

# Formulas in Control Engineering

Type	Step Response h(t)	Equation for Step Response	Transfer Function G(s)	Nyquist Curve G(jω)	Bode Plot  G(jω)  <sub>dB</sub> and φ (jω)	Poles (x) and Zeros (o)	Block Diagram
P		$h(t) = K_P \sigma(t)$	$G(s) = K_P$				
I		$h(t) = \frac{t}{T_I}$	$G(s) = \frac{1}{T_I s}$				
D		$h(t) = T_D \cdot \delta(t)$	$G(s) = T_D s$				
P - T <sub>1</sub>		$h(t) = K \left[ 1 - e^{-\frac{t}{T_1}} \right]$	$G(s) = \frac{K}{1 + T_1 s}$				
P - T <sub>2</sub>		$h(t) = K \left[ 1 - \frac{T_1}{T_1 - T_2} e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} e^{-\frac{t}{T_2}} \right]$	$G(s) = \frac{K}{(1 + T_1 s)(1 + T_2 s)}$				
P - T <sub>2s</sub>		$h(t) = K \left[ 1 \pm \frac{e^{-\frac{\vartheta t}{T_0}}}{\sqrt{1 - \vartheta^2}} \sin \left( \sqrt{1 - \vartheta^2} \frac{t}{T_0} + \psi \right) \right]$ $\psi = \arccos \vartheta$	$G(s) = \frac{K}{(1 + 2\vartheta T_0 s + T_0^2 s^2)}$ $T_0 = \frac{1}{\omega_0}; \vartheta < 1$				
I - T <sub>1</sub>		$h(t) = \frac{t}{T_I} - \frac{T_1}{T_I} \left( 1 - e^{-\frac{t}{T_1}} \right)$	$G(s) = \frac{1}{T_I s (1 + T_1 s)}$				
D - T <sub>1</sub>		$h(t) = \frac{T_D}{T_1} e^{-\frac{t}{T_1}}$	$G(s) = \frac{T_D s}{1 + T_1 s}$				
PI		$h(t) = K_P \left( 1 + \frac{t}{T_n} \right)$	$G(s) = K_P \frac{1 + T_n s}{T_n s}$				
PD		$h(t) = K_P [1 + T_v \delta(t)]$	$G(s) = K_P (1 + T_v s)$				
PD - T <sub>1</sub>		$T_{vp} > T_1$ $T_v = T_{vp} - T_1 > 0$	$G(s) = K_P \left[ 1 + \frac{T_v s}{1 + T_1 s} \right]$				
PP - T <sub>1</sub>		$T_{vp} < T_1$ $T_v = T_{vp} - T_1 < 0$	$= K_P \frac{1 + T_{vp} s}{1 + T_1 s}$ $T_{vp} = T_v + T_1$				
PID		$h(t) = K_P \left[ 1 + \frac{t}{T_n} + T_v \delta(t) \right]$	$K_P [1 + \frac{1}{T_n s} + T_v s] = K_{PP} \frac{(1 + T_{np} s)(1 + T_{vp} s)}{T_{np} s}$ $T_n = T_{np} + T_{vp}; T_v = T_{vp} \frac{T_{np}}{T_n}; K_P = K_{PP} \frac{T_n}{T_{np}}; \frac{T_n}{T_{np}} \geq 4$				
PID - T <sub>1</sub>		$h(t) = K_P \left[ 1 + \frac{t}{T_n} + \frac{T_v}{T_d} e^{-\frac{t}{T_d}} \right]$	$K_P \left[ 1 + \frac{1}{T_n s} + \frac{T_v s}{1 + T_d s} \right] = K_{PP} \frac{(1 + T_{np} s)(1 + T_{vp} s)}{T_{np} s (1 + T_d s)}$ $T_n = T_{np} + T_{vp} - T_d; T_v = T_{vp} \frac{T_{np}}{T_n} - T_d; K_P = K_{PP} \frac{T_n}{T_{np}}$				
T <sub>t</sub>		$h(t) = \sigma(t - T_t)$	$G(s) = e^{-T_t s}$				
A <sub>1</sub>		$h(t) = 1 - 2e^{-\frac{t}{T_1}}$	$G(s) = \frac{1 - T_1 s}{1 + T_1 s}$				

Check them out!

