# 性价比之王的宽度优先搜索

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课程版本 v7.0

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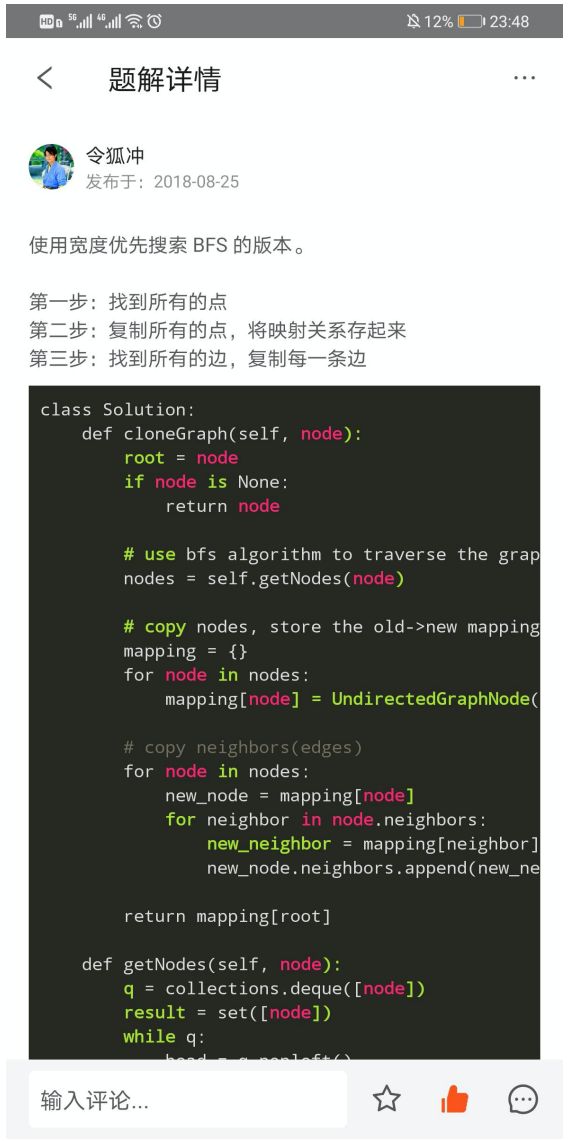
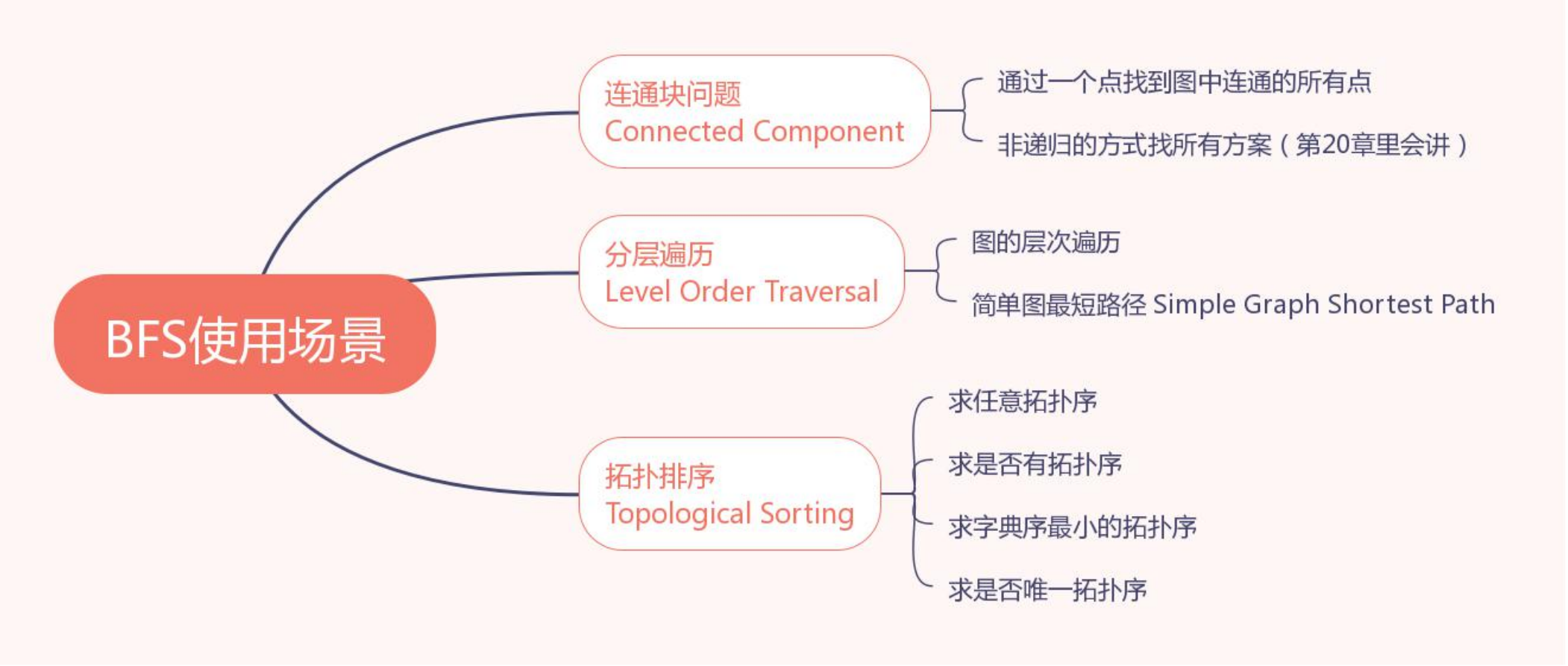
# 九章算法 App - IT口袋题库







