EE 202: Circuit Theory II

2007-2008 Spring Semester
Midterm I
March 27, 2008

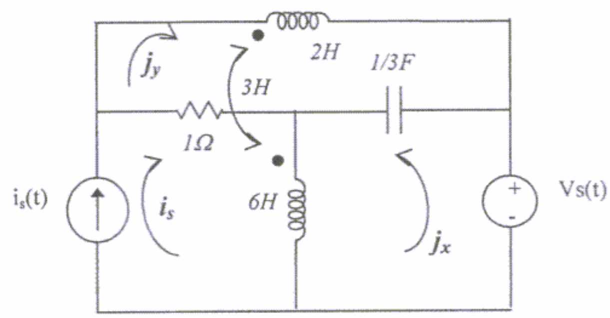
Q1) (25pts)

For the following circuit;

a) Write the mesh equations (in time domain) using the shown mesh currents. Express equations in the matrix form.

b) Find the natural frequencies.

(Assume that all initial conditions are zero.)

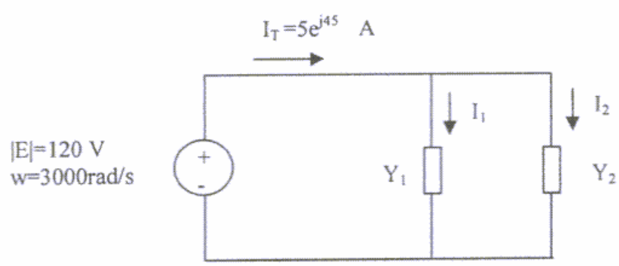


Q2) (15pts)

In the circuit given below, Y_1 and Y_2 blocks are a single circuit component either R or L or C. The branch current I_1 is in phase with E and its magnitude, $|I_1|$ is 3 A.

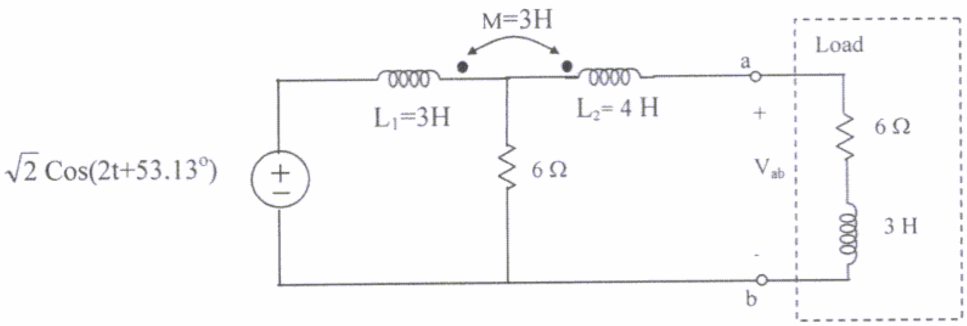
The source voltage E has magnitude, $|E|$, equal to 120 V. The phase of voltage source is unknown, but it is known that E leads the current $I_T=5e^{j45}$ A.

Find the value of each element and the equivalent simple series circuit at the given frequency.

