1.TERMS:

Fraction: A part of a whole or a quotient of two numbers, expressed as $\frac{a}{b}$. *a* and *b* are whole numbers. $b \neq 0$. $\frac{a}{b}$ is the same as $a \div b$ or a/b.

Proper fraction: A fraction in which the numerator is less than the denominator: $\frac{3}{5}$. Such a fraction has a value less than 1.

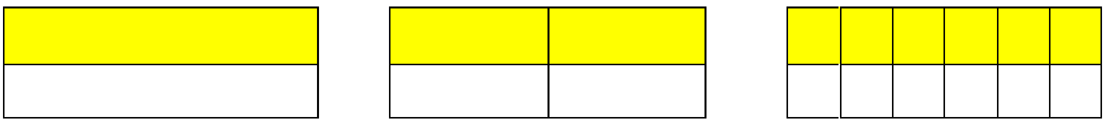
Improper fraction: A fraction in which the numerator is greater than or equal to the denominator: $\frac{5}{3}$. It has a value greater than or equal to 1.

Mixed number: A mixed number contains both a whole number part and a fraction part and can be written as an improper fraction: $2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}$.

2. PROPERTIES:

2.1. Equivalent Fraction (Cancellation Law): Two fractions are equal if they represent the same portion of a whole.

$$\frac{1}{2} = \frac{2}{4} = \frac{6}{12}$$



Examples: $\frac{38}{57} = \frac{2 \times \cancel{19}}{3 \times \cancel{19}} = \frac{2}{3}$; $\frac{38}{57} = \frac{38 \div 19}{57 \div 19} = \frac{2}{3}$

2.2. Fundamental Law of Fractions:

For any fraction $\frac{a}{b}$ and any number $c \neq 0$, $\frac{a}{b} = \frac{a \times c}{b \times c}$.

(The value of a fraction does not change if its numerator and denominator are multiplied by the same nonzero number).

Example. $\frac{2}{3}=\frac{2\times 5}{3\times 5}=\frac{10}{15}$.

2.3. Lowest (Reduced; Simplest) Term: A fraction in which the numerator and the denominator have no common terms except 1. The lowest terms are obtained by taking all the common factors out of the numerator and the denominator.

$\frac{10}{15}$ is not a fraction in the lowest term but $\frac{2}{3}$ is. ($\frac{10}{15}=\frac{2\times 5}{3\times 5}=\frac{2}{3}$).

2.4. Additon and Subtraction:

When working with fractions, only the numerators in fractions are added or subtracted.

(1). Two fractions having the same denominators:

We just add or subtract the numerators.

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \qquad \Rightarrow \qquad \frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}$$

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b} \qquad \Rightarrow \qquad \frac{3}{5}-\frac{1}{5}=\frac{3-1}{5}=\frac{2}{5}$$

(2). **Two fractions having the different denominators**:

We convert them to the same denominators first, and then add the numerators.

$$\frac{a}{b}+\frac{c}{d}=\frac{a\times d}{b\times d}+\frac{c\times b}{b\times d}=\frac{a\times d+c\times b}{b\times d} \Rightarrow$$

$$\frac{1}{2}+\frac{2}{5}=\frac{1\times 5}{2\times 5}+\frac{2\times 2}{5\times 2}=\frac{5+4}{10}=\frac{9}{10}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\times d}{b\times d}-\frac{c\times b}{b\times d}=\frac{a\times d-c\times b}{b\times d} \Rightarrow$$

$$\frac{1}{2}-\frac{2}{5}=\frac{1\times 5}{2\times 5}-\frac{2\times 2}{5\times 2}=\frac{5-4}{10}=\frac{1}{10}$$

2.5. Multiplication of Fractions

The numerator of the product is obtained by multiplying together the numerators. The denominator of the product is obtained by multiplying together the denominators.

