Inequalities with Two Variables

Inequalities with Two Variables: The graph of a linear equation separates the coordinate plane into two regions, one above and one below the graph of the equation.

A linear inequality in two variables, such as $2x - 3y < 6$, is the result of replacing the equal sign in a linear equation with $<$, $\leq$, $>$, $\geq$.

You can check if an ordered pair is a solution to an inequality by substituting the x-value and y-value into the inequality and seeing if the inequality is true. An ordered pair (x, y) is a solution of a linear inequality if substituting the values of x and y into the inequality produces a true statement.

Example: Tell whether the ordered pair is a solution of $2x - 3y < 6$.

$(0, 1)$

$2x - 3y < 6$

$2(0) - 3(1) \stackrel{?}{<} 6$

$-3 < 6$ ✓

$(0, 1)$ is a solution.

$(4, -2)$

$2x - 3y < 6$

$2(4) - 3(-2) \stackrel{?}{<} 6$

$14 \not< 6$

$(4, -2)$ is not a solution.

1. Tell whether the ordered pair is a solution of $4x + y > -1$

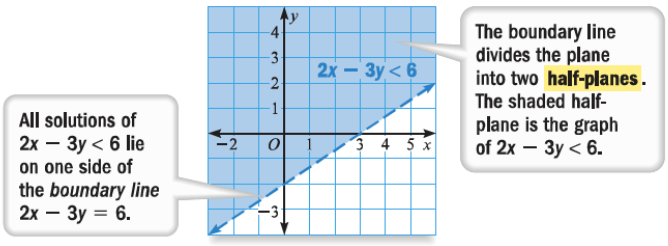
   a. $(-2, 5)$ b. $(0, 0)$ c. $(4, -4)$ d. $(-1, 3)$

2. Write two different ordered pairs of the solutions for each given inequality

   a. $y \geq \frac{2}{3}x - 5$ b. $2x - 2y > -5$

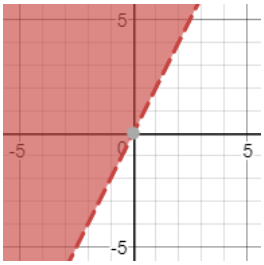
Graph of a Linear Inequality

Graphing Inequalities with Two Variables: The graph of a linear inequality in two variables is the set of points in a coordinate plane that represent all the inequality's solution.



1. Find the equation of the boundary line by replacing the inequality symbol =. Graph this equation. Use a dashed line for $<$ *or* $>$. Use a solid line for $\leq$ *or* $\geq$.
2. Shade the region "above" the line if the order sign says the y-value is greater ($y >$ or $y \geq$).
3. Shade the region "below" the line if the order sign says the y-value is less ($y <$ or $y \leq$).
4. Test a point in one of the half-planes to determine whether it is a solution of the inequality. If the test point is a solution, shade the half-plane that contains the points. If not, shade the other half-plane.

Example: Graph the inequality $y - 2x > 0$



Step 1: Write in slope-intercept form. $y > 2x$

Step 2: Graph the boundary equation $y = 2x$.
Draw a dashed line because the boundary line is not part of the graph of $y > 2x$.

Step 3: Shade the region "above" the line because the order sign says the y-value is greater.

Step 4: Pick a point in the shading and substitute its coordinates in the inequality to check if the shading is correct.

Pick (0, 3) and check the inequality.
$y > 2x$
$3 > 2(0)$
$3 > 0$

Thus the graph is correct.

