

Prime Patterns and Relations

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For this paper, I examine the relation of prime numbers to the primes that occur after it. I seek to identify interesting patterns in relations. This paper is based after the research of Dr. Kannan Soundararajan of Stanford University. He identified a pattern in the final digit of the prime numbers that I find rather counter intuitive. It is well known that primes are very random. If you examine the distribution of final digits of prime numbers in base 10, you find that the 1's, 3's, 7's and 9's are distributed relatively evenly regardless of the scale. The intuitive assumption would be that primes are random and that there is no influence from one prime to the next. Interestingly, Dr Soundararajan found this to not be the case at all. Infact, the final digit of a prime number seems to directly influence the final digit of a prime number that occurs next. In this paper, I check the results he finds and attempt to delve deeper into this phenomenon in an attempt to find additional interesting patterns.

First we check the results that Dr. Soundararajan finds relating to the final digits of prime distributions. We can see from the following chart that Dr. Soundararajan's results for a depth of 2 in base 10 match my own. Unlike him, I used total percentages instead of percents based on each section of 1's, 3's, 7's and 9's.

Depth	Count	Percent	Pattern
2	446808	4.4681%	1 \rightarrow 1
2	756071	7.5607%	1 \rightarrow 3
2	769923	7.6992%	1 \rightarrow 7
2	526954	5.2695%	1 \rightarrow 9
2	593195	5.9320%	3 \rightarrow 1
2	422302	4.2230%	3 \rightarrow 3
2	714795	7.1480%	3 \rightarrow 7
2	769915	7.6992%	3 \rightarrow 9
2	639384	6.3938%	7 \rightarrow 1
2	681759	6.8176%	7 \rightarrow 3
2	422289	4.2229%	7 \rightarrow 7
2	756851	7.5685%	7 \rightarrow 9
2	820368	8.2037%	9 \rightarrow 1 MAX
2	640076	6.4008%	9 \rightarrow 3
2	593275	5.9328%	9 \rightarrow 7
2	446032	4.4603%	9 \rightarrow 9

What I find interesting about this is that the final digits of the primes in base 10 are very evenly distributed. The intuitive idea would be that each prime would be independent of any of the others when you get to large numbers. This finding by Dr. Soundararajan disproves this and shows a very counter intuitive notion that each prime influences the prime that comes after. The final digit of the primes tend to oscillate. This pattern is interesting

and made me wonder if there are possibly patterns of final digits that occur with a larger frequency than others.

Next we look at further patterns to see if there are some patterns the the primes that will always be more common than others. The code provided can analyze the first 2 billion primes up to a specified depth and provide the most common final digit patterns that occur. When analyzing the first 10 million primes in base 10, the max patterns at different depths are:

Depth	Number	Percent	Pattern
1	2500283	25.0028%	7
2	820368	8.2037%	9 → 1
3	256932	2.5693%	9 → 1 → 7
4	81324	0.8132%	3 → 9 → 1 → 7
5	24413	0.2441%	1 → 3 → 9 → 1 → 7
6	7872	0.0787%	9 → 1 → 3 → 9 → 1 → 7
7	2521	0.0252%	3 → 9 → 1 → 3 → 9 → 1 → 7
8	758	0.0076%	7 → 3 → 9 → 1 → 3 → 9 → 1 → 7

The 10 most common patterns at depth 5 over a million primes are

First 10 million

Depth	Number	Percent	Pattern
5	24413	0.2441%	1 → 3 → 9 → 1 → 7
5	24270	0.2427%	9 → 1 → 3 → 9 → 1
5	24183	0.2418%	9 → 1 → 7 → 9 → 1
5	24167	0.2417%	3 → 9 → 1 → 7 → 9
5	23069	0.2307%	3 → 9 → 1 → 3 → 9
5	22961	0.2296%	7 → 9 → 1 → 3 → 9
5	22904	0.2290%	1 → 7 → 9 → 1 → 3
5	22866	0.2287%	1 → 7 → 3 → 9 → 1
5	22821	0.2282%	9 → 1 → 7 → 3 → 9
5	22699	0.2270%	1 → 7 → 9 → 1 → 7

First 100 million

Depth	Number	Percent	Pattern
5	220752	0.2208%	9 → 1 → 3 → 9 → 1
5	220393	0.2204%	1 → 3 → 9 → 1 → 7
5	220347	0.2203%	9 → 1 → 7 → 9 → 1
5	218761	0.2188%	3 → 9 → 1 → 7 → 9
5	210841	0.2108%	1 → 7 → 9 → 1 → 3
5	210425	0.2104%	7 → 9 → 1 → 3 → 9
5	209231	0.2092%	1 → 7 → 3 → 9 → 1
5	208463	0.2085%	3 → 9 → 1 → 3 → 9
5	208337	0.2083%	9 → 1 → 7 → 3 → 9
5	207712	0.2077%	1 → 7 → 9 → 1 → 7

It is interesting to note that the most common patterns change slightly when analyzing more primes. However the changes are minimal and the same general patterns are still very common regardless of the number of primes analyzed.

