BASIC KNOWLEDGE REVIEW

Statements

Examples:

Boston is a city in USA.
1 + 1 = 3
A spider does not have six legs.

The following sentences are not statements:

Do your homework.	(a command)
How do you solve this math problem?	(a question)
SAT test is harder than ACT test.	(an opinion)
This sentence is false.	(a paradox)

Negations

The sentence "SAT math test consists of 54 problems" is a statement; the negation of this statement is
The negation of a true statement is false, and the negation of a false statement is true.

Statement	Negation
All do Some do	

Examples: Form the negation of each statement:

The moon is not a star. $\Rightarrow$
The moon is a star. $\Rightarrow$
A spider does not have six legs. $\Rightarrow$

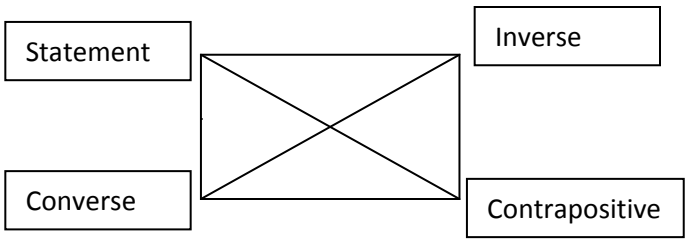
Some rabbits have short tails. $\Rightarrow$
Some rabbits do not have short tails. $\Rightarrow$
No rabbit has a short tail. $\Rightarrow$

Converse, Inverse, and Contrapositive

Direct statement	If p, then q.
Converse	If q, then p.
Inverse	If not p, then not q.
Contrapositive	If not q, then not p.

Direct statement	If I live in Boston, then I live in USA.
Converse	
Inverse	
Contrapositive	

Rectangle of logical equivalent



Logically equivalent pair of statements (diagonally opposite):
A statement and its contrapositive
The inverse and converse of the same statement

Not logically equivalent pair of statements (adjacent):
A statement and its inverse
A statement and its converse

The converse and contrapositive of the same statement
The inverse and contrapositive of the same statement

Examples:

Statement:	A square is a rectangle	(true)
Converse	A rectangle is a square	(false)
Inverse	A figure that is not a square is not a rectangle	(false)
Contrapositive	A figure that is not a rectangle s is not a square	(true)

Euler Diagram

Deductive reasoning consists of three steps as follows:

(1). Making a general statement (major premise).
(2). Making a particular statement (minor premise).
(3). Making a deduction (conclusion).

Example:

(1). The major premise is: All cats are animals
(2). The minor premise is: Jerry is a cat.
(3). The conclusion is: Jerry is an animal.

Procedures to draw the diagram:

(1) Draw a big circle to represent the first premise. This is the region for "animals".
(2) Draw a second circle to represent "all cats". Since all cats are animals, the second circle goes inside the first big circle.
(3) Put Jerry inside where it belongs. The second premise stated that Jerry is a cat. Put Jerry inside the region marked "Cats".

Example: Is the following argument valid? An argument is valid if that the premises are true and these premises force the conclusion to be true.

All apple trees have green leaves
That plant has green leaves.
That plant is an apple tree.

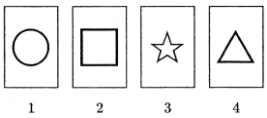
PROBLEM SOLVING SKILLS

(1). Find The Correct Order By Switching Positions

Example 1. Alexis, Britt, Carol, Danielle and Elizabeth are waiting in line. Alex is behind Carol but ahead of Danielle. Elizabeth is ahead of Britt, but behind Carol. Danielle is ahead of Britt. Who is first in line?

(2). Find The Contrapositive Of The Statement

Example 2. Each card has either a circle or a star on one side and either a triangle or a square on the other side. In order to verify the statement "every card with a star on it also has a triangle on it," which numbered card(s) must be turned over?



(3). Find Two Statements That Are Contradicted To Each Other

Example 3. There are three boxes with different colors: red, yellow and blue. One apple is in one of the three boxes. Only one of the following statements is true, and the others are false.

I: Apple is in the red box; II Apple is not in the yellow box, and III: Apple is not in the red box.

Which box is the apple in?

