3. PROBLEMS

Problem 1. Let the operations △ and □ be defined for all real numbers a and b as follows:

$a \triangle b = a + 3b$

$a \square b = a + 4b$

If $4 \triangle (5y) = (5y) \square 4$, what is the value of y?

(A) 6/5 (B) 1 (C) 2 (D) 3/5 (E) 3/5

☆**Problem 2.** For the positive integer n, let $<n>$ denote the sum of all the positive divisors of n with the exception of n itself. For example, $<4> = 1 + 2 = 3$ and $<12> = 1 + 2 + 3 + 4 + 6 = 16$. What is $<<<18>>>$?

(A) 1 (B) 11 (C) 21 (D) 6 (E) 3

Problem 3. If $A \otimes B = \frac{A}{B} + \frac{B}{A}$, what is the value of $(3 \otimes 2) - (2 \otimes 3)$?

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) 0 (D) 1 (E) $\frac{12}{13}$

Problem 4. If ☆ represents an operation defined by $a \star b = a^3 + b$, find $(1 \star 2) \star 3$.

(A) 3 (B) 27 (C) 30 (D) 9 (E) 732

Problem 5. If $a \diamondsuit b = \frac{1}{a} + \frac{1}{b}$, for what decimal value of a is $a \diamondsuit 0.2 = 10$?

(A) $\frac{1}{5}$ (B) $\frac{7}{10}$ (C) 0 (D) 1 (E) $\frac{1}{2}$

Problem 6. Given that $a \otimes b = (a^2 + b) \div 2$. What is the value of $5 \otimes 3$?

(A) 14 (B) 4 (C) 15 (D) 8 (E) 28

Problem 7. If $a \diamond b = a^2 + b$, evaluate $(4 \diamond 3) \diamond 18$.

(A) 19 (B) 361 (C) 380 (D) 324 (E) 379

☆**Problem 8.** Define $x \otimes y = x^3 - y$. What is $h \otimes (h \otimes h)$?

(A) $-h$ (B) -0 (C) h (D) $2h$ (E) h^4

Problem 9. If $x \diamond y = x^2 - y^2$, what is $(3 \diamond 2) \diamond 4$?

(A) 5 (B) 25 (C) 9 (D) 16 (E) 25

Problem 10. If $\langle ab \rangle = ab - a - b$, find the value of b in the equation $\langle 3b \rangle = 5$.

(A) 5 (B) 4 (C) 3 (D) 15 (E) 2

Problem 11. If $a \blacklozenge b = 2a - b$, what does $3 \blacklozenge 4$ equal?

(A) –2 (B) 2 (C) 10 (D) 5 (E) 12

Problem 12. Suppose that $a \clubsuit b = ab - b$ for all integers a and b. What is the value of $3 \clubsuit (-2)$?

(A) –4 (B) –5 (C) 8 (D) 4 (E) 7

Problem 13. If $A \star B = 3A^2 - 2B^3$, find $7 \star 3$.

(A) 179 (B) –93 (C) 93 (D) 21 (E) 10

Problem 14. If $y \triangleleft = y^2 - 1$, find $(9\triangleleft)\triangleleft$.

(A) 80 (B) 6400 (C) 6399 (D) 79 (E) 81

Problem 15. If $a \blacktriangledown b = 3a - b^2$, find $2 \blacktriangledown (3 \blacktriangledown 1)$.

Problem 16. If 4! means $4 \cdot 3 \cdot 2 \cdot 1$, express $\dfrac{8!}{6!2!2!}$ in simplest form.

(A) 40320 (B) 56 (C) 28 (D) 14 (E) 120

Problem 17. Given $a \diamondsuit b = a(a + b) + b(a + b)$, find $9 \diamondsuit 7$.

(A) 256 (B) 225 (C) 4 (D) 16 (E) 289

Problem 18. For all values a, b, c, and d, $\begin{vmatrix} a & c \\ d & b \end{vmatrix} = ab - cd$. If $\begin{vmatrix} 5 & x \\ -2 & 6 \end{vmatrix} = 8$, what is x?

(A) 14 (B) – 11 (C) – 14 (D) – 15 (E) 11

Problem 19. If $a \diamondsuit b = a^2b$, find $(3 \diamondsuit 2) - (2 \diamondsuit 3)$.

(A) 5 (B) 6 (C) 18 (D) 12 (E) – 6

Problem 20. If $x \circledcirc y = \frac{x}{y} + xy$, express $\frac{3}{8} \circledcirc \frac{3}{4}$ as a common fraction.

(A) $\frac{5}{32}$ (B) $\frac{9}{32}$ (C) $\frac{1}{2}$ (D) $\frac{32}{9}$ (E) $\frac{25}{32}$.

Problem 21. Given $a \diamondsuit b = \frac{(a^2 - b^2)}{ab}$, express $6 \diamondsuit 2$ as a common fraction.

(A) $\frac{8}{3}$ (B) $\frac{7}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{3}{8}$

Problem 22. Given the operations: $a \star b = 2a - b$ and $a \bigstar b = \frac{a+b}{b}$, evaluate: $6 \bigstar |3 \star 9|$.