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d} \qquad \Rightarrow \qquad \frac{2}{5}\times\frac{3}{7}=\frac{2\times 3}{5\times 7}=\frac{6}{35}$$

2.6. Division of Fractions

To divide by a fraction, we simply multiply by its reciprocal.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc} \qquad \Rightarrow \qquad \frac{2}{5}\div\frac{3}{7}=\frac{2}{5}\times\frac{7}{3}=\frac{2\times 7}{5\times 3}=\frac{14}{15}$$

The reciprocal of a number is obtained by switching the numerator and the denominator. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, and the reciprocal of 2 (note that 2 can be written as $\frac{2}{1}$) is $\frac{1}{2}$.

$$\frac{\;\frac{a}{b}\;}{\;\frac{c}{d}\;}=\frac{ad}{bc}$$

3. PROBLEM SOLVING SKILLS

3.1. Comparing Fractions

(1). Same Denominator:

The fraction with a larger numerator is larger: $\frac{3}{5}>\frac{1}{5}$

(2). Same Numerator:

The fraction with a larger denominator is smaller: $\frac{3}{7} < \frac{3}{5}$

(3). Both the numerator and denominator are not the same

$3 \times 11 = 33$ $\qquad$ $4 \times 8 = 32$

$\frac{3}{8} \;?\; \frac{4}{11} \quad \Rightarrow \quad 33 > 32 \quad \Rightarrow \quad \frac{3}{8} > \frac{4}{11}$

Example 1. Mary made two pies that were exactly the same size. The first pie was a cherry pie, which she cut into 6 equal slices. The second was a pumpkin pie, which she cut into 12 equal pieces. Mary takes her pies to a party. People eat 3 slices of cherry pie and 6 slices of pumpkin pie. Did people eat more cherry pie or pumpkin pie?

Example 2. Peter has two cakes that are the same size. The first cake was chocolate, which he cut 12 equal parts. The second cake was marble, which he cut into 6 equal parts. His family eats 5 slices of chocolate cake and 3 slices of marble cake. Did they eat more chocolate cake or marble cake?

☆**Example 3.** (AMC 8) What is the correct ordering of the three numbers 5/19, 7/21, and 9/23, in increasing order?

(A) $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$ $\qquad$ (B) $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$ $\qquad$ (C) $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$

(D) $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$ $\qquad$ (E) $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$

3.2. Sum of A Series of Fractions

Useful formulas:

$$\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1} \qquad \Rightarrow \qquad \frac{1}{3(3+1)}=\frac{1}{3}-\frac{1}{3+1}=\frac{1}{3}-\frac{1}{4}$$

$$\frac{1}{n}=\frac{1}{2n}+\frac{1}{2n} \qquad \Rightarrow \qquad \frac{1}{3}=\frac{1}{2\times 3}+\frac{1}{2\times 3}=\frac{1}{6}+\frac{1}{6}$$

$$\frac{1}{n(n+k)}=\frac{1}{k}(\frac{1}{n}-\frac{1}{n+k}) \qquad \Rightarrow \qquad \frac{1}{3(3+2)}=\frac{1}{2}(\frac{1}{3}-\frac{1}{5})$$

Example 4. Find the sum: $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\cdots+\frac{1}{11\times 13}$.

Example 5. Calculate: $\frac{1}{2\times 4}+\frac{1}{4\times 6}+\cdots+\frac{1}{98\times 100}$.

3.3. Continued Fractions

The simple continued fraction representation of a number is given by:

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \dots}}}}$$

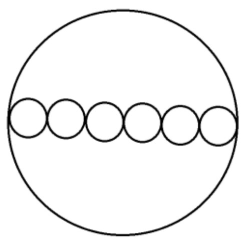
where a_0 is an integer, any other a_i members are positive integers, and n is a non-negative integer.

Example 6. Simplify: $1 + \cfrac{1}{1 + \cfrac{1}{1+1}}$. Express your answer as a common fraction.

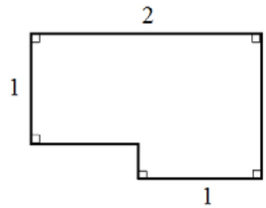
Example 7. Simplify: $\cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}$. Express your answer as a common fraction.

3.4. Fraction Related to Geometry

☆**Example 8.** Six pepperoni circles will exactly fit across the diameter of a 12-inch pizza when placed as shown. If a total of 20 circles of pepperoni are placed on this pizza without overlap, what fraction of the pizza is covered by pepperoni?
(A) 2/3 (B) 5/9 (C) 5/7 (D) 5/6 (E) 23/36.

