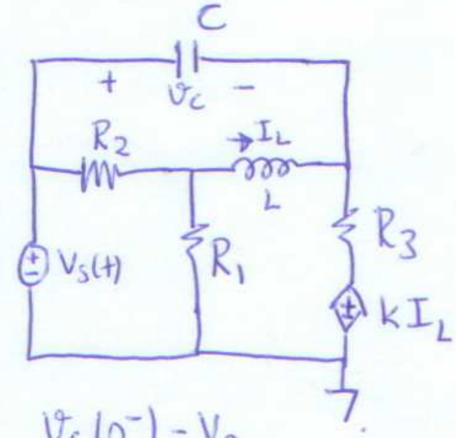
9th July, 2007.

EE 202
(Exam #1)
(50 min)

(1)



$v_C(0^-) = V_0$

$I_L(0^-) = I_0$

a) Apply a source-shift transformation so that circuit can be partitioned into generalized branches.

b) Write node equations using graph theoretical methods. (Show matrices; but do NOT multiply matrices at the last stage; just show which matrices should be multiplied). Find initial conditions for node voltages.

c) Find a tree such that whose fun. cut-set equations are exactly the same equations you have found in part b. How many such trees are there?

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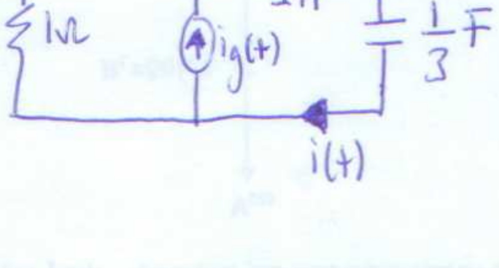
(2)



Find state equations.

EE 202, Exam #1. Solution

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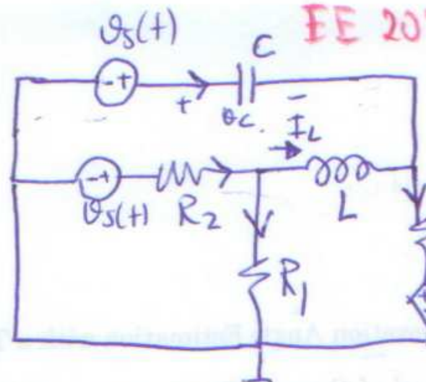
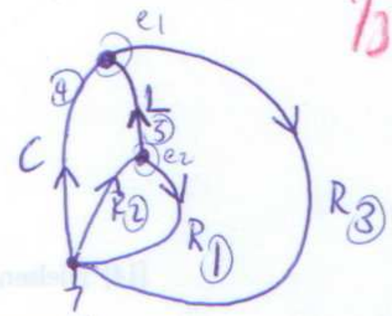
Q1. a)



b)



(1) $\underline{\underline{A}}\,\underline{J}=\underline{0} \longrightarrow \begin{bmatrix} 0 & 0 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(2) $\underline{\underline{A}}^T\underline{e}=\underline{v} \qquad \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$

(3) $J_1 = \dfrac{v_1}{R_1}$; $J_2 = \dfrac{v_2+v_s}{R_2}$, $J_3 = \dfrac{v_3 - kI_L(t)}{R_3} = \dfrac{v_3 - k\left(\overset{I_0}{I_L(0^-)}\right)+\frac{1}{L}\int_0^t v_5(\tau)d\tau}{R_3}$.

$J_5 = I_L(0^-) + \dfrac{1}{L}\int_0^t v_5(\tau)d\tau$, $J_4 = C\dfrac{d}{dt}\left[v_4(t) + v_s(t)\right]$.

$$\underbrace{\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{bmatrix}}_{\underline{J}} = \underbrace{\begin{bmatrix} 1/R_1 & 0 & 0 & 0 & 0 \\ 0 & 1/R_2 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_3} & 0 & -\frac{k}{LR_3}D^{-1} \\ 0 & 0 & 0 & CD & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{L}D^{-1} \end{bmatrix}}_{\underline{\underline{G}}(D)}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v_s}{R_2} \\ -\frac{kI_0}{R_3} \\ CDv_s(t) \\ I_0 \end{bmatrix}.$$

$\underline{J} = \underline{\underline{G}}(D)\,\underline{v} + \underline{\text{Source}}$