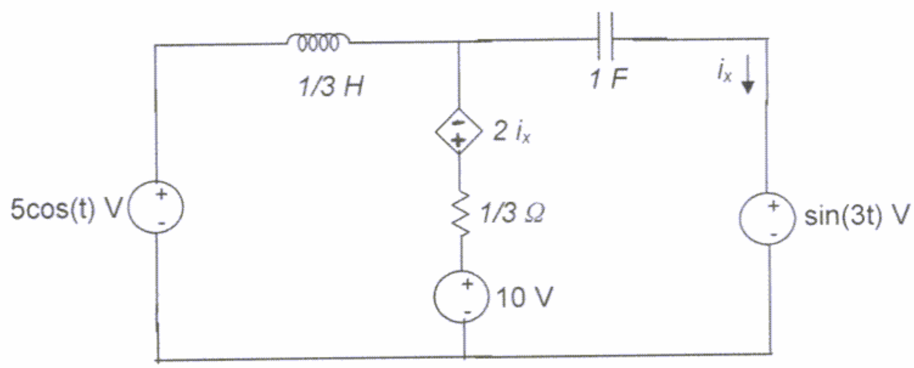


Q3) **(20pts)** For the following circuit,

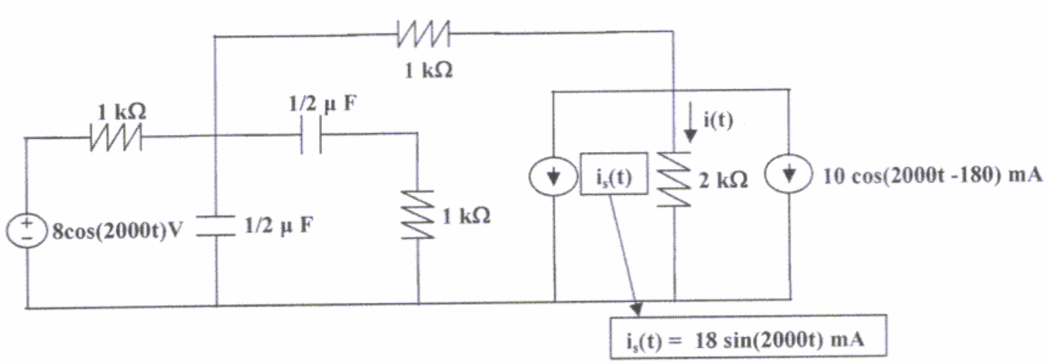
a) Find Thevenin equivalent circuit with respect to the a-b terminals.
b) Find the average power delivered to the load the load.



Q4) **(20pts)** Find $i_x(t)$ at steady-state.



Q5) **(20 pts)** Find $i(t)$ at steady-state.



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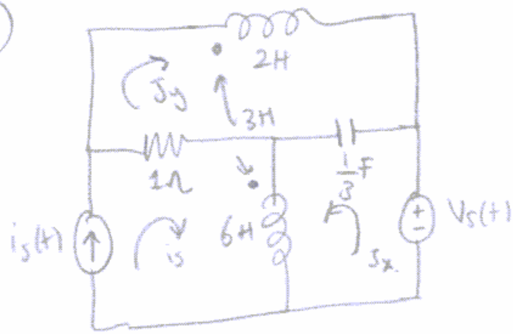
a) 15pts



Mesh J_y: $2DJ_y + 3D(J_x + i_s) + 3D^{-1}(J_x + J_y) + (J_y - i_s) = 0$

Mesh J_x: $-v_s(t) + 3D^{-1}(J_x + J_y) + 6D(J_x + i_s) + 3DJ_y = 0$.

$$\begin{bmatrix} 3D + 3D^{-1} & 2D + 1 + 3D^{-1} \\ 3D^{-1} + 6D & 3D^{-1} + 3D \end{bmatrix} \begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} i_s(t) - 3D\, i_s(t) \\ v_s(t) - 6D\, i_s(t) \end{bmatrix}$$

b) Assume 10pts.

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} e^{\lambda t} \longrightarrow \underbrace{\begin{bmatrix} 3\lambda + \frac{3}{\lambda} & 2\lambda + 1 + \frac{3}{\lambda} \\ \frac{3}{\lambda} + 6\lambda & \frac{3}{\lambda} + 3\lambda \end{bmatrix}}_{\underline{A}} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$|\underline{A}| = \left| \begin{bmatrix} 3\lambda + \frac{3}{\lambda} & 2\lambda + 1 + \frac{3}{\lambda} \\ 3\lambda & \lambda - 1 \end{bmatrix} \right| = \left| \begin{bmatrix} \lambda - 1 & 2\lambda + 1 + \frac{3}{\lambda} \\ 2\lambda + 1 & \lambda - 1 \end{bmatrix} \right|$$

$$= (\lambda - 1)^2 - (2\lambda + 1)^2 - \frac{3(2\lambda + 1)}{\lambda}$$

$$= 3\lambda(-\lambda - 2) - \frac{6\lambda + 3}{\lambda}$$

$$= -3\lambda^3 - 6\lambda^2 - 6\lambda - 3 \longrightarrow |A| = 0$$

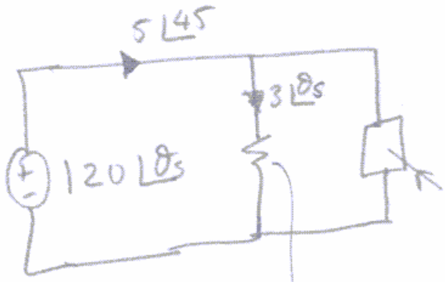
$$\Downarrow$$

$$\lambda^3 + 2\lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda + 1)(\lambda^2 + \lambda + 1) = 0$$

$$\lambda = \left\{-1, \frac{-1 \mp \sqrt{3}\,j}{2}\right\}$$

Problem 2 :



Y_2 : inductor since E leads the current. ($\theta_s > 45°$).

