Problem 20 [edit]

An integer between 1000 and 9999, inclusive, is chosen at random. What is the probability that it is an odd integer whose digits are all distinct?

(A) $\dfrac{14}{75}$ (B) $\dfrac{56}{225}$ (C) $\dfrac{107}{400}$ (D) $\dfrac{7}{25}$ (E) $\dfrac{9}{25}$
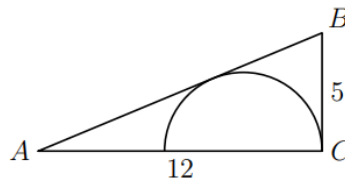
Solution

Problem 21 [edit]

Suppose a, b, and c are nonzero real numbers, and $a + b + c = 0$. What are the possible value(s) for $\dfrac{a}{|a|} + \dfrac{b}{|b|} + \dfrac{c}{|c|} + \dfrac{abc}{|abc|}$?

(A) 0 (B) $1 \text{ and } -1$ (C) $2 \text{ and } -2$ (D) $0, 2, \text{ and } -2$ (E) $0, 1, \text{ and } -1$

Solution

22   In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?


(A) $\frac{7}{6}$ (B) $\frac{13}{5}$ (C) $\frac{59}{18}$ (D) $\frac{10}{3}$ (E) $\frac{60}{13}$

23   Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips? (A) 10 (B) 15 (C) 25 (D) 50 (E) 82

24 Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren? **(A)** 78 **(B)** 80 **(C)** 144 **(D)** 146 **(E)** 152

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25 In the figure shown, $\overline{US}$ and $\overline{UT}$ are line segments each of length 2, and $m\angle TUS = 60°$. Arcs TR and SR are each one-sixth of a circle with radius 2. What is the area of the region shown?
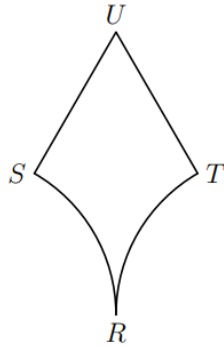
**(A)** $3\sqrt{3} - \pi$ **(B)** $4\sqrt{3} - \frac{4\pi}{3}$ **(C)** $2\sqrt{3}$ **(D)** $4\sqrt{3} - \frac{2\pi}{3}$ **(E)** $4 + \frac{4\pi}{3}$

20   Any three vertices of the cube $PQRSTUVW$, shown in the figure below, can be connected to form a triangle. (For example, vertices P, Q, and R can be connected to form $\triangle PQR$.) How many of these triangles are equilateral and contain P as a vertex?

**(A)** 0   **(B)** 1   **(C)** 2   **(D)** 3   **(E)** 6
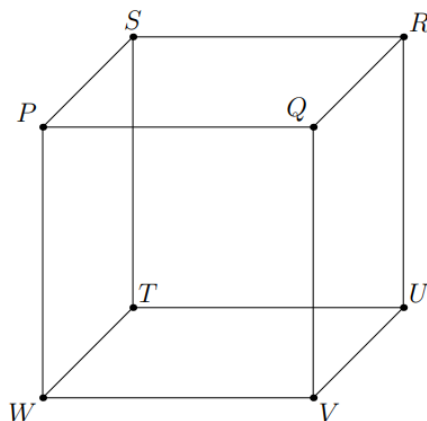
21   A group of frogs (called an army) is living in a tree. A frog turns green when in the shade and yellow when in the sun. Initially the ratio of green to yellow frogs was 3:1. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is 4:1. What is the difference between the number of green frogs and yellow frogs now?

(A) 10   **(B)** 12   **(C)** 16   **(D)** 20   **(E)** 24

22   A roll of tape is 4 inches in diameter and is wrapped around a ring that is 2 inches in diameter. A cross section of the tape is shown in the figure below. The tape is 0.015 inches thick. If the tape is completely unrolled, approximately how long would it be? Round your answer to the nearest 100 inches.
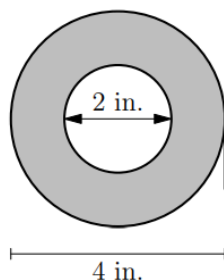

(A) 300   **(B)** 600   **(C)** 1200   **(D)** 1500   **(E)** 1800

23 Rodrigo has a very large sheet of graph paper. First he draws a line segment connecting point $(0, 4)$ to point $(2, 0)$ and colors the 4 cells whose interiors intersect the segment, as shown below. Next Rodrigo draws a line segment connecting point $(2000, 3000)$ to point $(5000, 8000)$. How many cells will he color this time?
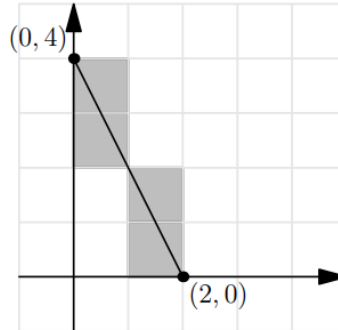
(A) 6000 (B) 6500 (C) 7000 (D) 7500 (E) 8000

