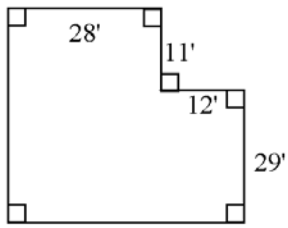
Problem 9. How many inches are in the perimeter of the following figure? All angles shown are right angles.

(A) 38 (B) 32 (C)3 (D) 48 (E) 24



Problem 10. How many square feet are there in the house with the dimensions shown in the figure?

(A) 1468 (B) 1600 (C) 900 (D) 1000 (E) 1100

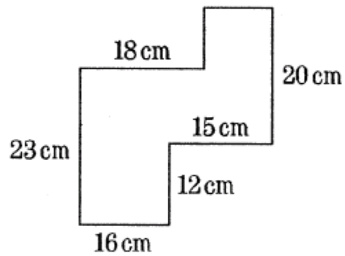


Problem 11. The perimeter of a rectangle is 46. The difference between the length and the width of the rectangle is 13. What is the area of the rectangle?

(A) 46 (B) 92 (C) 36 (D) 100 (E) 90

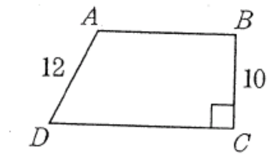
Problem 12. Given that all of the angles below are right angles, find the number of centimeters in the perimeter of the polygon.

(A) 136 (B) 129 (C) 125 (D) 126 (E) 128



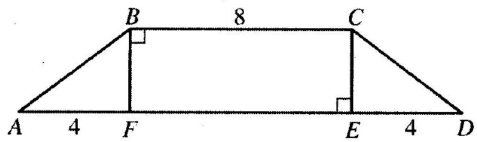
Problem 13. If the perimeter of trapezoid *ABCD* is 42 cm, what is the number of square centimeters in its area?

(A) 120 (B) 100 (C) 140 (D) 98 (E) 106



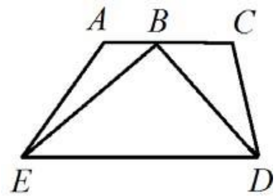
Problem 14. As shown in the figure, the area of trapezoid *ABCD* is 36. What is the length of *FB*?

(A) 8 (B) 2 (C) 4 (D) 3 (E) 5



Problem 15. Trapezoid *ACDE* has bases of lengths 16cm and 20 cm and area of 180 square centimeters. ΔBDE has the longer base of the trapezoid as one of its sides. *B* lies on the other base. Find the number of square centimeters in the area of ΔEBD.

(A) 200 (B) 140 (C) 100 (D) 180 (E) 120



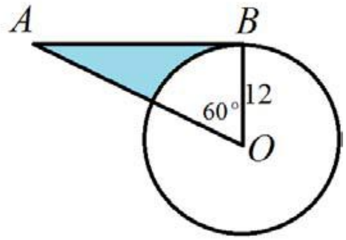
Problem 16. Find the area of a rhombus whose diagonals have length 4 and 9.

(A) 18 (B) 36 (C) 25 (D) 100 (E) 40

Problem 17. What is the radius of a circle whose perimeter is 64π cm?
(A) 64 (B) 32 (C) 16 (D) 8 (E) 128

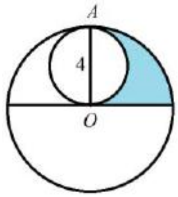
Problem 18. If the circumference of a circle is 8π, what is its area?
(A) 16π (B) 18π (C) 20π (D) 64π (E) 49π

Problem 19. In the figure, the center of the circle is O and $\overline{AB}$ is tangent to the circle at point B. What is the area of the shaded region?

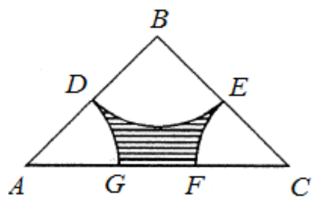


(A) $36\sqrt{3}-24\pi$ (B) $36\sqrt{3}-12\pi$
(C) $72\sqrt{3}-12\pi$ (D) $72\sqrt{3}-24\pi$
(E) $36\sqrt{3}-12\pi$

Problem 20. $\overline{OA}$ is the diameter of the smaller circle and the radius of the larger circle. How many square units are in the area of the shaded region?
(A) 16π (B) 8π (C) 4π (D) 2π (E) π



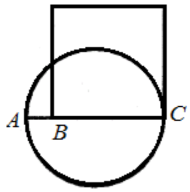
Problem 21. Isosceles right triangle ABC has legs of length 4cm with midpoints D and E. Three circles with centers A, B and C, respectively are drawn and the regions inside the triangle are shown. How many square centimeters are in the area of the shaded region?



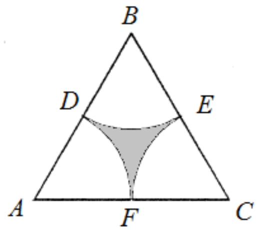
(A) $16-\pi$ (B) $16-2\pi$ (C) $8-2\pi$
(D) $8-\pi$ (E) 8

Problem 22. A square is constructed on diameter $\overline{AC}$ such that the area of the square is equal to the area of the circle. What percent of $\overline{AC}$ is $\overline{BC}$?

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\frac{\pi}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3\pi}{4}$ (E) $2-\frac{\pi}{2}$



Problem 23. The length of a side of equilateral triangle ABC is 2. D, E, and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively. A, B, and C are the centers of the circles that contain arcs DF, DE, and FE, respectively. What is the area of the shaded region?

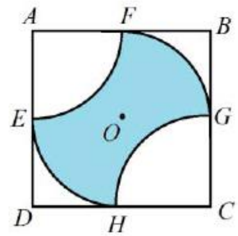


(A) $3\sqrt{2}-\frac{\pi}{2}$ (B) $\pi-\sqrt{3}$

(C) $2\sqrt{3}-\frac{\pi}{2}$ (D) $\sqrt{3}-\frac{\pi}{4}$

(E) $\sqrt{3}-\frac{\pi}{2}$

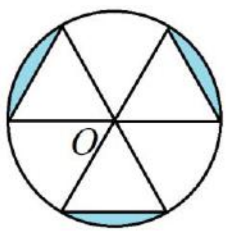
Problem 24. The figure shows a square with side of length 12. The center of the square is O, and E, F, G, and H are the midpoints of the sides. If the arcs shown have centers at A, O, and C, what is the area of the shaded region?



