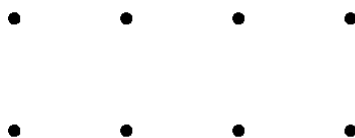


2009

- 20 How many non-congruent triangles have vertices at three of the eight points in the array shown below?



(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

- 21 Andy and Bethany have a rectangular array of numbers with 40 rows and 75 columns. Andy adds the numbers in each row. The average of his 40 sums is  $A$ . Bethany adds the numbers in each column. The average of her 75 sums is  $B$ . What is the value of  $\frac{A}{B}$ ?

(A)  $\frac{64}{225}$  (B)  $\frac{8}{15}$  (C) 1 (D)  $\frac{15}{8}$  (E)  $\frac{225}{64}$

- 22 How many whole numbers between 1 and 1000 do **not** contain the digit 1?

(A) 512 (B) 648 (C) 720 (D) 728 (E) 800

- 23 On the last day of school, Mrs. Wonderful gave jelly beans to her class. She gave each boy as many jelly beans as there were boys in the class. She gave each girl as many jelly beans as there were girls in the class. She brought 400 jelly beans, and when she finished, she had six jelly beans left. There were two more boys than girls in her class. How many students were in

her class?

(A) 26 (B) 28 (C) 30 (D) 32 (E) 34

- 24 The letters  $A$ ,  $B$ ,  $C$  and  $D$  represent digits. If 
$$\begin{array}{r} A \ B \\ + \ C \ A \\ \hline D \ A \end{array}$$
 and 
$$\begin{array}{r} A \ B \\ - \ C \ A \\ \hline A \end{array}$$
, what digit does  $D$  represent?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

- 25 A one-cubic-foot cube is cut into four pieces by three cuts parallel to the top face of the cube. The first cut is  $\frac{1}{2}$  foot from the top face. The second cut is  $\frac{1}{3}$  foot below the first cut, and the third cut is  $\frac{1}{17}$  foot below the second cut. From the top to the bottom the pieces are labeled A, B, C, and D. The pieces are then glued together end to end as shown in the second diagram. What is the total surface area of this solid in square feet?

- 20 In a room,  $\frac{2}{5}$  of the people are wearing gloves, and  $\frac{3}{4}$  of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove?
- (A) 3    (B) 5    (C) 8    (D) 15    (E) 20

- 21 Hui is an avid reader. She bought a copy of the best seller *Math is Beautiful*. On the first day,

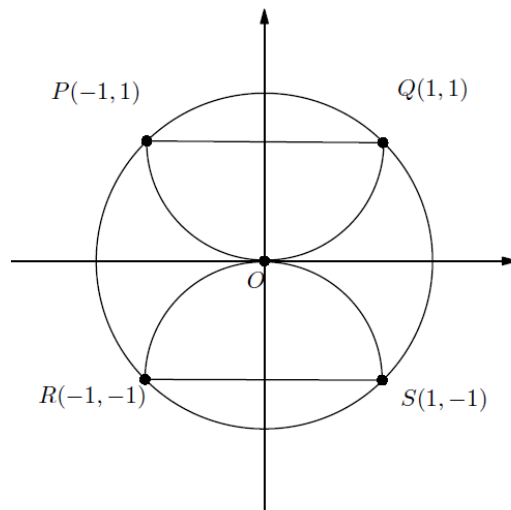
she read  $\frac{1}{5}$  of the pages plus 12 more, and on the second day she read  $\frac{1}{4}$  of the remaining pages plus 15 more. On the third day she read  $\frac{1}{3}$  of the remaining pages plus 18 more. She then realizes she has 62 pages left, which she finishes the next day. How many pages are in this book?

- (A) 120    (B) 180    (C) 240    (D) 300    (E) 360

- 22 The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

- (A) 0    (B) 2    (C) 4    (D) 6    (E) 8

- 23 Semicircles  $POQ$  and  $ROS$  pass through the center of circle  $O$ . What is the ratio of the combined areas of the two semicircles to the area of circle  $O$ ?



