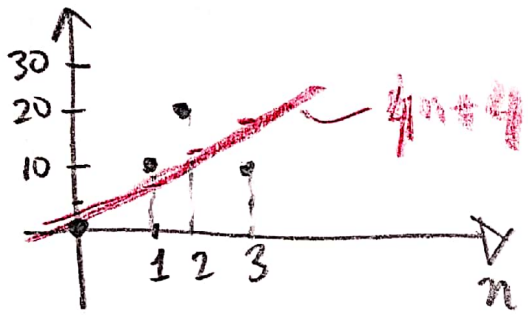
Quiz #1

The sequence $x[n]$ denotes the measurements of an experiment:

$$x[0] = 0$$
$$x[1] = 10$$
$$x[2] = 20$$
$$x[3] = 10$$

We would like to fit a line $\hat{x}[n] = an + b$ to this data.

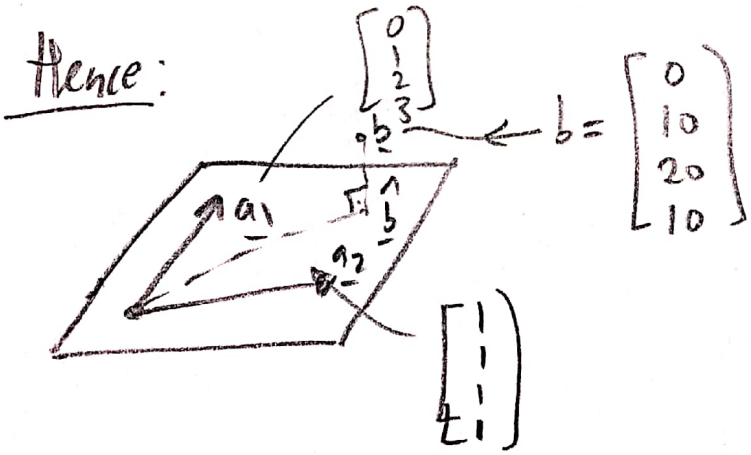
a) Determine the parameters a and b that minimizes the least squares error.
b) Calculate the error vector and verify the orthogonality condition for the optimal error vector.



$\hat{x}[n] = an + b, \quad n = \{0,1,2,3\}$

$$\begin{bmatrix} \hat{x}[0] \\ \hat{x}[1] \\ \hat{x}[2] \\ \hat{x}[3] \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}}_{a_1} a + \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{a_2} b$$

Hence:



Then.

$$\underline{A}\,\underline{x} = \underline{b} \rightarrow \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} a_{LS} \\ b_{LS} \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 0 \\ 10 \\ 20 \\ 10 \end{bmatrix}}_{\underline{b}} \rightarrow$$

$$\underline{x}_{LS} = (\underline{\underline{A}}^T \underline{\underline{A}})^{-1} \underline{\underline{A}}^T \underline{b}$$

$$= \left(\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} .10$$

$$\left(\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \right)^{-1} \qquad \begin{bmatrix} 8 \\ 4 \end{bmatrix} .10$$

$$= \frac{1}{\Delta} \begin{bmatrix} 4 & -6 \\ -6 & 14 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} 40 \; 2$$

$\Delta = 56 - 36 = 20$

$$\begin{bmatrix} a_{LS} \\ b_{LS} \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \longrightarrow \hat{b} = \underline{a}_1 4 + \underline{a}_2 4 = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 16 \end{bmatrix}$$

b) $\underline{e} = \underline{b} - \hat{\underline{b}} = \begin{bmatrix} -4 \\ 2 \\ +8 \\ -6 \end{bmatrix}$

① $\underline{e}^T \underline{a}_1 \overset{?}{=} 0$ ✓

$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$

② $\underline{e}^T \underline{a}_2 \overset{?}{=} 0$ ✓

$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$