IntelliChoice SAT Math Camp

Graphing

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June 29th–July 3rd

The goal of this guided activity is to show you how to look at any function and translate it into a graph. This is a powerful skill and enforces a conceptual understanding of various families of functions. It takes practice but it is very worth your time, on the SAT and for your further studies. If you're doing it right, you should never have to draw a table of values.

$y = x^2$ is the parent function of the parabola. Here are some points that lie on that parabola:

x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4

Graph this parabola below, connecting the points:

Now consider $y = x^2 + 1$.

Claim: The $+1$ *will shift the graph of* $y = x^2$ *up one unit*

Fill out the table below

x	$y = x^2 + 1$
-2	
-1	
0	
1	
2	

Graph this below to verify that it is displaced 1 unit above $y = x^2$:

From this example you can see that when "+1" was written on the *opposite* side of the equation that y was on, it had the effect of shifting the graph up one unit.

Now rewrite $y = x^2 + 1$ as $y - 1 = x^2$. *Notice that these two are the same thing!!*
Fill out the table below. Do not solve the right-hand column for y! Instead ask yourself, "What value does y need to be to satisfy the equation $y - 1 = x^2$?"

x	$y - 1 = x^2$
-2	
-1	
0	
1	
2	

Graph this below to verify that it is displaced 1 unit above $y = x^2$:

From this example you can see that when "-1" was written on the *same* side of the equation that y was on, it had the effect of shifting the graph up one unit.

Here's another example. Consider $y = 2x^2$. Fill out the table below

x	$y = 2x^2$
-2	
-1	
0	
1	
2	

Graph this below to verify that this graph takes the parent function $y = x^2$ and stretches vertically.

From this example you can see that when "$*2$" was written on the *opposite* side of the equation that y was on, it had the effect of stretching the graph vertically by a factor of 2.

Now rewrite $y = 2x^2$ as $\frac{1}{2}y = x^2$. *Notice that these two are the same thing!!*
Fill out the table below. Do not solve the right-hand column for y!
Instead ask yourself, "What value does y need to be to satisfy the equation $\frac{1}{2}y = x^2$?"

x	$\frac{1}{2}y = x^2$
-2	
-1	
0	
1	
2	

Graph this below to verify that this graph takes the parent function $y = x^2$ and stretches vertically.

From this example you can see that when "$*\frac{1}{2}$" was written on the *same* side of the equation that y was on, it had the effect of stretching the graph vertically by a factor of 2.

Now rewrite $y = 2x^2$ as $y = (\sqrt{2}x)^2$. *Notice that these two are the same thing!!*

Think for yourself before continuing: does the $\sqrt{2}$ *compress* or *stretch* the graph horizontally?

Answer:
The same rules apply to both y and x. **The dilation factor "$\sqrt{2}$" was written on the *same* side of the equation that y was on. So it had the effect of compressing the graph horizontally.** Note that this is the same as stretching vertically.

If you like to memorize "rules," here they are:

Given some the graph of some function $y = f(x)$,

1. For $y = \alpha f(x)$, α stretches the function vertically if $|\alpha| > 1$ and compresses the function vertically if $|\alpha| < 1$

2. For $\beta y = f(x)$, β stretches the function vertically if $|\beta| < 1$ and compresses the function vertically if $|\beta| > 1$

3. For $y = f(\gamma x)$, γ stretches the function horizontally if if $|\gamma| < 1$ and compresses the function horizontally if $|\gamma| > 1$

4. For $y = f(x) + C$ (where $C > 0$), C shifts the function upward.

5. For $y - C = f(x)$ (where $C > 0$), C shifts the function upward.

6. For $y = f(x - C)$ (where $C > 0$), C shifts the function rightward.

Please don't memorize these rules. Study dilations of functions enough so that the rules make intuitive sense.

Graph each of the following. Label axes x and y. Provide the important points of the graphs, including any x- or y-intercepts. Avoid writing tables to figure out the shapes of the graphs. Instead, think of transformations of the parent function. *Only create a table if in doubt.*

1. $y = 2x$

2. $y < -2x - 1$

3. $y > \frac{1}{2}x + 4$

4. $y = \left|\frac{x}{2}\right|$

5. $y = 1 + \left|\frac{x}{2}\right|$

6. $y = 1 + \left|\frac{x-1}{2}\right|$

7. $y = \frac{1}{2}x^2$

8. $y = \left|x^2\right|$

9. $y = -\left|x^2\right|$

10. $y < 1 + \left|x^2\right|$

11. $y = 1 - \left|x^2\right|$

12. $y = (x - 1)^2$

13. $y = (x+1)^2$

14. $y = -(x-3)^2$

15. $y = 5 - (x+2)^2$

16. $y = \left|5 - (x+2)^2\right|$

17. $y = 10^x$

18. $y = 1 + 10^x$

19. $y = 10^{(x-3)}$

20. $y = e^x$

21. $y = 10^{|x|}$

22. $y = \log_{10} x$

23. $y = 2\log_{10} x$

24. $y = \log_{10} \frac{x}{2}$

25. $y = 1 + \log_{10} x$

26. $y = |\log_{10} x|$