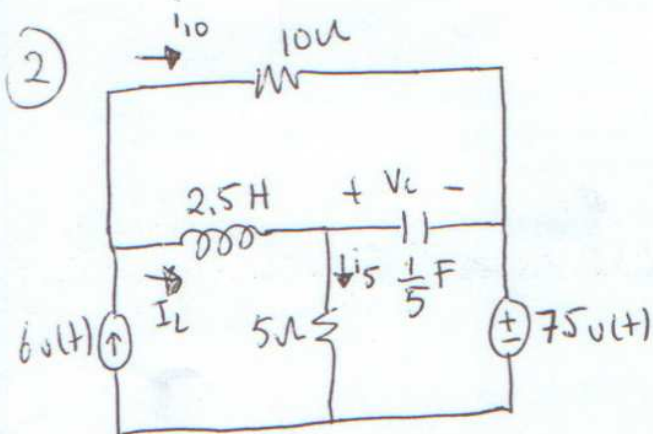


① Find inverse Laplace transform of:

$$F(s) = \frac{10(s^2 + 119)}{(s+5)(s^2 + 10s + 169)}$$



a) Assume there is no initial energy in the circuit.

Find value of $I_L(0^+)$ and $V_C(0^+)$, $i_{10}(0^+)$, $i_5(0^+)$

b) Using s-domain analysis find $I_L(t)$. (Assume zero initial conditions.)

c) What should be an initial condition set for $\{V_C(0^-), I_L(0^-)\}$ so that only the mode with the pole at -2 is excited, in the zero-input solution?

Exam 2 Solutions

①

$$\textcircled{1} \mathcal{L}^{-1}\{F(s)\} = ? \quad F(s) = \frac{10(s^2 + 119)}{(s+5)(s^2 + 10s + 169)} = \frac{A}{s+5} + \frac{Bs+C}{(s+5)^2 + 12^2}$$

$$A = \frac{10 \cdot (119 + 25)}{144} = 10$$

Now, let's find B comparing coef's of s^2 after equalization of denom.

$$\frac{10(s^2 + 119)}{(s+5)(s^2 + 12^2)} = \frac{10}{s+5} + \frac{Bs+C}{(s+5)^2 + 12^2}$$

$$\text{then } 10s^2 = 10s^2 + Bs^2 \rightarrow B = 0$$

then compare coef of "1".

$$1190 = 169 \cdot 10 + 5 \cdot C \rightarrow C = 2.50 = -100$$

$$\text{then } F(s) = \frac{10}{s+5} - \frac{100}{(s+5)^2 + 12^2} = \frac{10}{s+5} - \frac{100}{12} \frac{12}{(s+5)^2 + 12^2}$$

$$f(t) = \left(10e^{-5t} - \frac{100}{12} \cdot e^{-5t} \sin 12t \right) u(t)$$

Method 2:

2)

$$\frac{10(s^2 + 119)}{(s+5)((s+5)^2 + 12^2)} = \frac{10}{s+5} + \frac{K}{s+5+j12} + \frac{K^*}{s+5-j12}$$

$$K = \frac{10[(-5-j12)^2 + 119]}{(-j12)(-j24)} = \frac{10[25 - 144 + 5120 + 119]}{-144 \cdot 2}$$

$$= \frac{-j 120 \cdot 10}{144 \cdot 2} = -j \frac{25}{6} = \frac{25}{6} e^{-j\frac{\pi}{2}}$$

$$F(s) = \frac{10}{s+5} + \frac{\frac{25}{6} e^{-j\frac{\pi}{2}}}{s+5+j12} + \frac{\frac{25}{6} e^{+j\frac{\pi}{2}}}{s+5-j12}$$

$$f(t) = \left[10e^{-5t} + \frac{25}{6} e^{-j\frac{\pi}{2}} e^{-(5+j12)t} + \frac{25}{6} e^{+j\frac{\pi}{2}} e^{-(5-j12)t} \right] u(t)$$

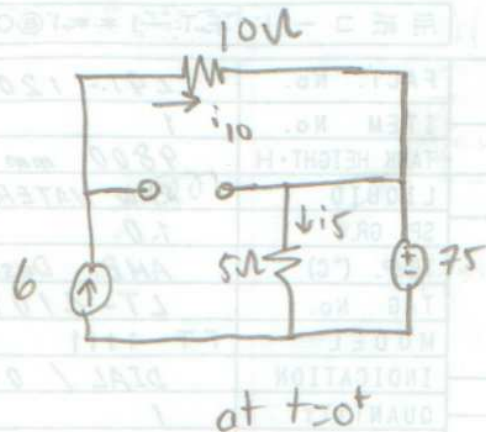
$$f(t) = \left(10e^{-5t} + 2 \cdot \operatorname{Re} \left\{ \frac{25}{6} e^{-j\frac{\pi}{2}} e^{-(5+j12)t} \right\} \right) u(t)$$

$$= \left(10e^{-5t} + \frac{50}{6} e^{-5t} \cos \left(12t + \frac{\pi}{2} \right) \right) u(t)$$

$$= \left(10e^{-5t} - \frac{50}{6} e^{-5t} \sin(12t) \right) u(t)$$

DATE	19/09/21	TOKYO KEISO CO., LTD.
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APPROVED BY	[Signature]	TRANSMITTER
SCALE		
NO.	PARTS NAME	MATERIAL
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2	GASKET	ACSA
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(2) a)



