

**STATISTICS 641 - FALL 2017 - ASSIGNMENT 8**

**DUE DATE: NOON, THURSDAY, DECEMBER 7**

Name \_\_\_\_\_

Email Address \_\_\_\_\_

**Make this cover sheet the first page of your Solutions.**

## STATISTICS 641 - ASSIGNMENT 8

- **Due Noon Thursday, December 7**

- Read Handout 13
- Supplemental Reading from Textbook: Chapters 8, 9, 14.2

(P1) (8 points) In each of the following studies, state whether the study uses an independent samples design or a matched pairs design:

- (A) In an evaluation of the efficiency of algorithms, two algorithms are evaluated in terms of CPU times required to complete the same six test problems.
- (B) A survey is conducted of 16 year old students from inner city public schools and suburban public schools to compare the proportion who had experimented with marijuana.
- (C) A psychologist designs a study to assess whether a visual or audio stimulus produces a more rapid response. A group of 250 undergraduates are randomly assigned to the order in which they are exposed to the two stimuli, audio then visual or vice versa. The response times to the stimuli are then recorded.
- (D) The effect of two types of viruses on tobacco leaves was studied by rubbing a preparation containing one of the viruses onto a different half of each of 8 tobacco leavers. The number of lesions counted on the two halves of these leavers were recorded.

- In Problems P2 - P7

- (A) State the null and alternative hypotheses,
- (B) Give the formula for the test statistic or the name associated with the test,
- (C) Set-up the rejection region but DO NOT Carry out the test of hypotheses.
  - DO NOT make any computations.

(P2) (5 points) An experiment is run to study the effects of PCB, an industrial contaminant, on the reproductive ability of owls. The shell thickness of eggs produced by owls exposed to PCB are compared to the shell thickness of eggs produced by owls which did not have PCB exposures. From previous studies, the shell thickness of eggs has been normally distributed.

| Owl          | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|
| PCB-Exposed: | 0.6 | 6.2 | 0.8 | 6.0 | 6.8 | 0.8 | 0.2 | 0.4 |
| UnExposed:   | 3.3 | 3.4 | 3.6 | 3.5 | 3.4 | 3.5 | 3.4 | 3.6 |

Is there significant( $\alpha = .05$ ) evidence that the PCB exposed owls have thinner egg shells than those of the unexposed owls?

(P3) (5 points) Scientists think that robots will play a crucial role in factories in the next 20 years. Suppose that in an experiment to determine whether the use of robots to weave computer cables is feasible, a robot was used to assemble 10 cables and an experienced worker also assembled 10 cables. The cables were examined and the number of defectives on each cable was recorded. The 10 data points for each method were plotted in a normal probability plot and the plot indicated a very heavy-tailed distribution. The data is given here:

| Cable | 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8  | 9  | 10 |
|-------|----|----|----|----|----|----|---|----|----|----|
| Robot | 2  | 8  | 9  | 1  | 18 | 19 | 7 | 8  | 7  | 8  |
| Human | 15 | 14 | 15 | 13 | 26 | 25 | 8 | 15 | 14 | 9  |

Does this data support the assertion that the average number of detectives per assembled cable is less for robots than for humans? Use  $\alpha = 0.05$ .

- (P4) (5 points) Production lines in a manufacturing plant are set to make steel ball bearings with a diameter of 1 micron. Ten ball bearings were randomly selected from the two production lines. The diameters of the ball bearings measured in microns are given in the following table:

| Bearing     | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
|-------------|------|------|------|------|------|------|------|------|------|------|
| First Line  | 1.18 | 1.42 | 0.69 | 0.88 | 1.62 | 1.09 | 1.53 | 1.02 | 1.19 | 1.32 |
| Second Line | 1.72 | 1.62 | 1.69 | 0.79 | 1.79 | 0.77 | 1.44 | 1.29 | 1.96 | 0.99 |

Does this data support the assertion that the variation in the diameter of the ball bearings is different from the two production lines? Use  $\alpha = 0.01$ .