1. Write the boundary equation in slope-intercept form. Specify whether the line should be dashed or solid.

   a. $y - \frac{x}{2} \leq 2$

   b. $5 - 2y \geq 4x$

   c. $y - 3x > 3$

   d. $2y - 5x - 5 > 0$

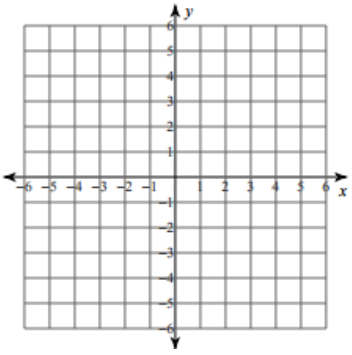
   e. $x + y > 5$

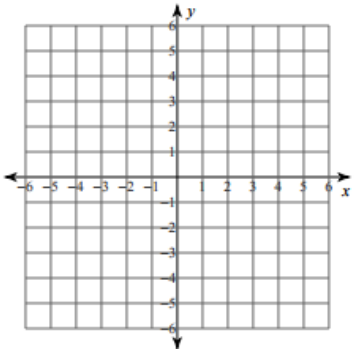
   f. $2(x + y) \leq 4$

2. Sketch the graph of each linear inequality. Then check your answer.

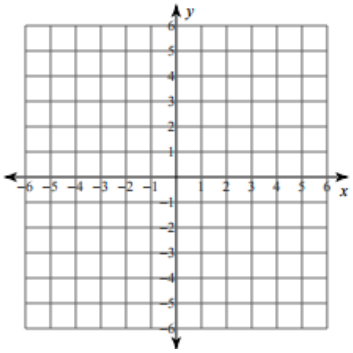
   a. $y \geq -3x + 4$

   b. $y \leq \frac{3}{5}x - 5$

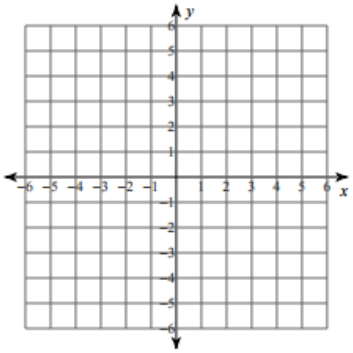




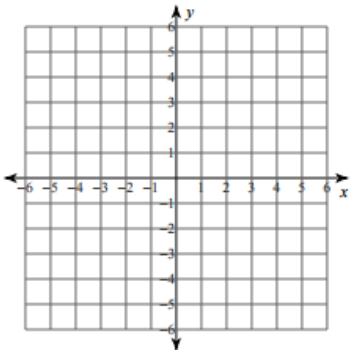
c. $y > 2x - 5$



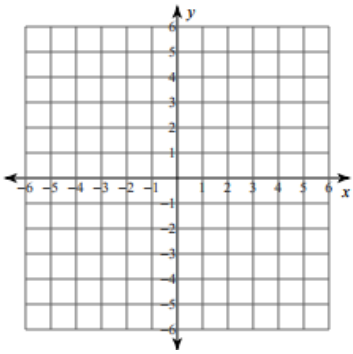
d. $y \geq \frac{7}{4}x + 2$



e. $3x - 2y < 10$



f. $5x - 3y \leq -15$



3. Check if each ordered pair is a solution of the inequality.

   a. $2x + 2y \leq 0$ for (-1, -1)

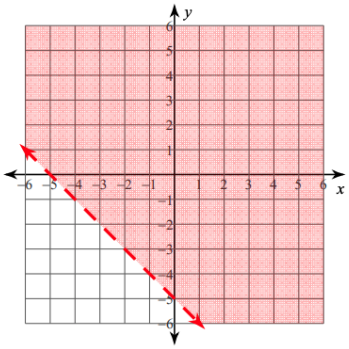
   b. $2x + 5y > 10$ for (6, 1)

   c. $\frac{5}{6}x + \frac{5}{3}y > 4$ for (6, -12)

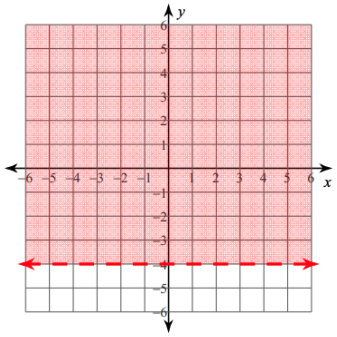
   d. $1\frac{2}{3}x - 1\frac{1}{6}y < 5$ for (10, 5)

4. Write the inequality for each graph.

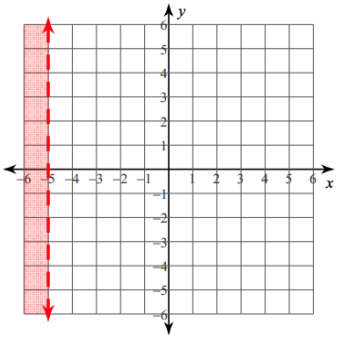
   a.



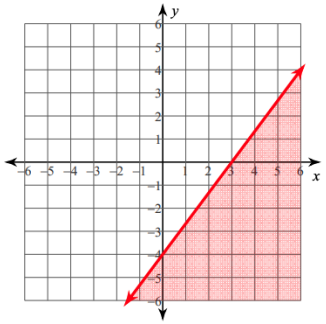
   b.



   c.



   d.