(A) $\frac{1}{3}$ (B) 3 (C) $\frac{1}{4}$ (D) 4 (E) – 3

Problem 23. Evaluate $3 \circledast 4$ if $a \circledast b = \frac{a^2 + b^2}{a + b}$. Express your answer as a common fraction.

(A) $\frac{5}{7}$ (B) $\frac{25}{7}$ (C) $\frac{9}{7}$ (D) $\frac{16}{7}$ (E) 1

Problem 24. The operation $*$ is defined to be $a * b = \frac{8a-2b}{2ab}$. Express $3 * (3 * 3)$ as a common fraction.

(A) $\frac{31}{3}$ (B) $\frac{31}{9}$ (C) $\frac{3}{11}$ (D) $\frac{11}{3}$ (E) 1

Problem 25. If $[abc] = \frac{a+b}{c}$, what is the value of $[[123][231][312]]$?

(A) 3 (B) 4 (C) 2 (D) 1 (E) 5

Problem 26. If $A \star B = \frac{2A-B}{2}$, what is the value of $(3 \star 4) \star 5$? Express your answer as a common fraction.

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$ (E) 1

Problem 27. If a ☆ b = $\frac{a+b}{2}$, what is the value of (7 ☆ 9)☆(30 ☆ 17)?

(A) $15\frac{1}{4}$ (B) $\frac{47}{2}$ (C) $15\frac{3}{4}$ (D) $\frac{11}{41}$ (E) $14\frac{3}{4}$

Problem 28. Given $a \bigstar b = \frac{a+b}{2}$, find $(7 \bigstar 9) \bigstar 12$.

(A) 10 (B) 8 (C) 20 (D) 96 (E) 6

Problem 29. If $a \triangle b = (ab)^a$, find $5 \triangle 2$. Express the answer as a whole number.

(A) 100,000 (B) 50,000 (C) 20,000 (D) 96,000 (E) 6,000

Problem 30. If $a \bigstar b = a^{b^a}$, then what value is associated with 2 ★ 3?

(A) 64 (B) 512 (C) 8 (D) 128 (E) 1024

Problem 31. If $a \text{ ✪ } b$ is defined as $2a - b^a$, what value is associated with (5 ✪ 2) – (3 ✪ 2)?

(A) – 24 (B) – 20 (C) – 22 (D) 22 (E) 24

Problem 32. Given $a \text{ ✿ } b = b^a - ba + a^b$, find (2 ✿ 3) × (3 ✿ 2).

(A) 121 (B) 22 (C) 11 (D) 144 (E) 81

Problem 33. For natural numbers, a and b $a \boxtimes b = b^a - a + b$. Find the value of (4 ⊠ 2) – (2 ⊠ 4).

(A) 32 (B) – 4 (C) 18 (D) 14 (E) 22

Problem 34. Given the $a \text{ ✦ } b = a^b - b^a$, and $a \bigtriangledown b = (a + b)(a - b)$, what is the value of $a \text{ ✦ } (a \bigtriangledown b)$ if $a = 3$ and $b = 2$?

(A) 118 (B) 15 (C) 243 (D) 125 (E) 115

Problem 35. If $(a \text{ ✦ } b) = (a \times b) + a^b + b^a$, find 3 ✦ 5.

(A) 243 (B) 383 (C) 125 (D) 15 (E) 115

Problem 36. If $a \clubsuit b = (\frac{1}{a})^b + (\frac{1}{b})^a$, find 2 ♣ 3.

(A) $\frac{17}{72}$ (B) $\frac{2}{17}$ (C) $1\frac{1}{9}$ (D) $\frac{1}{8}$ (E) $\frac{5}{6}$

Problem 37. If $a \diamondsuit b = \sqrt{a^2 + b^2}$, find the value of $(2\frac{1}{2}) \diamondsuit 6$ and express the result as a common fraction.

(A) $\frac{13}{2}$ (B) $\frac{17}{2}$ (C) $\frac{5}{2}$ (D) $\sqrt{13}$ (E) $\sqrt{61}$

Problem 38. Let $\triangledown$ be defined as $\triangledown(a, b) = \sqrt{a^2 + b^2}$, for all real numbers a and b. Find $\triangledown$ ($\triangledown(8, 5)$, 144).

(A) 85 (B) 125 (C) 135 (D) 145 (E) 165

Problem 39. Given $a \bigstar b = \sqrt{a^2 + b^2}$, find ((13 $\bigstar$ 84) $\bigstar$ (36 $\bigstar$ 77)).

(A) 85 (B) $85\sqrt{2}$ (C) $83\sqrt{2}$ (D) 135 (E) $58\sqrt{2}$

Problem 40. Given that $x \diamondsuit y = \sqrt{x + y}$, find (6 $\diamondsuit$ 10) $\diamondsuit$ 5.

(A) 4 (B) 3 (C) 16 (D) $\sqrt{15}$ (E) 7

Problem 41. The symbols ✦ and ∗ represent different operations, either +, −, ×, or ÷, and x is a positive integer. Find x if 17 ✦ x = 54 ∗ x.

(A) 4 (B) 3 (C) 6 (D) 5 (E) 7