Problem 1:

ⓐ Zero-Input Analysis

ⓑ (I): State, (II): Scalar Diff., (III): Mesh, (IV): Node or Modified Node Analysis

ⓒ The natural frequencies of all solution should be the same.

$$\dot{x} = \begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix} x$$

Char Poly: $\boxed{\lambda^2 + 6\lambda + 5 = 0}$

$$(3D^2 + 18D + 15)\, i_1 = 0$$

Char Poly: $3\lambda^2 + 18\lambda + 15 = 0$

$\boxed{\lambda^2 + 6\lambda + 5 = 0}$

$$\begin{bmatrix} D+2 & 1 \\ D-5 & D+5 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \underline{0}$$

Char Poly:

$(\lambda+2)(\lambda+5) - (\lambda-5) = 0$

$\boxed{\lambda^2 + 6\lambda + 15 = 0}$

$$\underbrace{\begin{bmatrix} D & 5 & -\frac{1}{4}D \\ -5 & 2D & D \\ 6 & 0 & 1 \end{bmatrix}}_{A(D)} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\det(A(D)) = 2D^2 + 30D + 3D^2 + 25 = 0$

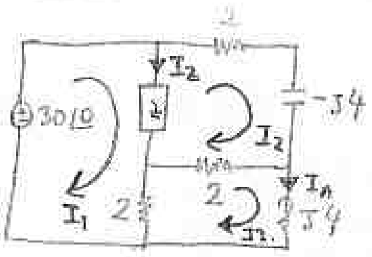
Char Poly: $5\lambda^2 + 30\lambda + 25 = 0$

$\boxed{\lambda^2 + 6\lambda + 5 = 0}$

The mesh analysis has the error. If the I_2 coefficient, in the 2nd equation is changed to $(D+5)$, Char. Poly becomes:

$$(\lambda+2)(\lambda+5) - (\lambda+5) = \lambda^2 + 6\lambda + 5 = 0.$$

Problem 2: (Part a)



Solution by Mesh Analysis:

$I_3 = 10\angle 0$

$\circlearrowleft_{I_1}: Z(I_1 - I_2) + 2(I_1 - 10) = 30$

$\circlearrowleft_{I_2}: I_2(2 - j4) + 2(I_2 - 10) + Z(I_2 - I_1) = 0$

$\circlearrowleft_{I_3}: j40 + 2(10 - I_1) + 2(10 - I_2) = 0$

$\circlearrowleft_{I_1} + \circlearrowleft_{I_2}: 2(I_1 - 10) + 2(I_2 - 10) + I_2(2 - j4) = 30$

Solving $\circlearrowleft_{I_3}$ and last equation:

$$\begin{bmatrix} 2 & 4-j4 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 70 \\ 40+j40 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 9+j18 \\ 11+j2 \end{bmatrix}$$

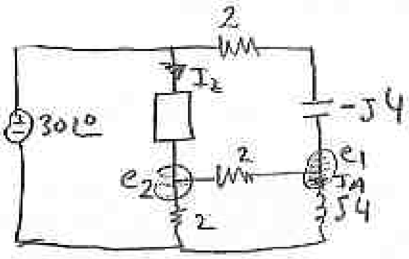
From $\circlearrowleft_{I_1}$ loop:

$Z(I_1 - I_2) + 2(I_1 - 10) = 30$

$Z(-2 + j16) + 2(-1 + j18) = 30$

$Z = \frac{32 - j36}{-2 + j16}$

$Z = -2.46 - j1.69$



Solution by Node Analysis:

KCL at ①: $\frac{e_1}{j4} + \frac{e_1 - 30}{2 - j4} + \frac{e_1 - e_2}{2} = 0$

KCL at ②: $\frac{e_2}{2} + \frac{e_2 - e_1}{2} + \frac{e_2 - 30}{Z} = 0$

$$\boxed{e_1 = I_A \cdot j4 = j40}$$

Substitute e_1 in 1st KCL eq:

$$10 + \underbrace{\frac{j40 - 30}{2 - j4}} + \frac{j40 - e_2}{2} = 0$$

$$\frac{\overset{j20\ -15}{\cancel{j40} - \cancel{30}}}{\underset{1-j2}{\cancel{2-j4}}} = \frac{(j\overset{4}{\cancel{20}} - \overset{3}{\cancel{15}})(1+j2)}{\cancel{5}} = -11-2j$$