(A) 72 (B) $36+\frac{36\pi}{7}$ (C) $18\pi - 18$

(D) 12π (E) $36 - 12\pi$

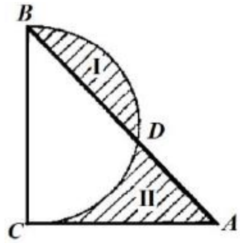
Problem 25. Circle O has a diameter of 20 cm and the triangles shown are equilateral. Find the percent of the circle that is shaded.



(A) $\frac{1}{2}-\frac{3\sqrt{3}}{2\pi}$ (B) $\frac{1}{2}-\frac{\sqrt{3}}{4\pi}$ (C) $\frac{1}{2}-\frac{\sqrt{3}}{\pi}$

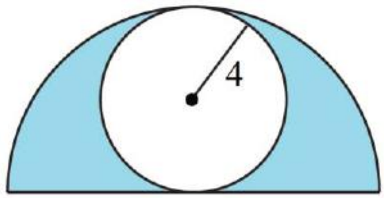
(D) $\frac{1}{2}-\frac{3\sqrt{3}}{4\pi}$ (E) $\frac{1}{2}-\frac{3\sqrt{3}}{\pi}$

Problem 26. As shown in the figure, right triangle ABC with $BC = 20\ cm$. BDC is a half circle with the diameter BC. The difference between two shaded areas I and II is 23. Find AC in terms of π.



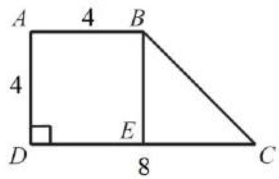
☆**Problem 27.** A circle of radius 4 is inscribed in a semicircle, as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?

(A) $\frac{1}{2}$ (B) $\frac{5\pi}{6}$ (C) $\frac{2}{\pi}$ (D) $\frac{2\pi}{3}$ (E) $\frac{3}{\pi}$.



☆ **Problem 28.** In trapezoid $ABCD$, AD is perpendicular to DC, $AD = AB = 4$, and $DC = 8$. In addition, E is on DC, and BE is parallel to AD. Find the area of ΔBEC.

(A) 4 (B) 8 (C) 12 (D) 18 (E) 10



BASIC KNOWLEDGE AND TERMS

Each pattern is created and arranged following a rule or rules. The key for solving pattern problems is to identify the core of the patterns.

Typical AMC 8/Mathcounts pattern problems can be classified as the following:

Growing patterns: Growing patterns have a sequence of elements that increase or decrease systematically when viewed as a recursive pattern.

Sequences pattern: Sequences pattern is a pattern of an ordered set of numbers or mathematical entities.

Repeating patterns: Repeating patterns can be generalized by recognizing pattern families that can look different but have the same core.

Geometric Patterns: A geometric pattern is a pattern that has repeating shapes such as dots, lines, triangles, circle, rectangles, and polygons.

USEFUL FORMULAS

Arithmetic sequence:

If any two consecutive terms in a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots,$ have the same difference, the sequence is an arithmetic sequence.
The difference is called the common difference.

$$d = a_{n+1} - a_n$$

The nth term is expressed as

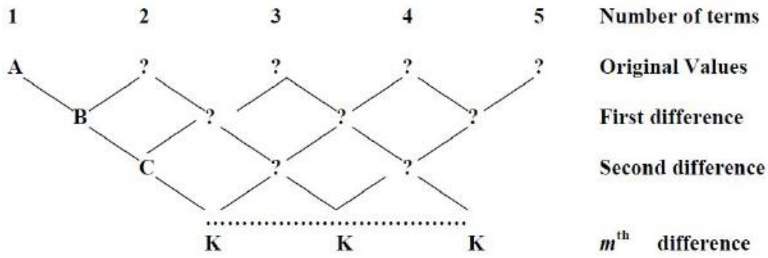
$$a_n = a_1 + (n-1)d$$

The sum of n terms in the sequence:

$$S = na_1 + \frac{(n-1)d}{2}n \quad \text{or} \quad S = \frac{(a_1 + a_n)n}{2}.$$

Newton's Little Formula for *n*th term:

For an arithmetic sequence of high order:



$$a_n = A\binom{n-1}{0} + B\binom{n-1}{1} + C\binom{n-1}{2} + \cdots\cdots + K\binom{n-1}{m}$$

Geometric sequence

If any two consecutive terms in a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots,$ have the same ratio, the sequence is called a geometric sequence (or geometric progression).

a_1 is the **first term**.

a_n is the **general term** or n^{th} term. $a_n = a_1 \cdot q^{n-1}$

The same ratio is called the **common ratio** (q or r).

The sum of the first n terms is expressed as S_n. For example, S_{12} means the sum of the first twelve terms.

$$S_n = \frac{a_1(1-q^n)}{1-q}.$$

GROWING PATTERNS

Example 1. Consider the following pattern:

$$\sqrt{1+1\cdot 2\cdot 3\cdot 4}=5$$
$$\sqrt{1+2\cdot 3\cdot 4\cdot 5}=11$$
$$\sqrt{1+3\cdot 4\cdot 5\cdot 6}=19$$
$$\sqrt{1+4\cdot 5\cdot 6\cdot 7}=29$$

Find $\sqrt{1+50\cdot 51\cdot 52\cdot 53}$

(A) 2550 (B) 2651 (C) 2652 (D) 2756 (E) 2703

Example 2. If the same pattern is continued, what is the number of 1's in the result of the calculation in the eighth line of the pattern?

1 × 9 + 2 = ________

12 × 9 + 3 = ________

123 × 9 + 4 = ________

(A) 4 (B) 6 (C) 8 (D) 9 (E) 10

Example 3. Look for a pattern in the following and then determine the value of *n*:

$$121 = \frac{22 \times 22}{1+2+1}$$

$$12321 = \frac{333 \times 333}{1+2+3+2+1}$$

$$n =$$

The sum of the digits of *n* is:

(A) 14 (B) 16 (C) 18 (D) 19 (E) 20

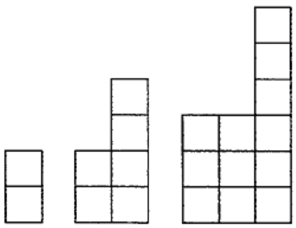
Example 4. Use the pattern given to express 100^2 in the form $a^2 + b^2 - c^2$. What is the value $a + b + c$?

