Formulas in Control Engineering

Туре	Step Response h(t)	Equation for Step Response	Transfer Function G(s)	Nyquist Curve G(jω)	Bode Plot G(jω) _{dB} and φ (jω)	Poles (x) and Zeros (o)	Block Diagram
P	h	$h(t) = K_P \sigma(t)$	$G(s) = K_P$	$Im\{G\}$ $K_P \longrightarrow Re\{G\}$	$\begin{matrix} \uparrow \\ G \end{matrix} \qquad \begin{matrix} K_P \\ \varphi \end{matrix}$ $0dB \qquad \qquad$	no poles no 0 σ zeros	$u \longrightarrow K_P \longrightarrow v$
I		$h(t) = \frac{t}{T_I}$	$G(s) = \frac{1}{T_I s}$	$jIm\{G\}$ $\omega = \infty$ $Re\{G\}$	$ \begin{array}{c c} & \downarrow \\ & \downarrow \\$	- jω σ	$\frac{u}{T_I}$ $\frac{1}{s}$ $\frac{y}{T_I}$
D	T _D	$h(t) = T_D \cdot \delta(t)$	$G(s) = T_D s$	$\omega = 0$ $Re\{G\}$	$\begin{array}{c c} & & & & \downarrow \\ & & & \varphi \\ & & & +90^{\circ} \\ & & & +45^{\circ} \end{array}$	jω σ	$u \longrightarrow T_D \longrightarrow s$
$P-T_1$		$h(t) = K \left[1 - e^{-\frac{t}{T_1}} \right]$	$G(s) = \frac{K}{1 + T_1 s}$	$\omega = \infty$ $\omega = \infty$ $\omega = \frac{1}{T_1}$ $Re\{G\}$	$ \begin{array}{c c} & \uparrow \\ & \downarrow K & \hline{T_1} \\ \hline 0dB & & & & & \\ \hline & & & \\ \hline & & &$	$-\frac{1}{T_1}$ σ	$\frac{u}{T_1}$ $\frac{1}{s}$ $\frac{1}{s}$
$P-T_2$	h	$h(t) = K \left[1 - \frac{T_1}{T_1 - T_2} e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} e^{-\frac{t}{T_2}} \right]$	$G(s) = \frac{K}{(1 + T_1 s)(1 + T_2 s)}$	$\omega = \infty$ $\omega = \frac{1}{\sqrt{T_1 T_2}}$ $\omega = 0$ $Re\{G\}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c c} \times & \times \\ -\frac{1}{T_2} - \frac{1}{T_1} \end{array} $	$\frac{u}{T_1} + \frac{1}{s} + \frac{1}{T_2} + \frac{1}{s} + \frac{1}{T_2}$
$P-T_{2S}$		$h(t) = K \left[1 \pm \frac{e^{\frac{-\vartheta t}{T_0}}}{\sqrt{1 - \vartheta^2}} \sin\left(\sqrt{1 - \vartheta^2} \frac{t}{T_0} + \psi\right) \right]$ $\psi = \arccos\vartheta$	$G(s) = \frac{K}{(1 + 2\vartheta T_0 s + T_0^2 s^2)}$ $T_0 = \frac{1}{\omega_0}; \ \vartheta < 1$	$\omega = \infty$ $\omega = \infty$ $\omega = \frac{1}{T_0}$ $\kappa = 0$ $Re\{G\}$	$ \begin{array}{c c} & \downarrow \\ & \downarrow \\$	$ \begin{array}{c c} & j\omega \\ \hline -\vartheta\omega_0 & j\sqrt{1-\vartheta^2}\omega_0 \\ \hline & -j\sqrt{1-\vartheta^2}\omega_0 \end{array} $	$\frac{u}{T_0} + \frac{1}{s} + \frac{K}{T_0} + \frac{1}{s} $
$I-T_1$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$h(t) = \frac{t}{T_I} - \frac{T_1}{T_I} \left(1 - e^{-\frac{t}{T_1}} \right)$	$G(s) = \frac{1}{T_I s (1 + T_1 s)}$	$-\frac{T_1}{T_I}$ $\omega = \infty$ $Re\{G\}$	$0dB \qquad \qquad$	$\frac{1}{-\frac{1}{T_1}}$	$\frac{u}{T_1} \xrightarrow{\frac{1}{s}} \frac{1}{T_l}$