- (P5) (5 points) A study evaluated the urinary-thromboglobulin excretion in 12 normal and 12 diabetic patients. The excretions were summarized with a value of 20 or less labeled as "Low" and values above 20 as "High".

|          | Excretion |      |
|----------|-----------|------|
|          | Low       | High |
| Normal   | 10        | 2    |
| Diabetic | 4         | 8    |

The researcher wanted to determine if there was substantial evidence of a difference in the urinary-thromboglobulin excretion between normal and diabetic patients. Use  $\alpha = .05$

- (P6) (5 points) A study was conducted to compare two topical anesthetic drugs for use in dentistry. The two drugs were applied on the oral mucous membrane of the two sides of each patient's mouth and after a fixed period of time it was recorded whether or not the membrane remained anesthetized. Data from the 45 patients is recorded below:

|          |                  | Drug 2 Response |                  |
|----------|------------------|-----------------|------------------|
|          |                  | Anesthetized    | Not Anesthetized |
| Drug 1   | Anesthetized     | 15              | 13               |
| Response | Not Anesthetized | 3               | 14               |

The experimental was to assess whether there is substantial evidence of a difference between the two drugs. Use  $\alpha = .05$

- (P7) (5 points) A genetics experiment on the characteristics of tomato plants provided the following data on the number of offspring expression four phenotypes.

| Phenotype | Tall, cut-leaf | Dwarf, cut-leaf | Tall, potato-leaf | Dwarf, potato-leaf | Total |
|-----------|----------------|-----------------|-------------------|--------------------|-------|
| Frequency | 926            | 293             | 288               | 104                | 1611  |

The researcher wants to determine if there is substantial evidence that the tomato plants deviate from the current theory that the four phenotypes will appear in the proportion 9:3:3:1. Use  $\alpha = .05$

• Show detailed solutions to Questions P8 and P9

- (P8) (20 points) A marine biologist was interested in the relationship between different coastline habitats and the populations of Hermit crabs inhabiting the site. The biologist counted Hermit crabs on 25 transects randomly located in each of three different sites of a coastline habitat. The data and summary statistics are given in the following tables.

**Number of Crabs per Transect at 3 Habitats (H)**

|      | North to South Orientation of Transects |   |   |   |   |   |   |    |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|------|---|---|---|---|---|---|---|----|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Site | 1                                       | 2 | 3 | 4 | 5 | 6 | 7 | 8  | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 1    | 0                                       | 0 | 0 | 4 | 2 | 2 | 5 | 4  | 2 | 1  | 0  | 12 | 1  | 30 | 0  | 3  | 28 | 2  | 21 | 8  | 82 | 12 | 10 | 2  | 0  |
| 2    | 0                                       | 1 | 1 | 2 | 2 | 1 | 2 | 29 | 2 | 2  | 0  | 13 | 0  | 19 | 1  | 3  | 26 | 30 | 5  | 4  | 94 | 1  | 9  | 3  | 0  |
| 3    | 0                                       | 0 | 0 | 2 | 3 | 0 | 0 | 4  | 0 | 5  | 4  | 22 | 0  | 64 | 4  | 4  | 43 | 3  | 16 | 19 | 95 | 6  | 22 | 0  | 0  |

- (A) Do the Crab counts from the three Sites appear to have a normal distribution? Use  $\alpha = .05$ . Justify your answer.
- (B) Do the Crab counts from the three Sites appear to have the same level of variability? Use  $\alpha = .05$ . Justify your answer.
- (C) Do the Crab counts within each Site appear to be correlated? Use  $\alpha = .05$ . Justify your answer.
- (D) Do the three Sites appear to have different average Crab counts? Use  $\alpha = .05$ . Justify your answer.
- (P9) (20 points) The following table gives the racial characteristics of 326 individuals convicted of homicide and whether or not they received the death penalty. Social scientists were interested in the relationship between Defendant's Race and the probability that the defendant would receive the Death Penalty. A possible confounding variable is the Race of the homicide victim

|                  | Victim's Race |     |               |    |
|------------------|---------------|-----|---------------|----|
|                  | White         |     | Black         |    |
|                  | Death Penalty |     | Death Penalty |    |
| Defendant's Race | Yes           | No  | Yes           | No |
| White            | 19            | 132 | 0             | 9  |
| Black            | 11            | 52  | 6             | 97 |

