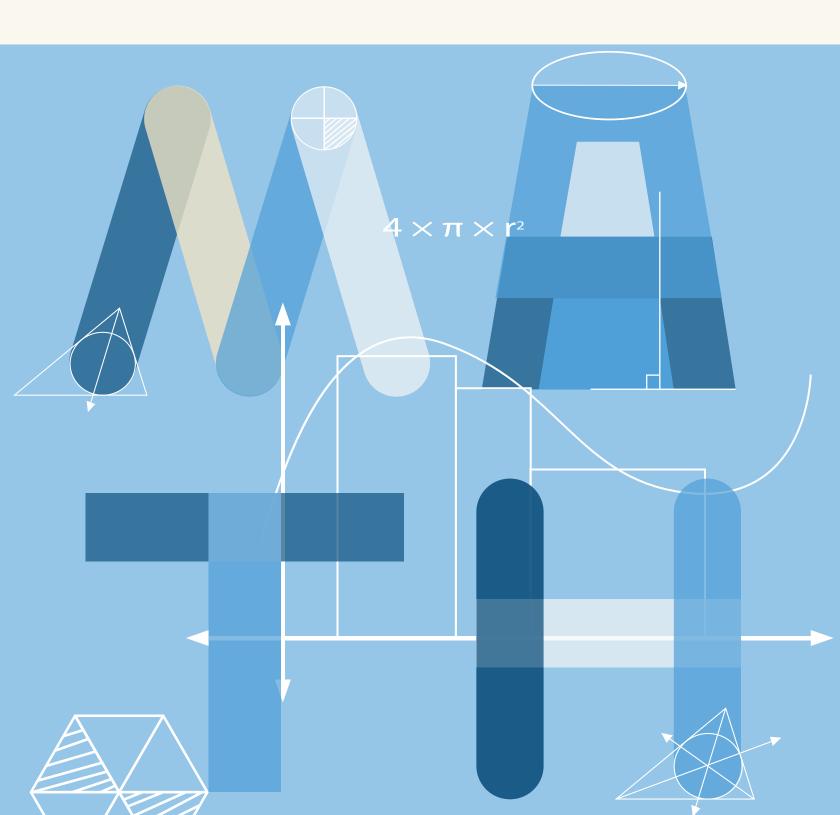
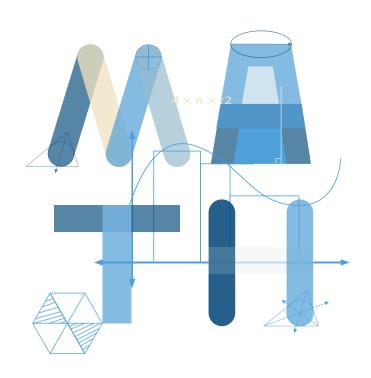


AMC8 PREP





NAME:

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Lesson 1 Number & Operations



Integers & Operations

- 1 What is the 100th number in the arithmetic sequence: 1, 5, 9, 13, 17, 21, 25, ...? (). (1988 AMC 8 Problem, Question #19)
 - A. 397
- B. 399
- C. 401
- D. 403
- E. 405

- Which of the following integers cannot be written as the sum of four consecutive odd integers? (). (2015 AMC 8 Problem, Question #14)
 - A. 16
- B. 40
- C. 72
- D. 100
- E. 200

- 3 The sum of 25 consecutive even integers is 10,000. What is the largest of these 25 consecutive integers? (). (2016 AMC 8 Problem, Question #19)
 - A. 360
- B. 388
- C. 412
- D. 416
- E. 424

- 4 Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference? (). (2007 AMC 8 Problem, Question #19)
 - A. 2
- B. 64
- C. 79
- D. 96
- E. 131



Fractions & Operations

- 1) What is the product of $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \cdots \times \frac{2006}{2005}$? (). (2006 AMC 8 Problem, Question #9)
 - A. 1
- B. 1002
- C. 1003
- D. 2005
- E. 2006

2 $2\left(1-\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)+4\left(1-\frac{1}{4}\right)+\cdots+10\left(1-\frac{1}{10}\right)=($).

(1998 AMC 8 Problem, Question #12)

- A. 45
- B. 49
- C. 50
- D. 54
- E. 55

3 What is the value of the expression $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}$? (). (2017 AMC 8) Problem, Question #5)

- A. 1020
- B. 1120
- C. 1220
- E. 3360

4 What is the value of the product?

$$\left(\frac{1\cdot 3}{2\cdot 2}\right)\!\!\left(\frac{2\cdot 4}{3\cdot 3}\right)\!\!\left(\frac{3\cdot 5}{4\cdot 4}\right)\!\!\cdots\!\!\left(\frac{97\cdot 99}{98\cdot 98}\right)\!\!\left(\frac{98\cdot 100}{99\cdot 99}\right) (2019 \text{ AMC 8 Problem, Question #17)}$$

- A. $\frac{1}{2}$ B. $\frac{50}{99}$ C. $\frac{9800}{9801}$ D. $\frac{100}{99}$
- E. 50

- 5 What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order? (). (2012 AMC 8 Problem, Question #20)

 - A. $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$ B. $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$ C. $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$ D. $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$ E. $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$

Exponents & Square Root

- 1 What is the value of the expression $\sqrt{16\sqrt{8\sqrt{4}}}$? (). (2017 AMC 8 Problem, Question #3
 - A. 4
- B. $4\sqrt{2}$
 - C. 8
- D. $8\sqrt{2}$
- E. 16

- 2 How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$? (). (1986 AMC 8 Problem, Question #7)
 - A. 5
- B. 6
- C. 7
- D. 8
- E. 9

- 3 If $3^p + 3^4 = 90$, $2^r + 44 = 76$, and $5^3 + 6^s = 1421$, what is the product of p, r, and s? (2013 AMC 8 Problem, Question # 15)
 - A. 27
- B. 40
- C. 50
- D. 70
- E. 90

- 4 What is the largest power of 2 that is a divisor of 13⁴ 11⁴? (). (2016 AMC 8 Problem, Question # 15)
 - A. 8
- B. 16
- C. 32
- D. 64
- E. 128

- 5 What is the correct ordering of the three numbers 10^8 , 5^{12} , and 2^{24} ? (). (2010 AMC 8 Problem, Question # 24)
 - A. $2^{24} < 10^8 < 5^{12}$
- B. $2^{24} < 5^{12} < 10^8$ C. $5^{12} < 2^{24} < 10^8$

- D. $10^8 < 5^{12} < 2^{24}$
- E. $10^8 < 2^{24} < 5^{12}$



Special Symbols & Operations

- 1 If $a \otimes b = \frac{a+b}{a-b}$, then $(6 \otimes 4) \otimes 3 = ($). (2001 AMC 8 Problem, Question # 12)
 - A. 4
- B. 13
- C. 15 D. 30
- E. 72

- 2 Suppose that a * b means 3a b. What is the value of x if 2 * (5 * x) = 1? (). (2016) AMC 8 Problem, Question # 10)
 - A. $\frac{1}{10}$
- B. 2 C. $\frac{10}{3}$
- D. 10
- E. 14

- 3 The operation \otimes is defined for all non-zero numbers by $\mathbf{a} \otimes \mathbf{b} = \frac{a^2}{b}$. Determine [(1 \otimes 2) \otimes 3] [1 \otimes (2 \otimes 3)] (). (2000 AMC 8 Problem, Question # 17)

 - A. $-\frac{2}{3}$ B. $-\frac{1}{4}$
- C. 0
- D. $\frac{1}{4}$ E. $\frac{2}{3}$



1) There is an arithmetic sequence: 66, 63, 60, 57, 54, ··· Given the last term of the sequence is 0, then there are in total ______ terms in this sequence.

