1. BASIC KNOWLEDGE

Even integer:

An integer with the last digit of 0, 2, 4, 6, or 8. General form: $2n$ or $2n + 2$, where n is any integer.

Examples: Even integers: 10, 12, 14, 16, and 18.

Odd integers:

An integer with the last digit of 1, 3, 5, 7, or 9. General form: $2n + 1$ or $2n - 1$.

Examples: Odd integers: 11, 13, 15, 17, and 19.

Parity:

An even number has even parity and an odd number has odd parity.

Properties:

even + even = even.	$12 + 14 = 26$.
even + odd = odd.	$12 + 13 = 25$
odd + odd = even.	$13 + 13 = 26$
odd × odd = odd.	$15 \times 15 = 225$
odd ÷ odd = odd.	$1001 \div 11 = 91$
odd × even = even.	$11 \times 12 = 132$
odd ≠ even.	$1 \neq 2$

2. PROBLEM SOLVING SKILLS

The sum of any even integer and 1 is odd: $4 + 1 = 5$.

The sum of two consecutive integers is odd: $n + (n + 1) = 2n + 1$; $12 + 13 = 25$.

The product of two consecutive integers is even: $n(n + 1)$; $12 \times 13 = 156$.

Any two consecutive integers have opposite parity: 12 is even and 13 odd.
$a + b$ and $a - b$ have the same parity: $15 - 5 = 10$ even; $15 + 5 = 20$ even.

If the product of n positive integers is even,

If the product of n positive integers is odd,

If the number of odd integers is even,

If the number of odd integers is odd,

Example 1. Add any 30 consecutive positive integers together. Is the sum even or odd?

Example 2. 300 is the sum of 15 consecutive even positive integers. What is the greatest even positive integer among them?

Example 3. The sequence 1, 1, 2, 3, 5, 8, 13, 21, … is formed like this: any term is the sum of the two terms before it, starting from the third term. How many are even numbers of the first 63 terms of the sequence?

Example 4. All the positive even integers greater than 0 are arranged in five columns (*A*, *B*, *C*, *D*, and *E*) as shown. Continuing the pattern, in what column will the integer 50 be?

A	B	C	D	E
	2	4	6	8
16	14	12	10	
	18	20	22	24
32	30	28	26	
.....				

Example 5. The sum of all multiples of 3 from 20 to 100 is *S*. Is *S* even or odd?

Example 6. Five lamps are arranged in a row as shown in the figure below. Each lamp has its own switch. All five lamps *A*, *B*, *C*, *D*, and *E* are now off. Ben starts to turn each switch from *A* to *E* and he repeats the pattern (always from *A* to *E* in order) until he turns the switches 2012 times. Which lamps are on finally?

Example 7. If x and y are integers and $x^2y^2 + x^3$ is odd, which of the following statements must be true?

I. x^2 is odd.

II. y is odd.

III. $x + y^2$ is odd.

(A) I only (B) III only (C) I and II (D) I and III (E) II and III

MORE EXAMPLES

Example 8. If a and b are positive integers and $a^2 - b^2 = 7$, what is the value of b?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Example 9. The lengths of the sides of a right triangle are consecutive even integers, and the hypotenuse of the right triangle is x. Which of the following equations could be used to find x?
(A) $x + x - 1 = x - 2$ (B) $x^2 + (x-1)^2 = (x-2)^2$
(C) $x^2 = (x-2)^2 + (x-4)^2$ (D) $x + x + 2 = x + 4$
(E) $x^2 = (x-2)(x-4)$

Example 10. If a and b are positive consecutive odd integers, which of the following must be a positive odd integer?
(A) $a + b$ (B) $a - b$ (C) $2a + b$ (D) $2a - 2b$ (E) $\frac{a+b}{2}$

Example 11. If x and y are positive consecutive odd integers, where $y > x$, which of the following is equal to $y^2 - x^2$?
(A) $6y$ (B) $8y$ (C) $4(y-2)$ (D) $2y - 1$ (E) $4(y-1)$

Example 12. The sequence 1, 1, 2, 4, 7, 13, 24, … is formed like this: any term is the sum of the three terms before it starting from the fourth term. Is the 100^{th} term even or odd in the sequence?

Example 13. The sequence 1, 1, 2, 3, 5, 8, 13, 21, … is formed like this: any term is the sum of the two terms before it, starting from the third term. How many odd numbers are there in the first 900 terms of the sequence?

Example 14. Mr. Mathis and his student Peter worked together to solve math problems last week. When each person solved a problem, that person put a marble in his own box. Mr. Mathis's problem solving speed was half of his student. At the end of the problem solving session, Peter had four boxes of marbles and Mr. Mathis had two boxes of marbles. Each box is labeled with the number of marbles inside it. The numbers are 78, 94, 86, 87, 82, and 80, respectively. Which two boxes belong to Mr. Mathis?

Example 15. All the positive integers greater than 1 are arranged in five columns (*A*, *B*, *C*, *D*, *E*) as shown. Continuing the pattern, in what column will the integer 800 be written?

	A	B	C	D	E
Row 1		2	3	4	5
Row 2	9	8	7	6	
Row 3		10	11	12	13
Row 4	17	16	15	14	
Row 5		18	19	20	21
⋮					

(A) *A* (B) *B* (C) *C* (D) *D* (E) *E*

Example 16. Five lamps are arranged in a row as shown in the figure below. Each lamp has its own switch. All five lamps *A*, *B*, *C*, *D*, and *E* are now off. Ben starts to turn each switch from *A* to *E* and he repeats the pattern (always from *A* to *E* in order) until he turns the switches 126 times. Which lamps are on in the end?