- (A) Test whether there is significant evidence that frequency of receiving the Death Penalty is related to Defendant's Race ignoring the Victim's Race. Use  $\alpha = .05$ .
- (B) For each category of Victim's Race, test whether there is significant evidence that frequency of receiving the Death Penalty is related to Defendant's Race. Use  $\alpha = .05$ .
- (C) Test whether there is significant evidence that frequency of receiving the Death Penalty is related to Defendant's Race after adjusting for the Victim's Race. Use  $\alpha = .05$ .
- (D) Are the odds-ratio for the death penalty for Black and White Defendants different when the victim is Black versus when the victim is White? Use  $\alpha = .05$

- (P10) (22 points) Multiple Choice Questions. **Select ONE** of the following letters (**A, B, C, D, or E**) corresponding to the **BEST** answer.

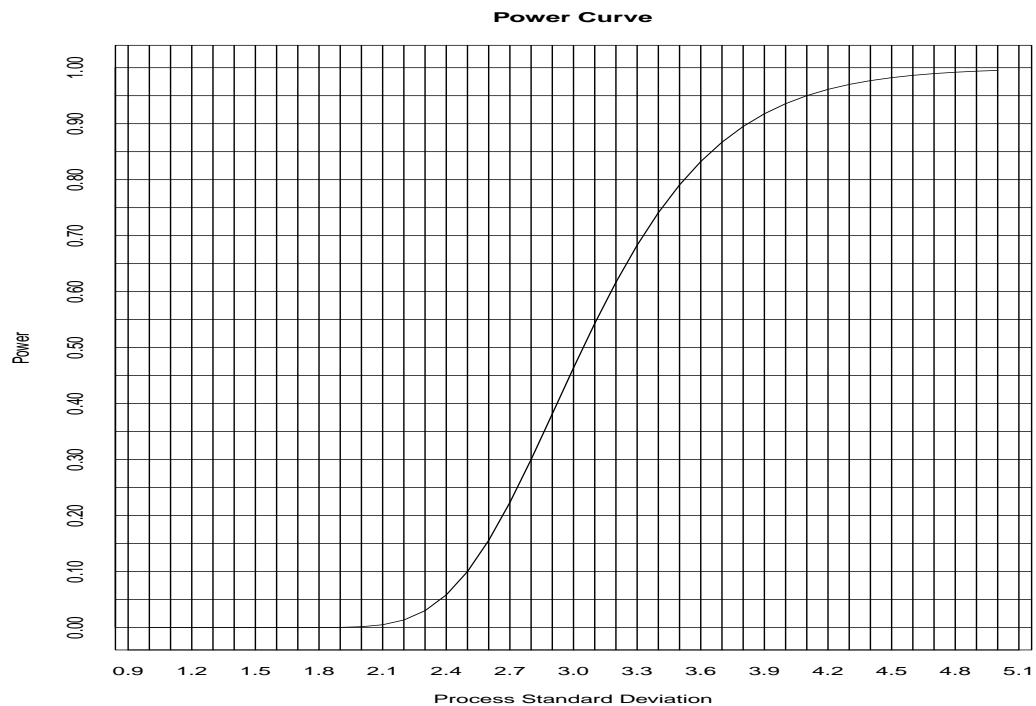
Use the following information for MC1 and MC2.