If the sum of seven consecutive even numbers is 182, the smallest of the seven numbers is ______.
 A. 20
 B. 23
 C. 26
 D. 32

3 Calculate: $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times \cdots \times 1\frac{1}{1000} =$

4 Compare $\frac{43}{44}$, $\frac{85}{87}$, $\frac{125}{128}$ and list them from least to greatest: ______.

5 Estimate the result of $\sqrt{5} - 1$: A. Between 1 and 2

B. Between 2 and 3

C. Between 3 and 4

D. Between 4 and 5

6 The difference between two square numbers is 51. What can these two numbers be?

7 Calculate: $(2^4 - 1) \times (2^4 + 1) =$ _____.

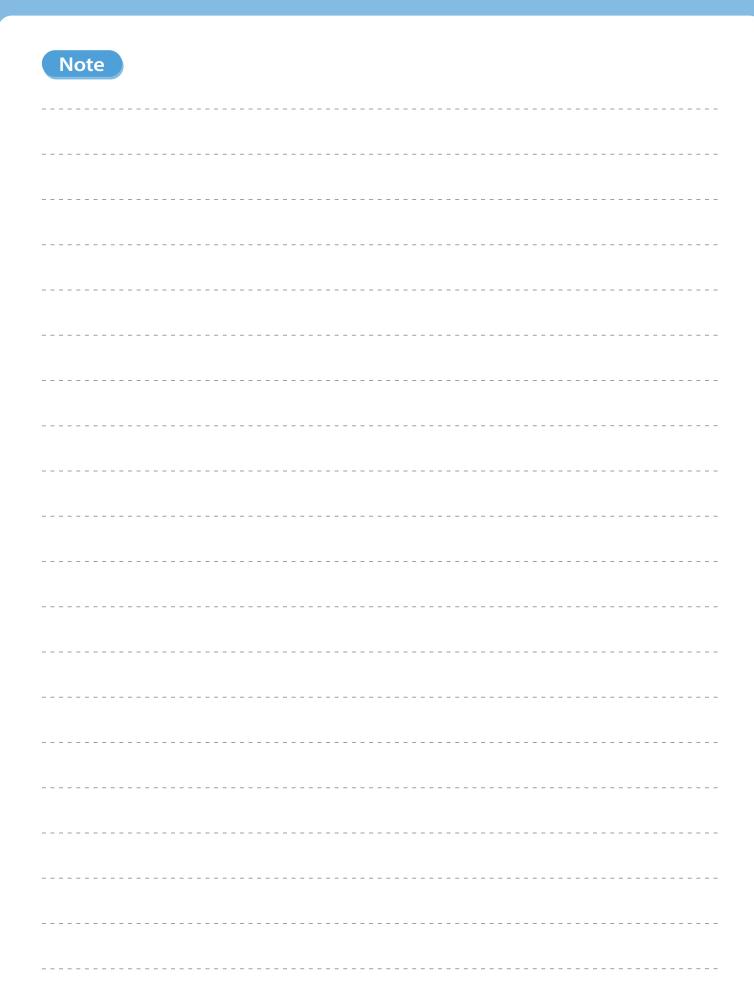
8 Compare the following numbers:

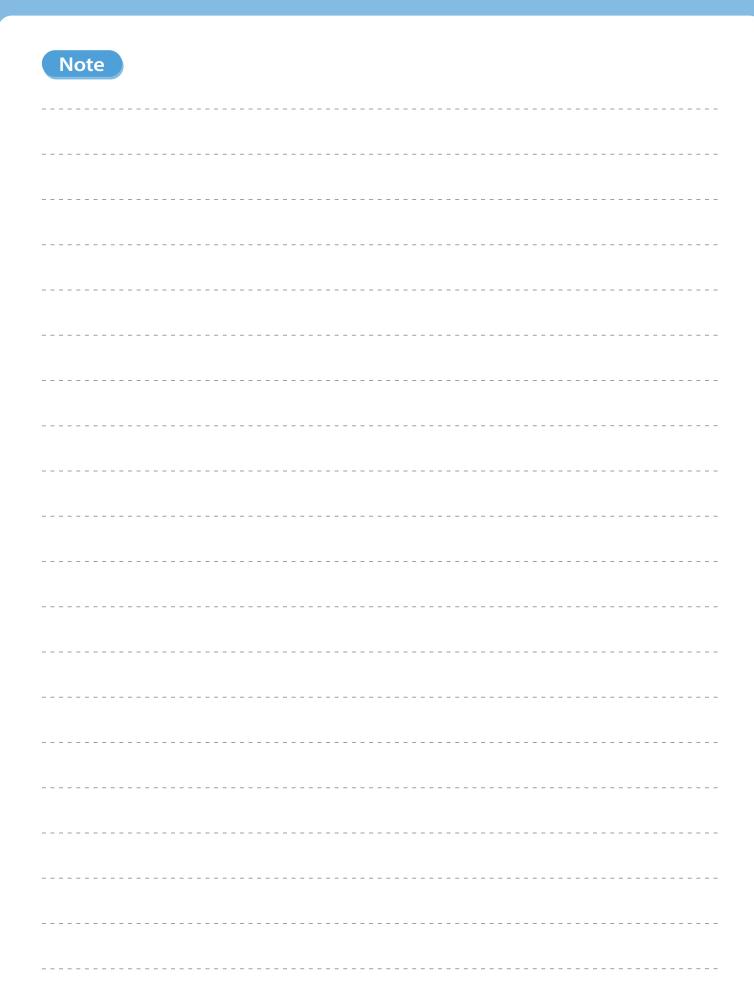
$$(1) 8^{21}, 16^{11}, 4^{31}$$

 $(2) 3^{105}, 23^{35}, 5^{70}$

9 Define the operation: $a * b = \frac{a+b}{2}$. If x * (x * 10) = x, x = ().

10 A new operation is defined as $a \odot b = \frac{a+1}{b}$. If $x \odot 4 = 1.35$, the value of x is _____.





Lesson 2 Word Problem



Fractions & Ratios

1 A number of students from Fibonacci Middle School are taking part in a community service project. The ratio of 8th-graders to 6th-graders is 5:3, and the ratio of 8thgraders to 7th-graders is 8:5. What is the smallest number of students that could be participating in the project? (). (2013 AMC 8 Problem, Question #16)

A. 16

B. 40

C. 55

D. 79

E. 89

2 In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixthgrade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy? (

(2015 AMC 8 Problem, Question #16)

A.
$$\frac{2}{15}$$

B. $\frac{4}{11}$ C. $\frac{11}{30}$

D. $\frac{3}{8}$

E. $\frac{11}{15}$

X)

Percentages

1 A mixture of 30 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of yellow tint are added to the original mixture. What is the percent of yellow tint in the new mixture? (). (2007 AMC 8 Problem, Question #17)

A. 25

B. 35

C. 40

D. 45

E. 50

2 Chloe and Zoe are both students in Ms. Demeanor's math class. Last night they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers? (). (2017 AMC 8 Problem, Question #14)

A. 89

B. 92

C. 93

D. 96

E. 98

3 Before district play, the Unicorns had won 45% of their basketbal	ll games. During
district play, they won six more games and lost two, to finish the sea	ason having won
half their games. How many games did the Unicorns play in all? (). (2007 AMC 8
Problem, Question #20)	

A. 48

B. 50

C. 52

D. 54

E. 60

4 A store increased the original price of a shirt by a certain percent and then decreased the new price by the same percent. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased? (2019 AMC 8 Problem, Question #22)

A. 16

B. 20

C. 28

D. 36

E. 40



Time & Distance

- 1 Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie? (). (2016 AMC 8 Problem, Question #16)
 - A. $1\frac{1}{4}$
- B. $3\frac{1}{3}$
- C. 4
- D. 5
- E. 25

- 2 Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take? (). (2005 AMC 8 Problem, Question #20)
 - A. 6
- B. 8
- C. 12
- D. 14
- E. 24

3 Wha	t is the measure of th	ie acute angle f	ormed by the ha	nds of the clock	at 4:20 PM?
(). (2003 AMC 8 Prob	lem, Question #	[‡] 20)		
A. 0	B. 5	C. 8	D. 10	E. 12	

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Assignment

1 The ratio of tigers to dogs to squirrels in the zoo is 4:5:12, while the ratio of squirrels to raccoons to rabbits is 10:3:6. What is the ratio of dogs to rabbits?

2 Alex paid \$945 to transport his animals by ferry. The costs are 3 dollars, 2 dollars and 1 dollar for each cat, dog, and squirrel, respectively. The ratio of cats to dogs is 2:9, and of dogs to squirrels is 3:7. How many cats were there?

A. 49

B. 47

C. 45

D. 43

E. 42

- 3 Two cylindrical wax candles of the same height are lit at the same time. The first burned all of its wax in 4 hours and the second in 3 hours. Assuming that each candle burns at a constant rate, in how many hours after being lit was the first candle three times the height of the second?
 - A. $2\frac{1}{3}$
- B. $2\frac{2}{3}$
- C. 2
- D. $1\frac{1}{3}$
- E. $3\frac{1}{3}$

4 On Monday, Euler's Bakery decreased the original price of the pie by 20%. On Tuesday, the bakery then decreased the new price by 70%. Compared with the original price, by what percent was the resulting price decreased?