# 问最短路径

除了 BFS 还有可能是什么算法？

# 问最短路径

简单图：BFS 

复杂图：Floyd, Dijkstra, Bellman-ford, SPFA

**面试中一般不考复杂图最短路径问题**

# 问最长路径

BFS 是否可以做？

应该用什么算法呢？

# 问最长路径

图可以分层：动态规划 Dynamic Programming

图不可以分层：深度优先搜索 DFS

分层的意思是：路径有一定方向性，不能绕圈

第i层的点只能走到第i+1层不能回到底 i-1 层

# 二叉树 vs 图的 BFS

有什么区别？

# 哈希表

图中存在环，同一个节点可能重复进入队列

Java: *HashMap* / *HashSet*

Python: *dict* / *set*

C++: *unordered_map* / *unordered_set*


# Clone Graph

http://www.lintcode.com/problem/clone-graph/

http://www.jiuzhang.com/solutions/clone-graph/

连通块问题

返回一个经过深度拷贝的新图. 新图和原图具有同样的结构, 并且对新图的任何改动不会对原图造成任何影响.

# 这份代码有什么问题



```python
def cloneGraph(self, node):
    if not node:
        return None
    queue = [node]
    start = 0
    mapping = {}
    while start < len(queue):
        curt_node = queue[start]
        start += 1
        if curt_node in mapping:
            new_node = mapping[curt_node]
        else:
            new_node = UndirectedGraphNode(curt_node.label)
            mapping[node] = new_node
        for neighbor in curt_node.neighbors:
            if neighbor in mapping:
                new_neighbor = mapping[neighbor]
            else:
                new_neighbor = UndirectedGraphNode(neighbor.label)
                mapping[neighbor] = new_neighbor
                queue.append(neighbor)
            new_node.neighbors.append(new_neighbor)
    return mapping[node]
```

```java
public class Solution {
    public UndirectedGraphNode cloneGraph(UndirectedGraphNode node) {
        if (node == null) {
            return null;
        }
        Queue<UndirectedGraphNode> queue = new ArrayDeque<>();
        int start = 0;
        queue.offer(node);
        Map<UndirectedGraphNode, UndirectedGraphNode> mapping = new HashMap<>();
        while (!queue.isEmpty()) {
            UndirectedGraphNode curtNode = queue.poll();
            UndirectedGraphNode newNode;
            if (mapping.containsKey(curtNode)) {
                newNode = mapping.get(curtNode);
            }
            else {
                newNode = new UndirectedGraphNode(curtNode.label);
                mapping.put(node, newNode);
            }
            UndirectedGraphNode newNeighbor;
            for (UndirectedGraphNode neighbor : curtNode.neighbors) {
                if (mapping.containsKey(neighbor)) {
                    newNeighbor = mapping.get(neighbor);
                }
                else {
                    newNeighbor = new UndirectedGraphNode(neighbor.label);
                    mapping.put(neighbor, newNeighbor);
                    queue.offer(neighbor);
                }
                newNode.neighbors.add(newNeighbor);
            }
        }
        return mapping.get(node);
    }
}
```

# 劝分不劝合

要把大象装冰箱，总共分几步？

第一步：把冰箱门打开

第二步：把大象装进去

第三步：把冰箱门关上

吃饭和出恭可以一起，但是不香，分开做更好

# 更好的实现方法



将整个算法分解为三个步骤：

1. 找到所有点
2. 复制所有点
3. 复制所有边

```python
def cloneGraph(self, node):
    if not node:
        return None

    # step 1: find nodes
    nodes = self.find_nodes_by_bfs(node)
    # step 2: copy nodes
    mapping = self.copy_nodes(nodes)
    # step 3: copy edges
    self.copy_edges(nodes, mapping)

    return mapping[node]
```

```python
def find_nodes_by_bfs(self, node):
    queue = collections.deque([node])
    visited = set([node])
    while queue:
        curt_node = queue.popleft()
        for neighbor in curt_node.neighbors:
            if neighbor in visited:
                continue
            visited.add(neighbor)
            queue.append(neighbor)
    return list(visited)

def copy_nodes(self, nodes):
    mapping = {}
    for node in nodes:
        mapping[node] = UndirectedGraphNode(node.label)
    return mapping

def copy_edges(self, nodes, mapping):
    for node in nodes:
        new_node = mapping[node]
        for neighbor in node.neighbors:
            new_neighbor = mapping[neighbor]
            new_node.neighbors.append(new_neighbor)
```