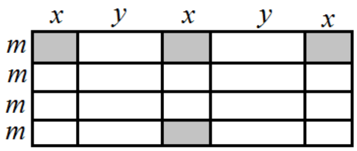


Example 9. The area of the figure shown is $\frac{11}{5}$. What is the perimeter of the figure?



Example 10. In the figure shown, all angles are right angles and $y = 2x$. If m, x, and y are lengths of the segments indicated, what fraction of the figure is shaded?

(A) $\frac{1}{7}$ (B) $\frac{1}{5}$ (C) $\frac{1}{14}$ (D) $\frac{3}{10}$ (E) $\frac{5}{14}$



3.5. Fraction Related to Numbers and Expressions

Example 11. What reduced common fraction is equivalent to $18\frac{1}{3}\%$?

Example 12. If 1/2 + 1/5 + 1/8 > 1/ x + 1/6 + 1/8, then x could not be which of the following?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 8

Example 13. If a and b are integers such that $a + b > 160$ and $a/b = 0.15$, what is the smallest possible value of a?

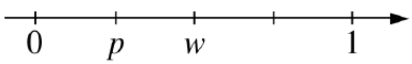
(A) 140 (B) 21 (C) 24 (D) 3 (E) 15

Example 14. Which of the following numbers is between $\frac{1}{6}$ and $\frac{1}{5}$?

(A) 0.14 (B) 0.15 (C) 0.16 (D) 0.17 (E) 0.26

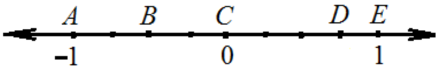
3.6. Fraction Related to Number Lines

Example 15. On the number line shown, the tick marks are equally spaced. What is the value of $w + p$?



(A) 3/4 (B) 2/3 (C) 1/2 (D) 1/3 (E) 1/4

Example 16. Dots are equally spaced on the number line shown. Which of the lettered points has a coordinate equal to $1-(-\frac{1}{2})^2$?



(A) A (B) B (C) C (D) D (E) E

3.7. Fraction Applications

Example 17. An hour-long television program included 20 minutes of commercials. What fraction of the hour-long program was not commercials?

Example 18. A container is $\frac{3}{5}$ full of water. If 16 gallons of the water were removed from the container, it would be $\frac{1}{3}$ full. How many gallons of water does this container hold when it is completely full?

(A) 20 (B) 35 (C) 40 (D) 60 (E) 90.

Example 19. Roy planted corn on $\frac{2}{7}$ of his land. If he planted 60 acres of corn, how many acres of land does he have?

(A) 90 (B) $112\frac{1}{2}$ (C) 135 (D) 210 (E) $337\frac{1}{2}$

☆**Example 20.** Peter's family ordered a 12-slice pizza for dinner. Peter ate two slices and shared another slice equally with his brother Tom. What fraction of the pizza did Peter eat?

(A) $\frac{1}{8}$ (B) $\frac{1}{7}$ (C) $\frac{5}{24}$ (D) $\frac{1}{6}$ (E) $\frac{1}{24}$

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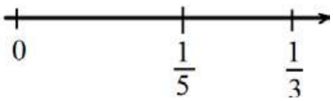
MORE EXAMPLES

Example 21. The fraction halfway between $\frac{1}{5}$ and $\frac{1}{3}$ (on the number line) is



(A) $\frac{1}{4}$ (B) $\frac{2}{15}$ (C) $\frac{4}{15}$ (D) $\frac{53}{200}$ (E) $\frac{8}{15}$

Example 22. How many more degrees of arc are there in $\frac{1}{5}$ of a circle than in $\frac{1}{6}$ of circle?
(A) 9° (B) 12° (C) 24° (D) 30° (E) 36°

Example 23. Write the common fraction equivalent to $2\frac{1}{2}$%.