$12^2 = 8^2 + 9^2 - 1^2$
$14^2 = 10^2 + 10^2 - 2^2$
$16^2 = 12^2 + 11^2 - 3^2$
$18^2 = 14^2 + 12^2 - 4^2$

(A) 198 (B) 153 (C) 145 (D) 196 (E) 194

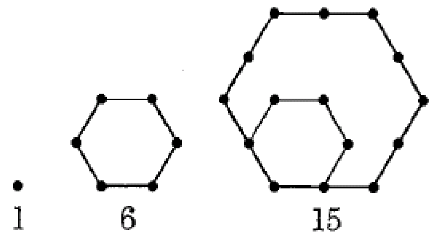
Example 5. The first three towers in a sequence are shown. The *n*th tower is formed by stacking *n* blocks on top of an $n \times n$ square of blocks. How many blocks are in the 99th tower?

(A) 9900 (B) 9816 (C) 9818 (D) 9919 (E) 9801



Example 6. The first three hexagonal numbers are represented as shown. Find the sum of the first five hexagonal numbers.

(A) 44 (B) 45 (C) 48 (D) 39 (E) 50



SEQUENCES PATTERN

☆**Example 7.** Terri produces a sequence of positive integers by following three rules. She starts with a positive integer, then applies the appropriate rule to the result, and continues in this fashion.
Rule 1: If the integer is less than 10, multiply it by 9.
Rule 2: If the integer is even and greater than 9, divide it by 2.
Rule 3: If the integer is odd and greater than 9, subtract 5 from it.

A sample sequence: 23, 18, 9, 81, 76, ...
Find the 198*th* term of the sequence that begins 49, 44. . . .
(A) 54 (B) 6 (C) 22 (D) 49 (E) 11

Example 8. All positive integers appear in the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, . . ., and each positive integer *k* appears in the sequence *k* times. In the sequence, each term after the first is greater than or equal to each of the terms before it. If the integer 12 first appears in the sequence as the *n*th term, what is the value of *n*?
(A) 64 (B) 67 (C) 65 (D) 66 (E) 62

Example 9. Complete the pattern: 10, 15, 22.5, 33.75, ———
(A) 44.85 (B) 55.95 (C) 40.675 (D) 50.625 (E) 50

Example 10. The first term of a sequence is 5 and each subsequent term is 5 less than twice the preceding term. What is the eighth term?
(A) 5 (B) 6 (C) 2 (D) 4 (E) 8

Example 11. What is the 50^{th} letter in this pattern: *ABCAABBCCAAABBBCCC* . ...?
(A) *A* (B) *B* (C) *C* (D) *D* (E) *E*

Example 12. A sequence is formed by writing the word COMPETITIONS over and over again. What is the 496th letter in this sequence?

(A) C (B) O (C) M (D) P (E) E

Example 13. The sequence 0, 1, 2, 2, 3, 3, 0, 1, 2, 2, 3, 3, . . .repeats every six terms. The first term is 0. What is the 998th term?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 998

Example 14. The first six terms of a sequence are 1, –2, 3, –4, 5, –6. . . .. The odd-numbered terms are increasing consecutive positive odd integers starting with 1. The even-numbered terms are decreasing consecutive negative even integers starting with –2. What is the sum of the 50th and 51st terms of the sequence?

(A) –101 (B) –1 (C) 0 (D) 1 (E) 101

REPEATING PATTERNS

Repeating patterns can be generalized by recognizing pattern families that can look different but have the same core.

Example 15. What is the 100th digit of the decimal representation of $\frac{1}{7}$?

(A) 1 (B) 4 (C) 2 (D) 8 (E) 7

Example 16. What is the 17th digit after the decimal point in the decimal expansion of $\frac{11}{7}$?

(A) 5 (B) 4 (C) 2 (D) 8 (E) 7

Example 17. What is the 123,999th digit after the decimal in the decimal expansion of $\frac{123}{999}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 9

Example 18. Starting with a green bead, colored beads are placed on a string according to the pattern green, red, blue, yellow, white, orange. If this pattern is repeated, what is the color of the 51st bead?

(A) Green (B) Red (C) Blue (D) Yellow (E) White

Example 19. The table shown shows Pythagorean triples for which $c = b + 1$. Find the value of c when $a = 15$.

a	b	c
3	4	5
5	12	13
7	24	25
9	40	41

(A) 110 (B) 111 (C) 112 (D) 113 (E) 115

Example 20. The whole numbers are written consecutively in rows as shown. Each row contains two more numbers than the previous row. What is the number of the row in which the number 1,300 is listed?

Row 1					0				
Row 2				1	2	3			
Row 3			8	7	6	5	4		
Row 4		9	10	11	12	13	14	15	
Row 5	24	23	22	21	20	19	18	17	16

(A) 35 (B) 36 (C) 37 (D) 38 (E) 39

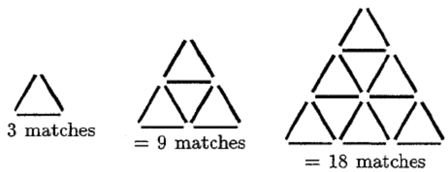
Example 21. The lattice shown is continued for 100 rows. What will be the third number in the 100^{th} row?

Row 1:	1	2	3	4	5	6	7
Row 2:	8	9	10	11	12	13	14
Row 3:	15	16	17	18	19	20	21
Row 4:	22	23	24	25	26	27	28

(A) 696 (B) 695 (C) 697 (D) 694 (E) 99

GEOMETRIC PATTERNS

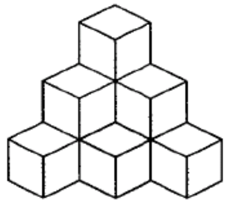
Example 22. Referring to the sketches, it is seen that 3, 9, and 18 matches are required to make the triangular patterns depicted, respectively. How many matches would be needed to construct a similar figure with a ten match-stick base?



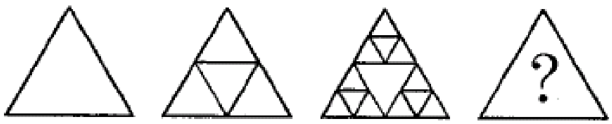
(A) 108 (B) 162 (C) 163 (D) 165 (E) 167

Example 23. The diagram shows an arrangement of 10 cubes in 3 layers. How many cubes will it take to make 8 layers?

(A) 116 (B) 118 (C) 120 (D) 124 (E) 144



Example 24. By continuing the pattern shown, how many non-overlapping triangles would appear in the last figure?