A process engineer samples a continuous flow of the company's product 200 times per day and obtains the following pH levels in the product :  $X_1, \dots, X_{200}$ . He determines that the daily pH levels are related by  $X_t = \theta + \rho X_{t-1} + e_t$ , where the  $e_t$ s have independent  $N(0, \sigma_e^2)$  distributions and  $\rho > .9$ .

- MC1 The engineer constructs a nominal 95% confidence interval for the average daily pH level,  $\mu$ , using the formula  $\bar{X} \pm t_{.025, 199}(s/\sqrt{200})$ , where  $\bar{X}$  and  $s$  are the sample mean and standard deviation for a given days pH levels. The true coverage probability of this confidence interval
- A. is 0.95.
  - B. is much less than 0.95.
  - C. is very close to 0.95.
  - D. is much greater than 0.95.
  - E. may be greater than 0.95 or less than 0.95 depending on the distribution of the  $X_t$ 's.
- MC2 Refer to MC1. The nominal pH level of the product is 5.3. The process engineer wants to test if the pH on a given day is different from 5.3. He uses  $t = \frac{\bar{X} - 5.3}{s/\sqrt{200}}$  as his test statistic. Next, he uses the t-distribution with df=199 to compute the p-value of the observed data. The computed p-value will be
- A. correct because the sample size is large.
  - B. much smaller than the correct p-value.
  - C. much larger than the correct p-value.
  - D. very close to the correct p-value because the sample size is large.
  - E. may be greater or less than the correct value depending on the size of *sigma*.
- MC3 A psychologist is investigating the IQ level of young children who have been in a head start program. She wants to determine if the variation in IQ scores for the population of head start students is smaller than the variation in the general population of children under the age of 6 which has a variation of  $\sigma = 10.2$ . She also informs you that the distribution of IQ scores is highly right skewed. Suppose she uses the test: reject  $H_o$  is  $\frac{(n-1)S^2}{(10.2)^2} < \chi_{.95, n-1}^2$ , where  $S$  is the standard deviation from a random sample of  $n$  head start students, to test whether  $\sigma$  is less than 10.2 with an  $\alpha$  value of 0.05.
- A. the actual level of significance will be greater than 0.05.
  - B. the actual level of significance will be less than 0.05.
  - C. the actual level of significance will be very close to 0.05.
  - D. the actual level of significance will be exactly 0.05.
  - E. it is impossible to determine the effect of skewness on the actual level of significance.
- MC4 In testing  $H_o : \mu \geq \mu_o$  versus  $H_1 : \mu < \mu_o$ , where  $\mu$  is the mean of a population having a symmetric pdf, f,
- A. the power of the t-test is greater than the power of the Wilcoxon signed-rank test.
  - B. the power of the t-test is greater than the power of the sign test.
  - C. the power of the t-test is less than the power of the Wilcoxon signed-rank test.
  - D. the power of the t-test is less than the power of the sign test.
  - E. the power of the t-test depends on the shape of f.

The following discussion will supply the information for Questions MC5, MC6, and MC7.

A company that manufactures silicon wafers for computer chips is concerned with both the mean thickness of the chips and the fluctuation in the thickness of the chips. In order to monitor the thickness, a random sample of  $n$  chips is selected every hour and the thickness is measured on each of the chips. The process is considered to be in control provided the process mean,  $\mu$ , is 200 mm and the process standard deviation,  $\sigma$  is less than or equal to 2.5 mm. The company's process engineer develops a test to evaluate whether the process standard deviation is greater than 2.5 mm. She plots the power curve of the test in order to evaluate its performance. The curve is given here:



MC5 What is the maximum probability of a Type I error when using the test whose power curve is depicted above?

- A. .05
- B. .10
- C. .15
- D. .90
- E. .95

MC6 What is the probability of a Type II error of the test in MC5 if  $\sigma = 3.5$ ?

