Polynomial Optimization: Theoretical and Applied

Polynomial optimization is a framework that is used in fields ranging from electrical and chemical engineering, to theoretical computer science. This optimization framework models a system in terms of polynomial equations, and then optimizes some value function on the possible configurations of that system. As a concrete example, consider a physical system such as an electrical network, where the voltages, currents and power consumption are related by polynomial equations. One might want to minimize the power consumption of that system or ensure that the system does not exceed some critical load threshold; these are instances of polynomial optimization problems, which can be very complicated as the problems grow larger. Polynomial equations govern a wide variety of physical systems, and by tackling problems at this level of generality, I hope to provide value to a wide range of fields at the same time.

My work has mostly been focused on semidefinite programming, which is a technique that can be used to approximate polynomial optimization both in theory and in practice. My work has drawn from a rich theory of convex optimization, algebraic geometry and graph theory and resulted in a number of publications. My experience in both pure mathematics and as a professional software engineer enable me to tackle these problems in novel ways.

One instance of my work is in sparse semidefinite programming. In many applications, the underlying polynomial system exhibits sparsity, meaning that the problem can be expressed using far fewer variables than the naïve formulation would require. By connecting sparse polynomial systems with some algebraic geometry theory and using some algorithmic methods for finding solutions in certain key examples, we were able to give good approximations for such sparse polynomial systems using fewer variables. This will lead to savings on the memory and time needed to solve such systems. Early instances of this work have been accepted for publication in the SIAM Journal of Applied Algebra and Geometry.

I have also proposed applications of this framework to causality theory, which is a branch of probability concerned with reasoning about the effect that one random variable has on another in the presence of hidden variables. By considering all possible ways that a collection of observed variables can be correlated with a potential confounder (these are also governed by polynomial equations), and then optimizing a cost function over this space, we are able to find causal connections between variables even without performing experiments. This work was the basis of a winning proposal for the NSF GRFP grant.

Optimization frameworks such as this are vital to solving complex problems, and to solve such optimization problems, the underlying geometry of the problem is crucial. There is a rich geometric theory for polynomials that dates back centuries, and we still have not found all of the applications of this theory yet. My broad interest is in bridging these gaps between pure and applied mathematics in impactful ways.