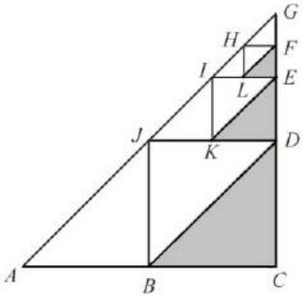
(A) 36 (B) 38 (C) 40 (D) 44 (E) 99

Example 25. Each time a change occurs, the central one-fourth of every white equilateral triangle is shaded. What fractional part of the original equilateral triangle would be shaded after four changes? Express your answer as a common fraction.

(A) (B) $\frac{173}{256}$ (C) $\frac{43}{64}$ (D) $\frac{177}{256}$ (E) $\frac{41}{256}$

Example 26. Thirty-six cannon balls are placed on a flat surface in the shape of a square to form the base of a display beside the cannon. How many additional cannonballs are needed to form a "pyramid" with the given square base?
(A) 55 (B) 91 (C) 40 (D) 36 (E) 99

☆**Example 27.** Points B, D, and J are midpoints of the sides of right triangle ACG. Points K, E, I are midpoints of the sides of triangle JDG, etc. If the dividing and shading process is done 101 times (the first three are shown) and $AC = CG = 12$, then the total area of the shaded triangles is nearest to
(A) 24 (B) 12 (C) 18 (D) 19 (E) 20



Example 28. A grocer stacks apples in the shape of a square pyramid. The bottom layer is a 10×10 square, the top layer is one apple, and the nth layer is an $n \times n$ square. How many apples does she have in the pyramid?

(A) 368 (B) 385 (C) 340 (D) 440 (E) 399

PROBLEMS

Problem 1. Find the numerical value x_8, if

$x_0 = 1^0$

$x_1 = 2^0 + 2^1$

$x_2 = 4^0 + 4^{\frac{1}{2}} + 4^1$

$x_3 = 8^0 + 8^{\frac{1}{3}} + 8^{\frac{2}{3}} + 8^1$

$x_4 = 16^0 + 16^{\frac{1}{4}} + 16^{\frac{1}{2}} + 16^{\frac{3}{4}} + 16^1$

(A) 512 (B) 511 (C) 256 (D) 1024 (E) 1023

Problem 2. Look for a pattern:

$$11 \times 11 = 121$$
$$111 \times 111 = 12321$$
$$1111 \times 1111 = 1234321$$

Find the value of n: $111111 \times 111111 = n$

Problem 3. Look for a pattern:

$1^3 = 1^2 - 0^2$

$2^3 = 3^2 - 1^2$

$3^3 = 6^2 - 3^2$

$\vdots$

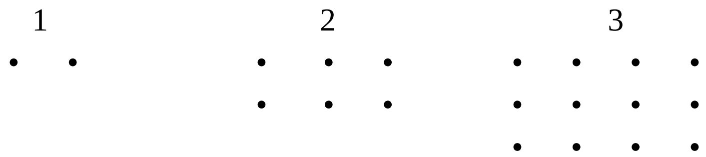
$6^3 = n^2 - m^2$

What is the value of $m + n$?

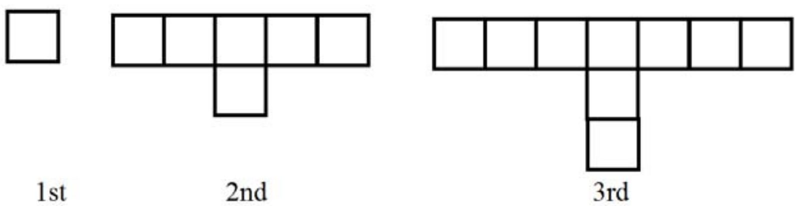
Problem 4. Follow the pattern to determine the value of 8(23456789) + 9.

$$8(2) + 2 = 18$$
$$8(23) + 3 = 187$$
$$8(234) + 4 = 1876$$
$$8(2345) + 5 = 18765$$
$$8(23456) + 6 = 187654$$

Problem 5. The first figure contains 2 dots, the second 6 dots, and the third 12 dots. If the pattern continues, how many dots would the tenth figure contain?



Problem 6. Each arrangement of squares is formed from the preceding arrangement by adding two additional squares to each end of the horizontal row and one square to the vertical column. How many squares will be in the sixth figure in the sequence?



Problem 7. If the pattern continues, what is the next term in the sequence 1, 7, 25, 61, 121, . . . ?

Problem 8. Complete the pattern: 40.5, 9, 2, ———

Problem 9. Find the next decimal term in the sequence:

$0, 0.5, 0.\overline{6}, 0.75, \ldots$

Problem 10. What is the 100th letter in the pattern *ABCABCABC* . . . ?

Problem 11. The sequence shown was formed by writing the first letter of the alphabet followed by writing the first two letters of the alphabet and continuing the pattern by writing one more letter of the alphabet each time. Continuing this pattern, what letter is the 280^{th} letter in this sequence?

A, A, B, A, B, C, A, B, C, D, A, B, C, D, E, . . .

Problem 12. A sequence of letters is formed by writing 1 A, 2 B's, 3 C's, and so forth, increasing the number of letters written by one each time the next letter of the alphabet is written. What is the 200^{th} letter in the sequence?

Problem 13. Begin with the 200-digit number 987654321098765 . . . 543210, which repeats the digits 0-9 in reverse order. From the left, choose every third digit to form a new number. Repeat the same process with the new number. Continue the process repeatedly until the result is a two-digit number. What is the resulting two-digit number?

Problem 14. What is the 1997^{th} digit to the right of the decimal point in the decimal expansion of $\frac{1}{7}$?

Problem 15. What is the 199^{th} digit of the decimal representation of $\frac{3}{37}$?

Problem 16. What is the 125^{th} digit beyond the decimal point in the decimal representation of $\frac{4}{7}$?

Problem 17. The positive odd integers are arranged in 5 columns, A, B, C, D, and E, continuing the pattern shown. In which column will 1599 appear?

A	B	C	D	E
	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	. . .	

Problem 18. The multiples of 3 are arranged in the following manner:

Column 1	Column 2	Column 3	Column 4
3	6	9	12
21	18	15	12
21	24	27	30
39	36	33	30
39	42	…	…

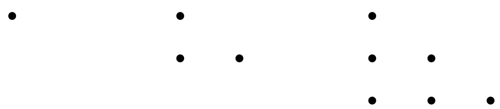
In which column will the number 1992 appear?

Problem 19. The natural numbers from 1 to 1,000 are arranged consecutively from left to right in a triangle as shown. Each row contains one more number than the row below. What number is directly above 723?

