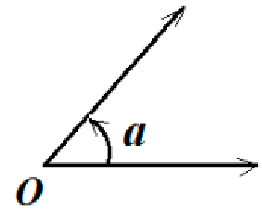
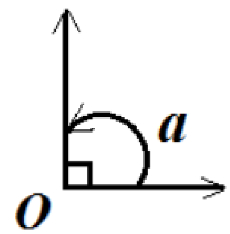
1.BASIC KNOWLEDGE

1.1. Terms

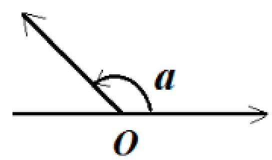
Acute angle: between 0 and 90°. $0° < a < 90°$.



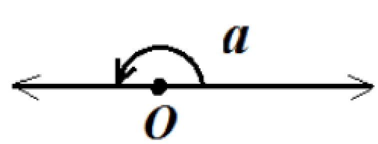
Right angle: $a = 90°$



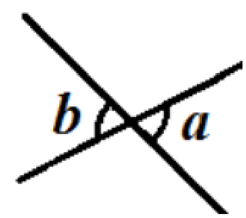
Obtuse angle: between 90 and 180°. $90° < a < 180°$.



Straight angle: $a = 180°$

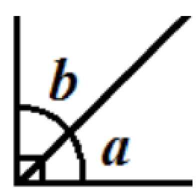


Vertical angles have equal measures. $a = b$.



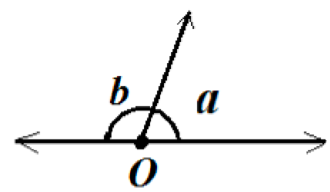
Complementary angles

If sum of the measures of two acute angles is 90°, the angles are said to be **complementary.** $a + b = 90°$.



Supplementary angles

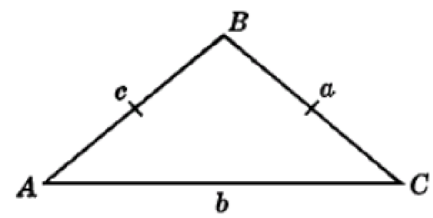
If sum of the measures of two angles is 180°, the angles are said to be **supplementary.** $a + b = 180°$.



Isosceles triangle

An isosceles triangle is a triangle with at least two congruent sides.

$AB = BC$. $\angle A = \angle C$

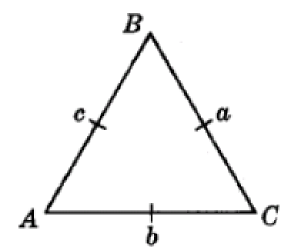


Isosceles Triangle

Equilateral triangle

An equilateral triangle is a triangle having three congruent sides.

$AB = BC = CA$. $\angle A = \angle B = \angle C = 60°$.



Equilateral Triangle

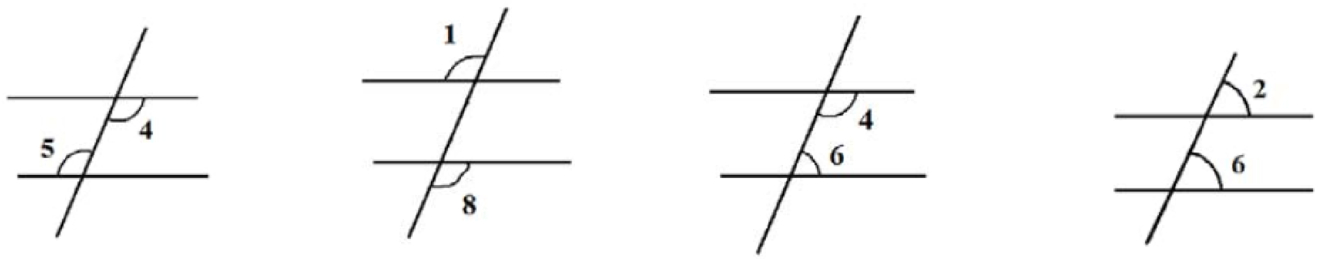
1.2. Relationship of Angles Formed by Parallel Lines

Alternate interior angles $\angle 4 = \angle 5$.