# 更好的实现方法(Java Part 1)



将整个算法分解为三个步骤：

1. 找到所有点
2. 复制所有点
3. 复制所有边

```java
public UndirectedGraphNode cloneGraph(UndirectedGraphNode node) {
    if (node == null) {
        return null;
    }
    // step 1: find nodes
    List<UndirectedGraphNode> nodes = findNodesByBFS(node);
    // step 2: copy nodes
    Map<UndirectedGraphNode, UndirectedGraphNode> mapping = copyNodes(nodes);
    // step 3: copy edges
    copyEdges(nodes, mapping);
    return mapping.get(node);
}
List<UndirectedGraphNode> findNodesByBFS(UndirectedGraphNode node) {
    Queue<UndirectedGraphNode> queue = new ArrayDeque<>();
    Set<UndirectedGraphNode> visited = new HashSet<>();
    queue.offer(node);
    visited.add(node);
    while (!queue.isEmpty()) {
        UndirectedGraphNode cUndirectedGraphNode = queue.poll();
        for (UndirectedGraphNode neighbor : cUndirectedGraphNode.neighbors) {
            if (visited.contains(neighbor)) {
                continue;
            }
            visited.add(neighbor);
            queue.offer(neighbor);
        }
    }
    return new LinkedList<>(visited);
}
```

# 更好的实现方法(Java Part 2)



将整个算法分解为三个步骤：

1. 找到所有点
2. 复制所有点
3. 复制所有边

```
Map<UndirectedGraphNode, UndirectedGraphNode> copyNodes(List<UndirectedGraphNode> nodes) {
    Map<UndirectedGraphNode, UndirectedGraphNode> mapping = new HashMap<>();
    for (UndirectedGraphNode node : nodes) {
        mapping.put(node, new UndirectedGraphNode(node.label));
    }
    return mapping;
}

void copyEdges(List<UndirectedGraphNode> nodes, Map<UndirectedGraphNode, UndirectedGraphNode> mapping) {
    for (UndirectedGraphNode node : nodes) {
        UndirectedGraphNode newNode = mapping.get(node);
        for (UndirectedGraphNode neighbor : node.neighbors) {
            UndirectedGraphNode newNeighbor = mapping.get(neighbor);
            newNode.neighbors.add(newNeighbor);
        }
    }
}
```

# 80% 的人都可能会写错的 BFS 算法



- 应该在哪里做访问标记？

```python
queue = collections.deque([node])
visited = set([node])
while queue:
    curt_node = queue.popleft()
    for neighbor in curt_node.neighbors:
        if neighbor in visited:
            continue
        visited.add(neighbor)
        queue.append(neighbor)
return list(visited)
```

```java
queue.offer(node);
set.add(node);
while (!queue.isEmpty()) {
    UndirectedGraphNode head = queue.poll();
    for (UndirectedGraphNode neighbor : head.neighbors) {
        if (!set.contains(neighbor)) {
            set.add(neighbor);
            queue.offer(neighbor);
        }
    }
}
```

```python
queue = collections.deque([node])
# visited = set([node])
while queue:
    curt_node = queue.popleft()
    visited.add(curt_node)
    for neighbor in curt_node.neighbors:
        if neighbor in visited:
            continue
        # visited.add(neighbor)
        queue.append(neighbor)
return list(visited)
```

```java
queue.offer(node);
// set.add(node);
while (!queue.isEmpty()) {
    UndirectedGraphNode head = queue.poll();
    set.add(head);
    for (UndirectedGraphNode neighbor : head.neighbors) {
        if (!set.contains(neighbor)) {
            // set.add(neighbor);
            queue.offer(neighbor);
        }
    }
}
```

# 分层 vs 不分层

代码实现上有什么区别？

# Python



```python
def find_nodes_by_bfs(self, node):
    queue = collections.deque([node])
    visited = set([node])
    while queue:
        curt_node = queue.popleft()
        for neighbor in curt_node.neighbors:
            if neighbor in visited:
                continue
            visited.add(neighbor)
            queue.append(neighbor)
    return list(visited)
```

```python
def find_nodes_by_bfs(self, node):
    queue = collections.deque([node])
    visited = set([node])
    while queue:
        for _ in range(len(queue)):
            curt_node = queue.popleft()
            for neighbor in curt_node.neighbors:
                if neighbor in visited:
                    continue
                visited.add(neighbor)
                queue.append(neighbor)
    return list(visited)
```

# Java



```java
private ArrayList<UndirectedGraphNode> getNodes(UndirectedGraphNode node) {
    Queue<UndirectedGraphNode> queue = new ArrayDeque<UndirectedGraphNode>();
    HashSet<UndirectedGraphNode> set = new HashSet<>();

    queue.offer(node);
    set.add(node);
    while (!queue.isEmpty()) {
        UndirectedGraphNode head = queue.poll();
        for (UndirectedGraphNode neighbor : head.heighbors) {
            if (!set.contains(neighbor)) {
                set.add(neighbor);
                queue.offer(neighbor);
            }
        }
    }

    return new ArrayList<UndirectedGraphNode>(set);
}
```

```java
private ArrayList<UndirectedGraphNode> getNodes(UndirectedGraphNode node) {
    Queue<UndirectedGraphNode> queue = new ArrayDeque<UndirectedGraphNode>();
    HashSet<UndirectedGraphNode> set = new HashSet<>();

    queue.offer(node);
    set.add(node);
    while (!queue.isEmpty()) {
        int size = queue.size();
        for (int i = 0; i < size; i++) {
            UndirectedGraphNode head = queue.poll();
            for (UndirectedGraphNode neighbor : head.heighbors) {
                if (!set.contains(neighbor)) {
                    set.add(neighbor);
                    queue.offer(neighbor);
                }
            }
        }
    }

    return new ArrayList<UndirectedGraphNode>(set);
}
```