$D-T_1$	$\frac{T_D}{T_1}$	$h(t) = \frac{T_D}{T_1} e^{-\frac{t}{T_1}}$	$G(s) = \frac{T_D s}{1 + T_1 s}$	$\omega = 0$ $\omega = \frac{1}{T_1}$ $\omega = \infty$ $Re\{G\}$	$\begin{array}{c c} & & & & & & & \\ \hline \downarrow & & & & & & \\ \hline & & & & & \\ \hline & & & & &$	$-\frac{1}{T_1}$ σ	$\underbrace{u} \xrightarrow{T_D} \underbrace{1}_{T_1} \underbrace{v} \xrightarrow{T_1} \underbrace{v} \underbrace{v} \xrightarrow{T_1} \underbrace{v} \underbrace{v} \xrightarrow{T_1} \underbrace{v} \underbrace{v} \underbrace{v} \underbrace{v} \underbrace{v} \underbrace{v} \underbrace{v} v$
PI	K_p	$h(t) = K_P \left(1 + \frac{t}{T_n} \right)$	$G(s) = K_P \frac{1 + T_n s}{T_n s}$	$\omega = \infty$ $\omega = \infty$ $\omega = \frac{1}{T_n}$ $Re\{G\}$	$ \begin{array}{c c} & 1 & K_p & \varphi \\ \hline 0dB & & & & & & & & & & & & & & \\ \hline 0dB & & & & & & & & & & & & & \\ \hline & & & & & & & & & & & & & \\ \hline & & & & & & & & & & & & & \\ \hline & & & & & & & & & & & & \\ \hline & & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ \hline & & & & & & & & & & \\ \hline & & & & & & & & & \\ \hline & & & & & & & & & \\ \hline & & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \hline & & & & \\ \hline & & & & & \\ \hline$	$ \begin{array}{c c} & \downarrow & \downarrow \\ \hline -\frac{1}{T_n} & \downarrow & \sigma \end{array} $	u K_p $\frac{1}{T_n}$ $\frac{1}{s}$
PD	K_{P} T_{D} t	$h(t) = K_P[1 + T_v \delta(t)]$	$G(s) = K_P(1 + T_v s)$	$\omega = 0$ $\omega = \frac{1}{T_v}$ $Re\{G\}$	$0dB \xrightarrow{K_{P}} \frac{1}{T_{v}} \qquad $	$\begin{array}{c c} & & & \\ \hline & & \\ \hline & -\frac{1}{T_v} & & \\ \hline \end{array}$	U K_P T_v S
$PD-T_1$	$K_P \frac{T_{vP}}{T_1}$ $K_P = \frac{T_v}{T_1}$ $K_P = \frac{T_v}{T_1}$	$T_{vP} > T_1$ $T_v = T_{vP} - T_1 > 0$ $h(t) = K_P \left[1 + \frac{T_v}{T_1} e^{-\frac{t}{T_1}} \right]$	$G(s) = K_P \left[1 + \frac{T_v s}{1 + T_1 s} \right]$ $= K_P \frac{1 + T_v s}{1 + T_1 s}$	$\omega = 0$ $\omega = \frac{1}{T_1}$ $\omega = \infty$ $K_P \frac{K_P \frac{T_v}{T_1}}{K_P \frac{T_{vP}}{T_1}}$ $Re\{G\}$	$\begin{array}{c c} \downarrow & \downarrow & \downarrow \\ \downarrow & \downarrow & \downarrow \\ \downarrow & \downarrow & \downarrow \\ \downarrow & \downarrow &$	$\begin{array}{c c} & & & \\ \hline & \times & \\ \hline -\frac{1}{T_1} - \frac{1}{T_{vP}} & & \\ \hline \end{array}$	$K_{P}T_{vP}$ T_{1} $+$ T_{v} $+$ T_{v} $+$ T_{v} $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$