Alternate exterior angles $\angle 1 = \angle 8$.

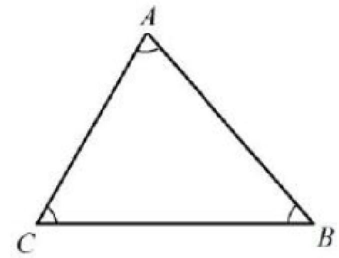
Interior angles on the same sides of transversal $\angle 4 + \angle 6 = 180$.

Corresponding angles $\angle 2 = \angle 6$.



1.3. Angle – Measure – Sum Principles

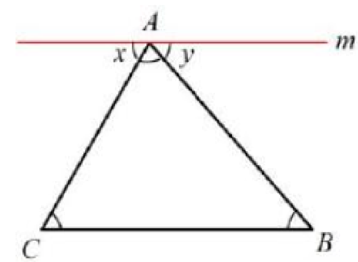
Theorem 1. The sum of the measures of the angles of a triangle equals the measure of a straight angle, or 180°. $\angle A + \angle B + \angle C = 180°$



Proof:
We draw the line m parallel to BC, the base of the triangle.

So $\angle C = \angle x$, $\angle B = \angle y$.

Since $\angle A + \angle x + \angle y = 180°$, $\angle A + \angle B + \angle C = 180°$.

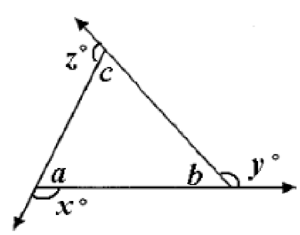


Theorem 2. The measure of each exterior angle of a triangle equals the sum of the measures of its two remote nonadjacent interior angles.

$z = a + b$

$x = b + c$

$y = c + a$



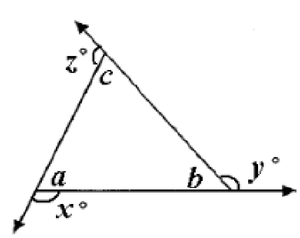
Proof:

$b + y = 180°$ (1)

$a + b + c = 180°$ (2)

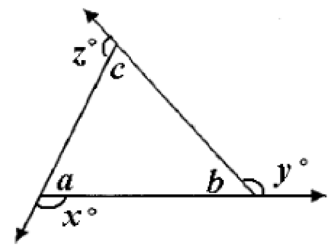
(1) – (2): $+ y - (c + a) = 0 \Rightarrow y = c + a$

Similarly we can prove $x = b + c$ and $z = a + b$.



Theorem 3. The sum of the measures of the exterior angles of a triangle equals 360°.

$x + y + z = 360°$



Proof:

$x = b + c$ (1)

$y = a + c$ (2)

$z = a + b$ (3)

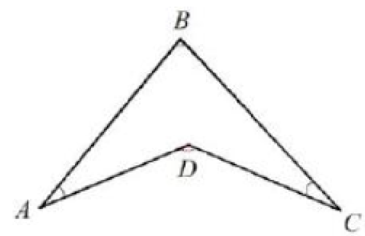
$a + b + c = 180°$ (4)

(1) + (2) + (3): $a + b + c + x + y + z = 540°$ (5)

Substituting (4) into (5): $x + y + z = 360°$.

Theorem 4. The four angles in the figure below have the following relationship:

$\angle D = \angle A + \angle B + \angle C$.

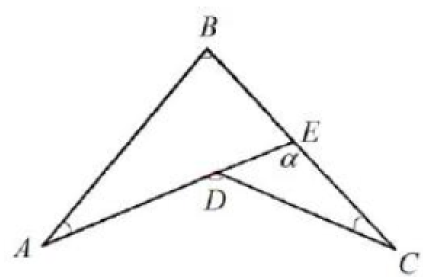


Proof:

Extend AD to meet BC at E.