# Word Ladder

http://www.lintcode.com/problem/word-ladder/

http://www.jiuzhang.com/solution/word-ladder/

简单图最短路径

给出两个单词（start和end）和一个字典，找出从start到end的最短转换序列，输出最短序列的长度。

# 最简洁的 BFS 算法的通用模板



```java
Queue<Node> queue = new ArrayDeque<>();
HashMap<Node, Integer> distance = new HashMap<>();

// step 1: 初始化
// 把初始节点放到 queue 里， 如果有多个就都放进去
// 并标记初始节点的距离为0， 记录在 distance 的 hashmap 里
// distance 有两个作用， 一是判断是否已经访问过， 二是记录离起点的距离
queue.offer(node);
distance.put(node, 0);

// step 2: 不断访问队列
// while 循环 + 每次 pop 队列中的一个点出来
while (!queue.isEmpty()) {
    Node node = queue.poll();
    // step 3: 拓展相邻节点
    // pop 出的节点的相邻节点， 加入队列并在 distance 中存储距离
    for (Node neighbor : node.getNeighbors()) {
        if (distance.containsKey(neighbor)) {
            continue;
        }
        distance.put(neighbor,  distance.get(node) + 1);
        queue.offer(neighbor);
    }
}
```

```python
# step 1 初始化
# 把初始节点放到 deque 里，如果有多个就都放进去
# 并标记初始节点的距离为0，记录在 distance 的 dict 里
# distance 有两个作用，一是判断是否已经访问过，二是记录节点距离
queue = collections.deque([node])
distance = {node : 0}

# step 2: 不断访问队列
# while 循环 + 每次 pop 队列中的一个点出来
while queue:
    node = queue.popleft()
    # step 3: 拓展相邻节点
    # pop 出的节点的相邻节点，加入队列并在 distance 中存储距离
    for neighbor in node.get_neighbors():
        if neighbor in distance:
            continue
        distance[neighbor] = distance[node] + 1
        queue.append(neighbor)
```

# 休息，休息一会儿

# 矩阵中的宽度优先搜索

BFS in Matrix

# 两个优化建议

Python 队列建议使用用 deque 不建议使用 Queue

（涉及多线程加锁会更慢）

Java 队列建议 new ArrayDeque 不建议 new LinkedList

（链表比数组慢）

# 矩阵 vs 图



图 Graph

N个点，M条边

M最大是 O(N^2) 的级别

图上BFS时间复杂度 = O(N + M)

- 说是O(M)问题也不大，因为M一般都比N大

所以最坏情况可能是 O(N^2)

矩阵 Matrix

R行C列

R*C个点，R*C*2 条边（每个点上下左右4条边，每条边被2个点共享）。

矩阵中BFS时间复杂度 = O(R * C)

# Number of Islands

http://www.lintcode.com/problem/number-of-islands/

http://www.jiuzhang.com/solutions/number-of-islands/

连通块问题

给一个 01 矩阵，求1构成的联通块的个数。

# 坐标变换数组

```
int[] deltaX = {1,0,0,-1};
int[] deltaY = {0,1,-1,0};
```

问：写出八个方向的坐标变换数组？

# DFS 的方法



```python
def numIslands(self, grid):
    if not grid or not grid[0]:
        return 0

    self.n, self.m = len(grid), len(grid[0])
    self.visited = [[False] * self.m for _ in range(self.n)]

    islands = 0
    for row in range(self.n):
        for col in range(self.m):
            if self.is_island(grid, row, col):
                self.visited[row][col] = True
                self.dfs(grid, row, col)
                islands += 1

    return islands

def is_island(self, grid, x, y):
    if not (0 <= x < self.n and 0 <= y < self.m):
        return False
    if not grid[x][y]:
        return False
    return not self.visited[x][y]
```

```python
def dfs(self, grid, x, y):
    dx = [1, 0, -1, 0]
    dy = [0, 1, 0, -1]
    for direction in range(4):
        newx = x + dx[direction]
        newy = y + dy[direction]

        if self.is_island(grid, newx, newy):
            self.visited[newx][newy] = True
            self.dfs(grid, newx, newy)
            # no backtracking
```

DFS 的方法有什么问题？

# 代码 - python



```python
def numIslands(self, grid):
    if not grid or not grid[0]:
        return 0

    islands = 0
    visited = set()
    for i in range(len(grid)):
        for j in range(len(grid[0])):
            if grid[i][j] and (i, j) not in visited:
                self.bfs(grid, i, j, visited)
                islands += 1

    return islands

def bfs(self, grid, x, y, visited):
    queue = deque([(x, y)])
    visited.add((x, y))
    while queue:
        x, y = queue.popleft()
        for delta_x, delta_y in DIRECTIONS:
            next_x = x + delta_x
            next_y = y + delta_y
            if not self.is_valid(grid, next_x, next_y, visited):
                continue
            queue.append((next_x, next_y))
            visited.add((next_x, next_y))
```

```python
def is_valid(self, grid, x, y, visited):
    n, m = len(grid), len(grid[0])
    if not (0 <= x < n and 0 <= y < m):
        return False
    if (x, y) in visited:
        return False
    return grid[x][y]
```