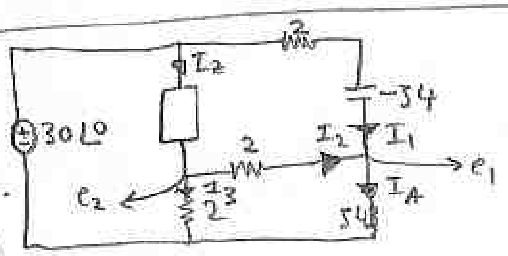
$$10 + (-11 - 2j) + j20 = \frac{e_2}{2}.$$

$$\boxed{e_2 = 2(-1 + j18)}$$

From 2nd KCL eq:

$$(-1 + j18) + (-1 + j18 - j20) + \frac{(-32 + j36)}{Z} = 0$$

$$Z = \frac{32 - j36}{-2 + j16}$$



Solution by Substitution:

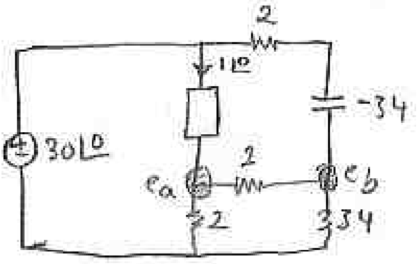
$I_A = 10\angle 0 \longrightarrow e_1 = I_A \cdot j4 = j40 \longrightarrow I_1 = \frac{30 - e_1}{2 - j4} = \frac{30 - 40j}{2 - j4} = 11 + j2 \longrightarrow I_2 = I_A - I_1 \longrightarrow$

$= -1 - j2$

$\longrightarrow e_2 = 2I_2 + e_1 = -2 + 36j \longrightarrow I_3 = \frac{e_2}{2} = -1 + j18 \longrightarrow I_Z = I_3 + I_2 = -2 + j16 \longrightarrow$

$\longrightarrow Z = \frac{30 - e_2}{I_Z} = \frac{32 - 36j}{-2 + 16j} = -2.46 - 1.7j$

Problem 2: (Part b)



Solution by Node Analysis:

KCL at

e_a: $\frac{e_a}{2} + \frac{e_a - e_b}{2} = 1\angle 0$

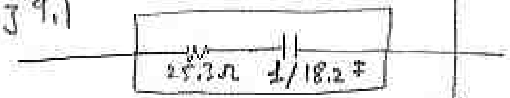
e_b: $\frac{e_b}{j4} + \frac{e_b - 30}{2-j4} + \frac{e_b - e_a}{2} = 0$

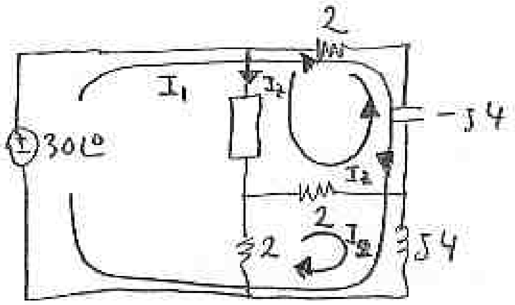
$$\begin{bmatrix} 1 & -1/2 \\ -1/2 & \underbrace{\frac{-j}{4}+\frac{1}{2}+\frac{\overset{1+j2}{2+j4}}{\underset{10}{20}}}_{0.6-j0.05} \end{bmatrix} \begin{bmatrix} e_a \\ e_b \end{bmatrix} = \begin{bmatrix} 1 \\ 3+j6 \end{bmatrix}$$

$$\begin{bmatrix} e_a \\ e_b \end{bmatrix} = \begin{bmatrix} 4.7+j9.1 \\ 7.4+j18.2 \end{bmatrix}$$