5 Thirty ounces of vinegar with a strength of 30% was mixed with 50 ounces of a 20% vinegar solution. What was the percentage of the resulting solution?
6 How many liters of a 20% alcohol solution must be added to 90 liters of a 50% alcohol
solution to form a 45% solution?
solution to form a 45% solution?

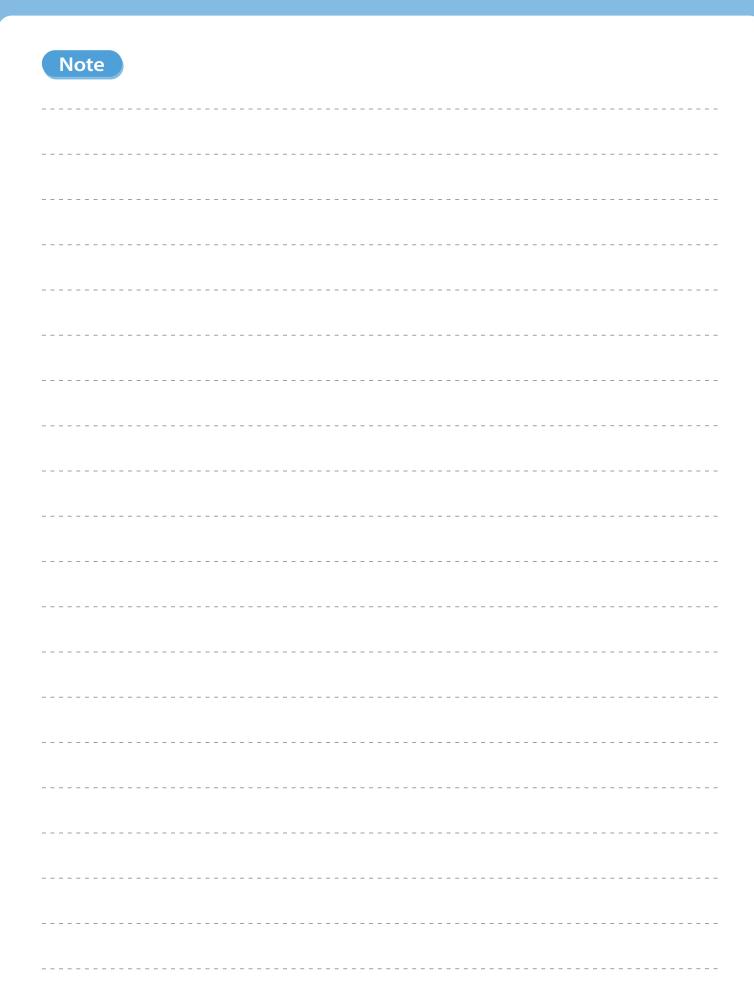
7 A test has two parts. The first part is worth 60% and the second part is worth 40%. If a student gets 95% of part one correct, what exact percentage must this student achieve on part two to average 90% for the whole test?

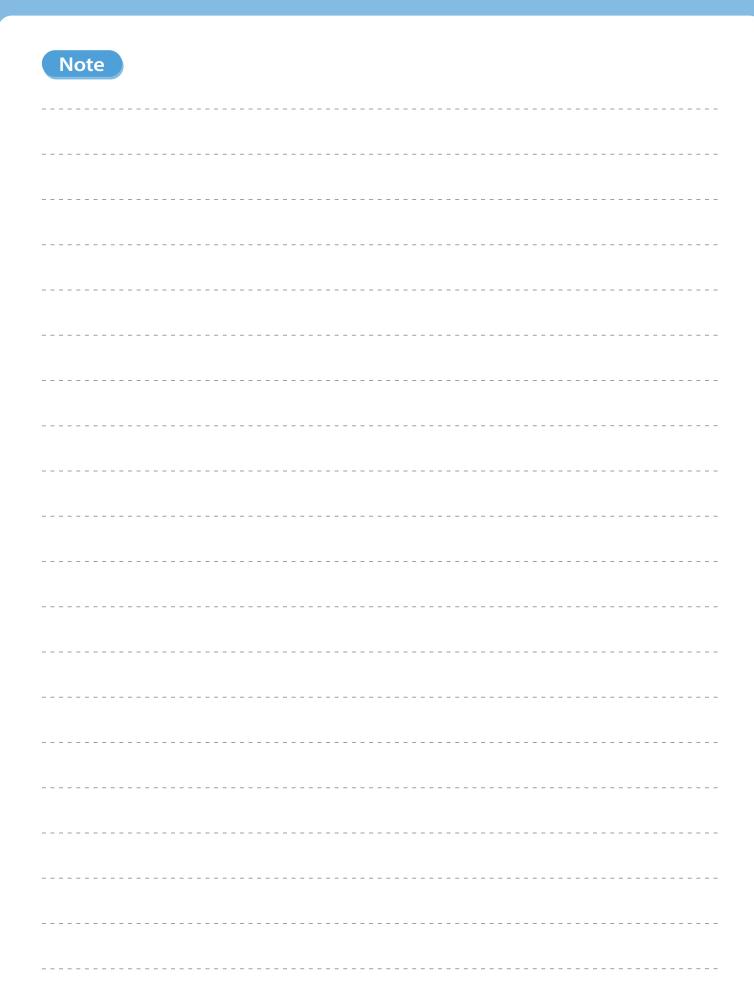
8 Tony and Cathy are running laps around a 500-meter oval track. They started together, and Tony is 40% faster than Cathy. How many laps will Tony have run when he passes Cathy the second time?

What is the degree of the acute angle formed by the hour hand and the minute hand of a clock at 7:20 p.m?

Answer: _____.

10 A new operation is defined as $a \odot b = \frac{a+1}{b}$. If $x \odot 4 = 1.35$, the value of x is _____.





Lesson 3 Number Theory



Divisibility Rules

- 1 Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker 1A2. What is the missing digit A of this 3-digit number? (2014 AMC 8 Problem, Question #8)
 - A. 0
- B. 1
- C. 2
- D. 3
- E. 4

- 2 The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number PQRST. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P? (2016 AMC 8 Problem, Ouestion #24)
 - A. 1
- B. 2
- C. 3
- D. 4
- E. 5

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Prime and Composite Numbers

1 In how many ways can 10001 be written as the sum of two primes? (2011 AMC 8 Problem, Question #24)

A. 0

B. 1

C. 2

D. 3

E. 4

2 What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50? (2012 AMC 8 Problem, Question #18)

A. 3127

B. 3133

C. 3137

D. 3139

E. 3149



Prime Factorization

1 How many positive factors does 23,232 have? (2018 AMC 8 Problem, Question #18)

A. 9

B. 12

C. 28

D. 36

E. 42

2 On June 1, a group of students is standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group? (2015 AMC 8 Problem, Question #22)

A. 21

B. 30

C. 60

D. 90



1 The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c? (2016 AMC 8 Problem, Question #20)

A. 20

B. 30

C. 60

D. 120

E. 180

2 Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren? (2017 AMC 8 Problem, Question #24)

A. 78

B. 80

C. 144

D. 146



Remainders

1 The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers? (2012 AMC 8 Problem, Question #15)

A. 40 and 50

B. 51 and 55

C. 56 and 60

D. 61 and 65

E. 66 and 99

2 How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11? (2018 AMC 8 Problem, Question #21)

A. 1

B. 2

C. 3

D. 4



Place Value

- 1 A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number? (2014 AMC 8 Problem, Question #22)
 - A. 1
- B. 3
- C. 5
- D. 7
- E. 9

- 2 The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result? (2010 AMC 8 Problem, Question #22)
 - A. 0
- B. 2
- C. 4
- D. 6
- E. 8



1 The 5-digit number <u>2018U</u> is divisible by 9. What is the remainder when this number is divided by 8? (2018 AMC 8 Problem, Question #7)

A. 1

B. 3

C. 5

D. 6

E. 7

2 The sum of two prime numbers is 85. What is the product of these two prime numbers? (2014 AMC 8 Problem, Question #4)

A. 85

B. 91

C. 115

D. 133

- 3 Let w, x, y, and z be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588$, then what does 2w + 3x + 5y + 7z equal? (2011 AMC 8 Problem, Question #17)
 - A. 21
- B. 25
- C. 27
- D. 35
- E. 56

- 4 The positive integers x and y are the two smallest positive integers for which the product of 360 and x is a square and the product of 360 and y is a cube. What is the sum of x and y? (2009 AMC 8 Problem, Question #17)
 - A. 80
- B. 85
- C. 115
- D. 165
- E. 610

(5) What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594? (2013 AMC 8 Problem, Question #10)