# 代码 - java



```java
public int numIslands(boolean[][] grid) {
    if (grid == null || grid.length == 0 || grid[0].length == 0) {
        return 0;
    }

    int n = grid.length;
    int m = grid[0].length;
    int islands = 0;

    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++) {
            if (grid[i][j]) {
                makeByBFS(grid, i, j);
                islands++;
            }
        }
    }

    return islands;
}


private boolean inBound(Coordinate coor, boolean[][] grid) {
    int n = grid.length;
    int m = grid[0].length;

    return coor.x >= 0 && coor.x < n && coor.y >= 0 && coor.y < m;
}
```

```java
private void markByBFS(boolean[][] gird, int x, int y) {
    // magic numbers!
    int[] directionX = {0, 1, -1, 0};
    int[] directionY = {1, 0, 0, -1};

    Queue<Coordinate> queue = new ArrayDeque<>();

    queue.offer(new Coordinate(x, y));
    gird[x][y] = false;

    while (!queue.isEmpty()) {
        Coordinate coor = queue.poll();
        for (int i = 0; i < 4; i++) {
            Coordinate adj = new Coordinate(
                coor.x += directionX[i],
                coor.y += directionY[i];
            );
            if (!inBound(adj, grid)) {
                continue;
            }
            if (grid[adj.x][adj.y]) {
                gird[adj.x][adj.y] = false;
                queue.offer(adj);
            }
        }
    }
}
```

# 更多 Union Find 有关的问题

将在《九章算法面试高频题冲刺班》中讲解

**并查集 Union Find**

# Knight Shortest Path

http://www.lintcode.com/problem/knight-shortest-path/

http://www.jiuzhang.com/solutions/knight-shortest-path/

简单图最短路径

八个方向坐标变换

# 代码 - java



```java
public static final int[] dx = {1, 1, -1, -1, 2, 2, -2, -2};
public static final int[] dy = {2, -2, 2, -2, 1, -1, 1, -1};

public int shortestPath(boolean[][] grid, Point source, Point destination) {
    if (grid == null || grid.length == 0) {
        return -1;
    }

    Queue<Point> queue = new ArrayDeque<>();
    Map<Integer, Integer> distance = new HashMap();

    int n = grid.length, m = grid[0].length;
    queue.offer(source);
    distance.put(source.x * m + source.y, 0);

    while (!queue.isEmpty()) {
        Point point = queue.poll();
        if (point.x == destination.x && point.y == destination.y) {
            return distance.get(point.x * m + point.y);
        }

        for (int i = 0; i < 8; i++) {
            int adjX = point.x + dx[i];
            int adjY = point.y + dy[i];
            if (!isValid(adjX, adjY, grid)) {
                continue;
            }
            if (distance.containsKey(adjX * m + adjY)) {
                continue;
            }

            distance.put(adjX * m + adjY, distance.get(point.x * m + point.y) + 1);
            queue.offer(new Point(adjX, adjY));
        }
    }

    return -1;
}
```

```java
private boolean isValid(int x, int y, boolean[][] grid) {
    if (x < 0 || x >= grid.length || y < 0 || y >= grid[0].length) {
        return false;
    }
    return !grid[x][y];
}
```

# 代码 - python



```python
def shortestPath(self, grid, source, destination):
    queue = collections.deque([(source.x, source.y)])
    distance = {(source.x, source.y): 0}

    while queue:
        x, y = queue.popleft()
        if (x, y) == (destination.x, destination.y):
            return distance[(x, y)]
        for dx, dy in DIRECTIONS:
            next_x, next_y = x + dx, y + dy
            if (next_x, next_y) in distance:
                continue
            if not self.is_valid(next_x, next_y, grid):
                continue
            distance[(next_x, next_y)] = distance[(x, y)] + 1
            queue.append((next_x, next_y))
    return -1

def is_valid(self, x, y, grid):
    n, m = len(grid), len(grid[0])

    if x < 0 or x >= n or y < 0 or y >= m:
        return False

    return not grid[x][y]
```

# 拓扑排序 Topological Sorting

几乎每个公司都有一道拓扑排序的面试题！

BFS or DFS?

# 独孤九剑——破剑式

能够用 BFS 解决的问题，一定**不要用** DFS 去做！

因为用 Recursion 实现的 DFS 可能造成 StackOverflow!

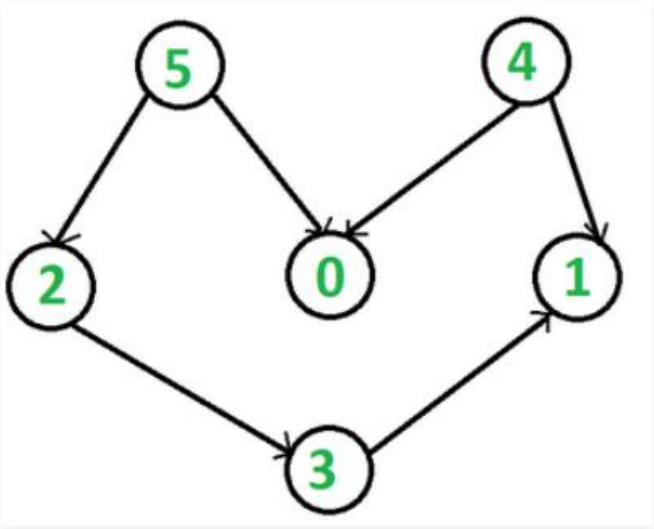
(Iteration 的 DFS 一来你不会写，二来面试官也看不懂)

# 拓扑排序 Topological Sorting



Topological sorting for Directed Acyclic Graph (DAG) is a linear ordering of vertices such that for every directed edge uv, vertex u comes before v in the ordering. Topological Sorting for a graph is not possible if the graph is not a DAG.

