

L2 Algebraic Thinking

Concept 1: Algebraic Expressions

1. The Basic of Algebraic Expression

The Definition of Variable:

A variable is a letter or symbol that represents one or more numbers.

The Definition of Algebraic Expression:

An **algebraic expression** is a combination of mathematical symbols (numbers or/and variables) using operations such as addition, subtraction, multiplication, and division.

Writing Rules of Algebraic Expression:

- **1**. Instead of writing " \times " to express the multiplication, we usually write the symbol " \cdot " or without the symbol to express it, and numbers are usually written in front of variables.
- **2.** When the numerical factor is **1 or -1**, **1** can be ignored.
- 3. Division is usually written in the form of a fraction instead of " ÷ "
- **4.** In multiplication, mixed numbers should be converted into **improper fractions**.
- **5.** Product of the same variable should be write in the form of exponent.
- **6.** If we need to write the unit after the expressions in word problems and the algebraic expression contains addition or subtraction, we have to put the expression in **parentheses** and write the units after it.

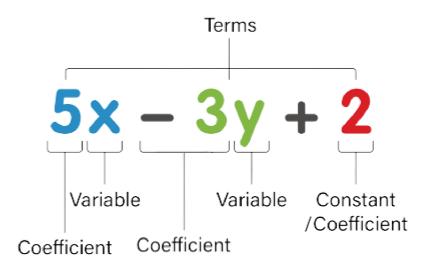
2. Components of Algebraic Expressions

Terms: The terms of an expression are the parts of the expressions separated by an addition or subtraction sign.

Constant: Constant is a term without variables (only with numbers).



Coefficient: A coefficient is the multiplier of the variable or constant in an algebraic expression.



Math Exploration 1

- 1 Write the algebraic expression.
 - (1) The product of x and 2 is _____.
 - (2) The product of m and -1 is _____ .
 - (3) The quotient of a by 6 is _____.
 - (4) The correct way of writing $2\frac{2}{5}x$ is ______.
 - (5) Jelly went to a candy shop. She spent 12 dollars on chocolates and y dollars on lolipops. Jelly paid _____ dollars for all the candies, including chocolates and lolipops.

2		xyz+12	3x+6+y	xy-4x-y
	coefficient of the last term			
	# of terms			
	# of variables			

Concept 2: Use Properties of Numbers to Generate Equivalent Expressions

Like terms are terms that have the same variables and exponents.

An algebraic expression is in its **simplest form** if all like terms are combined and there are no parentheses.

To combine like terms, use the distributive property to add or subtract the coefficients.

1. Commutative Property of Addition & Multiplication

$$a+b=b+a$$

$$a \times b = b \times a$$

2. Associative Property of Addition

$$a + (b + c) = (a + b) + c$$

3. Distributive Property of Multiplication

$$a \times (b+c) = ab + ac$$

Math Exploration 2

Simplify the following algebraic expressions:

(1)
$$3x + 1 - 2x =$$
_____.

(2)
$$9y + 2x - 7y =$$
_____.

(3)
$$\frac{2}{3}a - \frac{2}{3}b - \frac{3}{2}a = \underline{\hspace{1cm}}$$

(3)
$$\frac{2}{3}a - \frac{2}{3}b - \frac{3}{2}a = \underline{\hspace{1cm}}$$
.
(4) $2\frac{3}{5}a + 8 - 3 - \frac{3}{10}a = \underline{\hspace{1cm}}$.

Simplify the following algebraic expressions:

(1)
$$4(2x+1)-3(3-2x)=$$
____.

(2)
$$2(x+4)-3(2x+3y)-6=$$
____.

(3)
$$2x + 3 - 3(x + 3) =$$
_____.

Simplify the following algebraic expressions:

(1)
$$3(x+2)-x=$$
_____.

(2)
$$-(x+2)-2(3-2x)=$$
_____.

(3)
$$\frac{1}{2}(4x+6) - 2x + 3y - 6 =$$
_____.
(4) $\frac{3}{4}(2x+6) - 4(x+3) =$ ____.