$V_Z = 30 - e_a = (30-4.7) - j9.1 = 25.3 - j9.1$

$\frac{V_Z}{1\angle 0} = Z = 25.3 - j9.1$





Solution by Mesh Analysis:

KVL for

I_1 loop ⇒ $2(I_1-1) - j4(I_1-1) + j4(I_1+I_2) = 30$

I_2 loop ⇒ $2(I_2+1) + j4(I_2+I_1) + 2I_2 = 0$

$$\begin{bmatrix} 2 & j4 \\ j4 & 4+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 32-j4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6.9+j2.7 \\ -2.35-j4.55 \end{bmatrix}$$

I_2 loop:

$(2-j4)(1-I_1) + Z. + 2(1+I_2) = 0$

$Z = (2-j4)(I_1-1) - 2(I_2+1)$

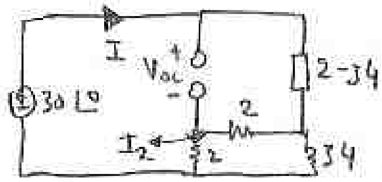
$= (2-j4)(5.9+j2.7) - 2(-1.35 - j4.55)$

$= (11.8+10.8+2.7) + j(-23.6+5.4+9.1)$

$Z = 25.3 + j(-9.1)$

Solution by Thevenin Equivalent:

V_{oc}:

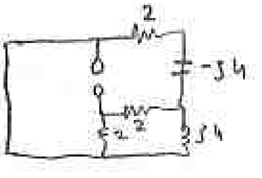


$I = \frac{30\angle 0}{(2-j4)+(4\|j4)} = \frac{30}{2-j4+\frac{16j}{\underbrace{4+4j}_{2+2j}}} = \frac{30}{4-2j} = 3(2+j)$

$I_2 = I \cdot \frac{j4}{4+j4} = \frac{3(2+j)j\cancel{4}}{\underset{1+j}{\cancel{4+j4}}} = \frac{3}{2}(2+j)(1+j) = \frac{3}{2}(1+3j)$

$V_{oc} = 30 - 2 \cdot I_2 = 30 - 3(1+3j) = 27 - 9j.$

Z_{Th}:



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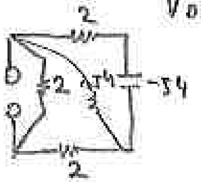
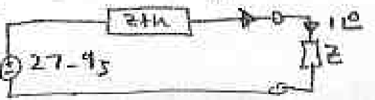


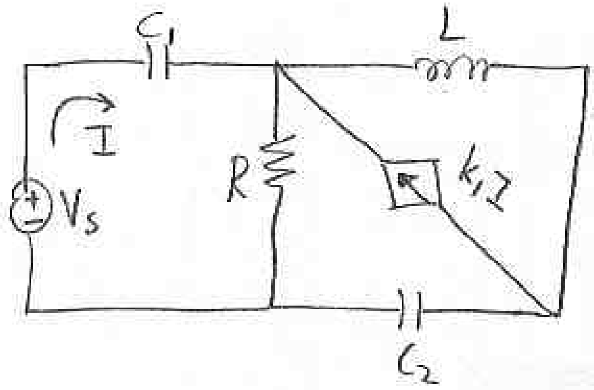
→ $Z_{th} = 2 \| [2 + [j4 \| 2-j4]] = 1.7 + 0.1j$

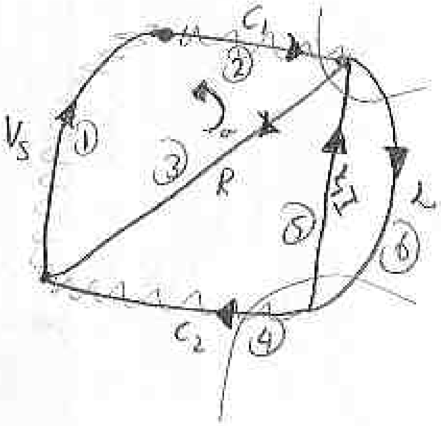


$27 - 9j = (Z_{th} + Z)1\angle 0$

$Z = V_{oc} - Z_{th}$

③





State Var's: $[V_{c_1}\ V_{c_2}\ I_L]$.