- (A)  $\frac{\sqrt{2}}{4}$     (B)  $\frac{1}{2}$     (C)  $\frac{2}{\pi}$     (D)  $\frac{2}{3}$     (E)  $\frac{\sqrt{2}}{2}$

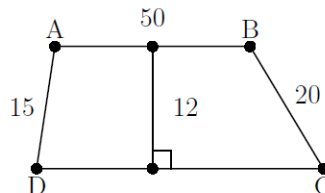
- 24 What is the correct ordering of the three numbers,  $10^8$ ,  $5^{12}$ , and  $2^{24}$ ?
- (A)  $2^{24} < 10^8 < 5^{12}$  (B)  $2^{24} < 5^{12} < 10^8$  (C)  $5^{12} < 2^{24} < 10^8$  (D)  $10^8 < 5^{12} < 2^{24}$  (E)  $10^8 < 2^{24} < 5^{12}$
- 

- 25 Everyday at school, Jo climbs a flight of 6 stairs. Joe can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

(A) 13      (B) 18      (C) 20      (D) 22      (E) 24

2011

- 20 Quadrilateral  $ABCD$  is a trapezoid,  $AD = 15$ ,  $AB = 50$ ,  $BC = 20$ , and the altitude is 12. What is the area of the trapezoid?



(A) 600    (B) 650    (C) 700    (D) 750    (E) 800

- 21 Students guess that Norb's age is 24, 28, 30, 32, 36, 38, 41, 44, 47, and 49. Norb says, "At least half of you guessed too low, two of you are off by one, and my age is a prime number." How old is Norb?

(A) 29    (B) 31    (C) 37    (D) 43    (E) 48

- 22 What is the tens digit of  $7^{2011}$ ?

(A) 0    (B) 1    (C) 3    (D) 4    (E) 7

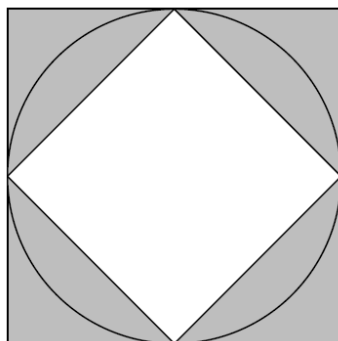
- 23 How many 4-digit positive integers have four different digits, where the leading digit is not zero, the integer is a multiple of 5, and 5 is the largest digit?

(A) 24    (B) 48    (C) 60    (D) 84    (E) 108

- 24 In how many ways can 10001 be written as the sum of two primes?

(A) 0    (B) 1    (C) 2    (D) 3    (E) 4

- 25 A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which fraction is closest to the ratio of the circle's shaded area to the area between the two squares?



(A)  $\frac{1}{2}$     (B) 1    (C)  $\frac{3}{2}$     (D) 2    (E)  $\frac{5}{2}$

2013

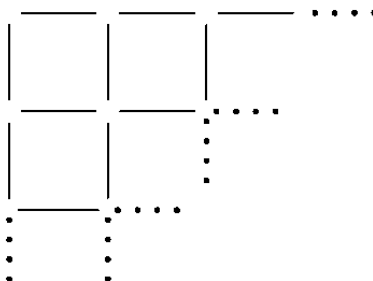
- 20 A  $1 \times 2$  rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle?

(A)  $\frac{\pi}{2}$     (B)  $\frac{2\pi}{3}$     (C)  $\pi$     (D)  $\frac{4\pi}{3}$     (E)  $\frac{5\pi}{3}$

- 21 Samantha lives 2 blocks west and 1 block south of the southwest corner of City Park. Her school is 2 blocks east and 2 blocks north of the northeast corner of City Park. On school days she bikes on streets to the southwest corner of City Park, then takes a diagonal path through the park to the northeast corner, and then bikes on streets to school. If her route is as short as possible, how many different routes can she take?

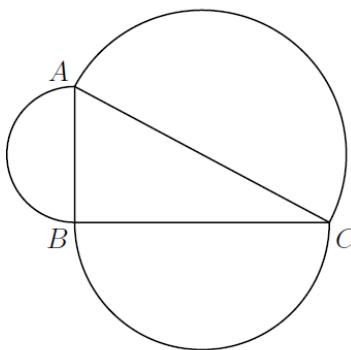
(A) 3    (B) 6    (C) 9    (D) 12    (E) 18

- 22 Toothpicks are used to make a grid that is 60 toothpicks long and 32 toothpicks wide. How many toothpicks are used altogether?



