

1. BASIC KNOWLEDGE

"Divisible by" means "when you divide one number by another number, the result is a whole number." "Divisible by" and "can be evenly divided by" mean the same thing.

The expressions \overline{abc} , \underline{abc} , and abc are the same. $\overline{abc} = \underline{abc} = abc = 100a + 10b + c$. They represent a three-digit number such as $\overline{234} = \underline{234} = 234$.

2.1. Divisibility rule for 2, 4, 8, and 16:

A number is divisible by 2 if the last digit of the number is divisible by 2 (2^1).

A number is divisible by 4 if the last two digits of the number are divisible by 4 (2^2).

A number is divisible by 8 if the last three digits of the number are divisible by 8 (2^3).

A number is divisible by 16 if the last four digits of the number are divisible by 16 (2^4).

2.2. Divisibility rule for 5, 25, 125, and 225:

A number is divisible by 5 if the last digit of the number is divisible by 5 (5^1).

A number is divisible by 25 if the last two digits of the number form a number that is divisible by 25 (5^2).

A number is divisible by 125 if the last three digits of the number form a number that is divisible by 125 (5^3).

A number is divisible by 625 if the last four digits of the number form a number that is divisible by 625 (5^4).

2.3. Divisibility rule for 3 and 9

A number is divisible by 3 if the sum of the digits of the number is divisible by 3.

A number is divisible by 9 if the sum of the digits of the number is divisible by 9.

2.4. Divisibility rule for 7, 11, and 13:

(1) *If you double the last digit and subtract it from the rest of the number and the answer is divisible by 7, the number is divisible by 7. You can apply this rule to that answer again if necessary.*

(2) *To find out if a number is divisible by 11, add every other digit, and call that sum "x." Add together the remaining digits, and call that sum "y." Take the positive difference of x and y. If the difference is zero or a multiple of eleven, then the original number is a multiple of eleven.*

(3) *Delete the last digit from the number, and then subtract 9 times the deleted digit from the remaining number. If what is left is divisible by 13, then so is the original number. Repeat the rule if necessary.*

(4) *If the positive difference of the last three digits and the rest of the digits is divisible by 7, 11, or 13, then the number is divisibly by 7, 11, or 13, respectively.*

2.5. Divisibility rule for 6, 10, 12, 14, 15, 18, 24, and 36:

A number is divisible by 6 if the number is divisible by both 2 and 3.

A number is divisible by 10 if the number is divisible by both 2 and 5.

A number is divisible by 12 if the number is divisible by both 3 and 4.

A number is divisible by 14 if the number is divisible by both 2 and 7.

A number is divisible by 15 if the number is divisible by both 3 and 5.

A number is divisible by 18 if the number is divisible by both 2 and 9.

A number is divisible by 24 if the number is divisible by both 3 and 8.

A number is divisible by 36 if the number is divisible by both 4 and 9.

NOTE: If a number is divisible by two numbers that are relatively prime, then it is divisible by the product of those two numbers. "Relatively prime" means two numbers have no common factor other than 1. For example, 3 and 4 are relatively prime.

2. PROBLEM SOLVING SKILLS**(1). Divisibility rule for 2, 4, 8, and 16:**

Example 1. If the three-digit number $\overline{78N}$ is divisible by 4, how many possible values of N are there?

- (A) 3 (B) 6 (C) 5 (D) 4 (E) 8

(2). Divisibility rule for 5, 25, 125, and 225:

Example 2. ☆ A three-digit integer contains one of each of the digits 3, 4 and 5. What is the probability that the integer is divisible by 5?

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$

Example 3. The six-digit number $\overline{713EF5}$ is divisible by 125. How many such six-digit numbers are there?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(3). Divisibility rule for 3 and 9

Example 4. What is the sum of all possible digits which could fill the blank in $47,\underline{\quad}21$ so that the resulting five-digit number is divisible by 3?

- (A) 3 (B) 6 (C) 9 (D) 12 (E) 18

Example 5. Find the least possible value of digit d so that $\underline{437,d03}$ is divisible by 9.

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(4). Divisibility rule for 7, 11, and 13:

Example 6. Which number is not divisible by 7? 616, 567, 798, or 878.