Fun. Cut. Set for C_1: $-C_1\dot{V}_{c_1} + V_3/R - k_1 I + I_L = 0$

$I \hookrightarrow C_1\dot{V}_{c_1}$

$V_3 = -V_{c_1} + V_s.$

$$\boxed{C_1(1+k_1)\dot{V}_{c_1} = -\frac{V_{c1}}{R} + I_L + \frac{V_s}{R}}$$

Fun. Cut Set for C_2: $C_2\dot{V}_{c_2} + k_1 I - I_L = 0$

$I \hookrightarrow C_1\dot{V}_{c_1}$

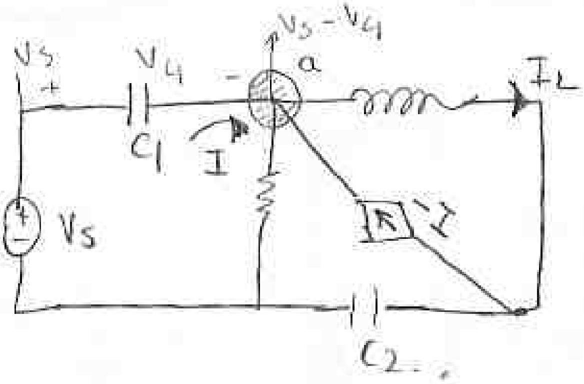
$$C_2\dot{V}_{c_2} + \frac{k_1}{1+k_1}\left[-\frac{V_{c1}}{R} + I_L + \frac{V_s}{R}\right] - I_L = 0$$

$$\boxed{C_2\dot{V}_{c_2} = \frac{k_1}{(1+k_1)R}V_{c_1} + \frac{1}{k_1+1}I_L - \frac{k_1}{(1+k_1)R}V_s}$$

Fun. loop for α: $\boxed{\alpha \dot{I}_L = -V_{c_1} + V_s - V_{c_2}}$

$$\begin{bmatrix}\dot{V}_{c_1}\\ \dot{V}_{c_2}\\ \dot{I}_L\end{bmatrix} = \begin{bmatrix}\frac{-1}{RC_1(1+k_1)} & 0 & \frac{1}{C_1(1+k_1)}\\ \frac{+k_1}{RC_2(1+k_1)} & 0 & \frac{1}{C_2(1+k_1)}\\ -1/L & -1/L & 0\end{bmatrix}\begin{bmatrix}V_{c_1}\\ V_{c_2}\\ I_L\end{bmatrix} + \begin{bmatrix}\frac{V_s}{RC_1(1+k_1)}\\ \frac{-V_s k_1}{RC_2(1+k_1)}\\ \frac{V_s}{L}\end{bmatrix}$$

3b)



1st State Eqn: $-I_L = \frac{V_{C_1} - V_S}{R}$ $(L = -1)$

2nd State Eqn: $C_2 \dot{V}_{C_2} - C_1 \dot{V}_{C_1} = I_L$

3rd State Eqn: $\boxed{L \dot{I}_L = -V_{C_1} + V_S - V_{C_2}}$

Insert I_L from 1st Eqn into 3rd.

$$\left(\frac{\dot{V}_{C_1} - \dot{V}_S}{R}\right) = -\frac{V_{C_1}}{L} + \frac{V_S}{L} - \frac{V_{C_2}}{L}.$$

$$\boxed{\dot{V}_{C_1} = -\frac{R}{L} V_{C_1} - \frac{R}{L} V_{C_2} + \frac{R}{L} V_S + \dot{V}_S}$$