By **Theorem 2**, $\angle D = \alpha + \angle C$ (1)

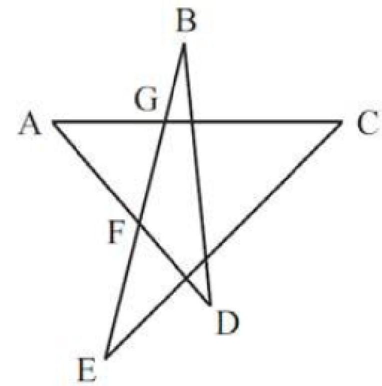
By **Theorem 2,** $\alpha = \angle A + \angle B$ (2)



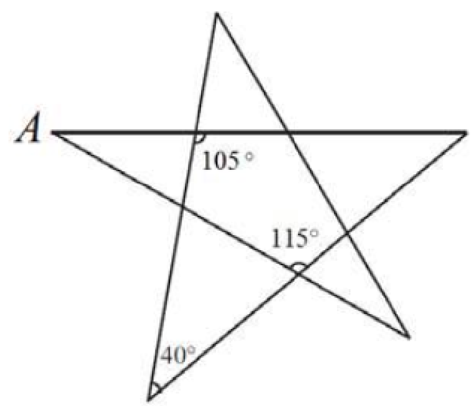
Substituting (2) into (1): $\angle D = \angle A + \angle B + \angle C$.

2. EXAMPLES

☆**Example 1.** If $\angle A = 22°$ and $\angle AFG = \angle AGF$, Then $\angle B + \angle D =$
(A) 54° (B) 66° (C) 79° (D) 88° (E) 100°

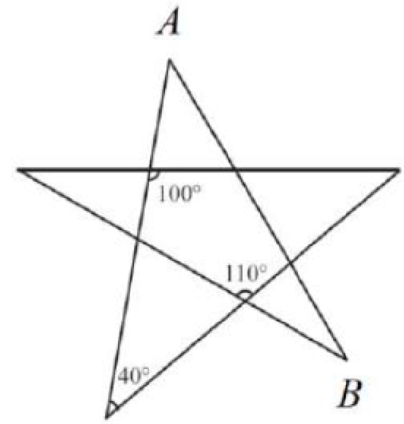


☆**Example 2.** The degree measure of angle A is
(A) 20° (B) 30° (C) 35° (D) 40° (E) 45°.



☆**Example 3.** The degree measure of angles $A + B$ is

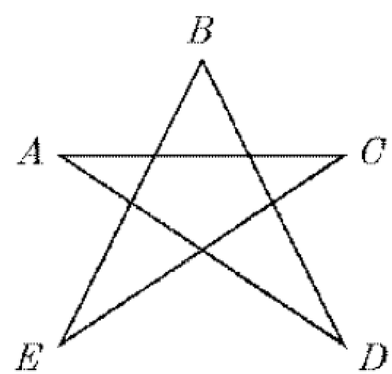
(A) 50° (B) 60° (C) 70° (D) 75° (E) 85°



Example 4. The sum of the measures of angles A, B, C, D, and E in the accompanying figure is:

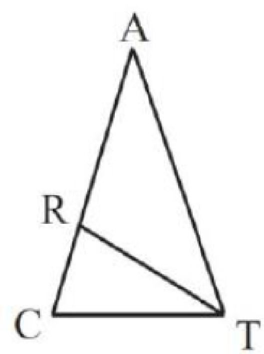
A. less than 180° B. 180° C. greater than 180° but less than 360°

D. 360° E . cannot be determined



☆**Example 5.** 19. Two angles of an isosceles triangle measure 70° and $x°$. What is the sum of the all possible values of x ?
(A) 95° (B) 125° (C) 140° (D) 165° (E) 180°

☆**Example 6.** In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 40°$. If TR bisects $\angle ATC$, then $\angle CRT =$
(A) 20° (B) 60° (C) 75° (D) 90° (E) 120°.