24 Jean made a piece of stained glass art in the shape of two mountains, as shown in the figure below. One mountain peak is 8 feet high and the other peak is 12 feet high. Each peak forms a $90°$ angle, and the straight sides of the mountains form $45°$ with the ground. The artwork has an area of 183 square feet. The sides of the mountains meet at an intersection point near the center of the artwork, h feet above the ground. What is the value of h?
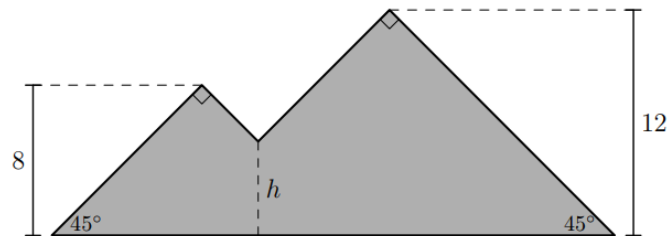
(A) 4 (B) 5 (C) $4\sqrt{2}$ (D) 6 (E) $5\sqrt{2}$

25 A small airplane has 4 rows of seats with 3 seats in each row. Eight passengers have boarded the plane and are distributed randomly among the seats. A married couple is next to board. What is the probability there will be 2 adjacent seats in the same row for the couple?

(A) $\dfrac{8}{15}$ (B) $\dfrac{32}{55}$ (C) $\dfrac{20}{33}$ (D) $\dfrac{34}{55}$ (E) $\dfrac{8}{11}$

20 The grid below is to be filled with integers in such a way that the sum of the numbers in each row and the sum of the numbers in each column are the same. Four numbers are missing. The number x in the lower left corner is larger than the other three missing numbers. What is the smallest possible value of x?

-2	9	5
		-1
x		8

(A) -1 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

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21 Steph scored 15 baskets out of 20 attempts in the first half of a game, and 10 baskets out of 10 attempts in the second half. Candace took 12 attempts in the first half and 18 attempts in the second. In each half, Steph scored a higher percentage of baskets than Candace. Surprisingly they ended with the same overall percentage of baskets scored. How many more baskets did Candace score in the second half than in the first?

	First Half	Second Half
Steph	$\dfrac{15}{20}$	$\dfrac{10}{10}$
Candace	$\dfrac{\square}{12}$	$\dfrac{\square}{18}$

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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22 A bus takes 2 minutes to drive from one stop to the next, and waits 1 minute at each stop to let passengers board. Zia takes 5 minutes to walk from one bus stop to the next. As Zia reaches a bus stop, if the bus is at the previous stop or has already left the previous stop, then she will wait for the bus. Otherwise she will start walking toward the next stop. Suppose the bus and Zia start at the same time toward the library, with the bus 3 stops behind. After how many minutes will Zia board the bus?

(A) 17 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 23

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23 A △ or ◯ is placed in each of the nine squares in a 3-by-3 grid. Shown below is a sample configuration with three △s in a line.
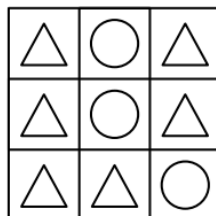
How many configurations will have three △s in a line and three ◯s in a line?

(A) 39 (B) 42 (C) 78 (D) 84 (E) 96

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24 The figure below shows a polygon $ABCDEFGH$, consisting of rectangles and right triangles. When cut out and folded on the dotted lines, the polygon forms a triangular prism. Suppose that $AH = EF = 8$ and $GH = 14$. What is the volume of the prism?
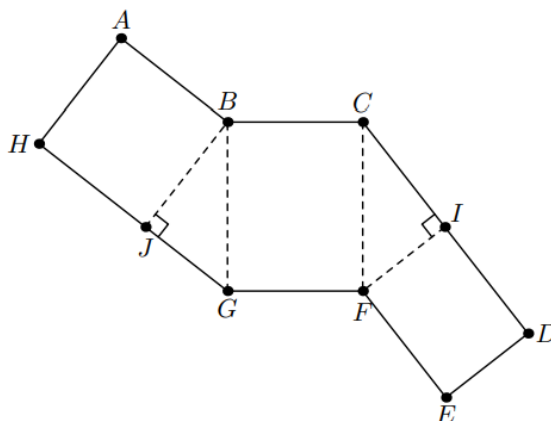
(A) 112 (B) 128 (C) 192 (D) 240 (E) 288

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25 A cricket randomly hops between 4 leaves, on each turn hopping to one of the other 3 leaves with equal probability. After 4 hops what is the probability that the cricket has returned to the leaf where it started?

(A) $\dfrac{2}{9}$ (B) $\dfrac{19}{80}$ (C) $\dfrac{20}{81}$ (D) $\dfrac{1}{4}$ (E) $\dfrac{7}{27}$