For example, a topological sorting of the following graph is “5 4 2 3 1 0”. There can be more than one topological sorting for a graph. For example, another topological sorting of the following graph is “4 5 2 3 1 0”. The first vertex in topological sorting is always a vertex with in-degree as 0 (a vertex with no incoming edges).




***Topological Sorting vs Depth First Traversal (DFS)***:
In DFS, we print a vertex and then recursively call DFS for its adjacent vertices. In topological sorting, we need to print a vertex before its adjacent vertices. For example, in the given graph, the vertex ‘5’ should be printed before vertex ‘0’, but unlike DFS, the vertex ‘4’ should also be printed before vertex ‘0’. So Topological sorting is different from DFS. For example, a DFS of the shown graph is “5 2 3 1 0 4”, but it is not a topological sorting

# 拓扑排序 Topological Sorting



入度（In-degree）：

有向图（Directed Graph）中指向当前节点的点的个数（或指向当前节点的边的条数）

算法描述：

1. 统计每个点的入度
2. 将每个入度为 0 的点放入队列（Queue）中作为起始节点
3. 不断从队列中拿出一个点，去掉这个点的所有连边（指向其他点的边），其他点的相应的入度 - 1
4. 一旦发现新的入度为 0 的点，丢回队列中

拓扑排序并不是传统的排序算法

一个图可能存在多个拓扑序（Topological Order），也可能不存在任何拓扑序

# 拓扑排序的四种不同问法



求任意一个拓扑序

问是否存在拓扑序

求是否存在且仅存在一个拓扑序

求字典序最小的拓扑排序

# 求任意一个拓扑排序

http://www.lintcode.com/problem/topological-sorting/

http://www.lintcode.com/problem/course-schedule/

一个个把点从图中抠出来

# 代码 - Topological Sorting



```python
def topSort(self, graph):
    node_to_indegree = self.get_indegree(graph)

    # bfs
    order = []
    start_nodes = [n for n in graph if node_to_indegree[n] == 0]
    queue = collections.deque(start_nodes)
    while queue:
        node = queue.popleft()
        order.append(node)
        for neighbor in node.neighbors:
            node_to_indegree[neighbor] -= 1
            if node_to_indegree[neighbor] == 0:
                queue.append(neighbor)

    return order

def get_indegree(self, graph):
    node_to_indegree = {x: 0 for x in graph}

    for node in graph:
        for neighbor in node.neighbors:
            node_to_indegree[neighbor] += 1

    return node_to_indegree
```

```java
public ArrayList<DirectedGraphNode> topSort(ArrayList<DirectedGraphNode> graph) {

    ArrayList<DirectedGraphNode> result = new ArrayList<DirectedGraphNode>();
    HashMap<DirectedGraphNode, Integer> map = new HashMap<>();
    for (DirectedGraphNode node : graph) {
        for (DirectedGraphNode neighbor : node.neighbors) {
            if (map.containsKey(neighbor)) {
                map.put(neighbor, map.get(neighbor) + 1);
            } else {
                map.put(neighbor, 1);
            }
        }
    }

    Queue<DirectedGraphNode> q = new ArrayDeque<DirectedGraphNode>();
    for (DirectedGraphNode node : graph) {
        if (!map.containsKey(node)) {
            q.offer(node);
            result.add(node);
        }
    }

    while (!q.isEmpty()) {
        DirectedGraphNode node = q.poll();
        for (DirectedGraphNode n : node.neighbors) {
            map.put(n,  map.get(n) - 1);
            if (map.get(n) == 0) {
                result.add(n);
                q.offer(n);
            }
        }
    }

    return result;
}
```

# 判断是否存在拓扑排序

http://www.lintcode.com/problem/course-schedule-ii/

http://www.jiuzhang.com/problem/course-schedule-ii/

所有节点均能从图中被删除进入拓扑序

# 代码



```python
def findOrder(self, numCourses, prerequisites):
    graph = [[] for i in range(numCourses)]
    in_degree = [0] * numCourses

    # 建图
    for node_in, node_out in prerequisites:
        graph[node_out].append(node_in)
        in_degree[node_in] += 1

    num_choose = 0
    queue = collections.deque()
    topo_order = []

    # 将入度为 0 的编号加入队列
    for i in range(numCourses):
        if in_degree[i] == 0:
            queue.append(i)

    while queue:
        now_pos = queue.popleft()
        topo_order.append(now_pos)
        num_choose += 1
        # 将每条邻边删去，如果入度降为 0，再加入队列
        for next_pos in graph[now_pos]:
            in_degree[next_pos] -= 1
            if in_degree[next_pos] == 0:
                queue.append(next_pos)

    if num_choose == numCourses:
        return topo_order
    return []
```

```java
public int[] findOrder(int numCourses, int[][] prerequisites) {
    List[] graph = new ArrayList[numCourses];
    int[] inDegree = new int[numCourses];

    for (int i = 0; i < numCourses; i++) {
        graph[i] = new ArrayList<Integer>();
    }

    // 建图
    for (int[] edge: prerequisites) {
        graph[edge[1]].add(edge[0]);
        inDegree[edge[0]]++;
    }

    int numChoose = 0;
    Queue queue = new LinkedList();
    int[] topoOeder = new int[numCourses];

    // 将入度为 0 的编号加入队列
    for (int i = 0; i < inDegree.length; i++) {
        if (inDegree[i] == 0) {
            queue.add(i);
        }
    }

    while (!queue.isEmpty()) {
        int nowPos = (int)queue.poll();
        topoOeder[numChoose] = nowPos;
        numChoose++;
        // 将每条边删去， 如果入度降为 0， 再加入队列
        for (int i = 0; i < graph[nowPos].size(); i++) {
            int nextPos = (int)graph[nowPos].get(i);
            inDegree[nextPos]--;
            if (inDegree[nextPos] == 0) {
                queue.add(nextPos);
            }
        }
    }

    if (numChoose == numCourses) {
        return topoOeder;
    }
    return new int[0];
}
```

