Problem 15.3: Michelle's Number Recycling Machine obeys two rules:

1. If an inserted number has exactly one digit, double the number.
2. If an inserted number has exactly two digits, compute the sum of the digits.

The first number Michelle inserts is 1. Then every answer she gets is inserted back into the machine until fifty numbers are inserted. What is the fiftieth number to be inserted? *(Source: MOEMS)*

15.1.3 In the table of numbers below, what number is directly above 119?

			1			
		2	3	4		
	5	6	7	8	9	
10	11	12	13	14	15	16
			⋮			

15.1.4 All of the even numbers from 2 through 288, except those ending in 0, are multiplied together. What is the units digit (ones digit) of the product? **Hints:** 100

Problem 15.9: A frog is at the bottom of a 12-meter well. Each morning he climbs up 5 meters. Each night he slides down 3 meters. If he starts climbing on a Sunday, on which day will he reach the top of the well and escape?

15.3.1 A lumberjack can cut a log into five pieces in 20 minutes. How long would it take to cut a log of the same size and shape into seven pieces?

15.3.3 Albert's house is 5 miles east of Belle's house and 3 miles west of Carnot's house. Dolly's house is 6 miles east of Carnot's house, and 4 miles east of Eli's house. Frank's house is 5 miles north of Eli's house and 8 miles north of Greta's house. To the nearest tenth of a mile, how far apart are Belle's house and Greta's house?

15.3.5$\star$ One line divides a plane into two regions. Two lines can divide a plane into at most four regions. What is the maximum number of regions possible using eight lines? **Hints:** 66

15.4.4 A list of 8 numbers is formed by beginning with two given numbers. Each new number in the list is the product of the two previous numbers. Find the first number if the last three are shown:

$$\underline{\ ?\ },\ \_\_\_,\ \_\_\_,\ \_\_\_,\ \_\_\_,\ \underline{16},\ \underline{64},\ \underline{1024}.$$

(Source: AMC 8)

15.23 Adnan began with a number. He divided his number by 2, subtracted 6 from the quotient, took the square root of the difference, added 1 to the square root, and took the square root of the sum. His final result was 3. What was Adnan's original number? *(Source: MOEMS)*

15.24 In the table below, the integers from 100 down to 0 are arranged in columns P, Q, R, S, and T. Write the letter of the column that contains the number 25.

P	Q	R	S	T
	100	99	98	97
93	94	95	96	
	92	91	90	89
85	86	87	88	
	84	83	82	81
77	78	79	80	
.	.	.	.	.

(Source: MOEMS)

15.27 Three generous friends, each with some cash, redistribute their money as follows: Ami gives enough money to Jan and Toy to double the amount of money that each has. Jan then gives enough to Ami and Toy to double their amounts. Finally, Toy gives Ami and Jan enough to double their amounts. If Toy has \$36 when they begin and \$36 when they end, what is the total amount that all three friends have? *(Source: AMC 8)*

15.30 Suppose there is a special key on a calculator that replaces the number x currently displayed with the number given by the formula $1/(1-x)$. For example, if the calculator is displaying 2 and the special key is pressed, then the calculator will display -1 since $1/(1-2) = -1$. Now suppose the calculator is displaying 5. After the special key is pressed 100 times in a row, what decimal number will the calculator display? *(Source: AMC 8)*

15.32 How many partitions of 7 are there? (See Problem 15.7 on page 554 for the definition of a partition.)

15.34 There are positive integers that have these properties:

1. the sum of the squares of their digits is 50, and
2. each digit is larger than the one to its left.

What is the largest integer with both properties? *(Source: AMC 8)*

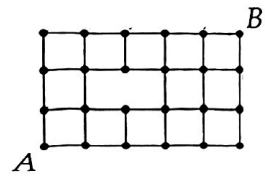
15.36 There are 24 four-digit positive integers that use each of the four digits 2, 4, 5, and 7 exactly once. Listed in numerical order from smallest to largest, what integer is in the 17^{th} position in the list? *(Source: AMC 8)*

15.39 A 2-by-2 square is divided into four 1-by-1 squares. Each of the small squares is to be painted either white or gray. In how many different ways can the painting be accomplished so that no gray square shares its top or right side with any white square? There may be as few as zero or as many as four small gray squares. *(Source: AMC 8)*

15.40 A certain calculator has only two keys [+1] and [×2]. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed [+1], it would display "10". If you then pressed [×2], it would display "20". Starting with the display "1", what is the fewest number of keystrokes you would need to reach "200"? *(Source: AMC 8)* **Hints:** 98, 127

15.41 The product of three positive integers (not necessarily different) is 40. How many sets of 3 integers have this property if the order of the 3 integers in a set does not matter?

15.42 How many paths with length 8 units are there from A to B along the grid at the right? Notice that one segment in the grid is missing! We cannot travel along the missing segment. **Hints:** 51



15.48⋆ Five married couples get together at a party. At the start of the party, each person shakes hands with everyone they didn't know before the party. After all the handshakes, Kyle, one of the husbands, asks everyone else how many hands they shook. He received each number from 0 to 8 as an answer once. How many hands did Kyle shake?