A. 110

B. 165

C. 330

D. 625

E. 660

6 The smallest positive integer greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 lies between which of the following pairs of numbers? (2017 AMC 8 Problem, Question #12)

A. 2 and 19

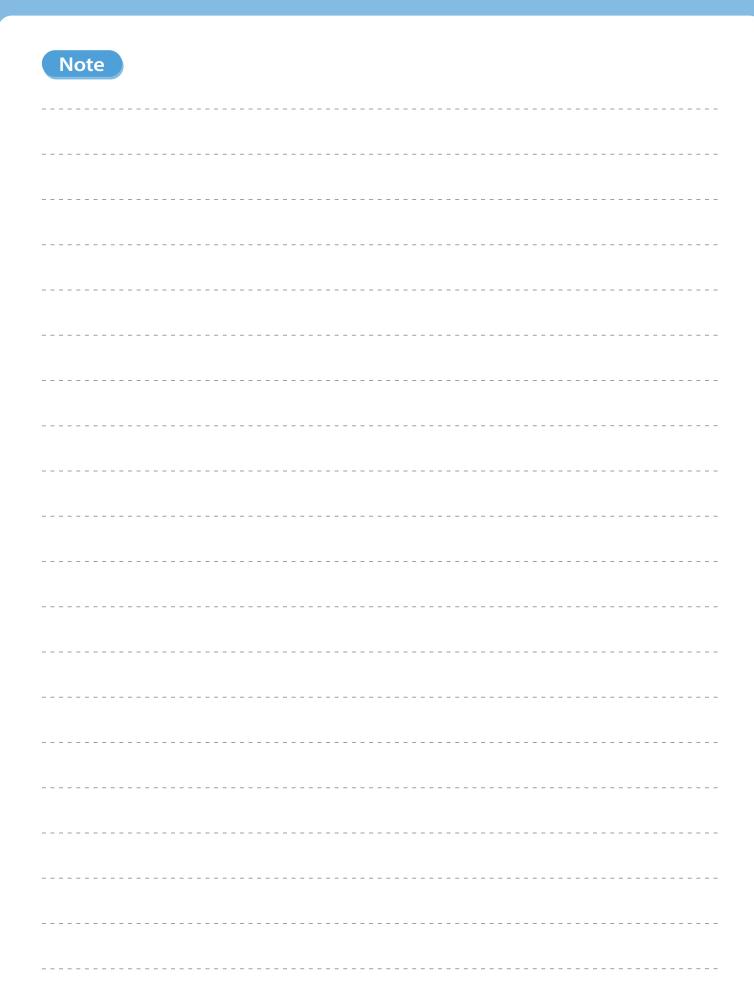
B. 20 and 39

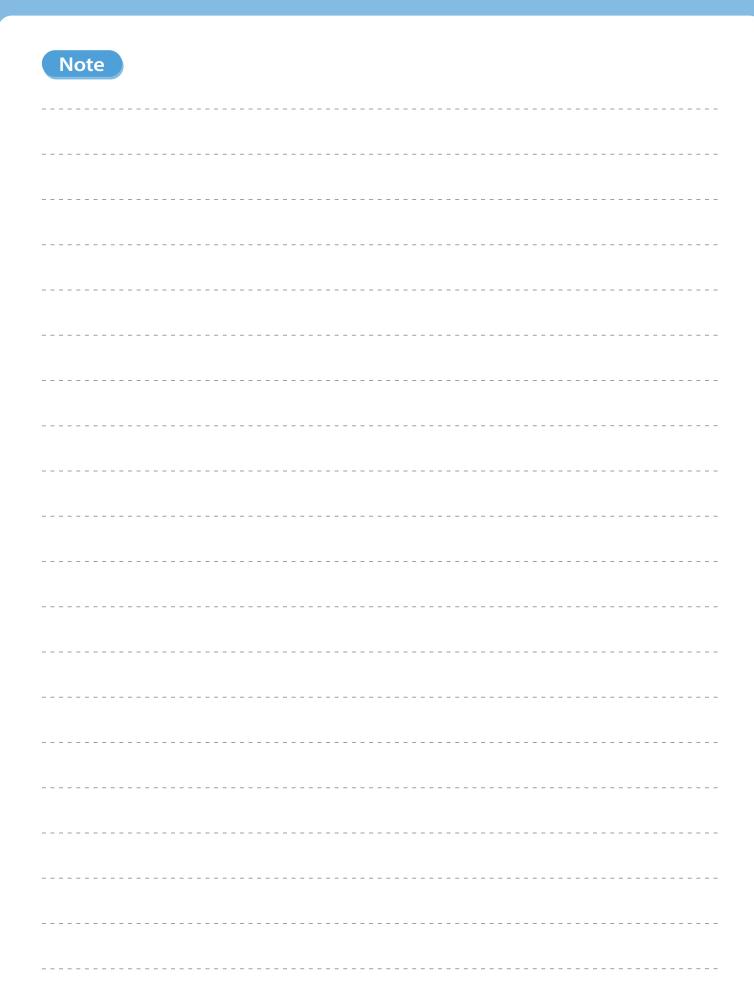
C. 40 and 59

D. 60 and 79

E. 80 and 124

7	When Clara totaled her scores, she inadvertently reversed the units digit and the tens digit of one score. By which of the following might her incorrect sum have differed from the correct one? (2013 AMC 8 Problem, Question #13)								
	A. 45	B. 46	C. 47	D. 48	E. 49				
8	 3 Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true." (1) It is prime. (2) It is even. (3) It is divisible by 7. (4) One of its digits is 9. This information allows Malcolm to determine Isabella's house number. What is its units digit? (2017 AMC 8 Problem, Question #8) A. 4 B. 6 C. 7 D. 8 E. 9 								





Lesson 4 Counting and Probability



Graphs & Stats

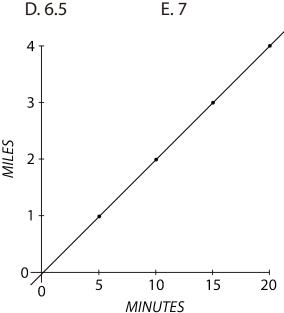
1 The graph shows the constant rate at which Suzanna rides her bike. If she rides a total of a half an hour at the same speed, how many miles would she have ridden? (2009) AMC 8 Problem, Question #3)

A. 5

B. 5.5

C. 6

D. 6.5



2 Let R be a set of nine distinct integers. Six of the elements are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of R? (2012 AMC 8 Problem, Question #22)

A. 4

B. 5

C. 6

D. 7



Venn Diagrams

- 1 In a room, $\frac{2}{5}$ of the people are wearing gloves, and $\frac{3}{4}$ of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove? (2010 AMC 8 Problem, Question #20)
 - A. 3
- B. 5
- C. 8
- D. 15
- E. 20

- 2 On a beach 50 people are wearing sunglasses and 35 people are wearing caps. Some people are wearing both sunglasses and caps. If one of the people wearing a cap is selected at random, the probability that this person is also wearing sunglasses is $\frac{2}{5}$. If instead, someone wearing sunglasses is selected at random, what is the probability that this person is also wearing a cap? (2019 AMC 8 Problem, Question #15)

- A. $\frac{14}{85}$ B. $\frac{7}{25}$ C. $\frac{2}{5}$ D. $\frac{4}{7}$ E. $\frac{7}{10}$



Enumerate

1 Abby, Bridget, and four of their classmates will be seated in two rows of three for a group picture, as shown.

X X X

X Χ X

If the seating positions are assigned randomly, what is the probability that Abby and Bridget are adjacent to each other in the same row or the same column? (2018 AMC 8 Problem, Question #11)

A. $\frac{1}{3}$ B. $\frac{2}{5}$ C. $\frac{7}{15}$ D. $\frac{1}{2}$ E. $\frac{2}{3}$

2 Samantha lives 2 blocks west and 1 block south of the southwest corner of City Park. Her school is 2 blocks east and 2 blocks north of the northeast corner of City Park. On school days she bikes on streets to the southwest corner of City Park, then takes a diagonal path through the park to the northeast corner, and then bikes on streets to school. If her route is as short as possible, how many different routes can she take? (2013 AMC 8 Problem, Question #21)

A. 3

B. 6

C. 9

D. 12



Coins & Dice

- 1 A fair coin is tossed 3 times. What is the probability of at least two consecutive heads? (2013 AMC 8 Problem, Question #8)
- A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. $\frac{3}{4}$

- 2 A fair 6 sided die is rolled twice. What is the probability that the first number that comes up is greater than or equal to the second number? (2011 AMC 8 Problem, Question #18)