# 问拓扑序是否唯一

http://www.lintcode.com/problem/sequence-reconstruction/

http://www.jiuzhang.com/problem/sequence-reconstruction/

保持队列中有且仅有一个元素

# 代码 - python



```python
def sequenceReconstruction(self, org, seqs):
    graph = self.build_graph(seqs)
    topo_order = self.topological_sort(graph)
    return topo_order == org

def build_graph(self, seqs):
    # initialize graph
    graph = {}
    for seq in seqs:
        for node in seq:
            if node not in graph:
                graph[node] = set()

    for seq in seqs:
        for i in range(1, len(seq)):
            graph[seq[i - 1]].add(seq[i])

    return graph

def get_indegrees(self, graph):
    indegrees = {
        node: 0
        for node in graph
    }

    for node in graph:
        for neighbor in graph[node]:
            indegrees[neighbor] += 1

    return indegrees
```

```python
def topological_sort(self, graph):
    indegrees = self.get_indegrees(graph)

    queue = []
    for node in graph:
        if indegrees[node] == 0:
            queue.append(node)

    topo_order = []
    while queue:
        if len(queue) > 1:
            # there must exist more than one topo orders
            return None

        node = queue.pop()
        topo_order.append(node)
        for neighbor in graph[node]:
            indegrees[neighbor] -= 1
            if indegrees[neighbor] == 0:
                queue.append(neighbor)

    if len(topo_order) == len(graph):
        return topo_order

    return None
```

# 代码 - java(part 1)



```java
public boolean sequenceReconstruction(int[] org, int[][] seqs) {
    Map<Integer, Set<Integer>> graph = buildGraph(seqs);
    List<Integer> topoOrder = getTopoOrder(graph);

    if (topoOrder == null || topoOrder.size() != org.length) {
        return false;
    }
    for (int i = 0; i < org.length; i++) {
        if (org[i] != topoOrder.get(i)) {
            return false;
        }
    }
    return true;
}

private Map<Integer, Set<Integer>> buildGraph(int[][] seqs) {
    Map<Integer, Set<Integer>> graph = new HashMap();
    for (int[] seq : seqs) {
        for (int i = 0; i < seq.length; i++) {
            if (!graph.containsKey(seq[i])) {
                graph.put(seq[i], new HashSet<Integer>());
            }
        }
    }
    for (int[] seq : seqs) {
        for (int i = 1; i < seq.length; i++) {
            graph.get(seq[i - 1]).add(seq[i]);
        }
    }
    return graph;
}
```

```java
private List<Integer> getTopoOrder(Map<Integer, Set<Integer>> graph) {
    Map<Integer, Integer> indegrees = getIndegrees(graph);
    Queue<Integer> queue = new ArrayDeque();
    List<Integer> topoOrder = new ArrayList();

    for (Integer node : graph.keySet()) {
        if (indegrees.get(node) == 0) {
            queue.offer(node);
            topoOrder.add(node);
        }
    }

    while (!queue.isEmpty()) {
        if (queue.size() > 1) {
            return null;
        }

        Integer node = queue.poll();
        for (Integer neighbor : graph.get(node)) {
            indegrees.put(neighbor, indegrees.get(neighbor) - 1);
            if (indegrees.get(neighbor) == 0) {
                queue.offer((neighbor));
                topoOrder.add(neighbor);
            }
        }
    }

    if (graph.size() == topoOrder.size()) {
        return topoOrder;
    }

    return null;
}
```

# 代码 - java(part 2)



```java
private Map<Integer, Integer> getIndegrees(Map<Integer, Set<Integer>> graph) {
    Map<Integer, Integer> indegrees = new HashMap();
    for (Integer node : graph.keySet()) {
        indegrees.put(node, 0);
    }
    for (Integer node : graph.keySet()) {
        for (Integer neighbor : graph.get(node)) {
            indegrees.put(neighbor, indegrees.get(neighbor) + 1);
        }
    }
    return indegrees;
}
```

