Homework 8, CSE 232

Due October 29 Note: On most of the problem sets through the semester, I'll put a horizontal line with "Optional" under it. Any problems below this section are encouraged - I think they're interesting and will help you learn the subject - but not necessary to complete in order to get credit for the homework.

Problem 1

This week's problem is Google Code Jam Qualification Round 2008 - Fly Swatter. Here's the problem statement:

> What are your chances of hitting a fly with a tennis racquet?
>
> To start with, ignore the racquet's handle. Assume the racquet is a perfect ring, of outer radius R and thickness t (so the inner radius of the ring is R−t).
>
> The ring is covered with horizontal and vertical strings. Each string is a cylinder of radius r. Each string is a chord of the ring (a straight line connecting two points of the circle). There is a gap of length g between neighbouring strings. The strings are symmetric with respect to the center of the racquet i.e. there is a pair of strings whose centers meet at the center of the ring.
>
> The fly is a sphere of radius f. Assume that the racquet is moving in a straight line perpendicular to the plane of the ring. Assume also that the fly's center is inside the outer radius of the racquet and is equally likely to be anywhere within that radius. Any overlap between the fly and the racquet (the ring or a string) counts as a hit.

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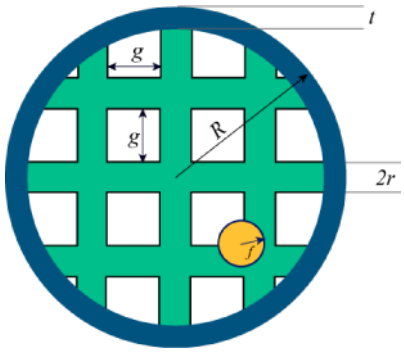
In [1]: from IPython.display import Image, display
        display(Image(filename='./week8_hwimg.png'))
```



Input

> One line containing an integer N, the number of test cases in the input file.
>
> The next N lines will each contain the numbers f, R, t, r and g separated by exactly one space. Also the numbers will have at most 6 digits after the decimal point.

Output

> N lines, each of the form "Case #k: P", where k is the number of the test case and P is the probability of hitting the fly with a piece of the racquet.
>
> Answers with a relative or absolute error of at most 10^{-6} will be considered correct.

Limits

f, R, t, r and g will be positive and smaller or equal to 10000.

$t < R$

$f < R$

$r < R$

Small dataset

$1 \leq N \leq 30$

The total number of strings will be at most 60 (so at most 30 in each direction).

Large dataset

$1 \leq N \leq 100$

The total number of strings will be at most 2000 (so at most 1000 in each direction).

a) The problem as stated requires us to work with a large fly of radius f. Suppose we had a problem where $f = 0$. How could we answer the probability of the zero-radius fly being hit by the racquet in terms of areas? We can make simultaneous adjustments to multiple parameters without changing the final probability. How can we do this so that $f' = 0$ after the change? What are the adjusted values of all the other parameters?

b) We can solve this problem by adding together the areas of the gaps between the strings. There is a symmetry in the problem that will simplify our computation. Describe how to use this symmetry to restrict the number of sections to examine.

c) For the sections of area to be added together, there are two trivial cases to consider and four non-trivial cases. In the trivial cases, either the full square of the gap is inside the inner rim (for area g'^2) or it is fully outside the inner rim (for area 0). Enumerate the four non-trivial cases and provide a formula to calculate their areas. You may find it useful to draw a diagram, and a formula to calculate the areas of circular segments will be of interest.

d) Using the formulas you came up with in part c, write an $O(N^2)$ solution to the problem by iterating through all squares within your region of interest. Test it on the Google Code Jam website and attach your code.

Optional

e) Suppose you weren't sure about the exact geometric formulas in part c for each possible case. Write an approximate recursive solution to finding the area of each gap by repeatedly dividing the square into four pieces, making sure that you are within the error margin required.