$$i_{10}(0^+) = 6 \text{ A}$$

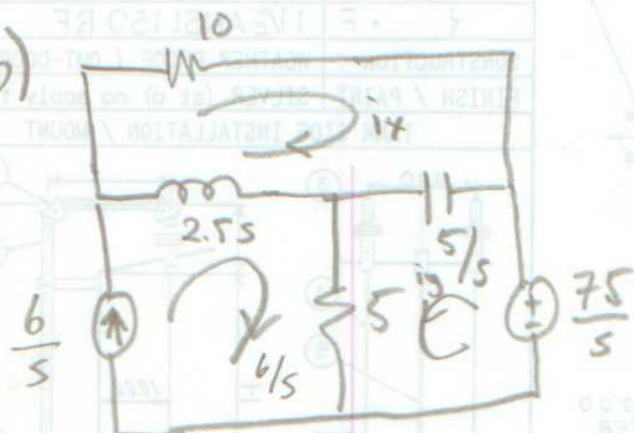
$$i_5(0^+) = 15 \text{ A}$$

$$V_c(0^+) = V_c(0^-) = 0 \text{ V}$$

$$I_L(0^+) = I_L(0^-) = 0 \text{ A}$$

(3)

b)



$$\text{KVL: } i_x$$

$$0 = 10i_x + \frac{5}{s}(i_x + i_y) + 2.5s(i_x - \frac{6}{s})$$

$$\text{KVL: } i_y$$

$$\frac{75}{s} = \frac{5}{s}(i_y + i_x) + 5(i_y + \frac{6}{s})$$

$$\begin{bmatrix} 10 + \frac{5}{s} + \frac{5s}{2} & \frac{5}{s} \\ \frac{5}{s} & 5 + \frac{5}{s} \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} = \begin{bmatrix} 15 \\ \frac{45}{s} \end{bmatrix}$$

$$\Delta(s) = 50 + \frac{25}{s} + \frac{25s}{2} + \frac{50}{s} + \frac{25}{s^2} + \frac{25}{2} - \frac{25}{s^2}$$

$$= 62.5 + \frac{75}{s} + 12.5s$$

$$= 12.5(5 + \frac{6}{s} + s) = \frac{12.5}{s}(s^2 + 5s + 6)$$

$$\left| \begin{array}{c|c} 15 & 5/s \\ \hline 45/s & s + \frac{5}{s} \end{array} \right| = 15 \left(s + \frac{5}{s} \right) - \frac{5}{s} \cdot \frac{45}{s} \quad (4)$$

$$= 75 \left[1 + \frac{1}{s} - \frac{3}{s^2} \right]$$

$$\Delta_1(s) = \frac{75}{s^2} [s^2 + s - 3]$$

$$i_x(s) = \frac{\Delta_1(s)}{\Delta(s)} = \frac{75/s^2 [s^2 + s - 3]}{\frac{12.5}{s} \cdot (s^2 + 5s + 6)}$$

$$i_x(s) = \frac{6}{s} \cdot \frac{(s^2 + s - 3)}{(s^2 + 5s + 6)}$$

$$= \frac{6}{s} \cdot \frac{s^2 + s - 3}{(s+2)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = -3 ; B = 3 ; C = +6.$$

$$i_x(t) = (-3 + 3e^{-2t} + 6e^{-3t}) u(t)$$

$$i_L(t) = 6u(t) - i_x(t)$$

$$= [9 - 3e^{-2t} - 6e^{-3t}] u(t)$$

Note

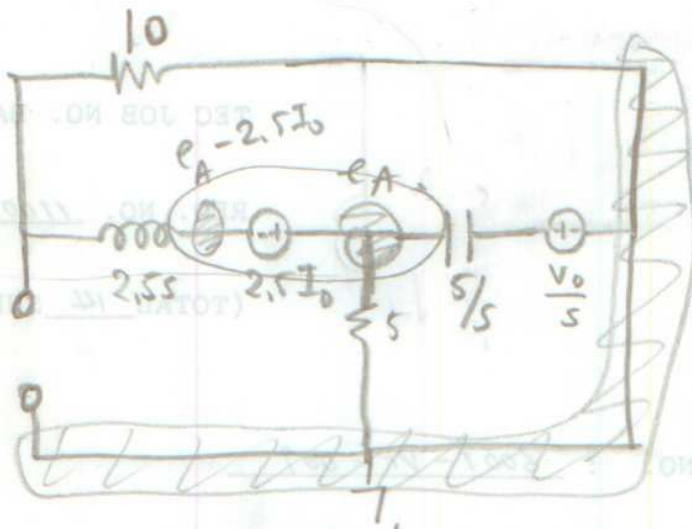
$$i_x(0^+) = 6 \text{ A}$$

$$i_L(0^+) = 0 \text{ A}$$

as found in part a

c) Zero input.

5



$$\frac{e_A - V_0/s}{5/s} + \frac{e_A}{5} + \frac{e_A - 2.5I_0}{2.5s + 10} = 0$$

$$e_A \left(\frac{s}{5} + \frac{1}{5} + \frac{2}{5(s+4)} \right) = \frac{V_0}{5} + \frac{I_0}{s+4}$$

$$\frac{e_A (s^2 + 5s + 6)}{5(s+4)} = \frac{V_0 s + 4V_0 + 5I_0}{5(s+4)}$$

$$e_A = \frac{V_0 s + 4V_0 + 5I_0}{s^2 + 5s + 6} = \frac{-(V_0 + 5I_0)}{s+3} + \frac{2V_0 + 5I_0}{s+2}$$

When $V_0 = -5I_0 \rightarrow e_A = \frac{V_0}{s+2}$

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \end{bmatrix} k.$$