$PP-T_1$	$K_{P} \frac{T_{vP}}{T_{1}} = \infty$ $T_{1} \qquad t$	$T_{vP} < T_1$ $T_v = T_{vP} - T_1 < 0$	$T_{vP} = T_v + T_1$	$K_{P} \frac{T_{vP}}{T_{1}}$ $\omega = \infty$ $\omega = 0 Re\{G\}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c c} & & \downarrow \\ & -\frac{1}{T_{vP}} - \frac{1}{T_1} \end{array} $	$\frac{1}{T_1} \left(1 - \frac{T_1}{T_1}\right)$
PID	K_pT_v T_n t	$h(t) = K_P \left 1 + \frac{1}{T} + T_v \delta(t) \right $	$K_{P}[1 + \frac{1}{T_{n}s} + T_{v}s] = K_{PP} \frac{(1 + T_{nP}s)(1 + T_{vP}s)}{T_{nP}s}$ $T_{n} = T_{nP} + T_{vP}; T_{v} = T_{vP} \frac{T_{nP}}{T_{n}}; K_{P} = K_{PP} \frac{T_{n}}{T_{nP}}; \frac{T_{n}}{T_{v}} \ge 4$	$\omega = \frac{1}{\sqrt{T_n T_v}}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c c} & & & \\ \hline & & \\ & -\frac{1}{T_{nP}} - \frac{1}{T_1} \end{array} $	U K_p T_v S
$PID-T_1$	$h \wedge K_P \left(1 + \frac{T_v}{T_d}\right)$ $T_n T_d \qquad t$	$h(t) = K_P \left[1 + \frac{t}{T_n} + \frac{T_v}{T_d} e^{-\frac{t}{T_d}} \right]$	$K_{P}\left[1 + \frac{1}{T_{n}s} + \frac{T_{v}s}{1 + T_{d}s}\right] = K_{PP}\frac{(1 + T_{nP}s)(1 + T_{vP}s)}{T_{nP}s(1 + T_{d}s)}$ $T_{n} = T_{nP} + T_{vP} - T_{d}; T_{v} = T_{vP}\frac{T_{nP}}{T_{n}} - T_{d}; K_{p} = K_{PP}\frac{T_{n}}{T_{nP}}$	$K_{P}\left(1 + \frac{T_{v}}{T_{d}}\right)$ $K_{P}\left(1 + \frac{T_{d}}{T_{d}}\right)$ $K_{P}\left(1 + \frac{T_{d}}{T_{d}}\right)$ $K_{P}\left(1 + \frac{T_{d}}{T_{d}}\right)$ $K_{P}\left(1 + \frac{T_{v}}{T_{d}}\right)$	$ \begin{array}{c c} \hline & 1 \\ \hline & T_{nP} \\ \hline & T_{vP} \\ \hline & +90^{\circ} \\ \hline & K_{PP} \\ \hline & K_{P} \\ \hline & 1 \\ \hline & T_{n} \\ \hline & T_{d} \\ \hline & -45^{\circ} \\ \hline & -90^{\circ} \\ \hline \end{array} $	$ \begin{array}{c c} -\frac{1}{T_d} \\ \hline \times \bigcirc \bigcirc \\ -\frac{1}{T_{vP}} - \frac{1}{T_{nP}} \end{array} $	u K_P $\frac{1}{T_n}$ $\frac{1}{T_d}$ $\frac{1}{T_d}$
T_t		$h(t) = \sigma(t - T_t)$	$G(s) = e^{-T_t s}$	$\omega = \frac{2k\pi}{T_t}$ $\omega = \frac{1}{T_t}$ $Re\{G\}$	$ \begin{array}{c c} & 1 & \varphi \\ \hline & 1 & \varphi \\ \hline & 0dB & & 0^{\circ} \\ \hline & -57,3^{\circ} & & -45^{\circ} \\ \hline & -90^{\circ} \end{array} $	No poles or zeros in a finite graph	
A_1	T_1	$h(t) = 1 - 2e^{-\frac{t}{T_1}}$	$G(s) = \frac{1 - T_1 s}{1 + T_1 s}$	$\lim_{\omega = \infty} G$ $\omega = \infty$ $\omega = \frac{1}{T_1}$ $Re\{G\}$	$\begin{array}{c c} & & & & & & & & & & & & & & \\ \hline IG & & & & & & & & & & \\ \hline 0dB & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & &$	$\begin{array}{c c} & & j\omega \\ \hline & & \\ -\frac{1}{T_1} & & \frac{1}{T_1} \end{array}$	$\begin{array}{c c} u & \frac{1}{T_1} & \frac{1}{s} \\ \hline & \frac{1}{T_1} & \frac{1}{s} \\ \hline \end{array}$

Check them out!