. . .				
11	12	13	14	15
7	8	9	10	
4	5	6		
2	3			
1				

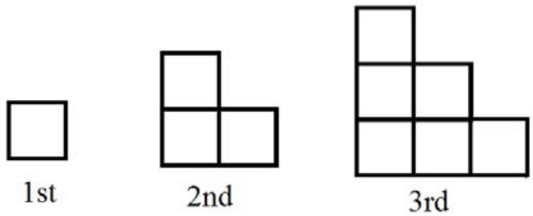
Problem 20. Triangular numbers can be represented by a triangular array. For example, 1, 3, and 6 can be represented as:

1 3 6

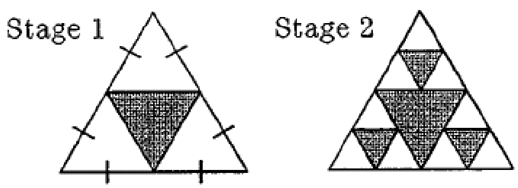


The difference of a pair of consecutive triangular numbers is 12. Find their sum.

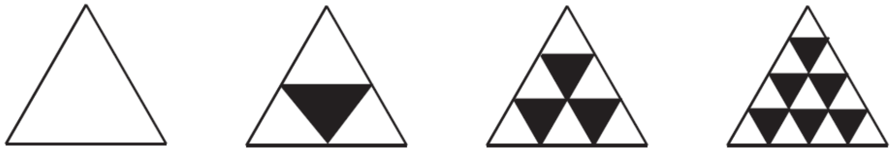
Problem 21. Squares are used to build the following sequence of drawings. If the length of a side of each square is one unit, how many units are in the perimeter of the 8^{th} drawing?



Problem 22. At each stage the midpoints of the sides of each unshaded equilateral triangle are connected and the triangle formed is shaded. Continuing in this process, what is the number of the stage when the shaded area is first larger than 90% of the area of the original equilateral triangle?

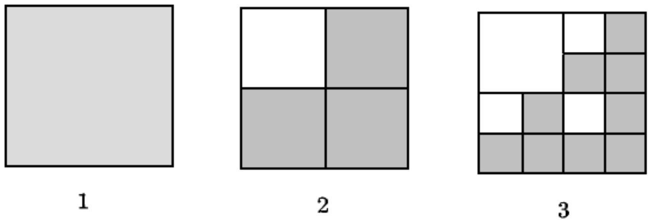


☆**Problem 23.** If the pattern in the diagram continues, what fraction of the interior would be shaded in the ninth triangle?

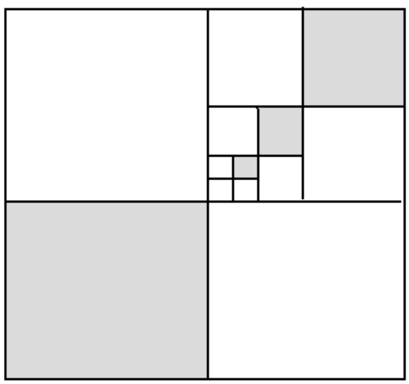


(A) 3/8 (B) 5/27 (C) 79/16 (D) 4/9 (E) 11/45

Problem 24. As you proceed from term to term, each shaded square is divided into four congruent squares and the upper left square of the four is painted white. By continuing the pattern, what fractional part of the tenth figure will be shaded? Express your answer as a common fraction in which the numerator and denominator are expressed in prime factored form using exponents.

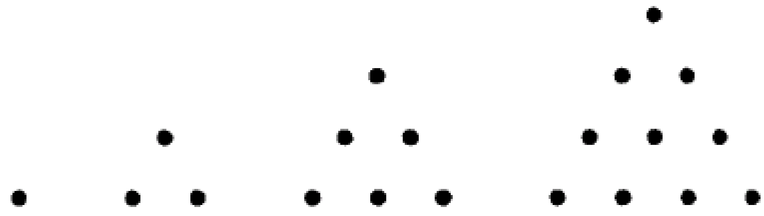


Problem 25. Each of the figures is a square formed by connecting midpoints of opposite sides of a larger square. What fraction of the largest square is shaded?

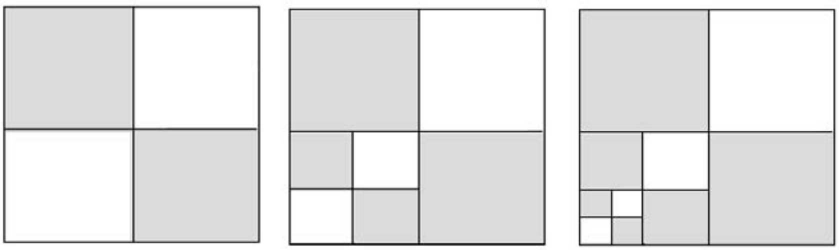


Problem 26. The first four triangular numbers are pictured. The *n*th triangular number is formed by drawing a row of *n* dots below the (n – 1)st triangular dot

pattern. The kth triangular number is represented by 120 dots. What is the value of k?

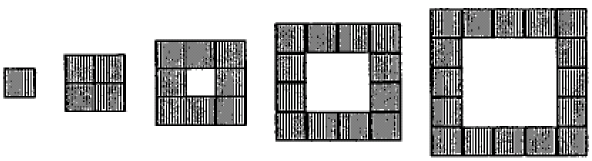


Problem 27. At each stage, the square at the lower left is divided into 4 congruent square regions, 2 of which are shaded. The area of the entire square (including shaded and unshaded parts) is 256 square units. How many square units are in the shaded area at the fifth stage? Express your answer as a decimal.



Stage 1 Stage 2 Stage 3

Problem 28. The "border number" of an $n \times n$ square is defined as the number of unit squares whose edges border the edges of a larger square. The border numbers of 1×1, 2×2, 3×3, 4×4, and 5×5 squares are illustrated. What is the border number of a 20 unit by 20 unit square?



1 4 8 12 16

BASIC KNOWLEDGE REVIEW

Statements

Examples:

Boston is a city in USA.
$1 + 1 = 3$
A spider does not have six legs.

The following sentences are not statements:

Do your homework.	(a command)
How do you solve this math problem?	(a question)
SAT test is harder than ACT test.	(an opinion)
This sentence is false.	(a paradox)

Negations

The sentence "SAT math test consists of 54 problems" is a statement; the negation of this statement is
The negation of a true statement is false, and the negation of a false statement is true.