Insert $\dot{V}_{C_1}$ into 2nd eq:

$$\dot{V}_{C_2} = \frac{I_L}{C_2} - \frac{RC_1}{LC_2} V_{C_1} - \frac{RC_1}{LC_2} V_{C_2} + \frac{RC_1}{LC_2} V_S +$$

$$\boxed{\begin{bmatrix} \dot{V}_{C_1} \\ \dot{V}_{C_2} \\ \dot{I}_L \end{bmatrix}} = \begin{bmatrix} -R/L & -R/L & 0 \\ -\frac{R}{L}\frac{C_1}{C_2} & -\frac{R}{L}\frac{C_1}{C_2} & 1/C_2 \\ -1/L & -1/L & 0 \end{bmatrix} \begin{bmatrix} V_{C_1} \\ V_{C_2} \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{R}{L} V_S + \\ \frac{C_1}{C_2}\frac{R}{L} V_S + \\ \frac{1}{L} V_S \end{bmatrix}$$

c) $\underline{\underline{A}} = \begin{bmatrix} -R/L & -R/L & 0 \\ -\frac{C_1}{C_2} R/L & -\frac{C_1}{C_2} R/L & 1/C_2 \\ -1/L & -1/L. & 0 \end{bmatrix}$ $|A| = 0$

since 1^{st} row $= R \times (3^{rd}$ Row$)$

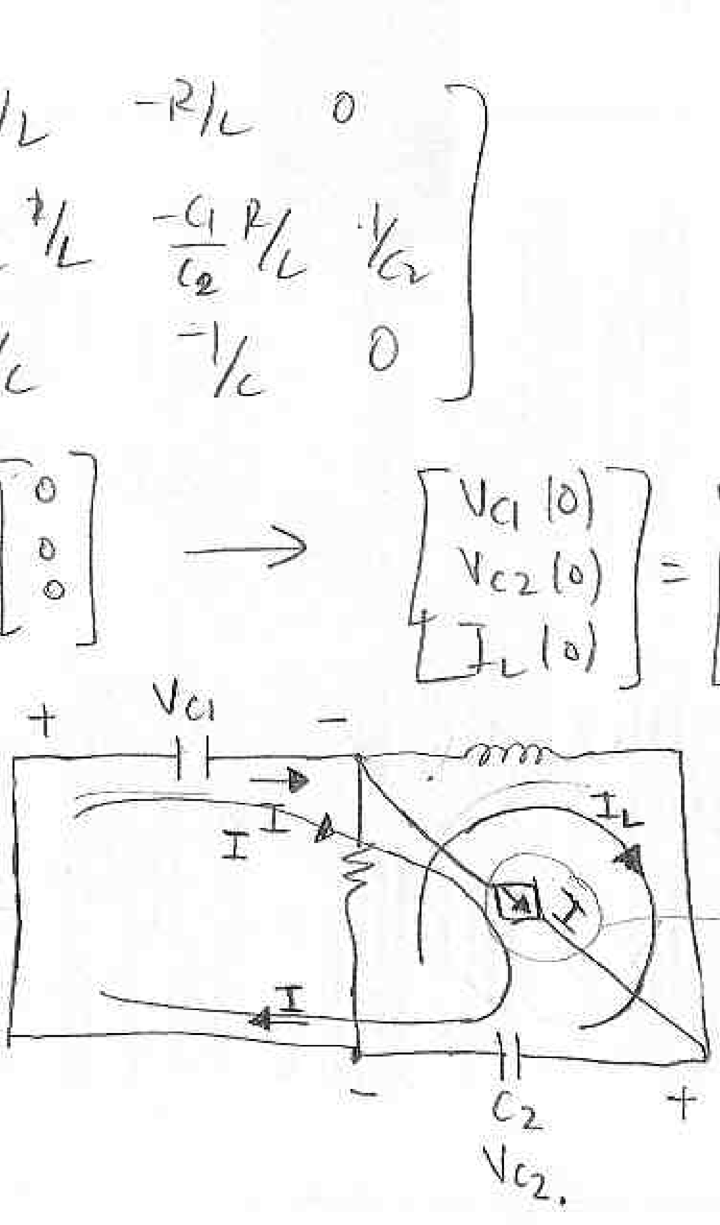
then $\lambda = 0 \longrightarrow$ Then

$$\begin{bmatrix} V_{C_1}(t) \\ V_{C2}(t) \\ I(t) \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} e^{\lambda t} \Big\downarrow \lambda = 0$$

$$= \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \rightarrow$$ determined (for $t \geq 0$) from initial conditions.

