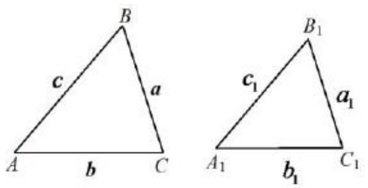
1. BASIC KNOWLEDGE

Similar triangles are triangles whose corresponding angles are congruent and whose corresponding sides are in proportion to each other. Similar triangles have the same shape but are not necessarily the same size.

The symbol for "similar" is ~. The notation $\Delta ABC \sim \Delta A'B'C'$ is read "triangle *ABC* is similar to triangle *A*-prime *B*-prime *C*-prime."

1.1. Principles of Similar Triangles

Principle 1. (*SSS*) Corresponding sides (segments) of similar triangles are in proportion to each other.



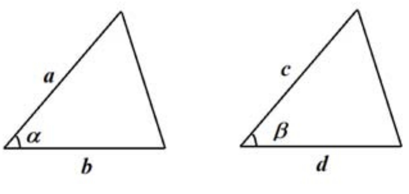
If $\Delta ABC \sim \Delta A_1B_1C_1$, then $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$.

If $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$, then $\Delta ABC \sim \Delta A_1B_1C_1$.

Principles 2. (*AA*) If two angles of one triangle are congruent respectively to two angles of the other triangle, the two triangles are similar by *AA* (angle, angle).

Corollary of Principle 2: Two right triangles are similar if they have one congruent cute angle.

Principles 3. (*SAS*) If two sides of one triangle are proportional to the corresponding parts of another triangle, and the ***included*** angles are congruent, the two triangles are similar by *SAS* (side, angle, side).



If $\frac{a}{c} = \frac{b}{d}$ and $\alpha = \beta$, then two triangles are similar.

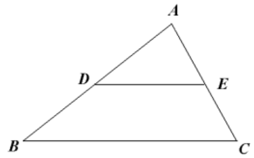
1.2. Important Theorems

Theorem 1. The ratio of the perimeters of two similar figures is:

$$\frac{P_{\Delta ABC}}{P_{\Delta A_1B_1C_1}} = \frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}.$$

The ratio of the areas of two similar figures is: $\frac{S_{\Delta ABC}}{S_{\Delta A_1B_1C_1}} = (\frac{a}{a_1})^2 = (\frac{b}{b_1})^2 = (\frac{c}{c_1})^2$.

Theorem 2. A line parallel to a side of a triangle cuts off a triangle similar to the given triangle. If $DE//BC$, then $\Delta ABC \sim \Delta ADE$.



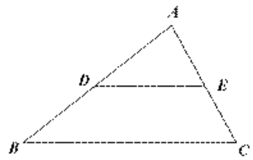
Proof:

Since $DE //BC$, $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$.

By the Principle 2 (AA), $\angle ADE = \angle ABC$.

If $\Delta ABC \sim \Delta AD$, then $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$; $\frac{AD}{DB} = \frac{AE}{EC}$; $\frac{AD}{AE} = \frac{DB}{EC}$.

Theorem 3. In ΔABC, if D is the midpoint of AB, E is the midpoint of AC, then $DE//BC$ and $DE = \frac{1}{2}BC$.



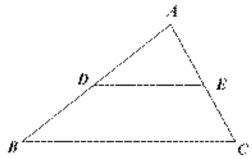
Proof:

We see that $\frac{AD}{AB} = \frac{1}{2}$, $\frac{AE}{AC} = \frac{1}{2}$, and $\angle A = \angle A$.

By the **Principles 3.** (SAS), we know that $\Delta ABC \sim \Delta ADE$. So $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$. Thus $DE//BC$.

$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{1}{2} \Rightarrow \qquad DE = \frac{1}{2}BC.$$

Theorem 4. In ΔABC, if D is the midpoint of AB, and $DE//BC$, then E is the midpoint of AC and $DE = \frac{1}{2} BC$.



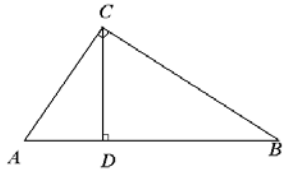
Theorem 5. If $\angle ACB = \angle ADC = 90°$, then $\Delta ABC \sim \Delta ACD \sim \Delta CBD$.