- A. .21
- B. .31
- C. .69
- D. .79
- E. cannot be determined from the power curve

- MC7 Refer to MC5. The process engineer decides to reduce the size of the test to  $\alpha = .01$  and use  $n=40$  chips in the study. What is the probability of a Type II error using an  $\alpha = .01$  test when  $\sigma = 2.775$ ?
- A. .99
  - B. .95
  - C. .90
  - D. .29
  - E. .71

- MC8 A biologist designs a study to determine if the average wing span of Mexican bats is less than the average wing span of South Carolina bats which is known to be 3.0 mm. She wants to determine how many Mexican bats to include in the study so that the probability of a Type II test at  $\mu = 2.8$  mm is at most 0.20 if she uses an  $\alpha=.05$  test. It is well known that the wing spans have a normal distribution. The biologist states she thinks that the wing span of Mexican bats have  $\sigma \approx .4$  mm. The sample size must be at least
- A. 16
  - B. 25
  - C. 35
  - D. 80
  - E. cannot be determined with the given information

- MC9 A random sample of 70 units was taken from a population yielding values  $Y_1, \dots, Y_{70}$ . From this data, the following confidence intervals for the population mean  $\mu$  were computed:  $\bar{Y} \pm t_{\alpha/2, 69} S / \sqrt{70}$

90% C.I. (22.5, 26.5);      95% C.I. (22.1, 26.9);      99% C.I. (21.4, 27.6)

In testing the hypotheses:  $H_o : \mu \leq 21$  versus  $H_1 : \mu > 21$ , the  $p - value$  computed from the data is

- A.  $p - value \leq .005$
  - B.  $.005 < p - value \leq .01$
  - C.  $.01 < p - value \leq .05$
  - D.  $p - value > .05$
  - E. can not be determined using confidence intervals
- MC10 You have been assigned to design an experiment to compare the mean responses from a placebo treatment and a new drug. You can either randomly assign  $n$  experimental units to each of the treatments or you can pair the  $2n$  experimental units based on severity of the disease and then randomly assign the two treatments within each pair of experimental units. Which of the following statements is **TRUE**?
- A. Pairing results in a large increase in the power of the paired t-test provided there is negative correlation within the pairs.
  - B. Pairing results in a large increase in the power of the paired t-test provided there is positive correlation within the pairs.
  - C. Pairing would reduce the power of the t-test if the  $2n$  experimental units have nearly the same level of severity of the disease.
  - D. Pairing is done to reduce the variance in the difference in the two sample means.
  - E. Pairing will always increase the power of the t-test, at least to some degree.

MC11 A biologist for a large pharmaceutical firm wants to determine the strength of a drug at various times after being injected into the patient. He injects the drug into 40 patient and then takes blood samples every 30 minutes for the next 10 hours yielding 20 strength values:  $X_{1i}, \dots, X_{20i}$  for each of the  $i = 1, \dots, 40$  patients. He uses the mean responses from the 40 patients,  $\bar{X}_1, \dots, \bar{X}_{40}$ , to test the hypotheses  $H_o : \mu \leq 53$  versus  $H_1 : \mu > 53$ , where  $\mu$  is the average strength of the drug during the 10 hours. An evaluation of the data reveals the following:

- The normal reference distribution plot has many data values above the line on the right end of the graph and many data values below the line on the left end of the graph.
- The Shapiro-Wilk test has p-value=0.003.
- The 20 data values have a lag one autocorrelation value  $\hat{\rho}_1 = 0.009$ .

A test was constructed using the following decision rule:  $\frac{\bar{\bar{X}} - 53}{S/\sqrt{40}} \geq t_{.05, 39}$ , where  $\bar{\bar{X}}$  and  $S$  are the mean and standard deviation of  $\bar{X}_1, \dots, \bar{X}_{40}$ . The true level of significance of this test would be

- A. 0.05.
- B. much less than 0.05.
- C. very close to 0.05.
- D. much greater than 0.05.
- E. may be greater or less than 0.05 depending on the value of  $\sigma$ .