(4). Find Two Statements That Are In Agreement With Each Other

Example 4. Each of three marbles *A*, *B*, and *C*, is colored one of the three colors. One of the marbles is colored white, one is colored red, and one is colored blue. Exactly one of these statements is true:

1) *A* is red. 2) *B* is not blue. 3) *C* is not red.

What color is marble B?

(5). Focus On The Step Before The Last

Example 5. A turtle crawls up a 12 foot hill after a heavy rainstorm. The turtle crawls 4 feet, but when it stops to rest, it slides back 3 feet. How many tries does the turtle make before it makes it up the hill?

(6). Dividing Into Three Groups

When you need to weigh a number of coins with counterfeit coin, divide the coins into three groups with the number of coins in each group: m, m, m, or m, m, $m - 1$ or m, m, $m + 1$.

Example 6. A jeweler has four small bars that are supposed to be gold. He knows that one is counterfeit and the other three are genuine. The counterfeit bar has a slightly different weight than a real gold bar. Using a balance scale, what is the minimum number of weighings necessary to guarantee that the counterfeit bar will be detected?

(7). Drawing Solid and Dash Lines

Example 7. Three friends – math teacher Mr. White, science teacher Mr. Black, and history teacher Mr. Redhead – met in a cafeteria. "It is interesting that one of us has white hair, another one has black hair, and the third has red hair, though no one's name gives the color of their hair" said the black-haired person. "You are right," answered White. What color is the history teacher's hair?

(8). Back one step and forward two

Example 8. There's a box of three hats: one black and two white. Andy and Betsy (each very smart and very logical) each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own head. If any of the two guesses correctly, everyone gets to go to the park!

First, Betsy removes her blindfold. She sees the hats that Andy is wearing, but admits that she is unable to discern her own hat color.

Then Andy says: “I can answer with my blindfold on! I know what color hat I am wearing." What color is Andy's hat?

Example 9. There's a box of five hats: two black and three white. Andy, Betsy, and Charles (each very smart and very logical) each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own head. If any of the three guesses correctly, everyone gets to go to the park!

First, Charles removes his blindfold. He sees the hats that the others are wearing, but admits that he is unable to discern his own hat color.

Next, Betsy removes her blindfold, and sadly reveals that she too is not able to determine the color of her own hat.

Finally, Andy pipes up and says “I can answer with my blindfold on! I know what color hat I am wearing." What color is Andy's hat?

(9). Squeezing method

Example 10. (2014 Mathcounts National) Larry tells Mary and Jerry that he is thinking of two consecutive integers from 1 to 10. He tells Mary one of the numbers, and he tells Jerry the other number. Then the following conversation occurs between Mary and Jerry:
Mary: I don’t know your number.
Jerry: I don’t know your number, either.

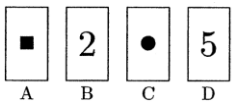
Mary: Ah, now I know your number.
Assuming both Mary and Jerry used correct logic, what is the sum of the possible numbers Mary could have?

MORE EXAMPLES

Example 11. Squares are faster than circles, hexagons are slower than triangles, and hexagons are faster than squares. Which of these shapes is the slowest?

(A) Squares (B) Circles (C) Hexagons (D) Triangles (E) None of them

Example 12. Four cards are constructed so that there is either a circle or a square on one side and an odd or even number on the other side. The cards are placed on a table as shown. Which cards must be turned to prove the following: Every square has an even number on the other side?



Example 13. Classroom window was broken. The principal had four students in his office. He knew that one of them did it, and he also knew that only one of the students told the truth, but not sure which one.
Alex said: Bob did;

Bob said: Dean did;
Cam said: not me;
Dean said: Bob lied.

Who broke the window?

Example 14. A sealed envelope contains a card with a single digit on it. Three of the following statements are true, and the other is false.
I. The digit is 1.
II. The digit is not 2.
III. The digit is 3.
IV. The digit is not 4.
Which one of the following must necessarily be correct?

(A) I is true. (B) I is false. (C) II is true. (D) III is true. (E) IV is false

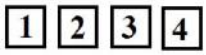
Example 15. A centipede climbs a 40-foot tree. Each day he climbs 5 feet, and each night he slides down 3 feet. In how many days will the centipede reach the top of the tree?

(A) 19 (B) 18 (C) 17 (D) 20 (E) 21

Example 16. Alex has 6 coins. Five of the 6 coins weigh the same and one coin is heavier. If Alex had a balance scale, what is the least number of times he could weigh coins to be sure he could determine which coin was heavier?

