

## Assignment-2

(Chabot) D

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① Keywords  $w_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $w_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $w_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

 $w_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

for first round key

Rot Word

$w_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{RotWord}} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

## ② SB (SubByte)

$t = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{SubWord}} \begin{bmatrix} 6 & 3 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$

→ Applying Rcon for first round.

$\begin{bmatrix} 6 & 3 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix} \oplus \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$

$63 = 011000011\text{bit}+2, \quad 01 = 0000 \quad 0001 \quad (\text{bitwise})$

$63 \oplus 01 = 011000010 \quad (\text{bitwise}) = 62 \quad (\text{Hexadecimal})$

$\text{and } 63 \oplus 00 = 63$

so finally  $t = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$

first Round by words are

$w_4 = t \oplus w_0 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$

$$\left\{ \begin{array}{l} \because x \oplus 0 = x \\ \therefore 62 \oplus 00 = 62 \end{array} \right\}$$

Ques

$$w_5 = w_0 \oplus w_1 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$$

$$w_6 = w_5 \oplus w_4 = \begin{bmatrix} 6 & 3 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$$

$$w_7 = w_6 \oplus w_3 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$$

so first and key words are

$$w_0 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}, w_5 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}, w_6 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}, w_7 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \end{bmatrix}$$

For first sound  $w_8 = w_{10}$  &  $w_3 = w_{11}$  firstly calculate  
for Round 2 by these steps

1.) Rotword

$$t_2 w_7 = \begin{bmatrix} 6 & 2 \\ 6 & 3 \\ 6 & 3 \\ 6 & 3 \end{bmatrix} \xrightarrow{\text{Rotword}} \begin{bmatrix} 6 & 3 \\ 6 & 2 \\ 6 & 3 \\ 6 & 2 \end{bmatrix}$$

2) Subword

$$t_2 \begin{bmatrix} 6 & 3 \\ 6 & 3 \\ 6 & 3 \\ 6 & 2 \end{bmatrix} \xrightarrow{\text{Subword}} \begin{bmatrix} F & B \\ F & B \\ F & B \\ A & B \end{bmatrix}$$

3) Applying ~~RCON~~ for second Round

$$\begin{bmatrix} F & B \\ F & B \\ F & B \\ A & B \end{bmatrix} \oplus \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} F & 9 \\ F & B \\ F & B \\ A & B \end{bmatrix}$$

$$\Rightarrow FB \oplus 02 \Rightarrow 11111011 \oplus 00000010$$

$$\Rightarrow 11111010$$

$$\Rightarrow F9$$

and  $x \oplus 00 = k$

$$\text{so } t_{2,10} = \begin{bmatrix} F & 9 \\ F & B \\ F & B \\ A & B \end{bmatrix}$$

words for Round 2

$$\omega_8 = t_{2/8} \oplus \omega_4 = \begin{bmatrix} F9 \\ FB \\ FB \\ AB \end{bmatrix} \oplus \begin{bmatrix} 62 \\ 63 \\ 63 \\ 63 \end{bmatrix} = \begin{bmatrix} 9B \\ 98 \\ 98 \\ C9 \end{bmatrix}$$

$$\Rightarrow F9 \oplus 62 = 9B, 1111001 \oplus 01100010 = 10011011 = 9B$$

$$\Rightarrow FB \oplus 63 = 98, 1111011 \oplus 01100011 = 10011000 = 9B$$

$$\Rightarrow FA \oplus 63 = C9, 11101010 \oplus 01100011 = 1101101 = C9$$

$$\omega_9 = \omega_8 \oplus \omega_5 = \begin{bmatrix} 9B \\ 98 \\ 98 \\ C9 \end{bmatrix} \oplus \begin{bmatrix} 62 \\ 63 \\ 63 \\ 63 \end{bmatrix} = \begin{bmatrix} F9 \\ FB \\ FB \\ FA \end{bmatrix}$$