- A. $\frac{1}{6}$ B. $\frac{5}{12}$ C. $\frac{1}{2}$ D. $\frac{7}{12}$ E. $\frac{5}{6}$



Forming Numbers

1 An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1, then how many passwords are possible? (2016 AMC 8 Problem, Question #17)

A. 30

B. 7290

C. 9000

D. 9990

E. 9999

2 How many 4-digit numbers greater than 1000 are there that use the four digits of 2012? (2012 AMC 8 Problem, Question #10)

A. 6

B. 7

C. 8

D. 9

3 A box contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn? (2016 AMC 8 Problem, Question #21)

A. $\frac{3}{10}$

B. $\frac{2}{5}$ C. $\frac{1}{2}$ D. $\frac{3}{5}$ E. $\frac{7}{10}$



Stars and Bars

1 Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the three people has at least two apples? (2019 AMC 8 Problem, Question #25) A. 105 B. 114 C. 190 D. 210 E. 380



Assignment

1 The mean, median, and unique mode of the positive integers 3, 4, 5, 6, 6, 7, and x are all equal. What is the value of x? (2012 AMC 8 Problem, Question #11)

A. 5

B. 6

C. 7

D. 11

E. 12

2 At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues? (2015 AMC 8 Problem, Question #15)

A. 49

- B. 70
- C. 79
- D. 99
- E. 149

3 How many non-congruent triangles have vertices at three of the eight points in the array shown below? (2009 AMC 8 Problem, Question #20)

• • •

• • • •

A. 5 B. 6 C. 7 D. 8 E. 9

4 Jack wants to bike from his house to Jill's house, which is located three blocks east and two blocks north of Jack's house. After biking each block, Jack can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach Jill's house by biking a total of five blocks? (2014 AMC 8 Problem, Question #11)

A. 4

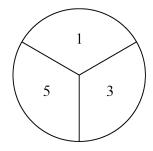
B. 5

C. 6

D. 8

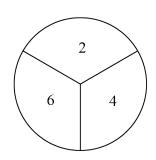
- 5 Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely? (2014 AMC 8 Problem, Question #18)
 - A. All 4 are boys
 - B. All 4 are girls
 - C. 2 are girls and 2 are boys
 - D. 3 are of one gender and 1 is of the other gender
 - E. All of these outcomes are equally likely

6 The two spinners shown are spun once and each lands on one of the numbered sectors. What is the probability that the sum of the numbers in the two sectors is prime? (2009 AMC 8 Problem, Question #12)



A. $\frac{1}{2}$

B. $\frac{2}{3}$

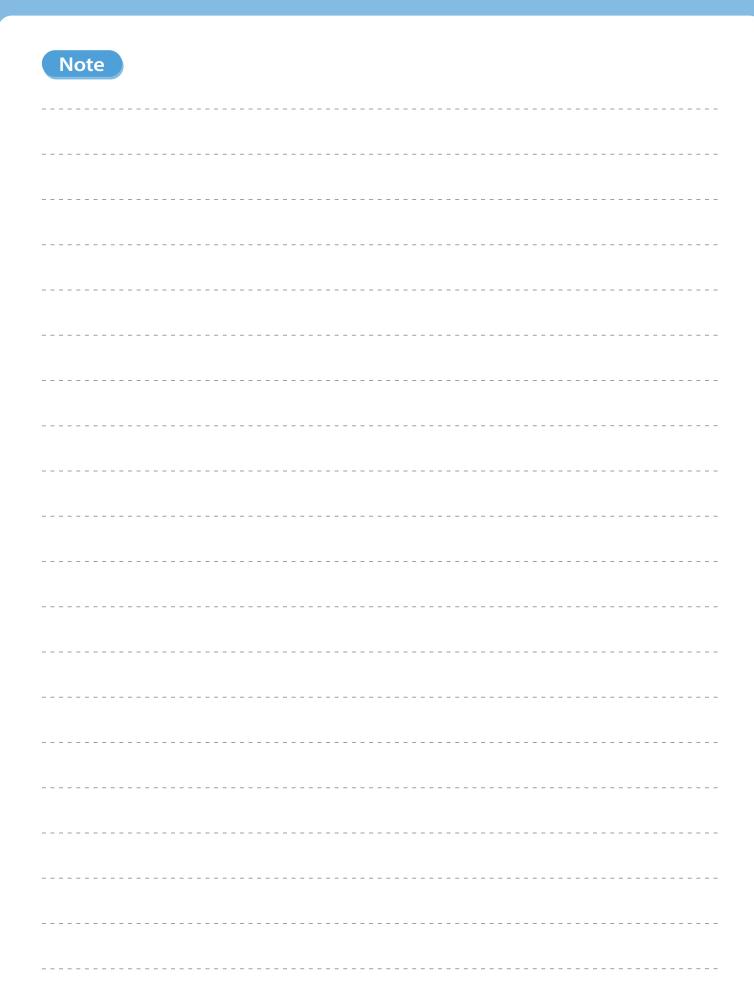


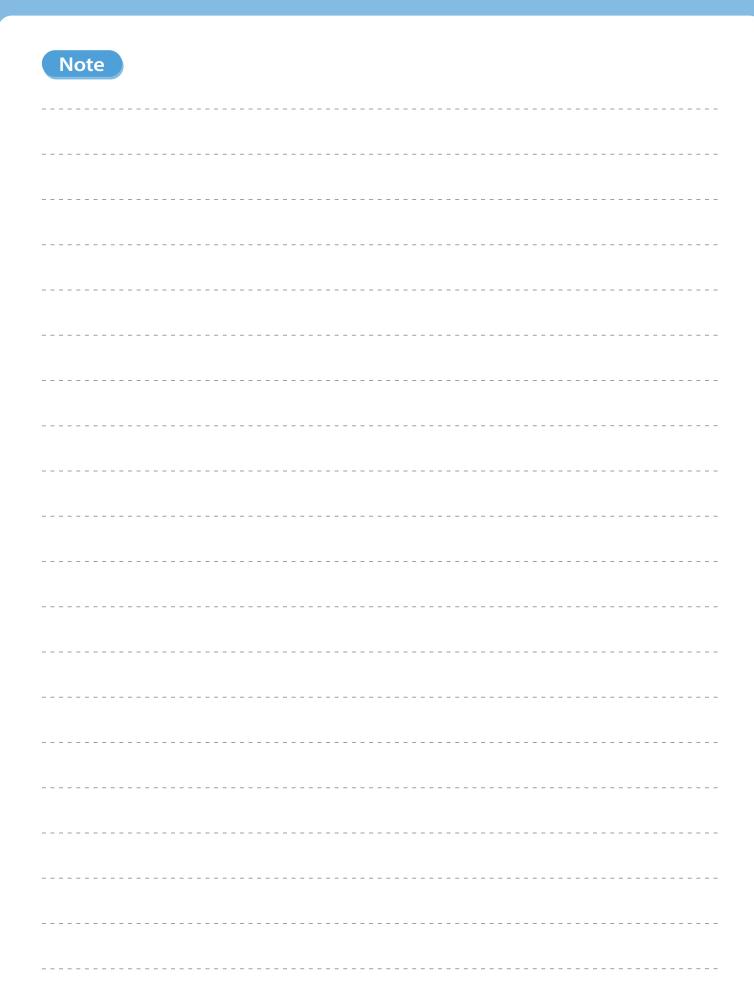
- C. $\frac{3}{4}$
- D. $\frac{7}{9}$
- E. $\frac{5}{6}$

- 7 A three-digit integer contains one of each of the digits 1, 3, and 5. What is the probability that the integer is divisible by 5? (2009 AMC 8 Problem, Question #13)
 - A. $\frac{1}{6}$

- B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. $\frac{5}{6}$

- 8 A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected? (2017 AMC 8 Problem, Question #10)
 - A. $\frac{1}{10}$
- B. $\frac{1}{5}$ C. $\frac{3}{10}$ D. $\frac{2}{5}$ E. $\frac{1}{2}$





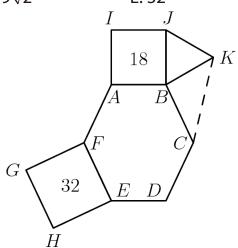
Lesson 5 Geometry



Pythagorean Theorem

- 1 In the given figure hexagon ABCDEF is equiangular, ABJI and FEHG are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and FE = BC. What is the area of \triangle *KBC*? (2015 AMC 8 Problem, Question #21)
 - A. $6\sqrt{2}$
- B. 9
- C. 12
- D. $9\sqrt{2}$