R: since current is in phase with source

$R = 40\Omega$.

$$|I_T| = \frac{120}{|Z_{Tot}|} = 5 \longrightarrow |Z_{Tot}| = 24 \longrightarrow |Y_{Tot}| = \frac{1}{24}$$

$R \,//\, j\omega L$

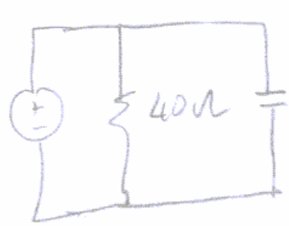
40Ω $\quad$ 3000 $\quad$?

$$Y_{Tot} = \frac{1}{40} + \frac{-j}{\omega L}$$

$$|Y_{Tot}| = \sqrt{\left(\frac{1}{40}\right)^2 + \left(\frac{1}{\omega L}\right)^2} = \frac{1}{24}$$

$$\left(\frac{1}{\omega L}\right)^2 = \left(\frac{1}{24}\right)^2 - \left(\frac{1}{40}\right)^2 = \left(\frac{1}{8}\right)^2\left(\frac{1}{9} - \frac{1}{25}\right) = \frac{16}{64 \cdot 9 \cdot 25}$$

$$\omega L = 30 \longleftarrow \frac{1}{\omega L} = \frac{4}{8 \cdot 3 \cdot 5}$$



$$\longrightarrow \quad L = \frac{30}{3000} = 10\,\text{mH}$$

$$40 \,\|\, -j30 = 10\,(4 \,\|\, +j3) = 10 \cdot \frac{+12j}{5\angle +37°} = 24\angle 53°$$

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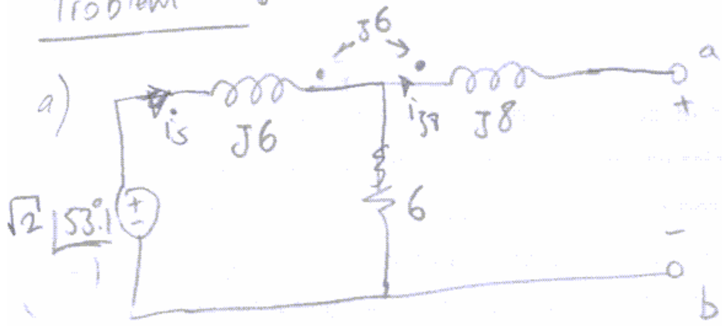
Problem 3:

a)

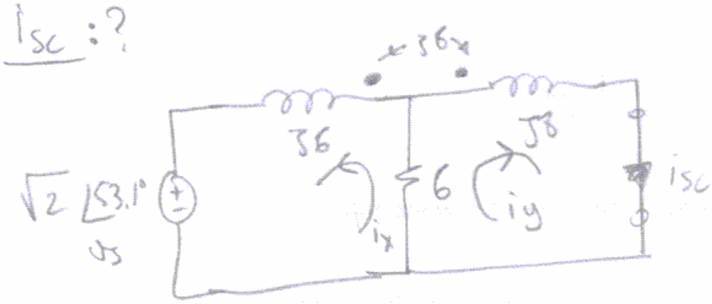


$\omega = 2$

V_{oc} : ? $\quad i_{j8} = 0 \rightarrow \quad i_s = \dfrac{\sqrt{2}\angle 53.1°}{j6+6} = \dfrac{\sqrt{2}\angle 53.1}{6\sqrt{2}\angle 45°} = \dfrac{1}{6}\angle 8.1°$

$$V_{oc} = \overbrace{6\, i_s}^{V_{6\Omega}} - \overbrace{(-j6\, i_s)}^{V_{j8}} = (6+j6)\, i_s = \sqrt{2}\angle 53.1°$$

i_{sc} : ?



Mesh i_x : $\quad 6(i_x + i_y) + j6 i_x + j6(i_y) = -v_s \qquad (1)$

Mesh i_y : $\quad 6(i_x + i_y) + j8 i_y + j6(i_x) = 0 \qquad (2)$

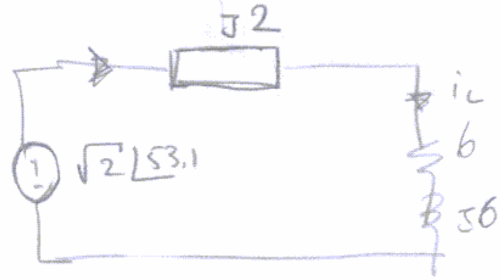
$(1)-(2) \longrightarrow \quad -j2 i_y = -v_s \rightarrow i_y = \dfrac{-\sqrt{2}\angle 53.1°}{-j2}$