1 Billion primes with depth 7

Depth	Number	Percent	Pattern
7	187110	0.0187%	$7 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1 \rightarrow 7$
7	185850	0.0186%	$3 \rightarrow 9 \rightarrow 1 \rightarrow 7 \rightarrow 9 \rightarrow 1 \rightarrow 3$
7	184150	0.0184%	$3 \rightarrow 9 \rightarrow 1 \rightarrow 7 \rightarrow 9 \rightarrow 1 \rightarrow 7$
7	184041	0.0184%	$3 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1 \rightarrow 7$
7	182959	0.0183%	$7 \rightarrow 9 \rightarrow 1 \rightarrow 7 \rightarrow 9 \rightarrow 1 \rightarrow 3$
7	182869	0.0183%	$7 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1 \rightarrow 3$
7	178559	0.0179%	$9 \rightarrow 1 \rightarrow 7 \rightarrow 3 \rightarrow 9 \rightarrow 1 \rightarrow 7$
7	178017	0.0178%	$3 \rightarrow 9 \rightarrow 1 \rightarrow 7 \rightarrow 3 \rightarrow 9 \rightarrow 1$
7	177663	0.0178%	$3 \rightarrow 7 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1$
7	177647	0.0178%	$1 \rightarrow 7 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1$

These results are interesting because it shows that there are infact some patterns that occur with much more regularity than others. The most common patterns are all very similar. They all oscillate as much as possilbe at every digit. What interests me is why some pat-terns that appear almost identical show up with a larger frequency.

Here is a list of the most common patterns at each depth up to 9 when analyzing the first 500 million primes.

Depth	Count	Percent	Pattern
1	125002060	25.0004%	7
2	39369475	7.8739%	$9 \rightarrow 1$
3	11785061	2.3570%	$3 \rightarrow 9 \rightarrow 1$
4	3562197	0.7124%	$3 \rightarrow 9 \rightarrow 1 \rightarrow 7$
5	1045672	0.2091%	$9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1$
6	321442	0.0643%	$9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1 \rightarrow 7$
7	96653	0.0193%	$7 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1 \rightarrow 7$
8	28313	0.0057%	$9 \rightarrow 1 \rightarrow 7 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1$
9	8649	0.0017%	$3 \rightarrow 9 \rightarrow 1 \rightarrow 7 \rightarrow 9 \rightarrow 1 \rightarrow 3 \rightarrow 9 \rightarrow 1$

I find the difference between depth 7 and 8 interesting. It looks like most of the patterns at lower depths are contained in the patterns at higher depths. However, this pattern breaks for depth 7 and 8. This makes me wonder if the pattern shown at depth 7 would be contained in a larger depth than 9. I also find it very interesting that every single pattern ends with either a 1 or a 7. There are no largest patterns that end in a 3 or a 9. This might be a coincidence or it might be a real pattern. I'm not sure why this is and it may change at larger depts. Unfortunately, analyzing larger depths takes an exponentially larger amount of time (especially with unoptimized code) so it would take a large amount of time to compute more that a depth of 9 for the first 500 million primes. Seeing these patterns made me wonder about other bases. Sadly, I realized too late that the main datastructure I was using to store information was taking advantage of primes in base 10 to perform optimizations. To change

this so that it would work for any base would require a rewrite of the main datastructure and consequently a rewrite of nearly all of the code. This is not possible with current time constraints.

The code used in this paper will be submitted with the project and can also be found at:

<https://github.com/cag51574/PrimePatterns>

Notice: you must have java 8 installed on your system to run this project as well as having downloaded and extracted the first 2 billion primes into a directory called primes. The first 2 billion primes can be downloaded from:

http://www.primos.mat.br/2T_en.html

Please follow the instructions in the README to run the code. There is a built in user interface that will allow you to perform your own analysis for any depth for the first two billion primes. Larger depths will take an exponentially larger amount of time, so I recommend staying below a depth of 10.