Example 17. In a horse race game on a computer, Secretariat, Man-Of-War, Affirmed and Citation finished in first through fourth places (not necessarily in that order), with no ties. Man-Of-War finished second or fourth. Affirmed did not win the race. Citation or Secretariat finished third. Man-Of-War beat Secretariat. What is the name of the horse that finished fourth?

Example 18. Below are the four labeled boxes. Each box is painted a different color. There is a red box, which is next to a blue box. There is a green box, which is next to the red box and a yellow box. Which box could be painted red?



(A) 1 only (B) 2 only (C) 3 only (D) 2 or 3 (E) 1 or 4

☆**Example 19.** (AMC 8) Amy, Bill and Celine are friends with different ages. Exactly one of the following statements is true.
I. Bill is the oldest.
II. Amy is not the oldest.
III. Celine is not the youngest.
Rank the friends from the oldest to the youngest.
(A) Bill, Amy, Celine (B) Amy, Bill, Celine (C) Celine, Amy, Bill
(D) Celine, Bill, Amy (E) Amy, Celine, Bill

☆**Example 20.** (AMC 8) Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?
(A) Alice (B) Ben (C) Cindy (D) Dave (E) Ellen

PROBLEMS

Problem 1. There are 9 apparently identical balls, except that one is heavier than the other 8. What is the smallest number of balance scale weighings required to ensure identification of the "odd" ball?
(A) 9 (B) 3 (C) 4 (D) 1 (E) 2

Problem 2. A kitchen pantry has five shelves, each containing a specific kind of food. The spices are on the shelf directly below the vegetables, the fruits are above the bread, and the vegetables are 3 shelves below the cereals. Which kind of food is on the third shelf?
(A) vegetables (B) fruits (C) bread (D) cereals (E) spices

Problem 3. At Hope Middle School, Mr. Eye, Mr. Love and Mr. Problems teach science, mathematics, and history—but not necessarily in that order. The history teacher, who was an only child, has the least experience. Mr. Problems, who married Mr. Eye's sister, has more experience than the science teacher. Who teaches science?

Problem 4. Five coins look the same, but one is a counterfeit coin with a different weight than each of the four genuine coins. Using a balance scale, what is the least number of weighings needed to ensure that, in every case, the counterfeit coin is found and is shown to be heavier or lighter?
(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

Problem 5. A centipede climbs a 40-foot tree. Each day he climbs 5 feet, and each night he slides down 2 feet. In how many days will the centipede reach the top of the tree?
(A) 14 (B) 13 (C) 12 (D) 8 (E) 20

Problem 6. Adam, Ben, Charles, David and Ed were waiting in line. Adam is between Ben and Chase. Ben is between David and Adam. Ed is also between David and Adam. Ben is between David and Ed. Who is in the middle of the line?

(A) Adam (B) Ben (C) Charles (D) David (E) Ed

Problem 7. Five cards are lying on a table as shown. Each card has a letter on one side and a whole number on the other side. Jane said, "If a vowel is on one side of any card, then an even number is on the other side." Mary showed Jane was wrong by turning over one card. Which card did Mary turn over?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

Problem 8. A centipede crawl a tree 75-inches high, starting from the ground. Each day it crawls 5 inches, and each night it slides down 4 inches. When will it first reach the top of the tree?

(A) 15 (B) 18 (C) 19 (D) 72 (E) 71.

Problem 9. There are 4 cards on the table with the symbols *a*, *b*, 4, and 5 written on their visible sides. What is the smallest number of cards we need to turn over to find out whether the following statement is true: "If an even number is written on one side of a card then a vowel is written on the other side?

Problem 10. Each of the cards shown below has a number on one side and a letter on the other. How many of the cards must be turned over to prove the correctness of the statement: Every card with a vowel on one side has a prime number on the other side.

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

Problem 11. Three kids are playing pitcher, catcher and infielder. Sam is not the catcher. The infielder lives next to Sam. The catcher and John go to the same school. What position does Alex play?

Problem 12. Cookies were missing, taken by either Alex, Bob, or Charles. Each person said:

Alex: I did not take the cookies.
Bob: Charles took the cookies.
Charles: That is true

If at least one of them lied and at least one told the truth, who took the cookies?