Statement	Negation
All do Some do	

Examples: Form the negation of each statement:

The moon is not a star. $\Rightarrow$
The moon is a star. $\Rightarrow$
A spider does not have six legs. $\Rightarrow$

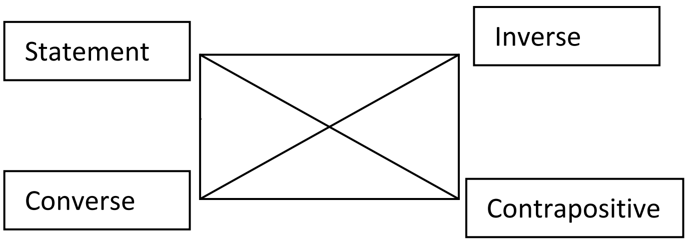
Some rabbits have short tails. $\Rightarrow$
Some rabbits do not have short tails. $\Rightarrow$
No rabbit has a short tail. $\Rightarrow$

Converse, Inverse, and Contrapositive

Direct statement	If p, then q.
Converse	If q, then p.
Inverse	If not p, then not q.
Contrapositive	If not q, then not p.

Direct statement	If I live in Boston, then I live in USA.
Converse	
Inverse	
Contrapositive	

Rectangle of logical equivalent



Logically equivalent pair of statements (diagonally opposite):

A statement and its contrapositive

The inverse and converse of the same statement

Not logically equivalent pair of statements (adjacent):

A statement and its inverse

A statement and its converse

The converse and contrapositive of the same statement
The inverse and contrapositive of the same statement

Examples:

Statement:	A square is a rectangle	(true)
Converse	A rectangle is a square	(false)
Inverse	A figure that is not a square is not a rectangle	(false)
Contrapositive	A figure that is not a rectangle s is not a square	(true)

Euler Diagram

Deductive reasoning consists of three steps as follows:

(1). Making a general statement (major premise).
(2). Making a particular statement (minor premise).
(3). Making a deduction (conclusion).

Example:

(1). The major premise is: All cats are animals
(2). The minor premise is: Jerry is a cat.
(3). The conclusion is: Jerry is an animal.

Procedures to draw the diagram:

(1) Draw a big circle to represent the first premise. This is the region for "animals".
(2) Draw a second circle to represent "all cats". Since all cats are animals, the second circle goes inside the first big circle.
(3) Put Jerry inside where it belongs. The second premise stated that Jerry is a cat. Put Jerry inside the region marked "Cats".

Example: Is the following argument valid? An argument is valid if that the premises are true and these premises force the conclusion to be true.

All apple trees have green leaves
That plant has green leaves.
That plant is an apple tree.

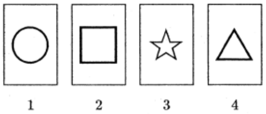
PROBLEM SOLVING SKILLS

(1). Find The Correct Order By Switching Positions

Example 1. Alexis, Britt, Carol, Danielle and Elizabeth are waiting in line. Alex is behind Carol but ahead of Danielle. Elizabeth is ahead of Britt, but behind Carol. Danielle is ahead of Britt. Who is first in line?

(2). Find The Contrapositive Of The Statement

Example 2. Each card has either a circle or a star on one side and either a triangle or a square on the other side. In order to verify the statement "every card with a star on it also has a triangle on it," which numbered card(s) must be turned over?



(3). Find Two Statements That Are Contradicted To Each Other

Example 3. There are three boxes with different colors: red, yellow and blue. One apple is in one of the three boxes. Only one of the following statements is true, and the others are false.

I: Apple is in the red box; II Apple is not in the yellow box, and III: Apple is not in the red box.

Which box is the apple in?

(4). Find Two Statements That Are In Agreement With Each Other

Example 4. Each of three marbles *A*, *B*, and *C*, is colored one of the three colors. One of the marbles is colored white, one is colored red, and one is colored blue. Exactly one of these statements is true:

1) *A* is red. 2) *B* is not blue. 3) *C* is not red.

What color is marble B?

(5). Focus On The Step Before The Last

Example 5. A turtle crawls up a 12 foot hill after a heavy rainstorm. The turtle crawls 4 feet, but when it stops to rest, it slides back 3 feet. How many tries does the turtle make before it makes it up the hill?

(6). Dividing Into Three Groups

When you need to weigh a number of coins with counterfeit coin, divide the coins into three groups with the number of coins in each group: m, m, m, or m, m, $m - 1$ or m, m, $m + 1$.

Example 6. A jeweler has four small bars that are supposed to be gold. He knows that one is counterfeit and the other three are genuine. The counterfeit bar has a slightly different weight than a real gold bar. Using a balance scale, what is the minimum number of weighings necessary to guarantee that the counterfeit bar will be detected?

(7). Drawing Solid and Dash Lines

Example 7. Three friends – math teacher Mr. White, science teacher Mr. Black, and history teacher Mr. Redhead – met in a cafeteria. "It is interesting that one of us has white hair, another one has black hair, and the third has red hair, though no one's name gives the color of their hair" said the black-haired person. "You are right," answered White. What color is the history teacher's hair?

(8). Back one step and forward two

Example 8. There's a box of three hats: one black and two white. Andy and Betsy (each very smart and very logical) each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own head. If any of the two guesses correctly, everyone gets to go to the park!

First, Betsy removes her blindfold. She sees the hats that Andy is wearing, but admits that she is unable to discern her own hat color.

Then Andy says: “I can answer with my blindfold on! I know what color hat I am wearing." What color is Andy's hat?

Example 9. There's a box of five hats: two black and three white. Andy, Betsy, and Charles (each very smart and very logical) each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own head. If any of the three guesses correctly, everyone gets to go to the park!

First, Charles removes his blindfold. He sees the hats that the others are wearing, but admits that he is unable to discern his own hat color.

Next, Betsy removes her blindfold, and sadly reveals that she too is not able to determine the color of her own hat.

Finally, Andy pipes up and says “I can answer with my blindfold on! I know what color hat I am wearing." What color is Andy's hat?

(9). Squeezing method

Example 10. (2014 Mathcounts National) Larry tells Mary and Jerry that he is thinking of two consecutive integers from 1 to 10. He tells Mary one of the numbers, and he tells Jerry the other number. Then the following conversation occurs between Mary and Jerry:
Mary: I don’t know your number.
Jerry: I don’t know your number, either.

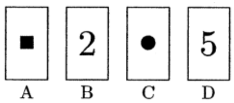
Mary: Ah, now I know your number.
Assuming both Mary and Jerry used correct logic, what is the sum of the possible numbers Mary could have?