Example 7. If the complements of angle A and B are complementary, then the supplements of angles A and B:

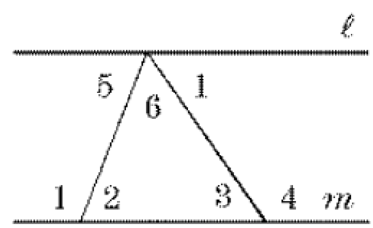
A. are congruent B. are supplementary C. are complementary
D. differ by 90° E. add up to 270°

Example 8. The measure of an angle for which the measure of the supplement is four times the measure of the complement is:

A. 20° B. 45° C. 60° D. 75° E. none of these

Example 9. In the figure, $\angle 1 = 7x + 10$. $\angle 5 = 3x$ and $l \parallel m$. The measure of $\angle 2$ equals:

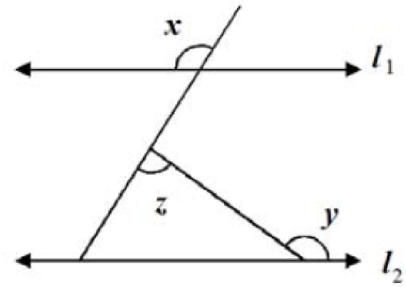
A. 17 B. 51 C. 87 D. 129 E. 139



$\angle 2 = 3x = 3 \times 17° = 51°$.

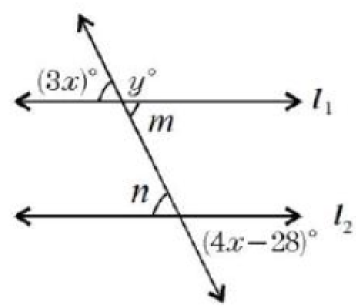
Example 10. x, y, and z are the measures of the angles shown. $l_1 // l_2$. The measure of x is:

A. $180° - y$ B. $180° - z$ C. $180° - z + y$
D. $180° + z - y$ E. $z + y - 180°$



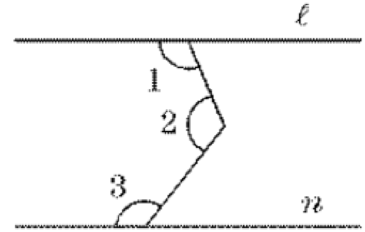
Example 11. If lines l_1 and l_2 are parallel, then find the value of y.

A. 84 B. 96 C. $90\frac{6}{7}$ D. $89\frac{1}{7}$ E. none of these

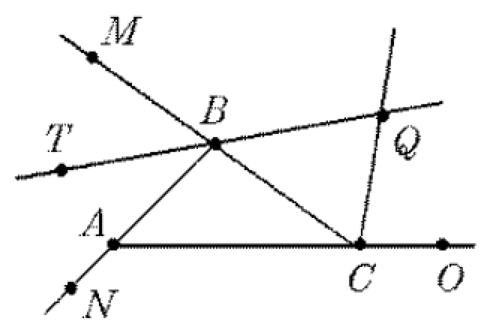


Example 12. In the figure $l // n$, $\angle 1 = 100°$, and $\angle 2 = 120°$. Find $\angle 3$.

A. 0° B. 100° C. 120° D. 140° E. 150°



Example 13. In the diagram, $\angle MBA$, $\angle NAC$, and $\angle OCB$ are exterior angles of triangle ABC. Lines TB and CQ intersect at point Q. Ray BT and ray CQ bisect $\angle MBA$ and $\angle OCB$ respectively. Then the measure of $\angle BQC$ is:



A. equal to the measure of $\angle CAN$.

B. equal to the measure of $\dfrac{\angle CAN}{2}$.

C. equal to the measure of $\dfrac{\angle CAN}{3}$.

D. equal to the measure of $\dfrac{\angle CAN}{4}$.

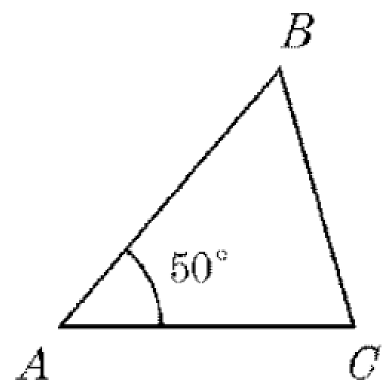
E. none of these.

☆**Example 14.** $\angle 1 + \angle 2 = 180°$. $\angle 3 = \angle 4$. Find $\angle 5$.