Example 24. If $\frac{3}{8}$ of a number is $\frac{21}{2}$, what is $\frac{1}{7}$ of the number?
(A) 3/2 (B) 9/2 (C) 4 (D) 6 (E) 28

Example 25. If 1/4 + 1/5 + 1/6 < 1/5 + 1/6 + 1/*y*, then *y* could be which of the following?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Example 26. Jenny had a pizza that was divided into 8 equal slices. She ate 3 of them. Alex has a pizza that is the same size, but his is divided into 4 equal slices. He ate 3 slices of his pizza. Who ate more pizza?

Example 27. Find the sum: $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{49\times 50}$.

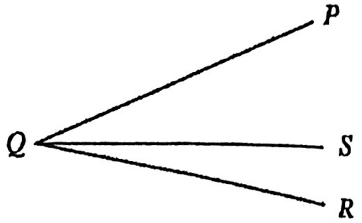
Example 28. Simplify: $\dfrac{1}{8+\dfrac{1}{8+\dfrac{1}{8}}}$.

Example 29. If $0 < a < b$, which of the following is greater than b/a?

(A) 1 (B) a/b (C) $1/(b/a)$ (D) $b/2a$ (E) $2b/a$

PROBLEMS

Problem 1. In the figure below, the measure of $\angle SQR$ is 2/5 the measure of $\angle PQR$. If the measure of $\angle PQR$ is 2/3 the measure of a right angle, what is the measure of $\angle SQP$?



Note: Figure not drawn to scale.

(A) 24° (B) 36° (C) 48° (D) 60° (E) 96°

Problem 2. If $1/7 + 1/8 + 1/9 > 1/x + 1/8 + 1/9$, then x could be which of the following?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Problem 3. If $1/4 + 1/5 + 1/6 < 1/5 + 1/y + 1/8$, then y could be which of the following?

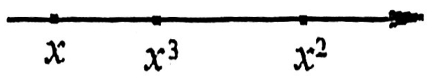
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Problem 4. If a and b are integers such that $a + b < 133$ and $a/b = 0.2$, what is the greatest possible value of b?

Problem 5. If $x = \frac{1}{3}$, what is the value of $\frac{1}{x} + \frac{1}{x-1}$?

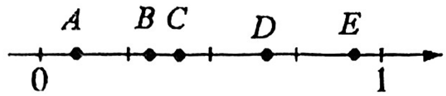
(A) $-3/2$ (B) $3/2$ (C) 1 (D) 2 (E) 3

Problem 6. If x, x^2, and x^3 lie on a number line in the order shown below, which of the following could be the value of x?



(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1 (E) $\frac{3}{2}$

Problem 7. If the tick marks on the number line below are equally spaced, which of the lettered points A through E is between 1/4 and 3/8?



(A) A (B) B (C) C (D) D (E) E

Problem 8. Bob is baking two pans of brownies that are the same size. One pan has nuts in it and the other pan does not. He cuts the pan with nuts into 8 equal pieces. He cuts the pan without nuts into 16 equal pieces. His friends ate 2 brownies with nuts and 3 brownies without nuts. Did they eat more of the brownies with nuts or without nuts?

Problem 9. On a blueprint, $\frac{1}{4}$ inch represents 16 feet, If a driveway is 80 feet long, what is its length, in inches, on the map?

(A) $\frac{3}{4}$ (B) $\frac{5}{8}$ (C) $\frac{5}{4}$ (D) $2\frac{1}{2}$ (E) 20

Problem 10. Of all the students in a high school, on a certain day $\frac{3}{5}$ rode the bus to school, $\frac{3}{10}$ rode in a car, and the remaining students walked. What fraction of the school's students walked to school on that day?

Problem 11. Find the sum $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{100\times 101}$.

Problem 12. Calculate: $\frac{3}{1\times 4}+\frac{3}{4\times 7}+\frac{3}{7\times 10}+\cdots+\frac{3}{19\times 22}$.

Problem 13. Find the sum: $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\cdots+\frac{1}{1997\times 1999}$.

Problem 14. Express as a simplified mixed number: $3+\cfrac{2}{3+\cfrac{2}{3+\cfrac{2}{3}}}$.