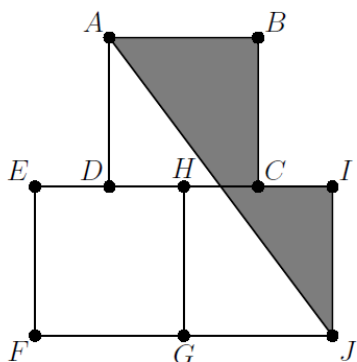
(A) 1920    (B) 1952    (C) 1980    (D) 2013    (E) 3932

- 23 Angle  $ABC$  of  $\triangle ABC$  is a right angle. The sides of  $\triangle ABC$  are the diameters of semicircles as shown. The area of the semicircle on  $\overline{AB}$  equals  $8\pi$ , and the arc of the semicircle on  $\overline{AC}$  has length  $8.5\pi$ . What is the radius of the semicircle on  $\overline{BC}$ ?

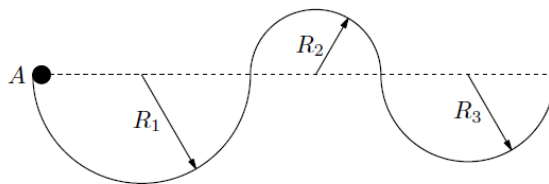


(A) 7    (B) 7.5    (C) 8    (D) 8.5    (E) 9

- 24 Squares  $ABCD$ ,  $EFGH$ , and  $GHIJ$  are equal in area. Points  $C$  and  $D$  are the midpoints of sides  $IH$  and  $HE$ , respectively. What is the ratio of the area of the shaded pentagon  $AJICB$  to the sum of the areas of the three squares?



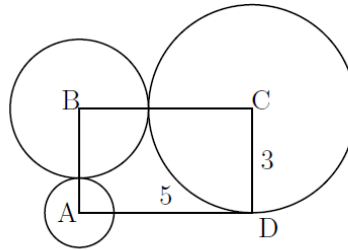
- (A)  $\frac{1}{4}$     (B)  $\frac{7}{24}$     (C)  $\frac{1}{3}$     (D)  $\frac{3}{8}$     (E)  $\frac{5}{12}$
- 25 A ball with diameter 4 inches starts at point A to roll along the track shown. The track is comprised of 3 semicircular arcs whose radii are  $R_1 = 100$  inches,  $R_2 = 60$  inches, and  $R_3 = 80$  inches, respectively. The ball always remains in contact with the track and does not slip. What is the distance the center of the ball travels over the course from A to B?



- (A)  $238\pi$     (B)  $240\pi$     (C)  $260\pi$     (D)  $280\pi$     (E)  $500\pi$

2014

- 20 Rectangle  $ABCD$  has sides  $CD = 3$  and  $DA = 5$ . A circle of radius 1 is centered at  $A$ , a circle of radius 2 is centered at  $B$ , and a circle of radius 3 is centered at  $C$ . Which of the following is closest to the area of the region inside the rectangle but outside all three circles?

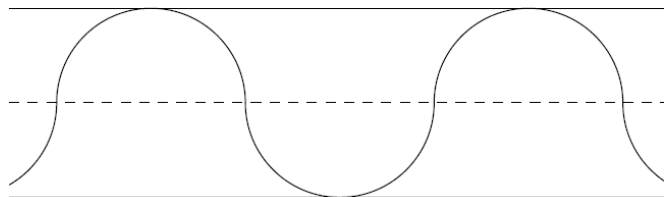


- (A) 3.5    (B) 4.0    (C) 4.5    (D) 5.0    (E) 5.5
- 
- 21 The 7-digit numbers  $\underline{74A52B1}$  and  $\underline{326AB4C}$  are each multiples of 3. Which of the following could be the value of  $C$ ?
- (A) 1    (B) 2    (C) 3    (D) 5    (E) 8
- 
- 22 A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number?
- (A) 1    (B) 3    (C) 5    (D) 7    (E) 9
- 
- 23 Three members of the Euclid Middle School girls' softball team had the following conversation.  
Ashley: I just realized that our uniform numbers are all 2-digit primes.  
Bethany: And the sum of your two uniform numbers is the date of my birthday earlier this month.  
Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.  
Ashley: And the sum of you two uniform numbers is today's date.  
What number does Caitlin wear?
- (A) 11    (B) 13    (C) 17    (D) 19    (E) 23
- 
- 24 One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?
- (A) 2.5    (B) 3.0    (C) 3.5    (D) 4.0    (E) 4.5



- 25 A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?

