EEMB METU
Fall 09/10

EE 503
HW #3
(Due: Nov. 17, 2009)

<u>Part 1.</u> Problems: 3.4, 3.6, 3.8, 3.9 of Hayes.

<u>Part 2.</u> Matlab Assignment:

During the lectures, we have discussed the conditions for which $x(t)$ given below

$$x(t) = \underset{\sim}{a}\cos(\omega t) + \underset{\sim}{b}\sin(\omega t)$$

is WSS.

1. We did not discuss the conditions for strict sense stationarity (SSS) of $x(t)$. Read Papoulis Example 10-13 (p.300 of 3rd edition) to learn the conditions for SSS. After reading the related section:
   a. Show that SSS condition (the circularly symmetric distribution for the joint pdf of $\underset{\sim}{a}$ and $\underset{\sim}{b}$) implies conditions for WSS.
   b. Using part 1a), find the joint pdf $x(t_1)$ and $x(t_2)$ and show that the pdf is a function of $\tau = t_1 - t_2$. (Hint: You may make use of conditioning, $x(t_2)$ given $x(t_1)$, as we did in the lectures.)

2. Let $\underset{\sim}{a}$ and $\underset{\sim}{b}$ be independent random variables. Let $\underset{\sim}{a} = \{+1,-1\}$ with equal probability and $\underset{\sim}{b} = \{+1,-1\}$ with equal probability.
   a. How many different realizations for $x(t)$ do we have?
   b. Write $x(t)$ in the form $x(t) = \underset{\sim}{A_x}\cos(wt - \underset{\sim}{\theta_x})$ where $\underset{\sim}{A_x}$ and $\underset{\sim}{\theta_x}$ are random variables. Find the joint pdf of $\underset{\sim}{A_x}$ and $\underset{\sim}{\theta_x}$. Are $\underset{\sim}{A_x}$ and $\underset{\sim}{\theta_x}$ independent ?
   c. Take $\omega = 2\pi$ and $T_s = \dfrac{1}{10}$ and form a discrete time signal $x[n] = x(nT_s)$. Show different realizations of $x[n]$ (in time span of [0,10] seconds) in the same figure.
   d. Find the *marginal* pdf of x(11/10) (which is x[11]) and x(2) (which is x[20]). . Do we have SSS and/or WSS in the first order ?
   e. Generate 100 realizations of x[n] and estimate the marginal pdf of x(11/10) and x(2) using histograms. Compare your estimate with part 2d.

f. Generate 200 realizations of x[n] and estimate the *joint* pdf of x(11/10) and x(2).

3. $h(t)$ is the impulse of a LTI filter which is defined as follows:

$$h(t)=\begin{cases}2 & 0<t<1/2\\ 0 & other\end{cases}$$

Let $y(t)=x(t)*h(t)$ and let y[n] and h[n] be the sampled version y(t) as discussed previously and assume that $y[n]=x[n]*h[n]$. (This is only approximately true since $h(t)$ is not band-limited. But $H(f)$ has little energy beyond Fs/2, which is 5 Hz, so aliasing is negligible.)

a. How many different realizations for $y(t)$ do we have?

b. Write $y(t)$ in the form $y(t)=\underset{\sim}{A_y}\cos(wt-\underset{\sim}{\theta_y})$ where $\underset{\sim}{A_y}$ and $\underset{\sim}{\theta_y}$ are random variables. Express $\underset{\sim}{A_y}$ and $\underset{\sim}{\theta_y}$ in terms of $\underset{\sim}{A_x}$ and $\underset{\sim}{\theta_x}$ and also find the joint pdf of $\underset{\sim}{A_y}$ and $\underset{\sim}{\theta_y}$.

c. Analytically calculate $r_{xy}(\tau)$ and $r_y(\tau)$. Note that the correlations are periodic with the period $\frac{\omega}{2\pi}$.

d. Analytically show that $r_y[k]=r_y(kT_s)$ and $r_{xy}[k]=r_{xy}(kT_s)$. Numerically approximate $r_{xy}[k]$ and $r_y[k]$ with

$$\hat{r}_{xy}[k]=\frac{1}{N-k}\sum_{n=k}^{N-1}x[n]y[n-k]$$

and

$$\hat{r}_{y}[k]=\frac{1}{N-k}\sum_{n=k}^{N-1}y[n]y[n-k]$$

where *x[n]* and *y[n]* are defined for *n={0,1, …, N-1}*. Try different values of N, i.e. $N_1,N_2, \ldots, N_k$. Present the true $r_{xy}[k]$ and $r_y[k]$ and their estimates found for different values of N. (A proper display format can be a table with 10 rows and (|Nset|+1) number of columns. Nset={$N_1,N_2, \ldots, N_k$} and |Nset| is the number of elements of the set Nset.)