(4)
$$\frac{3}{4}(2x+6)-4(x+3)=$$

Concept 3: Equations

An equation is a statement formed by placing an equal sign (=) between two expressions.

A solution of an equation in one variable is a number that produces a true statement when it is substituted for the variable.

Solve Equations:

- 1. Addition&Subtraction Properties of Equality: Adding or subtracting the same number on each side of an equation produces an equivalent equation.
- 2. Multiplication&Division Properties of Equality: Multiplying or dividing each side of an equation by the same nonzero number produces an equivalent equation.

Math Exploration 3

6 Which of the following is an equation? (select all correct answers)

A.
$$y = 2x + 5$$
 B. $4 = 7$

B.
$$4 = 7$$

C.
$$\frac{2}{x} = 4.5x$$
 D. $y + 2x - 3$

D.
$$y + 2x - 3$$

Solve the following equations:

(1)
$$x + \frac{1}{5} = \frac{7}{15}$$
. $x = \underline{\hspace{1cm}}$.

(2)
$$x + 9 = 8$$
 $x =$ _____.

(3)
$$x - 0.25 = -\frac{15}{16}$$
 $x =$ _____.

(4)
$$\frac{3}{4}x - 1 = \frac{5}{6}x - 2$$
 $x = \underline{\qquad}$.

8 Solve the following equations:

(1)
$$\frac{7}{3}x + 1 = \frac{13}{12}x$$
 $x = \underline{\hspace{1cm}}$.

(2)
$$\frac{15}{14}x = \frac{5}{7}x$$
 $x =$ _____.

(3)
$$x \div \frac{3}{4} = \frac{5}{8} \times 0.2 + 2x$$
 $x = \underline{\hspace{1cm}}$.

Quality Calculate:

(1)
$$\frac{x+7}{20} = \frac{x+1}{5}, x = \underline{\hspace{1cm}}$$
.

(2)
$$\frac{2x-1}{4} = 1 - \frac{3-x}{8}, x = \underline{\hspace{1cm}}$$



Concept 4: Inequalities

An inequality is a statement formed by placing an inequality symbol between two expressions. Some common inequality symbols include

≠: is not equal to

< : is less than

> : is greater than

≤: is less than or equal to

≥ : is greater than or equal to

The **solution to an inequality** is a value of a variable that makes the inequality true. Inequalities are not limited to one solution.

Inequality can have an infinite number of solutions. We can use a number line to represent all of the solutions to the inequality.

$$x \ge -2$$

$$x \leq 2$$

We can graph solutions to inequalities on number lines. Closed circles are used for inequalities with \leq or \geq . Open circles are used for inequalities with < or >.

Math Exploration 4

10 Which of the following is an inequality? (select all correct answers)

A.
$$3y = 5x$$

C.
$$\frac{2}{x} < 4.5x$$
 D. $5y + 2x - 1$

D.
$$5y + 2x - 1$$

- 11 Determine whether the given value of the variable is a solution of the inequality.
 - (1) x < 3.14; x = 3.14

A. Yes

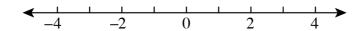
B. No

(2)
$$a \ge 5.5; a = \frac{5}{9}$$

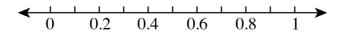
A. Yes

B. No

- 12 Graph the solution of the inequality on a number line.
 - (1) $m \le -1$



(2) x > 0.9



- 1. Addition & Subtraction: The inequality symbol still holds if we add & subtract the same number to both sides.
- 2. Multiplication & Division of Positive Number: If an inequality is multiplied or divided by the same positive number on both sides, the inequality symbol still holds.
- 3. Multiplication & Division of Negative Number: If an inequality is multiplied or divided by the same negative number on both sides, the inequality symbol must be reversed to make the inequality true.