Problem 15. In a poll, 45 people were in favor of building a new library, 27 people were against it, and 3 people had no opinion. What fraction of those polled were in favor of building a new library?

(A) $\frac{7}{10}$ (B) $\frac{3}{5}$ (C) $\frac{3}{7}$ (D) $\frac{1}{3}$ (E) $\frac{3}{10}$

Problem 16. Express as a common fraction: $\cfrac{1}{1+\cfrac{1}{5}}$.

Problem 17. Express the value of the following expression as a common fraction.

$$1+\cfrac{2}{3+\cfrac{4}{5}}$$

Problem 18. Simplify the following and express as a mixed number. $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}$.

Problem 19. By 7:00 A.M., 1/5 of the junior class had arrived at a school dance. By 8:00 A.M., 60 more juniors had arrived, raising attendance to 1/3 of the junior class. How many people are in the junior class?

(A) 90 (B) 120 (C) 180 (D) 380 (E) 450

Problem 20. On a hike, Ian walked downhill $\frac{3}{7}$ of the time and uphill $\frac{4}{7}$ of the time. His downhill walking rate was 5 miles per hour, and his uphill walking rate was 3 miles per hour. The distance that Ian walked downhill was what fraction of the total distance that he walked?

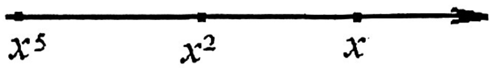
(A) $\frac{4}{7}$ (B) $\frac{3}{7}$ (C) $\frac{2}{5}$ (D) $\frac{5}{9}$ (E) $\frac{7}{9}$

Problem 21. On each of the days Monday through Thursday, Toni spent 2 hour commuting to work and 1 hour commuting back home. What fraction of the total number of hours in these four days did she spend commuting?

(A) $\frac{1}{12}$ (B) $\frac{1}{24}$ (C) $\frac{5}{12}$ (D) $\frac{5}{24}$ (E) $\frac{1}{8}$

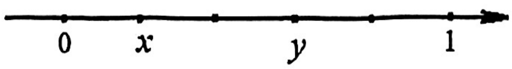
Problem 22. If the value of $1/t + 5$ is twice the value of $1/t - 1$, what is 7/2 of the value of t ?

Problem 23. If x, x^2, and x^5 lie on a number line in the order shown below, which of the following could be the value of x?



(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1 (E) $\frac{3}{2}$

Problem 24. On the number line below, the tick marks are equally spaced. What is the value of $x - y$?



(A) 4/5 (B) 3/4 (C) 2/5 (D) 1/4 (E) 1/5

Problem 25. If a cake is cut into thirds and each third is cut into sevenths, how many pieces of cake are there?

Problem 26. A company sells boxes of balloons in which the balloons are red, green, or blue. Luann purchased a box of balloons in which $\frac{1}{5}$ of them were red. If there were half as many green balloons in the box as red ones and 21 balloons were blue, how many balloons were in the box?

1. BASIC KNOWLEDGE

Even integer:

An integer with the last digit of 0, 2, 4, 6, or 8. General form: $2n$ or $2n + 2$, where n is any integer.

Examples: Even integers: 10, 12, 14, 16, and 18.

Odd integers:

An integer with the last digit of 1, 3, 5, 7, or 9. General form: $2n + 1$ or $2n - 1$.

Examples: Odd integers: 11, 13, 15, 17, and 19.

Parity:

An even number has even parity and an odd number has odd parity.

Properties:

even + even = even.	$12 + 14 = 26$.
even + odd = odd.	$12 + 13 = 25$
odd + odd = even.	$13 + 13 = 26$
odd × odd = odd.	$15 \times 15 = 225$
odd ÷ odd = odd.	$1001 \div 11 = 91$
odd × even = even.	$11 \times 12 = 132$
odd ≠ even.	$1 \neq 2$

2. PROBLEM SOLVING SKILLS

The sum of any even integer and 1 is odd: $4 + 1 = 5$.

The sum of two consecutive integers is odd: $n + (n + 1) = 2n + 1$; $12 + 13 = 25$.

The product of two consecutive integers is even: $n(n + 1)$; $12 \times 13 = 156$.