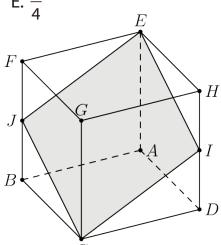




- 2 In the cube ABCDEFGH with opposite vertices C and E J and I are the midpoints of segments \overline{FB} and \overline{HD} respectively. Let R be the ratio of the area of the cross-section EJCI to the area of one of the faces of the cube. What is R^2 ? (2018 AMC 8 Problem, Ouestion #24)
 - A. $\frac{5}{4}$

- B. $\frac{4}{3}$ C. $\frac{3}{2}$ D. $\frac{25}{16}$





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1 Point E is the midpoint of side \overline{CD} in square ABCD, and \overline{BE} meets diagonal \overline{AC} at F. The area of quadrilateral AFED is 45. What is the area of ABCD? (2018 AMC 8 Problem, Question #22)

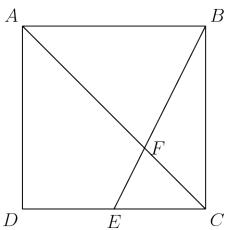
A. 100

B. 108

C. 120

D. 135

E. 144

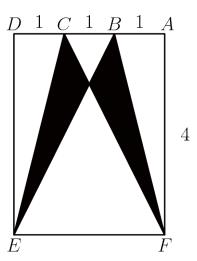


2 Rectangle *DEFA* below is a 3×4 rectangle with DC = CB = BA. What is the area of the "bat wings" (shaded area)? (2016 AMC 8 Problem, Question #22)

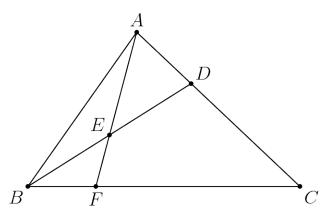
A. 2

B. $2\frac{1}{2}$

C. 3 D. $3\frac{1}{2}$



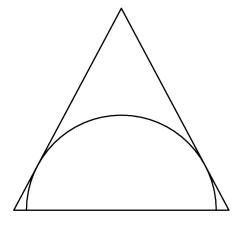
- 3 In triangle ABC, point D divides side \overline{AC} so that AD:DC = 1:2. Let E be the midpoint of \overline{BD} and let F be the point of intersection of line BC and line AE. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$? (2019 AMC 8 Problem, Question #24)
 - A. 24
- B. 30
- C. 32
- D. 36
- E. 40



S.

Circles & Tangents

- 1 A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle? (2016 AMC 8 Problem, Question #25)
 - A. $4\sqrt{3}$
- B. $\frac{120}{17}$
- C. 10
- D. $\frac{17\sqrt{2}}{2}$
- E. $\frac{17\sqrt{3}}{2}$



2 In the figure shown, \overline{US} and \overline{UT} are line segments each of length 2, and $m \angle TUS =$ 60°. Arcs \widehat{TR} and \widehat{SR} are each one-sixth of a circle with radius 2. What is the area of the region shown? (2017 AMC 8 Problem, Question #25)

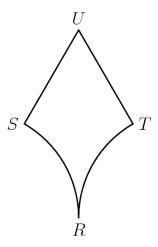
A.
$$3\sqrt{3} - \pi$$

B.
$$4\sqrt{3} - \frac{4\pi}{3}$$

C.
$$2\sqrt{3}$$

A.
$$3\sqrt{3} - \pi$$
 B. $4\sqrt{3} - \frac{4\pi}{3}$ C. $2\sqrt{3}$ D. $4\sqrt{3} - \frac{2\pi}{3}$ E. $4 + \frac{4\pi}{3}$

E.
$$4 + \frac{4\pi}{3}$$

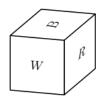


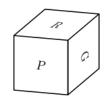


3D Figures

1) The faces of a cube are painted in six different colors: red (R), white (W), green (G), brown (B), agua (A), and purple (P). Three views of the cube are shown below. What is the color of the face opposite the agua face? (2019 AMC 8 Problem, Question #12) A. Red B. White C. Green D. Brown E. Purple







2 A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white. If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white? (2014 AMC 8 Problem, Question #19)

A. $\frac{5}{54}$

B. $\frac{1}{9}$

C. $\frac{5}{27}$

D. $\frac{2}{9}$

E. $\frac{1}{3}$

S.

More Geometry

1 What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19? (2015 AMC 8 Problem, Question #8)

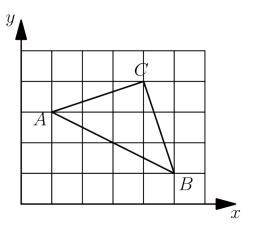
A. 24

B. 29

C. 43

D. 48

- 2 A triangle with vertices as A = (1, 3), B = (5, 1), and C = (4, 4) is plotted on a 6 × 5 grid. What fraction of the grid is covered by the triangle? (2015 AMC 8 Problem, Question #19)
- A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$

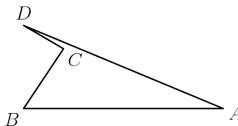


- 3 Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D. The circles intersect at two points, one of which is E. What is the degree measure of $\angle CED$? (2016 AMC 8 Problem, Question #23)
 - A. 90
- B. 105
- C. 120
- D. 135
- E. 150

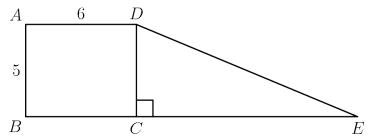


Assignment

- 1 In the non-convex quadrilateral ABCD shown below, $\angle BCD$ is a right angle, AB = 12, BC = 4, CD = 3, and AD = 13. What is the area of quadrilateral ABCD? (2017 AMC 8 Problem, Question #18)
 - A. 12
- B. 24
- C. 26
- D. 30
- E. 36

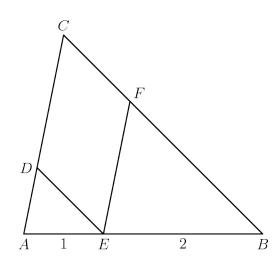


- 2 Rectangle ABCD and right triangle DCE have the same area. They are joined to form a trapezoid, as shown. What is DE? (2014 AMC 8 Problem, Question #14)
 - A. 12
- B. 13
- C. 14
- D. 15
- E. 16

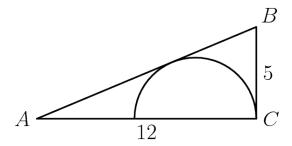


- 3 In $\triangle ABC$ a point E is on \overline{AB} with AE = 1 and EB = 2. Point D is on \overline{AC} so that $\overline{DE} \parallel \overline{BC}$ and point F is on \overline{BC} so that $\overline{EF} \parallel \overline{AC}$. What is the ratio of the area of CDEF to the area of $\triangle ABC$? (2018 AMC 8 Problem, Question #20)

- B. $\frac{1}{2}$ C. $\frac{5}{9}$ D. $\frac{3}{5}$



- 4 In the right triangle ABC, AC = 12, BC = 5, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle? (2017 AMC 8Problem, Question #22)
- B. $\frac{13}{5}$ C. $\frac{59}{18}$

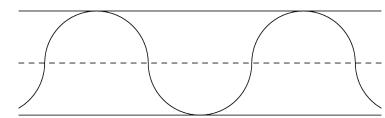


5 A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?

Note: 1 mile = 5280 feet

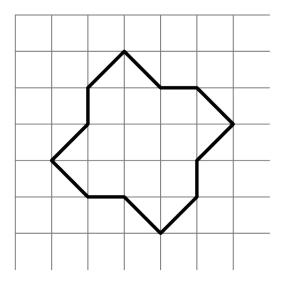
(2014 AMC 8 Problem, Question #25)

- A. $\frac{\pi}{11}$
- B. $\frac{\pi}{10}$
- $C.\frac{\pi}{5}$
- $D.\frac{2\pi}{5}$
- $E.\frac{2\pi}{3}$

