Quiz: Convolutions

This quiz is designed to test your knowledge of convolutions of 2π-periodic functions.

In this entire quiz, the expression $f * g$ denotes convolution of f and g, while the expression fg denotes the pointwise product of f and g. The expression $\hat{f}$ denotes the Fourier transform of f, thus $\hat{f}(n)$ is the n^{th} Fourier coefficient of f.

Discuss this quiz

(Key: correct, incorrect, partially correct.)

1. Let f and g be continuously differentiable 2π-periodic functions. The derivative $(f * g)'$ of the convolution $f * g$ is given by
   - A. ○ $(f') * g$

     This is one of two correct answers.
   - B. ○ $f * (g') + (f') * g$

     You're thinking of the product rule: $(fg)' = f'g + fg'$. Convolutions behave differently from products.
   - C. ○ $(f') * (g')$

     You're thinking of the sum rule: $(f+g)' = f' + g'$. Convolutions behave differently from sums.
   - D. ○ $(g') * (f')$

     You might be thinking of Abel's formula for matrices: $(AB)^{-1} = B^{-1}A^{-1}$. Convolutions behave differently from inverses.
   - E. ○ $f * (g')$

     This is one of two correct answers.
   - F. ◉ In general, there is no simple formula available.

2. Let f and g be continuously differentiable 2π-periodic functions, and let n be an integer. The n^{th} Fourier coefficient $\widehat{f * g}(n)$ of the convolution $f * g$ is given by
   - A. ○ $\hat{f} * g(n)$ (incorrect)
   - B. ○ $\hat{f}(n)\hat{g}(n)$ (correct)
   - C. ○ $\hat{f} * \hat{g}(n)$ (incorrect)
   - D. ○ $\hat{f}(n) + \hat{g}(n)$ (incorrect)
   - E. ○ $\hat{f}(n)g + f\hat{g}(n)$ (incorrect)
   - F. ◉ In general, there is no simple formula available.

3. Let f and g be continuously differentiable 2π-periodic functions. The average value of $f * g$ is equal to
   - A. ○ The difference between the average value of f and the average value of g.
   - B. ○ The average of the average value of f and the average value of g.
   - C. ○ The convolution of the average value of f and the average value of g.

     This is true if one thinks of the average values of f and g as constant functions rather than numbers, but this is a rather clumsy way to phrase the answer.
   - D. ○ The product of the average value of f and the average value of g.
   - E. ○ The sum of the average value of f and the average value of g.

F. ◉ In general, there is no simple formula available.

4. Let f, g, h be continuous 2π-periodic functions. The expression $f * (g + h)$ can also be written as

A. ○ $(f + g) * h$ (incorrect)

B. ○ $f * h + g * h$ (incorrect)

C. ○ $g * f + f * h$ (correct)

D. ○ $f * (g * h)$ (incorrect)

E. ○ $g * (f + h)$ (incorrect)

F. ◉ None of the above.

5. Let f, g, h be continuous 2π-periodic functions. The expression $(f + 3h) * (2g)$ can also be written as

A. ○ $2(f * g) + 6(h * g)$ (correct)

B. ○ $6 * f * g * h$ (incorrect)

C. ○ $2 * f * g + 3 * h * g$ (incorrect)

D. ○ $(2f) * g + (3h) * g$ (incorrect)

E. ○ $6 * h * g + 2 * f * g$ (incorrect)

F. ◉ None of the above.

6. Let f, g, h be continuous 2π-periodic functions. The expression $f * (gh)$ can also be written as

A. ○ $(fg) * h$ (incorrect)

B. ○ $(f * g)(f * h)$ (incorrect)

C. ○ $f * g + f * h$ (incorrect)

D. ○ $f(g + h)$ (incorrect)

E. ○ $f(g * h)$ (incorrect)

F. ◉ None of the above.

In general, there is no useful formula for pulling a product out of a convolution (or a convolution out of a product).

7. Let f, g be 2π-periodic functions. If f is continuously differentiable, and g is twice continuously differentiable, then the best we can say about $f * g$ is that it is 2π-periodic and

A. ○ Riemann integrable.

B. ○ Piecewise continuous.

C. ○ Continuous.

D. ○ Continuously differentiable.

E. ○ Twice continuously differentiable.

F. ○ Three times continuously differentiable.

Convolving two functions combines their orders of smoothness together.

G. ◉ Infinitely differentiable.

8. Let f, g be 2π-periodic functions. If f is continuously differentiable, and g is twice continuously differentiable, then the best we can say about $f + g$ is that it is 2π-periodic and

A. ○ Riemann integrable.

B. ○ Piecewise continuous.

C. ○ Continuous.

D. ○ Continously differentiable.

In general, the sum of two functions is only as smooth as the rougher of its two factors.

E. ○ Twice continuously differentiable.

F. ○ Three times continuously differentiable.

G. ◉ Infinitely differentiable.

9. Let f, g be 2π-periodic functions. If f is continuously differentiable, and g is twice continuously differentiable, then the best we can say about fg is that it is 2π-periodic and

A. ○ Riemann integrable.

B. ○ Piecewise continuous.

C. ○ Continuous.

D. ○ Continously differentiable.

In general, the product of two functions is only as smooth as the rougher of its two factors.

E. ○ Twice continuously differentiable.

F. ○ Three times continuously differentiable.

G. ◉ Infinitely differentiable.

10. Let f, g be 2π-periodic functions. If f and g are Riemann integrable, then the best we can say about $f * g$ is that it is 2π-periodic and

A. ○ Bounded.

B. ○ Riemann integrable.

While this true, more can be said.

C. ○ Piecewise continuous.

D. ○ Continuous.

E. ○ Continously differentiable.

F. ○ Twice continuously differentiable.

G. ◉ Infinitely differentiable.

11. Let f, g be 2π-periodic functions. If f and g are Riemann integrable, then the best we can say about fg is that it is 2π-periodic and

A. ○ Bounded.

While this true, more can be said.

B. ○ Riemann integrable.

C. ○ Piecewise continuous.

D. ○ Continuous.

E. ○ Continously differentiable.

F. ○ Twice continuously differentiable.

G. ◉ Infinitely differentiable.

12. Let f be a 2π-periodic function, and let 1 be the constant function 1. Then $f * 1$ is

A. ○ The same function as f.

B. ○ The constant function 1.

C. ○ The constant function with value equal to the mean of f.

D. ○ The value of $f(x)$ at the point $x = 0$.

E. ○ 0. (incorrect)

F. ◉ The constant function with value equal to $f(1)$.

13. Let f be a continuous 2π-periodic function, and let K_n be a family of approximations to the identity (a.k.a. good kernels). Which of the following statements is true?

A. ○ For each x, $f * K_n(x)$ converges to 1 as n goes to infinity.
B. ○ For each x, $K_n(x)$ converges to $f(x)$ as n goes to infinity.
C. ○ For each x, $f * K_n(x)$ converges to $f(x)$ as n goes to infinity.
D. ○ For each n, $f * K_n(x)$ converges to $f(x)$ as x goes to infinity.
E. ○ For each x and each n, we have $f * K_n(x) = f(x)$.
F. ◉ The functions $f * K_n$ converge to zero as n goes to infinity.

Score: 20/130

Expand all answers