$$\text{if } a \oplus b = x$$

$$\text{then } x \oplus b = a$$

as seen  $\omega_9$  &  $\omega_5$  are equal

$$\text{do } t_{2/8} \oplus \omega_0 \oplus \omega_5 = t_2(8)$$

and

$$\omega_{10} = \omega_9 \oplus \omega_6 = \begin{bmatrix} F9 \\ FB \\ FB \\ FA \end{bmatrix} \oplus \begin{bmatrix} 62 \\ 63 \\ 63 \\ 63 \end{bmatrix} = \begin{bmatrix} 9B \\ 98 \\ 98 \\ C9 \end{bmatrix}$$

same as in  $\omega_{10}$

$$\text{and } \omega_{11} = \omega_{10} \oplus \omega_7 = \begin{bmatrix} 9B \\ 98 \\ 98 \\ C9 \end{bmatrix} \oplus \begin{bmatrix} 62 \\ 63 \\ 63 \\ 63 \end{bmatrix} = \begin{bmatrix} F9 \\ FB \\ FB \\ F \end{bmatrix}$$

so  $\omega_8 = \omega_{10} \neq \omega_9 = \omega_{11}$  because of property of a xor.

In Round 1 all the words are same

$$\omega_0 = \omega_5 = \omega_6 = \omega_7$$

and

$$\omega_8 = t \oplus \omega_4 - ①$$

$$\omega_9 = \omega_8 \oplus \omega_5 = t \oplus \omega_0 \oplus \omega_5 = \omega_2 \quad [\because \omega_0 = \omega_5 \text{ & by xor property } x \oplus b \oplus b = b]$$

$$\omega_{10} = \omega_9 \oplus \omega_6 = t \oplus \omega_4 \quad (\because \omega_6 = \omega_4) - ③$$

$$\omega_{11} = \omega_{10} \oplus \omega_7 = t \oplus \omega_0 \oplus \omega_7$$

$$= t \quad (\because \omega_0 = \omega_4) - ④$$

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By equation ① & ② ③

$$\omega_8 = \omega_{10}$$

and by equation ② & ④

Ques 2 Show that  $sB(x_1 \oplus x_2) \neq sB(x_1) \oplus sB(x_2)$ . If  $x_1 = 57$  and  $x_2 = A2$ . (④)

Ans  $x_1 = 57 = 01010111$  (bitwise),  $x_2 = A2 = 10100010$  (bitwise)

$$x_1 \oplus x_2 = 11110101 = F5$$

$$LHS = sB(x_1 \oplus x_2) = sB(F5) = E6$$

$$RHS = sB(52) \oplus sB(A2) = 00 \oplus 3A = 3A$$

L.H.S  $\neq$  R.H.S

$$| sB(x_1 \oplus x_2) \neq sB(x_1) \oplus sB(x_2) |$$

Ques 3 Alice chooses AES-128 to encrypt the plain text using key  $k = [11 \dots 1]$  now determine the first key words of justify why  $\omega_8 = \text{Complement of } \omega_{10}$  &  $\omega_9 = \text{Complement of } \omega_{11}$

Ans key word

$$\omega_0 = \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix}, \omega_1 = \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix}, \omega_2 = \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix}, \omega_3 = \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix}$$

1. Firstly calculate  $t_{(10)}$  for first Round.

Rotword

$$t_{(10)} = \omega_3 = \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix} \xrightarrow{\text{Rotword}} \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix}$$

2. Sub Byte

$$t_{(10)} = \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix} \xrightarrow{\text{Sub Byte}} \begin{bmatrix} 16 \\ 16 \\ 16 \\ 16 \end{bmatrix}$$

3. Applying RCON for first Round

$$t_{(10)} = \begin{bmatrix} 16 \\ 16 \\ 16 \\ 16 \end{bmatrix} \oplus \begin{bmatrix} 01 \\ 00 \\ 00 \\ 00 \end{bmatrix} = \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix} \quad \because x \oplus 0 = x \text{ and } 16 \oplus 00 = 16$$

$$16 \oplus 01 = 00010111 = 17$$

$$\text{Finally } t_{(10)} = \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix}$$

first round words are

$$w_4 = t_{(10)} \oplus w_0 = \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix} \oplus \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix} = \begin{bmatrix} E8 \\ E9 \\ E9 \\ E9 \end{bmatrix}$$

