Example 2. 300 is the sum of 15 consecutive even positive integers. What is the greatest even positive integer among them?

Example 7. If x and y are integers and $x^2y^2 + x^3$ is odd, which of the following statements must be true?

I. x^2 is odd.

II. y is odd.

III. $x + y^2$ is odd.

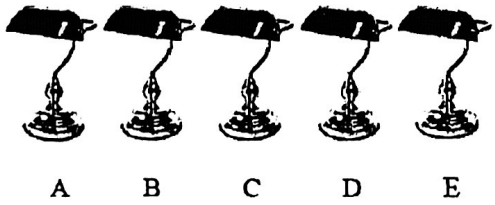
(A) I only (B) III only (C) I and II (D) I and III (E) II and III

MORE EXAMPLES

Example 8. If a and b are positive integers and $a^2 - b^2 = 7$, what is the value of b?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Example 16. Five lamps are arranged in a row as shown in the figure below. Each lamp has its own switch. All five lamps *A, B, C, D*, and *E* are now off. Ben starts to turn each switch from *A* to *E* and he repeats the pattern (always from *A* to *E* in order) until he turns the switches 126 times. Which lamps are on in the end?



PROBLEMS

Problem 1. If a and b are positive integers and $a^2 - b^2 = 143$, what is the value of a?

(A) 1 (B) 11 (C) 12 (D) 13 (E) 14

Problem 3. If a and b are positive odd integers, which of the following must be a positive even integer?

(A) $a + b$ (B) $a - b$ (C) $2a + b$ (D) $2a - b$ (E) $\frac{a+b}{2}$

Problem 7. If t represents an odd integer, which of the following expressions represents an even integer?

(A) $t+4$ (B) $2t-3$ ( ) $3t-6$ (D) $3t+8$ (E) $5t+5$

Problem 9. If k is a positive integer, which of the following must represent an odd integer that is twice the value of an odd integer?

(A) $4k+3$ (B) $2k+3$ (C) $2k+4$ (D) $4k+1$ (E) $4k+2$

Problem 15. Is $1 + 2 + 3 + 4 + \ldots + 2011 + 2012$ even or odd?

Problem 17. Seven lamps labeled *A* through *G* are arranged in a row. Each lamp has its own switch. Now lamps *A, C, E,* and *G* are on and other lamps are off. Ben starts to flip each switch from *A* to *G* the following way: if the lamp is on, he turns it off; if the lamp is off, he turns it on. He repeats the pattern until he flips the switches 2011 times. Which lamps are on finally?