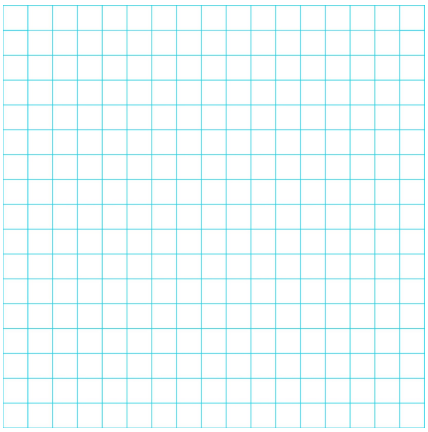


5. You have 100 pounds of clay to use for making bowls and vases. You need 5 pounds of clay for each bowl and 2 pounds for each vase.

   a. Write an inequality describing the possible numbers of bowls and vases that you can make.

   b. Graph the inequality from Part (a).

   c. Give 2 possible combinations of bowls and vases that you can make.

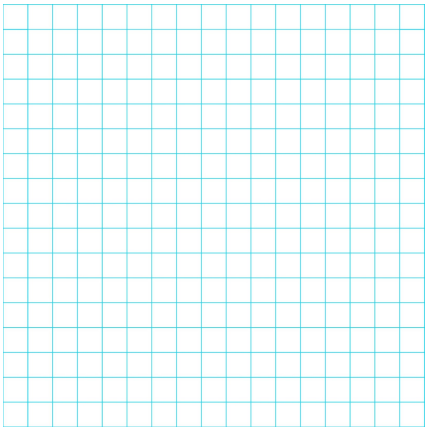


6. You have a gift certificate for $40 to use at a movie theater. Matinees cost $5 and evening shows cost $8. What are some possible combinations of matinees and evening shows that you can see?

   a. Write an inequality for this situation.

   b. Graph the inequality from part (a)

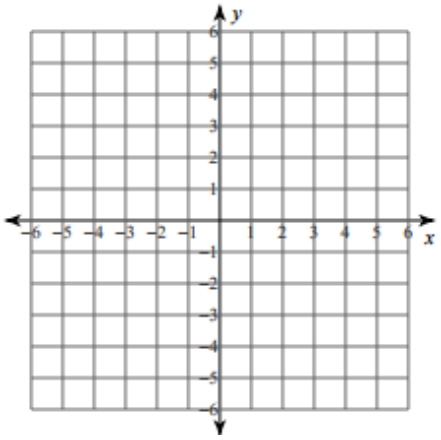
   c. Give two possible combinations of matinees and evening shows that you can see.



7. The system of linear inequalities is shown below. A solution of the system is an ordered pair that is a solution of each inequality.

$$y < x + 3$$
$$y \geq -2x - 3$$

   a. Graph the inequalities in the system. Draw both graphs in the same coordinate plane. If possible, use a different color for each graph.

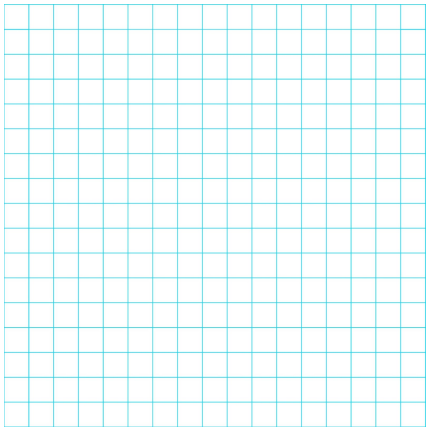


   b. Tell whether each ordered pair is a solution of the system.

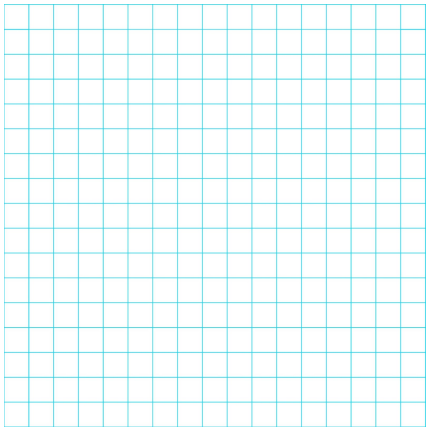
      i. (0, -4)

      ii. (-2, 1)

8. Suppose your class is raising money for the Red Cross. You make $3 on each basket of fruit and $5 on each box of cheese that you sell. Write and graph the inequality. How many items of each type can you sell to raise more than $150?