Math Exploration 5

- 13 Use "<" or ">" to answer the following questions.
 - (1) If a < b, then $a + 1 _b b + 1$.
 - (2) If a > b, then $a 1 _b = b 1$.
 - (3) If a > b, then $2a _ 2b$
 - (4) If a < b, then $-\frac{1}{2}a$ ______ $-\frac{1}{2}b$
- 14 Solve the following inequalities.
 - (1) x-4>4
 - (2) 2x + 1 < x 1

- 15 Solve the following inequalities.
 - (1) $3x \ge -9$
 - $(2) -2x \le -8$
 - (3) $-21x 10 \ge 30 x$



Concept 5: Word Problems

Math Exploration 6

16	There are 20 question	ns in an exam. Stude	ents get 5 points for e	each question they		
	answer correctly and 2 point for each question they answer wrong, but made a attempt. Anne complete all questions and gets 94 points from the exam, she					
	answer questions correctly.					
	A. 15	B. 16	C. 17	D. 18		

- 17 There are 25 questions in an exam. Students get 4 points for each question they answer correctly and 1 point for each question they answer wrong, but made an attempt. To get 70 points from the exam, Bob needs to answer _____ questions correctly.
 - A. 15
- B. **16**
- C. 17
- D. 18

18 The taxi fare is 3 dollars for the first mile with additional mileage charged at a rate of 2 dollars for each additional mile. How many miles can you ride for 61 dollars? _____.

Rick's toll pass account has a value of \$33. Each time he uses the toll road, \$0.9 is deducted from the account. When the value drops below \$5, he must add value to the toll pass. Write an inequality to represent and find out how many times he can use the toll road without having to add value to the toll pass.

Homework

20	Write	the	algebraic	expression.
----	-------	-----	-----------	-------------

- (1) The product of 5 and x is _____.
- (2) The product of a and -1 is _____.
- (3) The quotient of 8 by y is _____.
- (4) The correct way of writing $4\frac{1}{4}x$ is ______.
- (5) Owen went to a clothes store. A jecket cost him 188 dollars. A pair of pants cost him x dollars. Owen paid _____ dollars in total.

How many terms and variables does the expression
$$5ab - 6ac + 7b - 8 + a$$
 have, respectively?

A. 3,3

B. 3,4

C. 4,3

D. 5,3

22 Simplify the following algebraic expressions:

(1)
$$\frac{1}{4}x - 2x =$$
_____.

(2)
$$2a-2(3a-b)=$$
____.

(3)
$$2(3x-2)-3(x+2)$$
 _____.

23 Simplify the following algebraic expressions:

(1)
$$\frac{2}{3}(3x-9)-3x-9=$$
_____.

(1)
$$\frac{2}{3}(3x-9) - 3x - 9 = \underline{\hspace{1cm}}$$
.
(2) $\frac{1}{2}(4x+8) - 2(2x+4) = \underline{\hspace{1cm}}$.

24 Solve the following equations:

(1)
$$\frac{5}{2}x + 2 = \frac{5}{4}x$$
 $x = \underline{\hspace{1cm}}$.

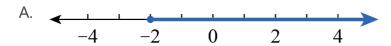
(2)
$$x \div \frac{4}{7} = \frac{5}{12}x + 4$$
 $x = \underline{\hspace{1cm}}$

25 Calculate:

(1)
$$\frac{4x+5}{6} = \frac{3x+7}{18}, x = \underline{\hspace{1cm}}$$

(2)
$$\frac{3x-2}{4} = \frac{1-2x}{3}, x = \underline{\hspace{1cm}}$$

26 For the inequality: $-2 \le m$. Which of the graph is correct? ()



27 Solve the following inequalities.

$$(1) 3x + 1 \ge -11$$

$$(2) -4x \le -12$$

$$(3) -8x - 10 \ge 11 - x$$

Tiara's toll pass account has a value of \$24. Each time she uses the toll road, \$1.4 is deducted from the account. When the value drops below \$10, she must add value to the toll pass. Assume there are x times Tiara can use the toll road without having to add value to the toll pass. Which inequality can represent the situation?

A.
$$24 - 1.4x \ge 10$$

B.
$$24 - 1.4x > 10$$

C.
$$24 + 1.4x \ge 10$$

D.
$$24 - 1.4x \le 10$$



The electricity rate of the apartment that Avril lives in is 5 dollars for the first kilowatt-hour with additional usage charged at a rate of 3 dollars for each additional kilowatt-hour. She uses _____ kilowatt-hours of electricity for 158 dollars.