$AC^2 = AB \times AD$ (1)

$BC^2 = AB \times BD$ (2)

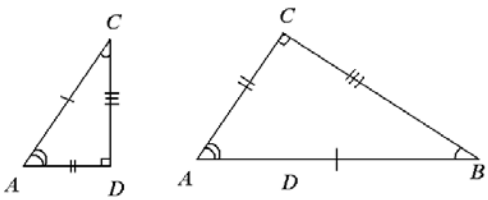
(3)

$CD \times AB = AC \times BC$ (4)



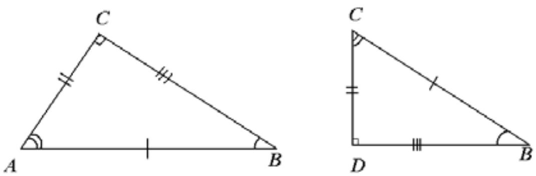
Proof:

(1). We separate two similar triangles as follows:



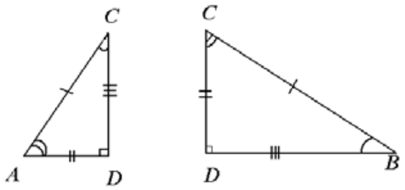
$$\frac{AC}{AB} = \frac{AD}{AC} \qquad \Rightarrow \qquad AC^2 = AB \times AD \qquad (1)$$

(2). We separate two similar triangles as follows:



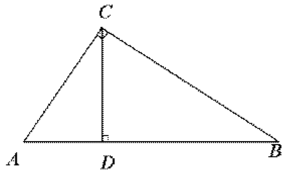
$$\frac{AB}{BC} = \frac{BC}{BD} \qquad \Rightarrow \qquad BC^2 = AB \times BD \qquad (2)$$

(3). We separate two similar triangles as follows:



$\frac{AD}{CD} = \frac{CD}{BD}$ $\Rightarrow$ (3)

(4). The area of triangle ABC is $S_{\Delta ABC} = \frac{1}{2} AC \times BC$

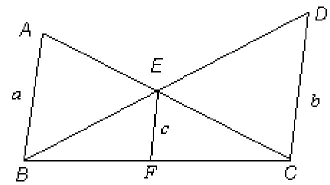


$S_{\Delta ABC}$ can also be written as $S_{\Delta ABC} = \frac{1}{2} AB \times CD$

$\frac{1}{2} AC \times BC = \frac{1}{2} AB \times CD \quad \Rightarrow \quad CD \times AB = AC \times BC$

Theorem 6. Given *AB*//*EF*//*CD*. *AB* = *a*, *CD* = *b*, and *EF* = *c*. Then

$\frac{1}{a} + \frac{1}{b} = \frac{1}{c} \quad \Rightarrow EF = c = \frac{ab}{a+b}$.



Proof:

$\Delta ABC \sim \Delta EFC.$ $\frac{c}{a} = \frac{FC}{BC}$ (1)

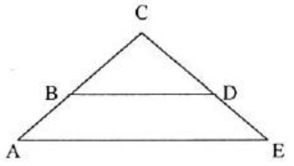
$\Delta DCB \sim \Delta EFB.$ $\frac{c}{b} = \frac{BF}{BC}$ (2)

(1) + (2): $\frac{c}{a} + \frac{c}{b} = \frac{FC + BF}{BC} = 1 \quad \Rightarrow \quad \frac{1}{a} + \frac{1}{b} = \frac{1}{c}$.

2. EXAMPLES

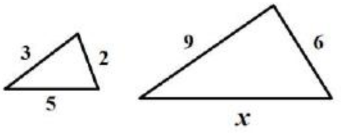
Example 1. Assume $\overline{BD}$ is parallel to $\overline{AE}$ in the figure shown. Which of the following segments corresponds to $\overline{CD}$ when we are considering the two similar triangles pictured?

A. $\overline{BD}$ B. $\overline{DE}$ C. $\overline{CE}$ D. $\overline{CA}$ E. $\overline{CE}$ and $\overline{CA}$



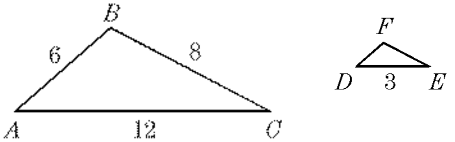
Example 2: These triangles are similar. Find x.

A. 9 B. 10 C. 15 D. 25 E. 12



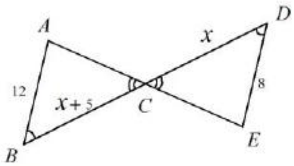
Example 3. Triangle *ABC* is similar to triangle *DEF* as sketched. The perimeter of triangle *DEF* is:

A. 6.5 B. 9 C. 6 D. 7.5 E. none of these



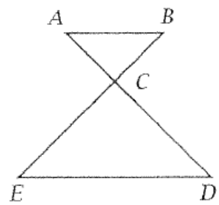
Example 4. In the figure shown, segment *AE* intersects segment *BD* at point *C*. Find the length of line segment *BD* if $\angle ABC = \angle CDE$.