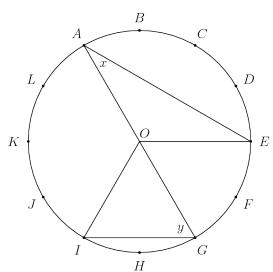


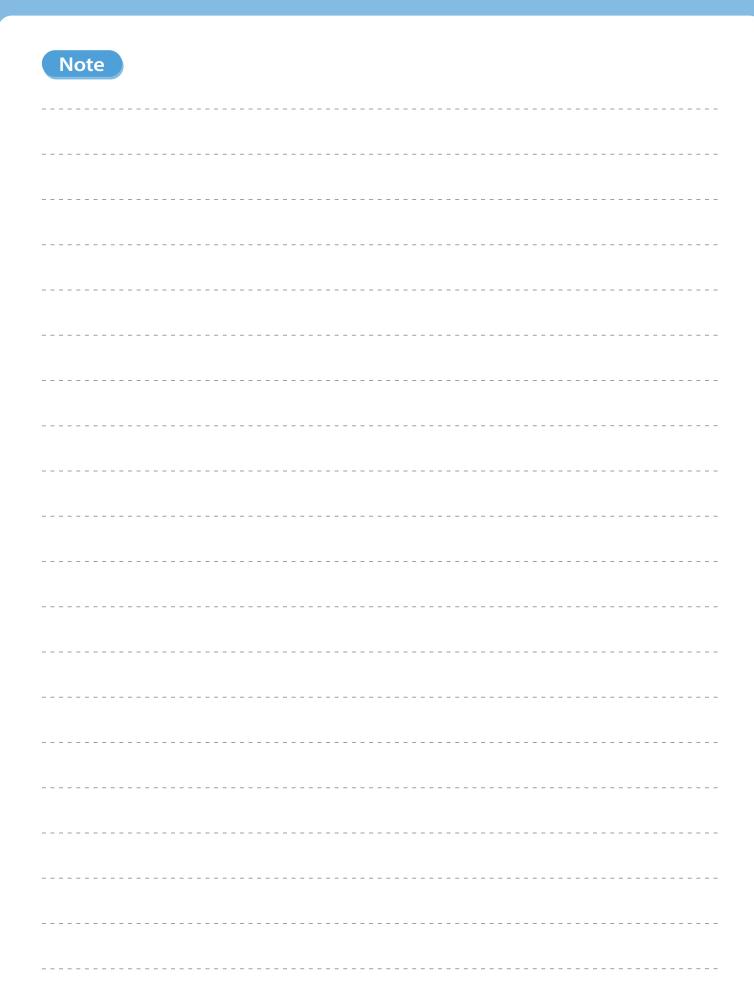
- 6 Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 6 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 6 cm high. What is the ratio of the volume of one of Alex's cans to the volume of one of Felicia's cans? (2019 AMC 8 Problem, Question #9)
 - A. 1:4
- B. 1:2
- C. 1:1
- D. 2:1
- E. 4:1

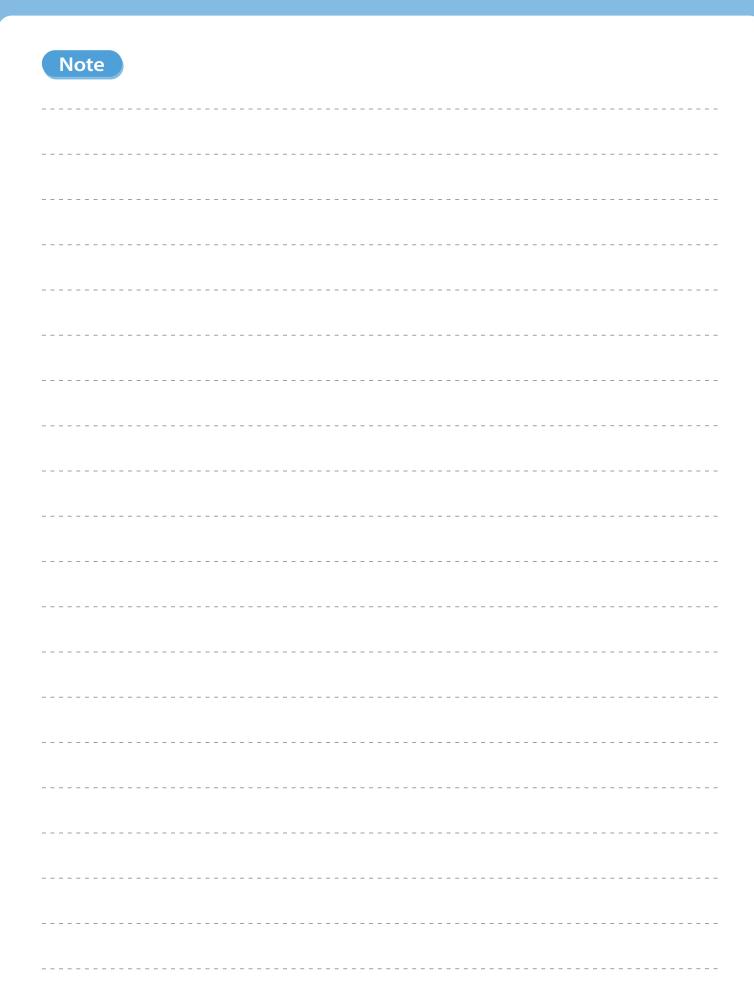
- 7 The twelve-sided figure shown has been drawn on 1 cm \times 1 cm graph paper. What is the area of the figure in cm²? (2018 AMC 8 Problem, Question #4)
 - A. 12
- B. 12.5
- C. 13
- D. 13.5
- E. 14



- 8 The circumference of the circle with center O is divided into 12 equal arcs, marked with the letters A through L as seen below. What is the number of degrees in the sum of the angles x and y? (2014 AMC 8 Problem, Question #15)
 - A. 75
- B. 80
- C. 90
- D. 120
- E. 150







Lesson 6 Combinatorics



Least & Greatest

1 A palindrome is a number that has the same value when read from left to right or from right to left. (For example 12321 is a palindrome.) Let N be the least three-digit integer which is not a palindrome but which is the sum of three distinct two-digit palindromes. What is the sum of the digits of N? (2019 AMC 8 Problem, Question #13)

A. 2

B. 3

C. 4

D. 5

E. 6

2 Each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 is used only once to make two five-digit numbers so that they have the largest possible sum. Which of the following could be one of the numbers? (2012 AMC 8 Problem, Question #16)

A. 76531

B. 86724

C. 87431

D. 96240

3	A five-legged	Martian has a	drawer full of s	ocks, each of wl	hich is red, white or blu	e,		
	and there are at least five socks of each color. The Martian pulls out one sock at a ti							
	without looking. How many socks must the Martian remove from the drawer to be							
	certain there will be 5 socks of the same color? (2005 AMC 8 Problem, Question #16)							
	A. 6	B. 9	C. 12	D. 13	E. 15			



Arranging & Ordering

1 Students Arn, Bob, Cyd, Dan, Eve, and Fon are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7, that person leaves the circle and the counting continues. Who is the last one present in the circle? (2018 AMC 8 Problem, Question #3) A. Arn B. Bob C. Cyd D. Dan E. Eve

2 Tiles I, II, III and IV are translated so one tile coincides with each of the rectangles A, B, C and D. In the final arrangement, the two numbers on any side common to two adjacent tiles must be the same. Which of the tiles is translated to Rectangle C? (2007 AMC 8 Problem, Question #11)

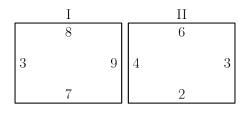
A. I

B. II

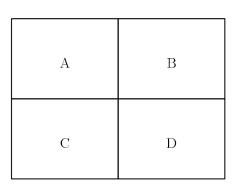
C. III

D. IV

E. Cannot be determined



	III			IV	
	7			2	
1		5	9		1
	0			6	



3 Tom has twelve slips of paper which he wants to put into five cups labeled A, B, C, D, E. He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from A to E. The numbers on the papers are 2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3, 5, 4, and 4.5. If a slip with 2 goes into cup E and a slip with 3 goes into cup E, then the slip with 3.5 must go into what cup? (2015 AMC 8 Problem, Question #23)

A. *A*

 $\mathsf{B.}\,B$

C. *C*

D. *L*

 $\mathsf{E}.E$



Sporting Events

1 Peter, Emma, and Kyler played chess with each other. Peter won 4 games and lost 2 games. Emma won 3 games and lost 3 games. If Kyler lost 3 games, how many games did he win? (2017 AMC 8 Problem, Question #13)

A. 0

B. 1

C. 2

D. 3

E. 4

2 In an All-Area track meet, 216 sprinters enter a 100-meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race, the five non-winners are eliminated, and the winner will compete again in a later race. How many races are needed to determine the champion sprinter? (2016 AMC 8 Problem, Ouestion #18)

A. 36

B. 42

C. 43

D. 60

3 The Little Twelve Basketball Conference has two divisions, with six teams in each
division. Each team plays each of the other teams in its own division twice and every
team in the other division once. How many conference games are scheduled? (2004)
AMC 8 Problem, Question #14)