$$\because x \oplus FF = \bar{x}$$

$$\therefore 17 \oplus FF = \overline{17} = \overline{00010111} = 11101000 = E8$$

$$16 \oplus FF = \overline{16} = \overline{00010110} = 11101001 = E9$$

$$w_5 = w_4 \oplus w_7 = t_{(4)} \oplus w_0 \oplus w_1 \quad (\because w_0 = w, \\ \therefore x \oplus b \oplus b = x)$$

$$= t_{(4)} = \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix}$$

$$w_6 = w_5 \oplus w_2 = t_{(4)} \oplus w_2 = \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix} \oplus \begin{bmatrix} FF \\ FF \\ FF \\ FF \end{bmatrix} = \begin{bmatrix} E8 \\ E9 \\ E9 \\ E8 \end{bmatrix}$$

$$w_7 = w_6 \oplus w_3 = t_{(4)} \oplus w_2 \oplus w_5 \quad (\because w_2 = w_3)$$

$$\therefore x \oplus 6 \oplus 6 = x$$

$$= t_{(4)}$$

$$= \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix}$$

so find keyword words are

$$w_4 = \begin{bmatrix} E8 \\ E9 \\ E9 \\ E9 \end{bmatrix}, \quad w_5 = \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix}, \quad w_6 = \begin{bmatrix} E8 \\ E9 \\ E9 \\ E9 \end{bmatrix}, \quad w_7 = \begin{bmatrix} 17 \\ 16 \\ 16 \\ 16 \end{bmatrix}$$

Now let  $t_3(8)$  is the first IV for second round so

$$w_8 = t_2(8) \oplus w_4 \quad \textcircled{1}$$

$$w_9 = w_8 \oplus w_5 = t_2(18) \oplus w_4 \oplus w_5 \quad \textcircled{2}$$

$$\omega_{10} = \omega_9 \oplus \omega_6 \quad - \textcircled{3}$$

$$\omega_{11} = \omega_{10} \oplus \omega_7 \quad - \textcircled{4}$$

and

$$\omega_4 \oplus \omega_5 = \begin{bmatrix} E & 8 \\ E & 9 \\ E & 9 \\ E & 9 \end{bmatrix} \oplus \begin{bmatrix} 1 & 7 \\ 1 & 6 \\ 1 & 6 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} F & F \\ F & F \\ F & F \\ F & F \end{bmatrix} \quad - \textcircled{5}$$

$$\omega_6 \oplus \omega_7 = \begin{bmatrix} E & 8 \\ E & 9 \\ E & 9 \\ E & 9 \end{bmatrix} \oplus \begin{bmatrix} 1 & 7 \\ 1 & 6 \\ 1 & 6 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} F & F \\ F & F \\ F & F \\ F & F \end{bmatrix} \quad - \textcircled{6}$$

$$\omega_4 = \omega_6 \text{ and } \omega_5 = \omega_7 \quad - \textcircled{7}$$

By equation  $\textcircled{2}$  &  $\textcircled{5}$

$$\omega_9 = \overline{t_2(8)} \oplus \begin{bmatrix} F & F \\ F & F \\ F & F \\ F & F \end{bmatrix} = \overline{t_2(18)} \quad - \textcircled{8}$$

$(\because x \oplus FF = \bar{x})$

By equation  $\textcircled{3}$ ,  $\textcircled{8}$  and  $\textcircled{7}$

$$\omega_{10} = \overline{t_2(8)} \oplus \omega_6$$

$$\Rightarrow \omega_{10} = \overline{t_2(18)} \oplus \omega_4 \quad (\text{by equation } \textcircled{1}) \quad - \textcircled{9}$$

By equation  $\textcircled{4}$ ,  $\textcircled{5}$  &  $\textcircled{9}$

$$\omega_{11} = \overline{t_2(8)} \oplus \omega_4 \oplus \omega_7$$

$$\therefore \cancel{\omega_{11}} \quad \omega_4 \oplus \omega_7 = \begin{bmatrix} F & F \\ F & F \\ P & P \\ F & F \end{bmatrix}$$

$$\therefore \omega_{11} = \overline{t_2(18)} \oplus \begin{bmatrix} P & P \\ F & R \\ F & R \\ F & F \\ F & F \end{bmatrix} = \overline{t_2(8)} \quad - \textcircled{10}$$

