Problem 14.4: How many multiples of 4 are between 25 and 101?

14.1.1 How many numbers are in each of the following lists?

(b) $-27, -23, -19, \ldots, 33, 37$

14.1.5 For how many positive integer values of n are both $\frac{n}{3}$ and $3n$ three-digit integers? (Source: *AMC 8*)

14.1.8★ How many two-digit positive numbers are divisible by 3 or 5? **Hints:** 36

Problem 14.10: In how many ways can we form a license plate using only digits (0–9) and capital letters (other than O and I), given that each plate has 6 characters, the first of which is a digit, and the second of which is a letter?

14.2.4 A shopkeeper sells house numbers. She has a large supply of the digits 1, 2, 7, and 8, but no other digits. How many different three-digit house numbers could be made using only the digits in her supply?

14.2.7★ Suppose that I have 5 different books, 2 of which are math books. In how many ways can I place my 5 books left-to-right on a shelf if I want a math book on both ends?

14.2.8★ How many integers between 99 and 999 contain exactly one 0? *(Source: AMC 8)*

Problem 14.15: How many pairs of positive integers (a, b) satisfy $a^2 + b < 24$?

14.3.2 I have two hats. In one hat are balls numbered 1 through 15. In the other hat are balls numbered 16 through 25. I first choose a hat, then from that hat, I choose 2 balls, without replacing the balls between selections. How many different ordered selections of 2 balls are possible? (By "ordered selections," we mean that "Ball 1 then ball 2" is considered different from "Ball 2 then ball 1.")

14.3.4 How many positive 3-digit numbers have the property that the first digit is at least three times the second digit?

14.3.5 How many positive 4-digit numbers have the last digit equal to the sum of the first two digits?

14.3.6 How many positive two-digit numbers have digits whose sum is a perfect square? *(Source: AMC 8)*

14.4.2 A club has 12 members and needs to choose 2 members to be co-presidents. How many different pairs of co-presidents are possible?

14.4.3 Two students are needed to work in the school store during the lunch hour every day, and four students volunteer for this work. What is the greatest number of days that can be arranged in which no pair of the four students work together more than once? *(Source: MOEMS)*

14.4.5 A sports conference has 12 teams in two divisions of 6. How many games are in a complete season for the conference if each team must play every other team in its own division twice and every team in the other division once?

14.4.7★ How many interior diagonals does an icosahedron have? (An **icosahedron** is a 3-dimensional figure with 20 triangular faces and 12 vertices, with 5 faces meeting at each vertex. An **interior diagonal** is a segment connecting two vertices that do not lie on a common face.) **Hints:** 12, 103, 93

Problem 14.25: Suppose we have 2 fair 6-sided dice numbered from ⚀ to ⚅. If we roll them both, then what is the probability that the two numbers shown sum to 7?

14.5.4 Suppose we flip four coins simultaneously: a penny, a nickel, a dime, and a quarter. What is the probability that

(c)★ at least 15¢ worth of coins come up heads?

14.5.6 Two dice are thrown. What is the probability that the product of the two numbers is a multiple of 5? *(Source: AMC 8)*

14.28 There are 40 students in Mrs. Rusczyk's first grade class. If there are three times as many students with blond hair as with blue eyes, 3 students with blond hair and blue eyes, and 15 students with neither blond hair nor blue eyes, how many students have blue eyes?

14.30 A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window? *(Source: AMC 8)*

14.32 How many 5-digit numbers have the second digit odd and the fifth digit at least four times the second digit?

14.33 Pat Peano has plenty of 0's, 1's, 3's, 4's, 5's, 6's, 7's, 8's, and 9's, but he has only twenty-two 2's. How far can he number the pages of his scrapbook with these digits? *(Source: AMC 8)*

14.36 Tyler has entered a buffet line in which he chooses one kind of meat, two different vegetables, and one dessert from the selections below. If the order of food items is not important, how many different meals might he choose?
Meat: beef, chicken, pork
Vegetables: baked beans, corn, potatoes, tomatoes
Dessert: brownies, chocolate cake, chocolate pudding, ice cream
(Source: AMC 8)

14.41 If one neglects the ":", certain times displayed on a digital watch are palindromes. Three examples are $\boxed{1:01}$, $\boxed{4:44}$, and $\boxed{12:21}$. How many times during a 12-hour period will be palindromes? *(Source: AMC 8)*

14.44 My classroom has 11 rows of chairs, with 11 chairs in each row. The chairs in each row are numbered from 1 to 11.

(b)$\star$ Suppose we replaced all of the occurrences of "11" in this problem with n, where n is an odd number. Can you find a formula in terms of n for the number of chairs with odd numbers? Is the formula different if n is even?

14.46 When writing the numbers from 1 to 500, how many times will you write the digit 3? **Hints:** 134, 71

14.47 In a sports league, there are 20 total teams, divided into 4 divisions of 5 teams each. Over the course of a season, each team plays each of the other teams in its own division 3 times, and each of the other teams in the other divisions twice. How many games does the league have in a complete season?

14.49 If two dice are tossed, what is the probability that the product of the numbers showing on the tops of the dice is greater than 10? *(Source: AMC 8)*

14.51 In Park School's 8th grade, 33 students like volleyball, 34 like softball, 39 like basketball, 20 like volleyball and softball, 10 like volleyball and basketball, 8 like softball and basketball, 3 like all three sports, and 12 like none of these sports. How many students are in Park School's 8th grade? *(Source: MOEMS)* **Hints:** 163, 114

14.53★ There are 120 seats in a row. What is the fewest number of seats that must be occupied so that the next person to be seated must sit next to someone? *(Source: AMC 8)*

14.54★ In how many ways can you spell the word NOON in the grid below? You can start on any letter, then on each step you can move one letter in any direction (up, down, left, right, or diagonal). You cannot visit the same letter twice. **Hints:** 80

14.56⋆ We connect dots with toothpicks in a grid as shown at right. If there are 10 horizontal toothpicks in each row and 20 vertical ones in each column, how many total toothpicks are there? *(Source: AMC)* **Hints:** 145