A. 80

B. 96

C. 100

D. 108

E. 192

4 In the BIG N, a middle school football conference, each team plays every other team exactly once. If a total of 21 conference games were played during the 2012 season, how many teams were members of the BIG N conference? (2012 AMC 8 Problem, Question #14)

A. 6

B. 7

C. 8

D. 9



Logical Reasoning

1 Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, 'I didn't get the lowest score in our class,' and Bridget adds, 'I didn't get the highest score.' What is the ranking of the three girls from highest to lowest? (2013 AMC 8 Problem, Question #19)

A. Hannah, Cassie, Bridget C. Cassie, Bridget, Hannah

B. Hannah, Bridget, Cassie D. Cassie, Hannah, Bridget

E. Bridget, Cassie, Hannah

2 To complete the grid below, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand square? (2007 AMC 8 Problem, Question #9)

A. 1

- B. 2
- C. 3
- D. 4
- E. Cannot be determined

1		2	
2	3		
			4



1 Soda is sold in packs of 6, 12 and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda? (2005 AMC 8 Problem, Question #5)

A. 4

B. 5

C. 6

D. 8

E. 15

2 Ms. Osborne asks each student in her class to draw a rectangle with integer side lengths and a perimeter of 50 units. All of her students calculate the area of the rectangle they draw. What is the difference between the largest and smallest possible areas of the rectangles? (2008 AMC 8 Problem, Question #17)

A. 76

B. 120

C. 128

D. 132

F. 136

3 Isabella has 6 coupons that can be redeemed for free ice cream cones at Pete's Sweet Treats. In order to make the coupons last, she decides that she will redeem one every 10 days until she has used them all. She knows that Pete's is closed on Sundays, but as she circles the 6 dates on her calendar, she realizes that no circled date falls on a Sunday. On what day of the week does Isabella redeem her first coupon? (2019 AMC 8 Problem, Ouestion #14)

A. Monday

B. Tuesday

C. Wednesday

D. Thursday

E. Friday

4 A certain calculator has only two keys [+1] and [x2]. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed [+1], it would display "10." If you then pressed [x2], it would display "20." Starting with the display "1," what is the fewest number of keystrokes you would need to reach "200"? (2005 AMC 8 Problem, Question #24)

A. 8

B. 9

C. 10

D. 11

5 In a tournament there are six teams that play each other twice. A team earns 3 points for a win, 1 point for a draw, and 0 points for a loss. After all the games have been played it turns out that the top three teams earned the same number of total points. What is the greatest possible number of total points for each of the top three teams? (2019 AMC 8 Problem, Question #19)

A. 22

B. 23

C. 24

D. 26

E. 30

6 A baseball league consists of two four-team divisions. Each team plays every other team in its division N games. Each team plays every team in the other division M games with N > 2M and M > 4. Each team plays a 76 game schedule. How many games does a team play within its own division? (2015 AMC 8 Problem, Question #24)

A. 36

B. 48

C. 54

D. 60

7 A singles tournament had six play	yers. Each player played every other player only
once, with no ties. If Helen won 4	games, Ines won 3 games, Janet won 2 games,
Kendra won 2 games and Lara won	2 games, how many games did Monica win? (2006
AMC 8 Problem, Ouestion #20)	

A. 0

B. 1

C. 2

D. 3

E. 4

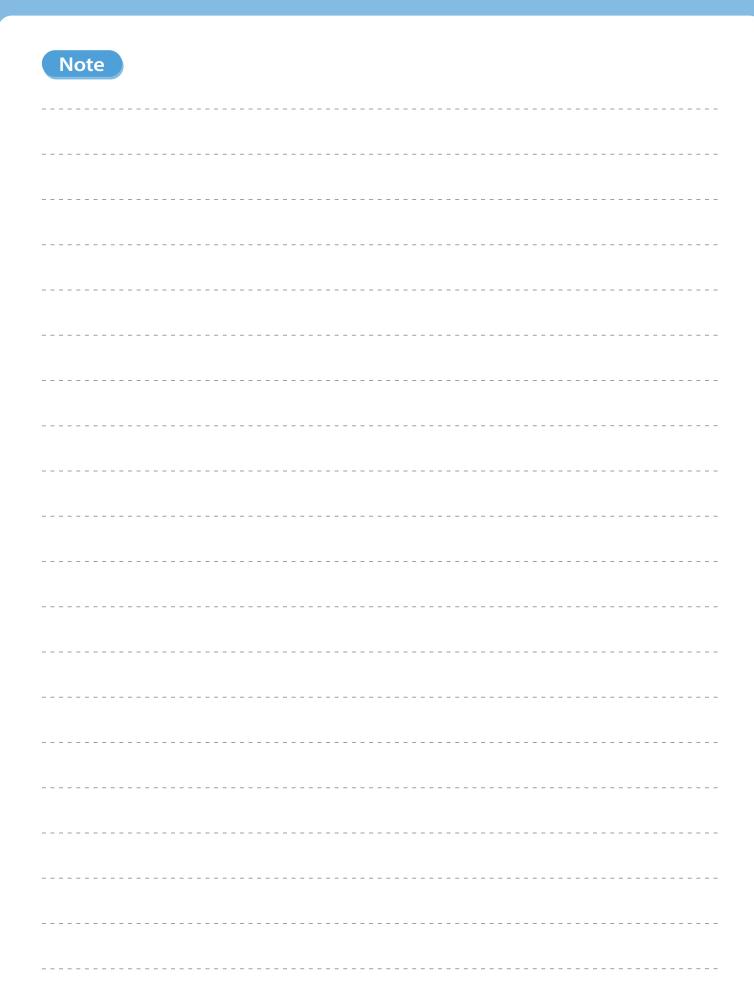
8 Students guess that Norb's age is 24, 28, 30, 32, 36, 38, 41, 44, 47, and 49. Norb says, "At least half of you guessed too low, two of you are off by one, and my age is a prime number." How old is Norb? (2011 AMC 8 Problem, Question #21)

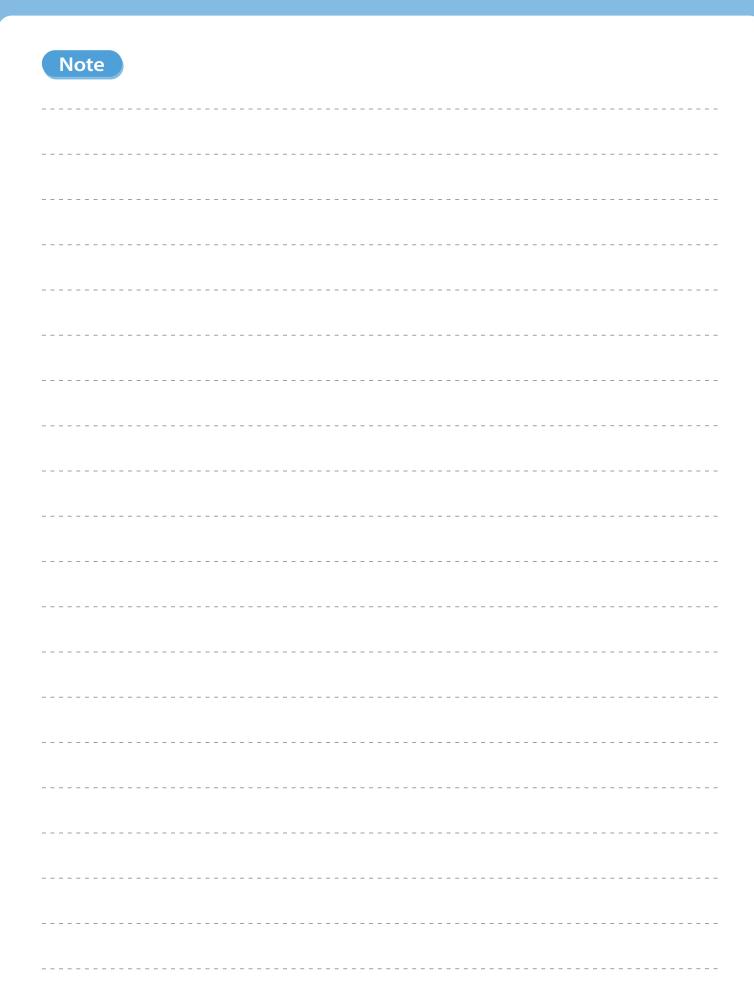
A. 29

B. 31

C. 37

D. 43















Email: contact@thethinkacademy.com

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