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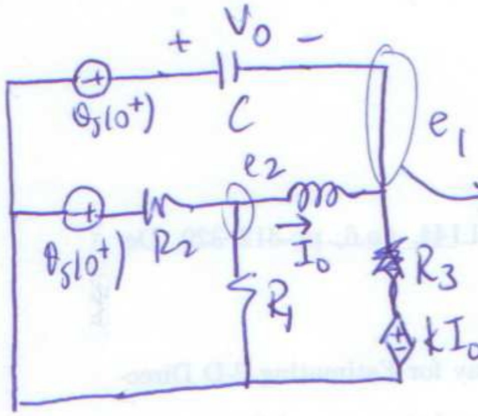
At $t=0^+$



$$\boxed{e_1(0^+) = v_s(0^+) - v_0}$$

$$\frac{e_2(0^+) - v_s(0^+)}{R_2} + \frac{e_2(0^+)}{R_1} + I_0 = 0.$$

$$\boxed{e_2(0^+) = \left(\frac{1}{R_2} + \frac{1}{R_1}\right)^{-1}\left[\frac{v_s(0^+)}{R_2} - I_0\right]}$$

c) F-Cut-sets include only one branch from tree.

then Fun Cut Set 1 has $\{C, L, R_3\}$ branches.

Fun Cut Set 2 has $\{R_1, R_2, L\}$ branchs.

then L can not be a tree branch (appears in two different fun. cut set), but all other combinations are feasible. $\{C, R_1\}$, $\{C, R_2\}$, $\{R_3, R_1\}$, $\{R_3, R_2\}$.

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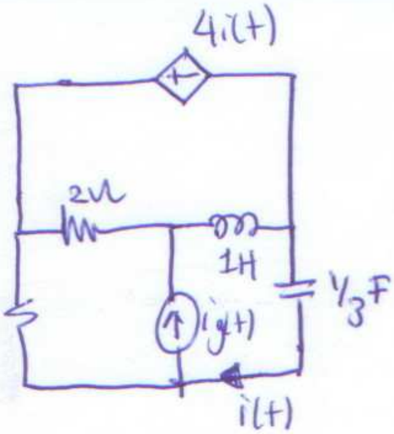
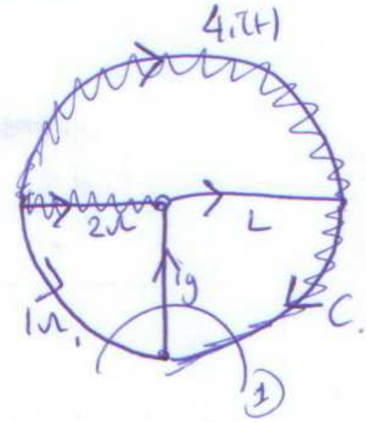
d) cont

$$\underline{\underline{A}}\,\underline{J} = 0 \longrightarrow \underline{\underline{A}}\,\underline{J} = \underline{\underline{A}}(\underline{\underline{G}}(D)\,\underline{v} + \text{source}) = 0.$$
$$= \underline{\underline{A}}(\underline{\underline{G}}(D)\,\underline{\underline{A}}^T\underline{e} + \text{source}) = 0.$$

$$\underbrace{\left(\underline{\underline{A}}\,\underline{\underline{G}}(D)\,\underline{\underline{A}}^T\right)}_{2\times 2}\underbrace{\underline{e}}_{[e_1\ e_2]^T} = \underbrace{-\underline{\underline{A}}\ \text{source}}_{2\times 1}$$

(2)





State Vars. $= \{ i_L, v_C \}$.

$$\text{(1)}\quad C\dot{v}_c = i_g(t) - i_{1\Omega} = i_g(t) - \frac{v_{1\Omega}}{1} \overset{\text{F.L.}}{=} i_g(t) - \frac{4\overset{C\dot{v}_c}{i(t)} + v_c}{1}$$

$$= i_g(t) - 4\underset{\frac{1}{3}}{C}\dot{v}_c - v_c.$$

$$\boxed{\dot{v}_c = -\frac{3}{5}v_c + \frac{3}{5}i_g(t)}$$

$$\text{(2)}\quad \underset{1H}{L}\,\dot{i}_L(t) = -v_{2\Omega} + 4i(t)$$

$$= -i_{2\Omega}\cdot 2 + 4\frac{1}{3}\dot{v}_c$$

$$= -(i_L - i_g)2 - \frac{4}{5}v_c + \frac{4}{5}i_g(t).$$

$$\boxed{\dot{I}_L(t) = -\frac{4}{5}v_c - 2I_L + \frac{14}{5}i_g(t).}$$