By equation  $\textcircled{1}$  &  $\textcircled{9}$

$\omega_8 = \text{Complement of } \omega_{10}$

By equation  $\textcircled{8}$  &  $\textcircled{10}$

$$\omega_9 = \overline{\omega_{11}}$$

$\omega_9 = \text{Complement of } \omega_{11}$

Ques ④ Let the group  $G_1 = \{1, 2, 3, 4, 5, 6\}$  and check  $G_1$  is cyclic under multiplication modulo 7 with generator 3. Opener 7

Ans Cyclic group are those group in which at least one exist such that all subgroup of generate

Let generator represent the whole group so

$$G_1 = [1, 2, 3, 4, 5, 6]$$

i)  $G_1$  is a cyclic group with generator 3 then 3 must represent the entire group, subgroup of  $g$  is:

$$\langle 3 \rangle = \{3, 6, 2, 5, 1, 4, 0\}$$

$\therefore$  generator 3 represent the entire group so  $G_1$  is a cyclic group

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Q-5 If Alice always uses the same IV instead of different IVs for all communication with Bob. Which mode of operation (CBC vs CTR) in block cipher is more secure against same IV and why?

Ans If Alice sends a message with IV two times (maybe more) by both CBC and CTR. So in IV same case these point describe season.

1. If IV same then in CTR bits  $(C_1, C_2, \dots, C_n)$  are same. It does not depend on the plaintext but in CBC the ~~bits~~ bits  $(C_1, C_2, \dots, C_n)$  is depend on plaintext. Hence for each plaintext CBC has different bit stream.

② If Eve knows the plaintext and ~~ciphertext~~ ciphertext so  
in CTR it can easily find  $\text{bitstream}(e, p, \text{IV})$  by knowing  
both cipher and plaintext. Because bitstream is same (due to  
IV same) In all future plaintext Eve can easily find  
the plain text by knowing ciphertext with bitstream.  
But in case of CBC if Eve knows plaintext and ciphertext  
then it cannot determine key for finding future  
plaintext.

So by above points it clear find CTR is more secure  
than CBC.

Ques 6 Let's consider the stream cipher RC4, but instead of the  
full 256 bytes we will use state vector  $S$  is  $8 \times 3 = 24$ -bit. Encrypt  
a 3 bit plaintext  $p = [1\ 5\ 3]$  if we use a  $4 \times 3$ -bit key of  
 $k = [2\ 0\ 1\ 3]$ . Also show the process of key stream generator.

Ans, RC4 is a byte oriented cipher. RC4 has several  
steps before ~~initialization~~ <sup>cipher</sup> (initial permutation, key generation  
process). As in question state is return of 8-bit. Then  
 $S = [0\ 1\ 2\ 3\ 4\ 5\ 6\ 7]$

After this generate a vector  $T$  by ~~get~~ separated key bytes  
until  $T$ 's get full.  $T$  has also key equal to state.  
 $T = [2\ 0\ 1\ 3\ 2\ 0\ 1\ 3]$

After this Initial permutation of state do by help of  $T$ .  
Also of initial permutation is

Iterate  $i=0\ t=0$  to 8

$$j = (f + s[r] + [i])$$

Swap  $[s[i], s[j]]$

at first iteration  $i=0, j=0$

$$j = (0+0+2) \% 8 = 2$$

swap  $[s[0], s[2]]$  (state is  $[2\ 1\ 0\ 3\ 4\ 5\ 6\ 7]$ )