MORE EXAMPLES

Example 11. Squares are faster than circles, hexagons are slower than triangles, and hexagons are faster than squares. Which of these shapes is the slowest?

(A) Squares (B) Circles (C) Hexagons (D) Triangles (E) None of them

Example 12. Four cards are constructed so that there is either a circle or a square on one side and an odd or even number on the other side. The cards are placed on a table as shown. Which cards must be turned to prove the following: Every square has an even number on the other side?



Example 13. Classroom window was broken. The principal had four students in his office. He knew that one of them did it, and he also knew that only one of the students told the truth, but not sure which one.
Alex said: Bob did;

Bob said: Dean did;
Cam said: not me;
Dean said: Bob lied.

Who broke the window?

Example 14. A sealed envelope contains a card with a single digit on it. Three of the following statements are true, and the other is false.
I. The digit is 1.
II. The digit is not 2.
III. The digit is 3.
IV. The digit is not 4.
Which one of the following must necessarily be correct?

(A) I is true. (B) I is false. (C) II is true. (D) III is true. (E) IV is false

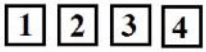
Example 15. A centipede climbs a 40-foot tree. Each day he climbs 5 feet, and each night he slides down 3 feet. In how many days will the centipede reach the top of the tree?

(A) 19 (B) 18 (C) 17 (D) 20 (E) 21

Example 16. Alex has 6 coins. Five of the 6 coins weigh the same and one coin is heavier. If Alex had a balance scale, what is the least number of times he could weigh coins to be sure he could determine which coin was heavier?

Example 17. In a horse race game on a computer, Secretariat, Man-Of-War, Affirmed and Citation finished in first through fourth places (not necessarily in that order), with no ties. Man-Of-War finished second or fourth. Affirmed did not win the race. Citation or Secretariat finished third. Man-Of-War beat Secretariat. What is the name of the horse that finished fourth?

Example 18. Below are the four labeled boxes. Each box is painted a different color. There is a red box, which is next to a blue box. There is a green box, which is next to the red box and a yellow box. Which box could be painted red?



(A) 1 only (B) 2 only (C) 3 only (D) 2 or 3 (E) 1 or 4

☆**Example 19.** (AMC 8) Amy, Bill and Celine are friends with different ages. Exactly one of the following statements is true.
I. Bill is the oldest.
II. Amy is not the oldest.
III. Celine is not the youngest.
Rank the friends from the oldest to the youngest.
(A) Bill, Amy, Celine (B) Amy, Bill, Celine (C) Celine, Amy, Bill
(D) Celine, Bill, Amy (E) Amy, Celine, Bill

☆**Example 20.** (AMC 8) Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?

(A) Alice (B) Ben (C) Cindy (D) Dave (E) Ellen

PROBLEMS

Problem 1. There are 9 apparently identical balls, except that one is heavier than the other 8. What is the smallest number of balance scale weighings required to ensure identification of the “odd” ball?

(A) 9 (B) 3 (C) 4 (D) 1 (E) 2

Problem 2. A kitchen pantry has five shelves, each containing a specific kind of food. The spices are on the shelf directly below the vegetables, the fruits are above the bread, and the vegetables are 3 shelves below the cereals. Which kind of food is on the third shelf?

(A) vegetables (B) fruits (C) bread (D) cereals (E) spices

Problem 3. At Hope Middle School, Mr. Eye, Mr. Love and Mr. Problems teach science, mathematics, and history—but not necessarily in that order. The history teacher, who was an only child, has the least experience. Mr. Problems, who married Mr. Eye’s sister, has more experience than the science teacher. Who teaches science?

Problem 4. Five coins look the same, but one is a counterfeit coin with a different weight than each of the four genuine coins. Using a balance scale, what is the least number of weighings needed to ensure that, in every case, the counterfeit coin is found and is shown to be heavier or lighter?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

Problem 5. A centipede climbs a 40-foot tree. Each day he climbs 5 feet, and each night he slides down 2 feet. In how many days will the centipede reach the top of the tree?

(A) 14 (B) 13 (C) 12 (D) 8 (E) 20

Problem 6. Adam, Ben, Charles, David and Ed were waiting in line. Adam is between Ben and Chase. Ben is between David and Adam. Ed is also between David and Adam. Ben is between David and Ed. Who is in the middle of the line?

(A) Adam (B) Ben (C) Charles (D) David (E) Ed

Problem 7. Five cards are lying on a table as shown. Each card has a letter on one side and a whole number on the other side. Jane said, “If a vowel is on one side of any card, then an even number is on the other side.” Mary showed Jane was wrong by turning over one card. Which card did Mary turn over?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

Problem 8. A centipede crawl a tree 75-inches high, starting from the ground. Each day it crawls 5 inches, and each night it slides down 4 inches. When will it first reach the top of the tree?
(A) 15 (B) 18 (C) 19 (D) 72 (E) 71.

Problem 9. There are 4 cards on the table with the symbols *a*, *b*, 4, and 5 written on their visible sides. What is the smallest number of cards we need to turn over to find out whether the following statement is true: “If an even number is written on one side of a card then a vowel is written on the other side?

Problem 10. Each of the cards shown below has a number on one side and a letter on the other. How many of the cards must be turned over to prove the correctness of the statement: Every card with a vowel on one side has a prime number on the other side.

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

Problem 11. Three kids are playing pitcher, catcher and infielder. Sam is not the catcher. The infielder lives next to Sam. The catcher and John go to the same school. What position does Alex play?

Problem 12. Cookies were missing, taken by either Alex, Bob, or Charles.
Each person said:

Alex: I did not take the cookies.
Bob: Charles took the cookies.
Charles: That is true

If at least one of them lied and at least one told the truth, who took the cookies?

PROBLEMS

Problem 1. There are 9 apparently identical balls, except that one is heavier than the other 8. What is the smallest number of balance scale weighings required to ensure identification of the “odd” ball?
(A) 9 (B) 3 (C) 4 (D) 1 (E) 2

Problem 2. A kitchen pantry has five shelves, each containing a specific kind of food. The spices are on the shelf directly below the vegetables, the fruits are above the bread, and the vegetables are 3 shelves below the cereals. Which kind of food is on the third shelf?
(A) vegetables (B) fruits (C) bread (D) cereals (E) spices

Problem 3. At Hope Middle School, Mr. Eye, Mr. Love and Mr. Problems teach science, mathematics, and history—but not necessarily in that order. The history teacher, who was an only child, has the least experience. Mr. Problems, who married Mr. Eye’s sister, has more experience than the science teacher. Who teaches science?