9. Suppose you intend to spend no more than \$60 buying books. Hardcover books cost \$12 and paperback cost \$5. Write and graph the inequality. How many books of each type can you buy?

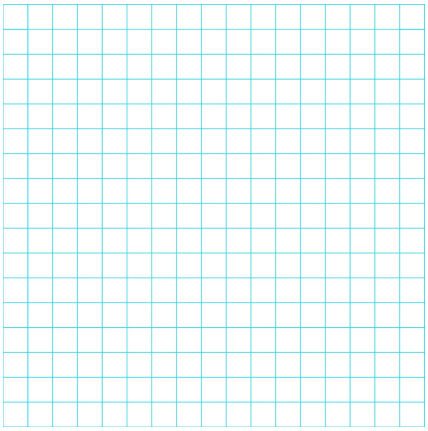


10. At a county fair, you buy 20 tickets that you can use for carnival rides and other attractions. Some rides require 1 ticket while others require 2 tickets.

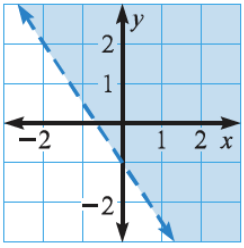
    a. Write an inequality describing the possible numbers of 1-ticket rides and 2-ticket rides that you can go on.

    b. Graph the inequality from part (a).

    c. Give two possible combinations of 1-ticket rides and 2-ticket rides that you can go on.



11. Multiple choice: The graph of which inequality is shown?



A. $y < \frac{3}{2}x - 1$

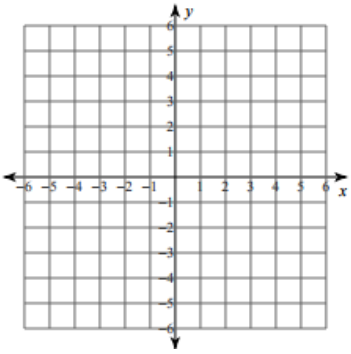
B. $2x + \frac{1}{3}y \geq -2$

C. $3x + 2y > -2$

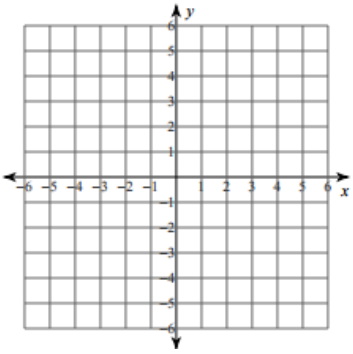
D. $3x + 2y < -2$

12. Sketch the graph of each linear inequality.

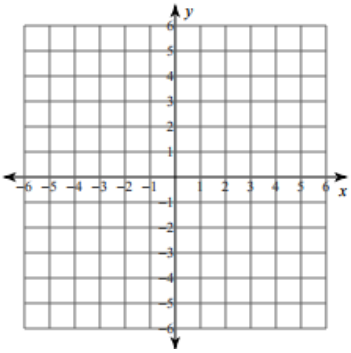
a. $y > -\frac{1}{3}x + 1$



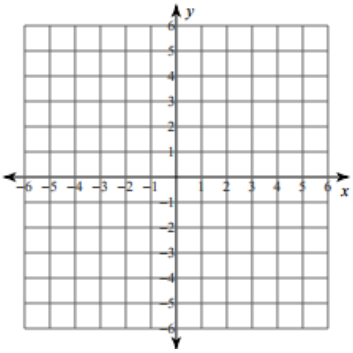
b. $y \geq -2x - 2$



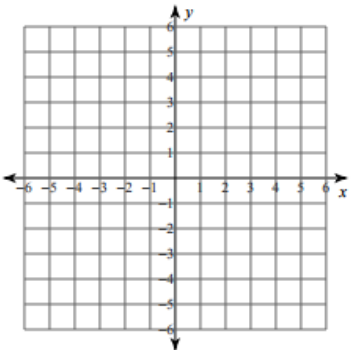
c. $y < 6x + 1$



d. $y \geq x - 2$



e. $5x - y > 5$



f. $x + 3y \geq 3$