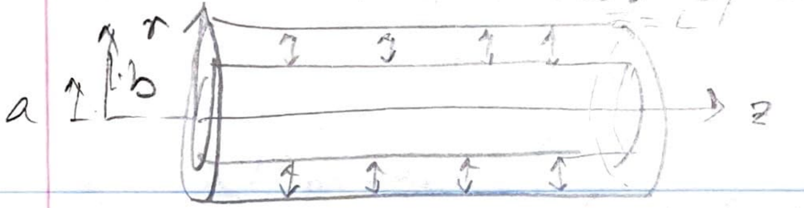
Sound between two cylinders



All boundaries are pressure-release.

General solution to Helmholtz equation is

$$p = \begin{Bmatrix} J_m(k_r r) \\ N_m(k_r r) \end{Bmatrix} \begin{Bmatrix} \cos m\theta \\ \sin m\theta \end{Bmatrix} \begin{Bmatrix} \cos k_z z \\ \sin k_z z \end{Bmatrix}$$

Axial: $p(z=0) = 0 \Rightarrow \cos k_z z$ is eliminated.

$p(z=L) = 0 \rightarrow \sin k_z L = 0 \Rightarrow k_z L = \underset{\uparrow \; 1,2,3\ldots}{l}\pi$

Radial: $A_m J_m(k_r a) + B_m N_m(k_r a) = 0.$

$A_m J_m(k_r b) + B_m N_m(k_r b) = 0.$

$$\begin{bmatrix} J_m(k_r a) & N_m(k_r a) \\ J_m(k_r b) & B_m(k_r b) \end{bmatrix} \begin{bmatrix} A_m \\ B_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For non-zero A_m and B_m to solve the above, the column space must have dimension d s.t. $d > 0$. By rank-nullity theorem, $d = n - r$. (n: # of col., r: rank)

Thus $n = 2 > r$, i.e., the matrix cannot be full rank. Thus matrix cannot be invertible: $\det A = 0$.

i.e., $J_m(k_r a) B_m(k_r b) - J_m(k_r b) N_m(k_r a) = 0.$

Transcendental eq. must be solved numerically.

$$p = \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \left[A_m J_m(k_r r) + B_m N_m(k_r r)\right] \times \sin\left(\frac{l\pi z}{L}\right) \cos(m\theta - \gamma_m).$$

If roots are $\beta_{mn}(b/a)$, then $k_r = \beta_{mn}(b/a)/a$.

and thus $f_{mnl} = \frac{c_0}{2\pi}\sqrt{\left(\frac{\beta_{mn}(b/a)}{a}\right)^2 + \left(\frac{l\pi}{L}\right)^2}.$