# 求字典序最小的拓扑排序

http://www.lintcode.com/problem/alien-dictionary/

http://www.jiuzhang.com/solution/alien-dictionary/

应使用什么数据结构？

# 代码 - java (part 1)



```java
public String alienOrder(String[] words) {
    Map<Character, Set<Character>> graph = constructGraph(words);
    if (graph == null) {
        return "";
    }
    return topologicalSorting(graph);
}
```

```java
private Map<Character, Integer> getIndegree(Map<Character, Set<Character>> graph) {
    Map<Character, Integer> indegree = new HashMap<>();
    for (Character u : graph.keySet()) {
        indegree.put(u, 0);
    }

    for (Character u : graph.keySet()) {
        for (Character v : graph.get(u)) {
            indegree.put(v, indegree.get(v) + 1);
        }
    }

    return indegree;
}
```

```java
private Map<Character, Set<Character>> constructGraph(String[] words) {
    Map<Character, Set<Character>> graph = new HashMap<>();

    // create nodes
    for (int i = 0; i < words.length; i++) {
        for (int j = 0; j < words[i].length(); j++) {
            char c = words[i].charAt(j);
            if (!graph.containsKey(c)) {
                graph.put(c, new HashSet<Character>());
            }
        }
    }

    // create edges
    for (int i = 0; i < words.length - 1; i++) {
        int index = 0;
        while (index < words[i].length() && index < words[i + 1].length()) {
            if (words[i].charAt(index) != words[i + 1].charAt(index)) {
                graph.get(words[i].charAt(index)).add(words[i + 1].charAt(index));
                break;
            }
            index++;
        }
        if (index == Math.min(words[i].length(), words[i + 1].length())) {
            if (words[i].length() > words[i + 1].length()) {
                return null;
            }
        }
    }

    return graph;
}
```

# 代码 - java (part 2)



```java
private String topologicalSorting(Map<Character, Set<Character>> graph) {
    // as we should return the topo order with lexicographical order
    // we should use PriorityQueue instead of a FIFO Queue
    Map<Character, Integer> indegree = getIndegree(graph);
    Queue<Character> queue = new PriorityQueue<>();

    for (Character u : indegree.keySet()) {
        if (indegree.get(u) == 0) {
            queue.offer(u);
        }
    }

    StringBuilder sb = new StringBuilder();
    while (!queue.isEmpty()) {
        Character head = queue.poll();
        sb.append(head);
        for (Character neighbor : graph.get(head)) {
            indegree.put(neighbor, indegree.get(neighbor) - 1);
            if (indegree.get(neighbor) == 0) {
                queue.offer(neighbor);
            }
        }
    }
    if (sb.length() != indegree.size()) {
        return "";
    }
    return sb.toString();
}
```

# 代码 - python



```python
def alienOrder(self, words):
    graph = self.build_graph(words)
    if not graph:
        return ""
    return self.topological_sort(graph)

def build_graph(self, words):
    # key is node, value is neighbors
    graph = {}

    # initialize graph
    for word in words:
        for c in word:
            if c not in graph:
                graph[c] = set()

    # add edges
    n = len(words)
    for i in range(n - 1):
        for j in range(min(len(words[i]), len(words[i + 1]))):
            if words[i][j] != words[i + 1][j]:
                graph[words[i][j]].add(words[i + 1][j])
                break
            if j == min(len(words[i]), len(words[i + 1])) - 1:
                if len(words[i]) > len(words[i + 1]):
                    return None

    return graph
```

```python
def topological_sort(self, graph):
    # initialize indegree
    indegree = {
        node: 0
        for node in graph
    }

    # calculate indegree
    for node in graph:
        for neighbor in graph[node]:
            indegree[neighbor] = indegree[neighbor] + 1

    # use heapq instead of regular queue so that we can get the
    # smallest lexicographical order
    queue = [node for node in graph if indegree[node] == 0]
    heapify(queue)

    # regular bfs algorithm to do topological sorting
    topo_order = ""
    while queue:
        node = heappop(queue)
        topo_order += node
        for neighbor in graph[node]:
            indegree[neighbor] -= 1
            if indegree[neighbor] == 0:
                heappush(queue, neighbor)

    # if all nodes popped
    if len(topo_order) == len(graph):
        return topo_order

    return ""
```

# 相关问题 Related Questions



- 图上的 BFS
  - 判断一个图是否是一棵树
    - http://www.lintcode.com/problem/graph-valid-tree/
  - 搜索图中最近值为target的点
    - http://www.lintcode.com/problem/search-graph-nodes/
  - 无向图连通块
    - http://www.lintcode.com/problem/connected-component-in-undirected-graph/
- 矩阵上的 BFS
  - 僵尸多少天吃掉所有人
    - http://www.lintcode.com/problem/zombie-in-matrix/
  - 建邮局问题 Build Post Office II
    - http://www.lintcode.com/problem/build-post-office-ii/

# 总结 Conclusion



- 能用 BFS 的一定不要用 DFS（除非面试官特别要求）
- BFS 的三个使用场景
  - 连通块问题
  - 层级遍历问题
  - 拓扑排序问题
- 是否需要层级遍历
  - 需要多一重循环
  - 或者使用 distance 哈希表记录到所有点的距离
- 矩阵坐标变换数组
  - deltaX, deltaY
  - inBound / isValid

# 在第 6 周的互动课中继续学习如下与 BFS 有关的内容



- 第三十二章 使用宽度优先搜索找出所有方案
  - 使用 BFS 求所有方案类问题
  - 使用 BFS 序列化二叉树
    - 什么是序列化与反序列化
- 第三十三章【互动】双向宽度优先搜索算法
  - 双向宽度优先搜索算法
    - 双向宽度优先搜索到底优化了多少
    - 如何优雅的实现双向宽度优先搜索