Any two consecutive integers have opposite parity: 12 is even and 13 odd.
$a + b$ and $a - b$ have the same parity: $15 - 5 = 10$ even; $15 + 5 = 20$ even.

If the product of n positive integers is even,

If the product of n positive integers is odd,

If the number of odd integers is even,

If the number of odd integers is odd,

Example 1. Add any 30 consecutive positive integers together. Is the sum even or odd?

Example 2. 300 is the sum of 15 consecutive even positive integers. What is the greatest even positive integer among them?

Example 3. The sequence 1, 1, 2, 3, 5, 8, 13, 21, … is formed like this: any term is the sum of the two terms before it, starting from the third term. How many are even numbers of the first 63 terms of the sequence?

Example 4. All the positive even integers greater than 0 are arranged in five columns (*A*, *B*, *C*, *D*, and *E*) as shown. Continuing the pattern, in what column will the integer 50 be?

A	B	C	D	E
	2	4	6	8
16	14	12	10	
	18	20	22	24
32	30	28	26	
…………………………………………………				

Example 5. The sum of all multiples of 3 from 20 to 100 is *S*. Is *S* even or odd?

Example 6. Five lamps are arranged in a row as shown in the figure below. Each lamp has its own switch. All five lamps *A, B, C, D*, and *E* are now off. Ben starts to turn each switch from *A* to *E* and he repeats the pattern (always from *A* to *E* in order) until he turns the switches 2012 times. Which lamps are on finally?

Example 7. If x and y are integers and $x^2y^2 + x^3$ is odd, which of the following statements must be true?

I. x^2 is odd.

II. y is odd.

III. $x + y^2$ is odd.

(A) I only (B) III only (C) I and II (D) I and III (E) II and III

MORE EXAMPLES

Example 8. If a and b are positive integers and $a^2 - b^2 = 7$, what is the value of b?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Example 9. The lengths of the sides of a right triangle are consecutive even integers, and the hypotenuse of the right triangle is x. Which of the following equations could be used to find x?
(A) $x + x - 1 = x - 2$ (B) $x^2 + (x - 1)^2 = (x - 2)^2$
(C) $x^2 = (x - 2)^2 + (x - 4)^2$ (D) $x + x + 2 = x + 4$
(E) $x^2 = (x - 2)(x - 4)$

Example 10. If a and b are positive consecutive odd integers, which of the following must be a positive odd integer?
(A) $a + b$ (B) $a - b$ (C) $2a + b$ (D) $2a - 2b$ (E) $\frac{a+b}{2}$

Example 11. If x and y are positive consecutive odd integers, where $y > x$, which of the following is equal to $y^2 - x^2$?
(A) $6y$ (B) $8y$ (C) $4(y - 2)$ (D) $2y - 1$ (E) $4(y - 1)$

Example 12. The sequence 1, 1, 2, 4, 7, 13, 24, … is formed like this: any term is the sum of the three terms before it starting from the fourth term. Is the 100th term even or odd in the sequence?

Example 13. The sequence 1, 1, 2, 3, 5, 8, 13, 21, … is formed like this: any term is the sum of the two terms before it, starting from the third term. How many odd numbers are there in the first 900 terms of the sequence?

Example 14. Mr. Mathis and his student Peter worked together to solve math problems last week. When each person solved a problem, that person put a marble in his own box. Mr. Mathis's problem solving speed was half of his student. At the end of the problem solving session, Peter had four boxes of marbles and Mr. Mathis had two boxes of marbles. Each box is labeled with the number of marbles inside it. The numbers are 78, 94, 86, 87, 82, and 80, respectively. Which two boxes belong to Mr. Mathis?

Example 15. All the positive integers greater than 1 are arranged in five columns (A, B, C, D, E) as shown. Continuing the pattern, in what column will the integer 800 be written?

	A	B	C	D	E
Row 1		2	3	4	5
Row 2	9	8	7	6	
Row 3		10	11	12	13
Row 4	17	16	15	14	
Row 5		18	19	20	21
⋮					

(A) A (B) B (C) C (D) D (E) E

Example 16. Five lamps are arranged in a row as shown in the figure below. Each lamp has its own switch. All five lamps A, B, C, D, and E are now off. Ben starts to turn each switch from A to E and he repeats the pattern (always from A to E in order) until he turns the switches 126 times. Which lamps are on in the end?