(c) Third iteration  $i=2, j=3$ 

$$i = (3+0+1)\%8 = 4$$

$$\text{swap}(s[2], s[4]) = (23410567)$$

(d) fourth iteration  $i=3, j=4$ 

$$j = (4+1+3)\%8 = 0$$

$$\text{swap}(s[3], s[0]) \quad (\text{state } [13420567])$$

(e) fifth iteration  $i=4, j=0$ 

$$j = (0+0+2)\%8 = 2$$

$$\text{swap}(s[4], s[2]) \quad (\text{state is } [13024567])$$

(f) sixth iteration  $i=5, j=2$ 

$$j = (2+5+0)\%8 = 7$$

$$\text{swap}(s[5], s[7]) \quad (\text{state is } [13024765])$$

(g) seventh iteration  $i=6, j=7$ 

$$j = (7+6+1)\%8 = 6$$

$$\text{swap}(s[6], s[6]) \quad (\text{state is } [13024765])$$

(h) eighth iteration  $j = (6+5+3)\%8 = 6$ 

$$\text{swap}(s[7], s[6]) \quad (\text{state } [13024756])$$

After this key seed generation  $\rightarrow$  start Algo of generation process  
 is  
 Our final state

$$S = [13024756]$$

Now generate key

(i) final iteration  $i=0, j=0$ 

$$i = (0+1)\%8 = 1$$

$$j = (0+3)\%8 = 3$$

$$\text{swap}(s[1], s[3])$$

Iterate ~~on~~ on the length of plaintext.

$$i = (i+1)\%8$$

$$j = (i + \cancel{s[i]})\%8$$

$$\text{swap } \cancel{s[i]}(s[i], s[j]),$$

$$t = [s[i] + s[j]]\%8$$

$$k = s[t]$$

(state is  $[1\ 2\ 0\ 3\ 4\ 7\ 5\ 6]$ )

$$t = (2+3) \% 8 = 5$$

$$k = S[5] = 7 \quad (k = [7])$$

(b) Second iteration  $i=1, j=3$ 

$$i = (1+1) \% 8 = 2$$

$$j = (3+0) \% 8 = 3$$

swap ( $S[2], S[3]$ )

$$t = [3+0] \% 8 = 3 \quad (\text{state } [1\ 2\ 3\ 0\ 4\ 7\ 5\ 6])$$

$$k = S[3] = 0 \quad (k = [7, 0])$$

(c) Third iteration  $i=2, j=3$ 

$$i = (2+1) \% 8 = 3$$

$$j = (3+0) \% 8 = 3$$

swap ( $S[3], S[3]$ )  $\Rightarrow$  (state  $[1\ 2\ 3\ 0\ 4\ 7\ 5\ 6]$ )

$$t = (0+0) \% 8 = 0$$

$$k = S[0] = 0 \quad (k = [7, \oplus, \phi])$$

(d) fourth iteration  $i=3, j=3$ 

$$i = (3+1) \% 8 = 4$$

$$j = (3+4) \% 8 = 7$$

swap ( $S[4], S[7]$ )

$$t = (6+4) \% 8 = 2 \quad (\text{state is } [1\ 2\ 3\ 0\ 6\ 7\ 5\ 0])$$

$$k = S[2] = 3 \quad (R = [7, 0, \phi, 3])$$

So for length key =  $[7\ 0\ 1\ 3]$ 

After key generation encryption starts

$$\text{plaintext} = [1\ 5\ 3\ 2]$$

$$\text{key} = [7\ 0\ 1\ 3]$$

Cipher = Plaintext  $\oplus$  key

$$C = [1 \oplus 7, 5 \oplus 0, 3 \oplus 1, 2 \oplus 3]$$

$$= [16, 5, 2, 1]$$

GATEKAS (11)

Ques 7. Find the value of  $x$  using Fermat's theorem when

$$x^{85} \equiv 6 \pmod{29}$$

Ans

$$x^{85} \equiv 6 \pmod{29}$$

$$(x^{28} \times x^{28} \times x^{29}) \pmod{29} \equiv 6 \pmod{29}$$

$$(x^{28} \pmod{29}) \times (x^{28} \pmod{29}) \times (x^{29} \pmod{29}) \equiv 6 \pmod{29}$$

[By module multiplication law] □

\*  $a^{p-1} \pmod{p} \equiv 1 \pmod{p}$   
and  $a^p \pmod{p} = a \pmod{p}$  { by Fermat's theorem }

∴ In equation ①

$$(1 \pmod{29}) \times (1 \pmod{29}) \times (x \pmod{29}) \equiv 6 \pmod{29}$$
$$\Rightarrow x \pmod{29} \equiv 6 \pmod{29}$$

By Comparing  $x = 6$

so at  $x = 6$   $x^{85} \equiv 6 \pmod{29}$

Q8. An integer gives remainders 3, 3, 0 on division by 7, 13, and 5 respectively. Find the integer value using Chinese remainder theorem.