$i_{sc} = i_y \quad \longleftarrow \quad i_y = \dfrac{1}{\sqrt{2}}\angle -37°$

$$Z_{Th} = \frac{\sqrt{2}\angle 53.1}{\frac{1}{\sqrt{2}}\angle -37°} = 2j$$

Problem 3,

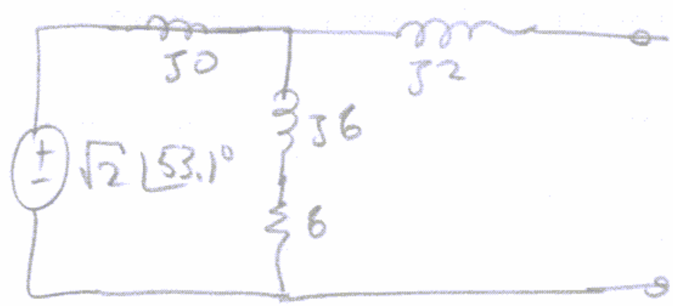
Part b)



$$i_L = \frac{\sqrt{2}\angle 53.1}{6+j8} = \frac{\sqrt{2}\angle 53.1}{10\angle 53.1} = \frac{\sqrt{2}}{10}$$

$$P^{Load}_{delivered} = \frac{1}{2}|i_L|^2 \cdot 6 = \frac{6}{100} \text{ watts.}$$

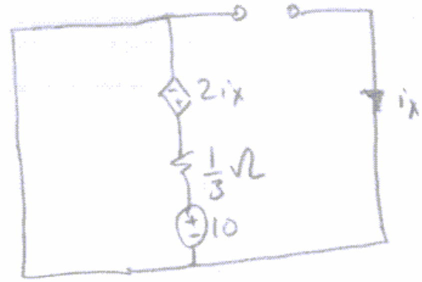
Easier way: Insert T-model for mutual inductor.



$$\left.\begin{array}{l} V_{oc} = \sqrt{2}\angle 53.1° \\ Z_{Th} = j2 \end{array}\right\} \text{ by inspection.}$$

Problem 4:

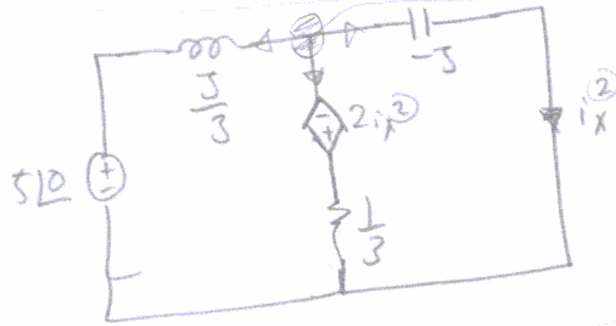
① DC: ON, $\omega=1$: OFF, $\omega=3$: OFF $\longrightarrow$



$\omega = 0$

$i_x^{(1)} = 0$

② DC: OFF, $\omega=1$: ON, $\omega=3$: OFF $\longrightarrow$



$e = -J i_x^{(2)}$

$$\frac{-J i_x^{(2)} + 2 i_x^{(2)}}{1/3} + i_x^{(2)} + \frac{-J i_x^{(2)} - 5}{J/3} = 0$$

$$i_x^{(2)}\left(-J + 2 + \frac{1}{3} + 1\right) = -5J$$

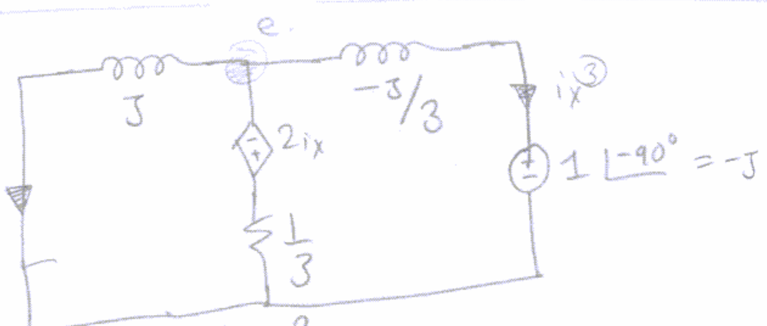
$$i_x^{(2)}\left(\frac{4 - 3J}{3}\right) = -5J$$