A B C D E

PROBLEMS

Problem 1. If a and b are positive integers and $a^2 - b^2 = 143$, what is the value of a?

(A) 1 (B) 11 (C) 12 (D) 13 (E) 14

Problem 2. The lengths of the sides of a right triangle are consecutive even integers, and the length of the longer leg is x. Which of the following equations could be used to find x?

(A) $x + x + 1 = x + 2$ (B) $x^2 + (x + 1)^2 = (x + 2)^2$
(C) $(x - 2)^2 + x^2 = (x + 2)^2$ (D) $x - 2 + x = x + 2$
(E) $x^2 = (x - 2)(x + 2)$

Problem 3. If a and b are positive odd integers, which of the following must be a positive even integer?

(A) $a + b$ (B) $a - b$ (C) $2a + b$ (D) $2a - b$ (E) $\frac{a+b}{2}$

Problem 4. If x and y are positive consecutive even integers, where $y > x$, which of the following is equal to $y^2 - x^2$?

(A) $2x$ (B) $4x$ (C) $2x + 2$ (D) $2x + 4$ (E) $4x + 4$

Problem 5. If x and y are positive consecutive odd integers, where $y > x$, which of the following is equal to $y^2 - x^2 + 8$?

(A) $6x$ (B) $8x$ (C) $2x + 2$ (D) $2x + 4$ (E) $4(x + 3)$

Problem 6. If a and b are odd integers, which of the following must also be an odd integer?

I. $(a + 2)b$ II. $(a + 2) + b$ III. $(a + 2) - b$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

Problem 7. If t represents an odd integer, which of the following expressions represents an even integer?

(A) $t + 4$ (B) $2t – 3$ (C) $3t – 6$ (D) $3t + 8$ (E) $5t + 5$

Problem 8. If $\frac{x+7}{2}$ is an integer. Then x must be
(A) a negative integer (B) a positive integer (C) a multiple of 3
(D) an even integer (E) an odd integer

Problem 9. If k is a positive integer, which of the following must represent an odd integer that is twice the value of an odd integer?
(A) $4k + 3$ (B) $2k + 3$ (C) $2k + 4$ (D) $4k + 1$ (E) $4k + 2$

Problem 10. If k is a negative even integer and n is a positive odd integer, which of the following could be equal to $n - k$?
I. 0 II. 1 III. 3.

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

Problem 11. The sum of the positive odd integers less than 1000 is subtracted from the sum of the positive even integers less than or equal to 1000. What is the resulting difference?

Problem 12. Each of the 75 children in a line was assigned one of the integers from 1 through 99 by counting off in order. Then, standing in the same order, the children counted off in the opposite direction, so that the child who was assigned the number 99 the first time was assigned the number 1 the second time. Which of the following is a pair of numbers assigned to the same child?
(A) 50 and 48 (B) 49 and 50 (C) 66 and 33 (D) 33 and 67 (E) 45 and 32

Problem 13. The counting numbers are arrange in four columns as shown below. Under which column letter will 2012 appear?

A	B	C	D
1	2	3	4
8	7	6	5
9	10	11	12
	…	14	13

Problem 14. Suppose all the counting numbers are arranged in columns as shown below. Under what column-letter will 2012 appear?

A	B	C	D	E	F	G
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	_	_

Problem 15. Is 1 + 2 + 3 + 4 +…+ 2011 + 2012 even or odd?

Problem 16. Is the expression $1\times2+3\times4+5\times6+...+99\times100$ even or odd?

Problem 17. Seven lamps labeled *A* through *G* are arranged in a row. Each lamp has its own switch. Now lamps *A*, *C*, *E*, and *G* are on and other lamps are off. Ben starts to flip each switch from *A* to *G* the following way: if the lamp is on, he turns it off; if the lamp is off, he turns it on. He repeats the pattern until he flips the switches 2011 times. Which lamps are on finally?