Note: 1 mile= 5280 feet



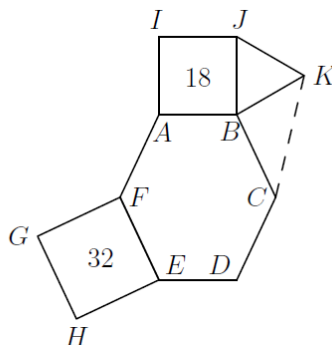
- (A)  $\frac{\pi}{11}$     (B)  $\frac{\pi}{10}$     (C)  $\frac{\pi}{5}$     (D)  $\frac{2\pi}{5}$     (E)  $\frac{2\pi}{3}$

- 20 Ralph went to the store and bought 12 pairs of socks for a total of \$24. Some of the socks he bought cost \$1 a pair, some of the socks he bought cost \$3 a pair, and some of the socks he bought cost \$4 a pair. If he bought at least one pair of each type, how many pairs of \$1 socks did Ralph buy?

(A) 4    (B) 5    (C) 6    (D) 7    (E) 8

- 21 In the given figure hexagon  $ABCDEF$  is equiangular,  $ABJI$  and  $FEHG$  are squares with areas 18 and 32 respectively,  $\triangle JBK$  is equilateral and  $FE = BC$ . What is the area of  $\triangle KBC$ ?

(A)  $6\sqrt{2}$     (B) 9    (C) 12    (D)  $9\sqrt{2}$     (E) 32



- 22 On June 1, a group of students is standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

(A) 21    (B) 30    (C) 60    (D) 90    (E) 1080

- 23 Tom has twelve slips of paper which he wants to put into five cups labeled  $A, B, C, D, E$ . He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from  $A$  to  $E$ . The numbers on the papers are 2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3.5, 4, and 4.5. If a slip with 2 goes into cup  $E$  and a slip with 3 goes into cup  $B$ , then the slip with 3.5 must go into what cup?

(A)  $A$     (B)  $B$     (C)  $C$     (D)  $D$     (E)  $E$

- 24 A baseball league consists of two four-team divisions. Each team plays every other team in its division  $N$  games. Each team plays every team in the other division  $M$  games with  $N > 2M$  and  $M > 4$ . Each team plays a 76 game schedule. How many games does a team play within its own division?

(A) 36    (B) 48    (C) 54    (D) 60    (E) 72

2012

- 20 What is the correct ordering of the three numbers  $\frac{5}{19}$ ,  $\frac{7}{21}$ , and  $\frac{9}{23}$ , in increasing order?
- (A)  $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$  (B)  $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$  (C)  $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$  (D)  $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$  (E)  $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$

- 21 Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Marla uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet?

(A)  $5\sqrt{2}$  (B) 10 (C)  $10\sqrt{2}$  (D) 50 (E)  $50\sqrt{2}$

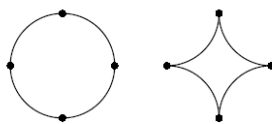
- 22 Let  $R$  be a set of nine distinct integers. Six of the elements are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of  $R$ ?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

- 23 An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon?

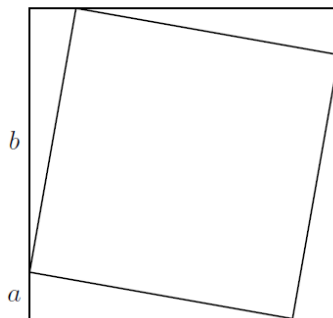
(A) 4 (B) 5 (C) 6 (D)  $4\sqrt{3}$  (E)  $6\sqrt{3}$

- 24 A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?



(A)  $\frac{4-\pi}{\pi}$  (B)  $\frac{1}{\pi}$  (C)  $\frac{\sqrt{2}}{\pi}$  (D)  $\frac{\pi-1}{\pi}$  (E)  $\frac{3}{\pi}$

- 25 A square with area 4 is inscribed in a square with area 5, with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length  $a$ , and the other of length  $b$ . What is the value of  $ab$ ?



(A)  $\frac{1}{5}$  (B)  $\frac{2}{5}$  (C)  $\frac{1}{2}$  (D) 1 (E) 4