$$i_x^{(2)} = -3J \angle \tan^{-1} 3/4 \rightarrow 37^\circ$$

$$i_x^{(2)} = 3 \angle -90 + 37^\circ = 3 \angle -53^\circ$$

$$i_x^{(2)}(t) = 3\cos(t - 53)\ V$$

③ DC: OFF, $\omega=1$: OFF, $\omega=3$: ON $\longrightarrow$



$\omega = 3$

$$\frac{e}{J} + \frac{e + 2 i_x^{(3)}}{1/3} + \frac{e - (-J)}{-J/3} = 0 \longrightarrow e(-J + 3 + 3J) = +3 - 6 i_x^{(3)}$$

$$-J + \frac{-J}{3} i_x^{(3)}$$

$$\left(-j - \frac{j}{3} i_x^{(3)}\right)(2j+3) = 3 - 6\, i_x^{(3)}$$

$$-j\,(3 + i_x)(2j+3) = 9 - 18\, i_x^{(3)}$$

$$i_x^{(3)}(20 - 3j) = 3 + 9j$$

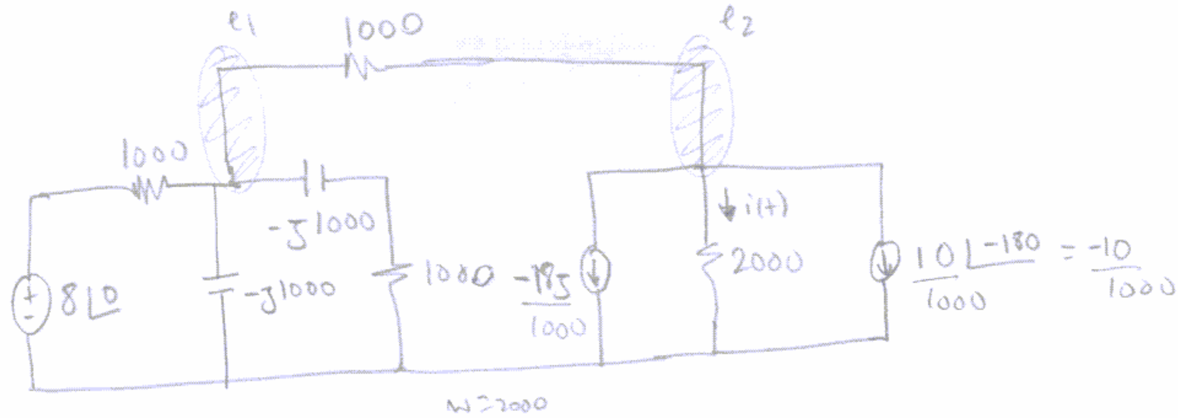
$$i_x^{(3)} = \frac{3+9j}{20-3j} = \frac{3\sqrt{10}\,\angle \tan^{-1} 3}{\sqrt{409}\,\angle -\tan^{-1} 3/20}$$

$$i_x^{(3)} = 3\sqrt{\frac{10}{409}}\cos(3t + \tan^{-1} 3 + \tan^{-1} 3/20)\ v.$$

$$i_x(t) = i_x^{(1)}(t) + i_x^{(2)}(t) + i_x^{(3)}(t)$$

$$= 3\cos(t - 53^\circ) + 3\sqrt{\frac{10}{409}}\cos\left(3t + \tan^{-1} 3 + \tan^{-1}\frac{3}{20}\right)\ v.$$

Problem 5:



$$\frac{e_1 - 8}{1000} + \frac{e_1}{1000 - j1000} + \frac{e_1 - e_2}{1000} + \frac{e_1}{-j1000} = 0 \qquad e_1\left(1 + \tfrac{1}{2}(1+j) + 1 + j\right) - e_2 = 8$$

$$\frac{e_2}{2000} + \frac{e_2 - e_1}{1000} = \frac{18j + 10}{1000} \longrightarrow e_1(-2) + e_2(1+2) = 36j + 20$$

$$\begin{bmatrix} 2.5 + 1.5j & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 20 + 36j \end{bmatrix}$$

$$e_2 = \frac{\left|\begin{bmatrix} 2.5 + 1.5j & 8 \\ -2 & 20 + 36j \end{bmatrix}\right|}{\left|\begin{bmatrix} +2.5 + 1.5j & -1 \\ -2 & 3 \end{bmatrix}\right|} = \frac{4(5 + 9j)\frac{1}{2}(5 + 3j) + 16}{7.5 + j4.5 - 2} = \frac{12 + j120}{5.5 + j4.5}$$

$$= \frac{12(1 + j10)}{\frac{1}{2}(11 + j9)} \longrightarrow \frac{24(101 + j101)}{202} = 12(1 + j)$$

$$\boxed{i(t) = 6\sqrt{2}\cos(2000t + 45^\circ)\ A}$$