How to solve a Linear System Equation using Python

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1 Problem

Consider the system of linear equations below:

$$\begin{cases} x + y = 6 \\ -3x + y = 2 \end{cases}$$

A system with linear equations is equivalent to a matrix equation of the form:

$$\mathbf{AX = B}$$

$$\begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

So, to solve a system of linear equations, we need to define the 3 elements:

$$A = \begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

Using some matrix properties, we can isolate the vector with unknown variables and solve the product between the inverse of matrix A and matrix B.

$$\mathrm{AX} = \mathrm{B}$$
$$\mathrm{A}^{-1}AX = A^{-1}B$$
$$\mathbf{X = A}^{-1}B$$