d) $\underline{\underline{A}} = \begin{bmatrix} -R/L & -R/L & 0 \\ -\frac{C_1}{C_2} R/L & -\frac{C_1}{C_2} R/L & 1/C_2 \\ -1/L & -1/L & 0 \end{bmatrix}$

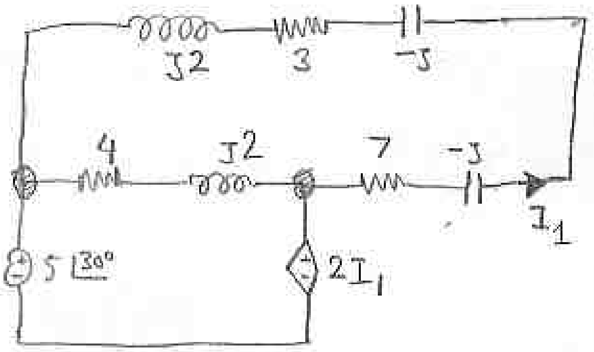
$$\underline{\underline{A}} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} V_{C1}(0) \\ V_{C2}(0) \\ I_L(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$



Acts as a Short circuit when $I_L = 0$

④ Solution:

V_g: on ; I_g: off



$$I_1 = \frac{2I - 5\angle 30}{7 - \cancel{j} - \cancel{j} + 3 + 2\cancel{j}}$$

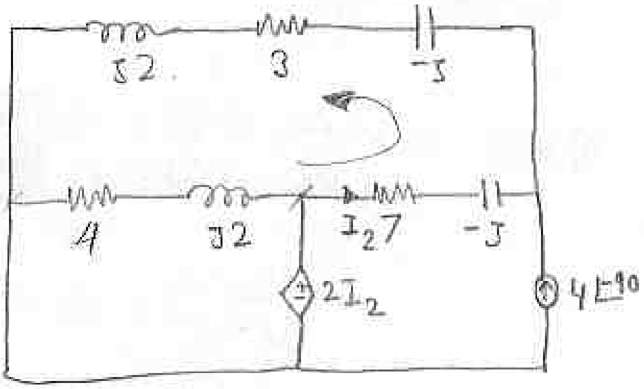
$$I_1 = \frac{2I - 5\angle 30}{10}$$

$$I_1 = \frac{5}{8}\angle 210 = -0{,}54 - j\,0.3125$$

$$P_{AV}^{I} = \frac{1}{2}|I_1|^2 7 = \frac{25{,}7}{128} = \frac{175}{128}\ \text{W}$$

$$\boxed{i_1(t) = \frac{5}{8}\cos(2t + 210^\circ)}$$

V_g: off ; I_g: on



KVL around the loop:

$$I_2(7 - j) + (I_2 - 4j)(3 + j) = 2I_2$$

$$I_2(10) + 4 - 12j = 2I_2$$

$$I_2 = -\frac{1}{2} + j\frac{3}{2}$$

$$= \frac{\sqrt{10}}{2}\angle -71$$

$$P_{AV}^{II} = \frac{1}{2}\cdot\frac{10}{4}\cdot 7 = \frac{35}{4}\ \text{W}.$$

$$\boxed{i_2(t) = \frac{\sqrt{10}}{2}\cos(2t - 71^\circ)}$$

$$I = I_1 + I_2 = -1.04 + j\,1.1875 = 1.57\angle 132^\circ$$

$$i(t) = \frac{5}{8}\cos(2t + 210^\circ) + \frac{\sqrt{10}}{2}\cos(2t - 71^\circ)$$

$$\boxed{i(t) = 1.57\cos(2t + 132^\circ)}$$

$$P_{AV}^{Tot} = \frac{1}{2}(1.57)^2 7 = 8.62\ \text{W}$$

c) $P_{AV}^{Tot} = 8.62$; $P_{AV}^{I} + P_{AV}^{II} = 10.1172$ W.

Suprposition principle does not hold for power calculations of AC sources of the same frequency.