A. 9 B. 10 C. 15 D. 25 E. 5



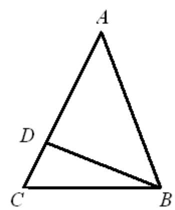
Example 5. If $AB \parallel DE$, $AB = 5$, $CE = 8$, and $DE = 12.5$, find the measure of $\overline{BC}$.

A. 20 B. 7.8 C. 15 D. 3.2 E. 4



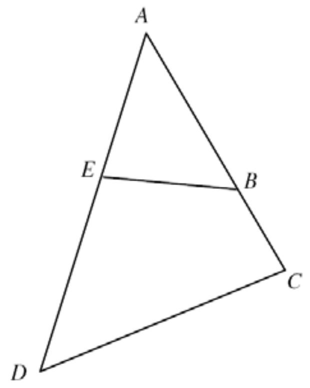
Example 6. If $AB = AC$, $DB = CB$, $AB = 12$ and $BC = 5$, find the measure of DC. Express your answer as a mixed number.

A. B. $\frac{7}{3}$ C. 3 D. $\frac{29}{12}$ E. $\frac{4\sqrt{13}}{7}$



Example 7. In the following diagram (not necessarily to scale), $\angle ABE = \angle ADC$, $AE = 6$, $BC = 2$, $BE = 3$, and $CD = 5$. $AB + DE$ is equal to

A. 46/3 B. 112/3 C. 13/2 D. 20 E. none of these

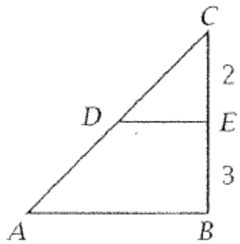


Example 8. A triangle with sides 9, 12 and 15 is similar to another triangle which has longest side 25. The area of the larger triangle is:

A. 54 B. 96 C. 105 D. 210 E. 150.

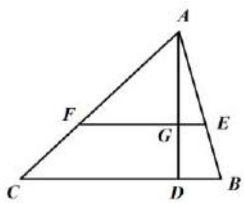
Example 9. In right triangle ABC, $\overline{DE}$ is parallel to $\overline{AB}$, $CE = 2$ cm, and $EB = 3$ cm. If the area of ΔABC is 30 cm^2, what is the number of square centimeters in the area of ΔCDE? Express your answer as a decimal number.

A. 5.4 B. 9.6 C. 4.8 D. 3.3 E. 5.

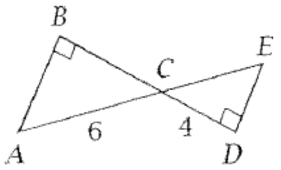


Example 10. A line parallel to the base of a triangle cuts the triangle into two regions of equal area. This line also cuts the altitude into two parts. Find the ratio of the two parts of the altitude.

A. 1 : 1 B. 1 : 2 C. 1 : $\sqrt{2}$ D. 1 : $\sqrt{2}+1$ E. none of these

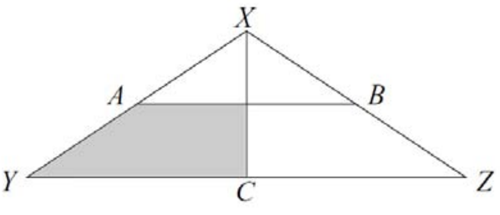


Example 11. In the figure, $AC = 6$ cm, $CD = 4$ cm, and $DE = 3$ cm. Find the number of square centimeters in the area of the triangle ABC .



A. $9\frac{1}{12}$ B. $7\frac{7}{3}$ C. 8 D. $8\frac{4}{5}$ E. $8\frac{16}{25}$

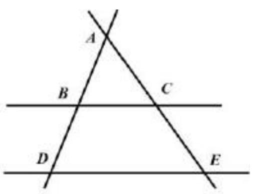
☆**Example 12.** (2002 AMC 8 problem 20) The area of triangle XYZ is 8 square inches. Points A and B are midpoints of congruent segments XY and XZ. Altitude XC bisects YZ. The area (in square inches) of the shaded region is



A. $1\frac{1}{2}$. B. 2 C. $2\frac{1}{2}$. D. 3. E. $3\frac{1}{2}$

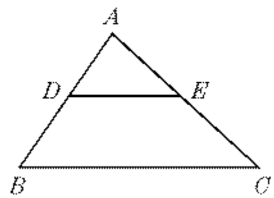
Example 13. In the figure, $\overline{BC}$ is parallel to $\overline{DE}$. The length of $\overline{AB}$ is 6, the length of $\overline{BD}$ is 4, and the length of $\overline{AC}$ is 9. What is the length of $\overline{CE}$?