Example 7. What is the largest integer less than 100 and evenly divisible by 7?

Example 8. Which digit should replace a in the units place so that $9867542a$ is divisible by 11?

- (A) 3 (B) 6 (C) 9 (D) 4 (E) 8

(5). Divisibility rule for 6, 10, 12, 14, 15, 18, 24, and 36:

*A number is divisible by 6 if the number is divisible by both 2 and 3.
A number is divisible by 10 if the number is divisible by both 2 and 5.
A number is divisible by 12 if the number is divisible by both 3 and 4.
A number is divisible by 14 if the number is divisible by both 2 and 7.
A number is divisible by 15 if the number is divisible by both 3 and 5.
A number is divisible by 18 if the number is divisible by both 2 and 9.
A number is divisible by 24 if the number is divisible by both 3 and 8.
A number is divisible by 36 if the number is divisible by both 4 and 9.*

NOTE: If a number is divisible by n numbers that are relatively prime, then it is divisible by the product of those n numbers.

Example 9. What is the greatest three-digit number that is divisible by 6?

- (A) 999 (B) 998 (C) 997 (D) 996 (E) 995

Example 10. Given the 4-digit base-ten number $\underline{77A4}$. For what value of the nonzero digit A will this 4-digit number be divisible by 3 and by 4?

- (A) 3 (B) 6 (C) 9 (D) 1 (E) 7

Example 11. Find the value of x such that the four-digit number $\underline{x15x}$ is divisible by 18.

- (A) 1 (B) 2 (C) 4 (D) 6 (E) 8

Example 12. Find distinct digits A and B such that $\underline{A47B}$ is as large as possible and divisible by 36. Name the number.

- (A) 5472 (B) 6471 (C) 5470 (D) 3474 (E) 6470

Example 13. If k is a positive integer divisible by 3, and if $k < 80$, what is the greatest possible value of k ?

- (A) 75 (B) 76 (C) 77 (D) 78 (E) 79

Example 14. which of the following numbers can be used to show that the statement below is FALSE?

All numbers that are divisible by both 2 and 6 are also divisible by 12.

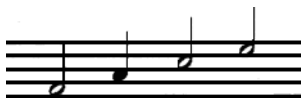
- (A) 8 (B) 12 (C) 18 (D) 24 (E) 36

Example 15. On a square gameboard that is divided into n rows of n squares each, k of these squares not lie along the boundary of the gameboard. If k is one of the four numbers below, what is the possible value for n ?

- (I) 10 (II) 25 (III) 34 (IV) 52

- (A) 10 (B) 12 (C) 14 (D) 16 (E) 18

Example 16. A student practices the four musical notes as shown, starting with the note furthest left and continuing in order from left to right. If the student plays these notes over and over according to this pattern and stops immediately after playing the shaded note, which of the following could be the total number of notes played?



- (A) 50 (B) 51 (C) 52 (D) 53 (E) 64

Example 17. Which of the following statement can be used to determine if a number is divisible by 54 or not?

- I. The number must be divisible by both 6 and 9.
II. The number must be divisible by both 3 and 18.
III. The number must be divisible by both 2 and 27.

- (A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

Example 18. For what digit(s) x will the 7-digit number $3xx6xx2$ be divisible by 4?

- (A) 3 (B) 6 (C) 2 (D) 5 (E) 8

Example 19. What is the largest digit which can replace b to make the number $437,b32$ divisible by 3?

- (A) 3 (B) 6 (C) 2 (D) 5 (E) 8

Example 20. The three-digit number $2a3$ is added to the number 326 to give the three-digit number $5b9$. If $5b9$ is divisible by 9, then $a + b$ equal:

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 9

Example 21. A and B are non-zero digits for which $\underline{A468B05}$ is divisible by 11. What is $A + B$?

- (A) 6 (B) 8 (C) 10 (D) 14 (E) 12

Example 22. If the 4-digit number $\underline{273X}$ is divisible by 12, what is the value of X ?

- (A) 3 (B) 6 (C) 5 (D) 4 (E) 8

Example 23. What value can a have to make $\underline{a74a}$ divisible by 36?

(A) 2

(B) 4

(C) 6

(D) 8

(E) 9