A. 140° B. 145° C. 150° D. 160° E. 165°

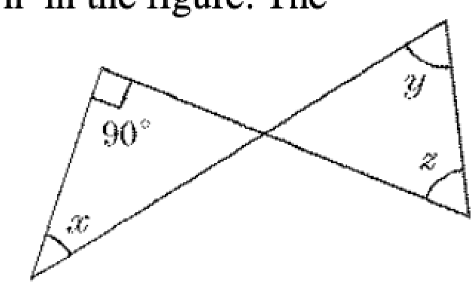
Example 15. Given that $\triangle ABC$ is a triangle such that $AB = AC$ and $\angle A = 50°$. Then $\angle B$ is:

A. 50° B. 55° C. 60° D. 65° E. 70°



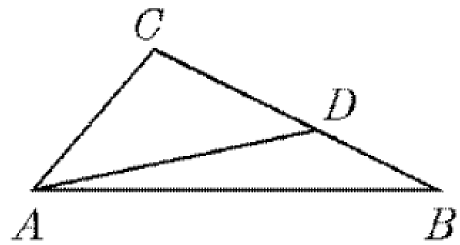
Example 16. x, y, and z are the measures of the angles shown in the figure. The sum of y and z in terms of x is:

A. $2x$ B. $90° + x$ C. $180° - x$ D. $180° - 2x$ E. $90° - x$



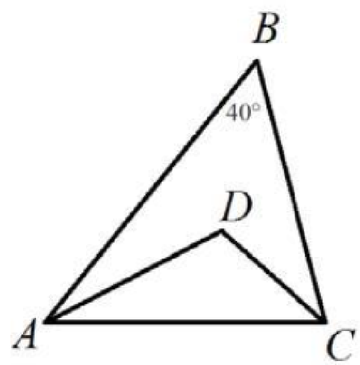
Example 17. In ΔABC, $\overline{AC} = \overline{CD}$ and $\angle CAB - \angle ABC = 40^\circ$. Then $\angle BAD$ equals:

A. 15° B. 20° C. 30° D. 35° E. 40°



☆**Example 18.** The measure of angle ABC is 40°. AD bisects angle BAC, and DC bisects angle BCA. The measure of angle ADC is

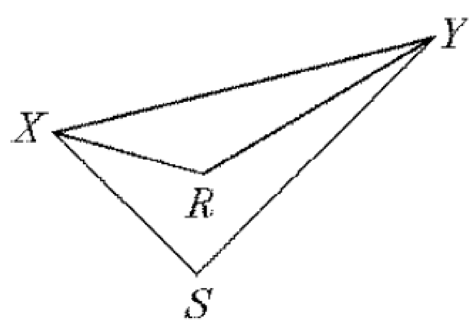
A. 90° B. 105° C. 110° D. 125.5° E. 130°



Example 19. In the given figure $\overrightarrow{XR}$ bisects $\angle YXS$, $\overrightarrow{YR}$ bisects $\angle XYS$, and $\angle S = a$. Express the measure of $\angle R$ in terms of a.

A. $90 + \frac{a}{2}$ B. $\frac{180 - a}{3}$ C. $\frac{2a + 90}{3}$

D. $180 + \frac{a}{2}$ E. $\frac{a - 90}{3}$

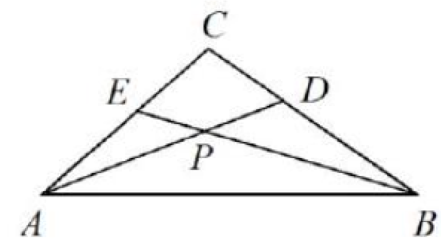


Example 21. In triangle ABC, $\overline{AD}$ and $\overline{BE}$ bisects angles A and B, respectively, and intersect in point P. The measure of angle ACB is 70°.

The measure of angle APE is:

A. 50° B. 55° C. 60° D. 67°

E. cannot be determined.



Example 22. If the measures of the angles of a triangle are in the ratio $4 : 5 : 6$, what is the measure of the smallest acute angle?