A. $\frac{9}{2}$. B. 5 C. 6. D. $\frac{27}{2}$. E. Cannot be determined.



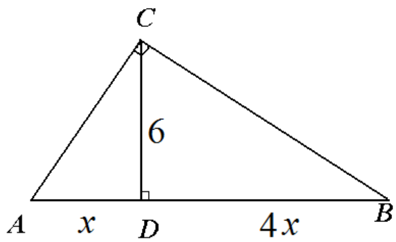
Example 14. In the sketch, *DE*//*BC* and *BD* is the square of *AD*. If *AC* = 21/4 and *EC* = 9/2, what is *BD* ?

A. 16 B. 36 C. 64 D. 144 E. none of these



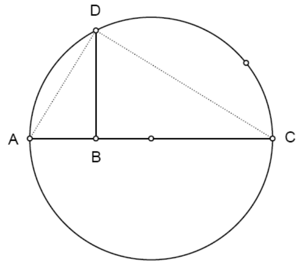
Example 15. In a right triangle, a perpendicular is dropped from the right angle to the hypotenuse and the segments of the hypotenuse have lengths of x inches and 4x inches. If the altitude is 6 inches in length, then x has length in inches, of:

A. 1/3 B. 2/3 C. 3/2 D. 3 E. cannot be determined



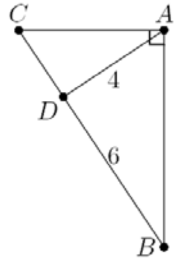
Example 16. *AC* is a diameter of a circle in which *AD* is a chord; *B* is a point on *AC* such that $DB \perp AC$. If $AB = 9$, and $BC = 16$, how long is *DB*?

A. 12 B. 13 C. 14 D. 15 E. 18.



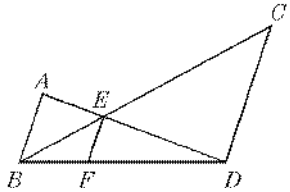
Example 17. What is length of *BC* in the right triangle $\triangle ABC$ if $AD \perp BC$?

(A) 26/3 (B) 52 (C) $\frac{10\sqrt{13}}{3}$ (D) $\frac{8\sqrt{13}}{3}$ (E) 1/3



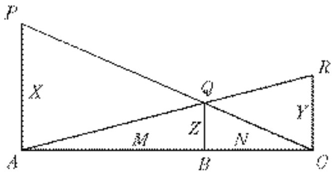
Example 18. In the diagram *AB//FE//DC*, and $AB = 2$ with $CD = 4$. Find the length of *EF*.

A. 4/3 B. 1 C. 3/4 D. 5/4 E. none of these

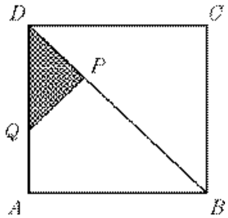


Example 19. In the figure, $\overline{PA}$, $\overline{QB}$ and $\overline{RC}$ are each perpendicular to $\overline{AC}$ .Which of the following is correct?

(A) $\frac{Z}{X}=\frac{N}{M}$ (B) $\frac{Z}{X}=\frac{M+N}{N}$ (C) $\frac{Z}{Y}=\frac{M}{N}$ (D) $\frac{Z}{Y}=\frac{M+N}{M}$ (E)

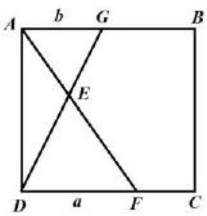


Example 20. What fraction of the area of square *ABCD* is represented by the area of ΔDPQ? $BP = BA$. $DP = QP$.



A. $\frac{1}{2}(\sqrt{2}-1)^2$ B. $\frac{1}{8}$ C. $\frac{\sqrt{2}}{2}$ D. $(\sqrt{2}-1)^2$ E. $\frac{\sqrt{2}}{6}$

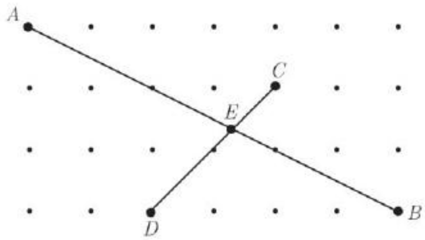
Example 21. In the unit square, find the distance from *E* to $\overline{AD}$ in terms of *a* and *b*, the lengths of $\overline{DF}$ and $\overline{AG}$, respectively.



A. $\frac{ab}{a+b}$ B. $\frac{b}{a+b}$ C. $\frac{a-b}{a+b}$ D. $\frac{a}{a+b}$ E. $\frac{2a-b}{a+b}$

☆**Example 22.** (2000 AMC 10 problem 16) The diagram shows 28 lattice points, each one unit from its nearest neighbors.

Segment AB meets segment CD at E. Find the length of segment AE.



A. $\frac{4\sqrt{5}}{3}$ B. $\frac{5\sqrt{5}}{3}$ C. $\frac{12\sqrt{5}}{7}$ D. $2\sqrt{5}$

E. $\frac{5\sqrt{65}}{9}$