Problem 4. Five coins look the same, but one is a counterfeit coin with a different weight than each of the four genuine coins. Using a balance scale, what is the least number of weighings needed to ensure that, in every case, the counterfeit coin is found and is shown to be heavier or lighter?
(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

Problem 5. A centipede climbs a 40-foot tree. Each day he climbs 5 feet, and each night he slides down 2 feet. In how many days will the centipede reach the top of the tree?
(A) 14 (B) 13 (C) 12 (D) 8 (E) 20

Problem 6. Adam, Ben, Charles, David and Ed were waiting in line. Adam is between Ben and Chase. Ben is between David and Adam. Ed is also between David and Adam. Ben is between David and Ed. Who is in the middle of the line?

(A) Adam (B) Ben (C) Charles (D) David (E) Ed

Problem 7. Five cards are lying on a table as shown. Each card has a letter on one side and a whole number on the other side. Jane said, “If a vowel is on one side of any card, then an even number is on the other side.” Mary showed Jane was wrong by turning over one card. Which card did Mary turn over?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

Problem 8. A centipede crawl a tree 75-inches high, starting from the ground. Each day it crawls 5 inches, and each night it slides down 4 inches. When will it first reach the top of the tree?
(A) 15 (B) 18 (C) 19 (D) 72 (E) 71.

Problem 9. There are 4 cards on the table with the symbols *a*, *b*, 4, and 5 written on their visible sides. What is the smallest number of cards we need to turn over to find out whether the following statement is true: “If an even number is written on one side of a card then a vowel is written on the other side?

Problem 10. Each of the cards shown below has a number on one side and a letter on the other. How many of the cards must be turned over to prove the correctness of the statement: Every card with a vowel on one side has a prime number on the other side.

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

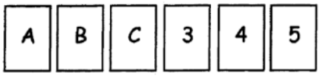
Problem 11. Three kids are playing pitcher, catcher and infielder. Sam is not the catcher. The infielder lives next to Sam. The catcher and John go to the same school. What position does Alex play?

Problem 12. Cookies were missing, taken by either Alex, Bob, or Charles.
Each person said:

Alex: I did not take the cookies.
Bob: Charles took the cookies.
Charles: That is true

If at least one of them lied and at least one told the truth, who took the cookies?

Problem 13. Each of the cards shown has a number on one side and a letter on the other. How many of the cards must be turned over to prove the correctness of this statement for these cards: "If a card has a vowel on one side, then it has a prime number on the other side?"



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 14. If all alligators are ferocious creatures and some creepy crawlers are alligators, which statement(s) must be true?

I. All alligators are creepy crawlers.
II. Some ferocious creatures are creepy crawlers.
III. Some alligators are not creepy crawlers.

(A) I only (B) II only (C) III only (D) II and III only
(E) None must be true

Problem 15. A number of bacteria are placed in a container. One second later each bacterium divides into two, the next second each of the resulting bacteria divided in two again, et al. After one minute the container is full. When was the container half full?
(A) 58 (B) 59 (C) 60 (D) 120 (E) 119

Problem 16. If the two statements below are true, which of the following statements must also be true?
(1) Alex sometimes goes to adventure movies.

(2) Betsy never goes to comedy movies.

I. Alex never goes to comedy movies.
II. Betsy sometimes goes to adventure movies.
III. Alex and Betsy never go to comedy movies together.

(A) I only (B) II only (C) III only (D) I and III (E) II and III

Problem 17. The four children in the Jones family are Alex, Bob, Cathy, and Debra. Bob is neither the youngest nor the oldest. Debra is one of the two younger children. Cathy is the oldest child. Alex is often taking care of his younger brother and sister. Who is the youngest child?

(A) Bob (B) Debra (C) Alex (D) Cathy
(E) It cannot be determined from the information

Problem 18. Sam is not a member of the math club, then from which of the following statements can it be determined whether or not Sam is in the science club?

(A) Anyone in the math club is not in the science club.
(B) No one is in both the math club and the science club.
(C) Anyone who is not in the math club is not in the science club.
(D) Everyone in the math club is in the science club.
(E) Some people who are not in the math club are not in the science club.

Problem 19. If the statement "If a number is in list A, it is not in list B" is true, which of the following statements must also be true?
(A) If a number is not in list A, it is in list B.
(B) If a number is not in list B, it is in list A.
(C) If a number is in list B, it is not in list A.
(D) If a number is in list B, it is in list A.
(E) If a number is in list A, it is also in list B.

Problem 20. The Hope Middle School has three clubs: math, reading, and writing. Five students from a family each participated in one club only. The statements below are about what these five students participated. If n is the number of students who participated in the reading club, which of the following statements is true?

The first student participated in the math club.
The second student did not participate in the math club.
The third student participated in the reading club.
The fourth student participated in the same club as the first student.
The fifth student participated in the same club as the second student.

(A) n must be 1. (B) n must be 2. (C) n must be 3. (D) n must be 1 or 2.
(E) n must be 1 or 3.

Problem 21. If the statement "Some integers in set X are odd" is true, which of the following must also be true?

(A) If an integer is odd, it is in set X.
(B) If an integer is even, it is in set X.
(C) All integers in set X are odd.
(D) All integers in set X are even.
(E) Not all integers in set X are even.

Problem 22. If all boys in the math club are good at math. Which of the following statements must be true?

(A) No boy whose math is not good is a member of the math club.
(B) All boys whose math is good are members of the math club.
(C) All boys who are not members of the math are not good at math.
(D) Every member of the math club whose math is good is a boy.
(E) There is one boy in the math club whose math is not good.

Problem 23. At Hope High School, some members of the math club are on the science team and no members of the science team are 9th graders. Which of the following must also be true?

(A) No members of the math club are 9th graders.
(B) Some members of the math club are 9th graders.

(C) Some member of the math club are not 9th graders.
(D) More 9th graders are on the science team than are in the math club.
(E) More 9th graders are in the math club than are on the science team.

Problem 24. The teacher whispers positive integer A to Anna, B to Brett, and C to Chris. The students don't know one another's numbers but they do know that the sum of their numbers is 14. Anna says, "I know that Brett and Chris have different numbers". Then Brett says, "I already knew that all three of our numbers were different". Finally, Chris announces, "Now I know all three of our numbers". What is the product ABC? (Mathcounts)