Ans

$$3 = x \pmod{7}$$

~~3~~

$$0 = x \pmod{5}$$

$$(m_1 = 7, m_2 = 13, m_3 = 5 \text{ and } a_1 = 3, a_2 = 3, a_3 = 0)$$

In Chinese Remainder Theorem these are four steps

$$M = m_1 \times m_2 \times m_3$$

$$M = 7 \times 13 \times 5 = 455$$

$$x \equiv 3 \pmod{7}$$

$$x \equiv 3 \pmod{13}$$

$$x \equiv 0 \pmod{5}$$

$$x \equiv a, \pmod{m_i}$$

$$\begin{array}{ll} a_1 = 3 & m_1 = 7 \\ a_2 = 3 & m_2 = 13 \\ a_3 = 0 & m_3 = 5 \end{array}$$

since 5, 7 & 13 all are point to one another we can find  $x$ .

$$\text{i.e. } \gcd(5, 7) = \gcd(7, 13) = \gcd(5, 13) = 1$$

$$M = m_1 m_2 m_3 = 5 \times 7 \times 13 = 455$$

$$M = 455$$

$$M_1 = m_2 m_3 = 65$$

$$M_2 = m_1 m_3 = 35$$

$$M_3 = m_1 m_2 = 91$$

Now we call  $x_i$  value

$$M_1 x_1 \equiv 1 \pmod{m_1} \quad \text{i.e. } M_1 x_1 \pmod{m_1} \equiv 1$$

$$65 x_1 \pmod{7} \equiv 1$$

$$2 x_1 \pmod{7} \equiv 1$$

$$\therefore x_1 = 4$$

similarity

$$M_2 x_2 \equiv 1 \pmod{m_2}$$

$$35 x_2 \pmod{13} \equiv 1$$

$$9 x_2 \pmod{13} \equiv 1$$

$\therefore$

$$x_2 = 3$$

111<sup>rd</sup>

$$M_3 x_3 \equiv 0 \pmod{m_3}$$

$$91 x_3 \pmod{5} \equiv 0$$

$$1 x_3 \pmod{5} \equiv 0$$

$$\therefore x_3 = 5$$

$$a_1 = 3$$

$$m_1 = 7$$

$$\bullet X_1 = 4$$

$$\textcircled{M}_1 = 65$$

$$a_2 = 3$$

$$m_2 = 13$$

$$X_2 = 3$$

$$M_2 = 35$$

$$a_3 = 0$$

$$m_3 = 5$$

$$X_3 = 5$$

$$M_3 = 291$$

$$m = 455$$

$$x = (M_1 x_1 a_1 + M_2 x_2 a_2 + M_3 x_3 a_3) \bmod m$$

$$= (65 \times 4 \times 3) + (35 \times 3 \times 3) + (291 \times 5 \times 0)) \bmod m$$

$$= (780 + 315 + 0) \bmod 455$$

$$= 1095 \bmod 455$$

$$\underline{\underline{x = 185}}$$

To verify

$$185 \bmod 7 = 3$$

$$185 \bmod 13 = 3$$

$$185 \bmod 5 = 0$$

Hence verified  $x = 185$

(10) Cipher text = 10

$$\text{Public key} = \{e, n\} = \{5, 35\}$$

$$\text{Encrypt} = C = M^e \bmod n$$

$$\text{Decrypt} = M = C^d \bmod n$$

To find d

$$d = e^{-1} \bmod \phi(n)$$

$$\textcircled{ed} \bullet ed \bmod \phi(n) = 1$$

$$5 \times d \bmod \phi(35) = 1$$

$$\phi(35) = \phi(7) \times \phi(5) = 6 \times 4 = 24$$

$$5 \times d \bmod 24 = 1$$

$$\therefore d = 5$$

In decryption

$$M = C^d \pmod{n}$$

$$M = 10^5 \pmod{35}$$

$$M = 10^5 \pmod{35}$$

$$\therefore M = 5$$