Multi-Ind. Beam Derivations:

1 Max-SINR:

Beam 1: $\underline{r}_1 = \alpha_1 \underline{s}_{\theta_1} + \underline{n}_1$ ← include clutter

Beam 2: $\underline{r}_2 = \alpha_2 \underline{s}_{\theta_2} + \underline{n}_2$

i) $\underline{n}_1$ and $\underline{n}_2$ are ind. vectors

ii) α_1, α_2 : non-random unknown

iii) $\underline{s}_{\theta_1}, \underline{s}_{\theta_2}$: known vectors.

Assume $\underline{n}_k \sim CN(\underline{0}; \underline{\underline{M}}_k)$ and $\underline{\underline{M}}_k$ also known.

For target present hypothesis (H_1), we have:

$$f_{\underline{r}_1,\underline{r}_2}(\underline{r}_1,\underline{r}_2) = f_{\underline{r}_1}(\underline{r}_1) f_{\underline{r}_2}(\underline{r}_2)$$
$$= K e^{-\sum_{k=1}^{2} (\underline{r}_k - \alpha_k \underline{s}_{\theta_k})^H \underline{\underline{M}}_k^{-1} (\underline{r}_k - \alpha_k \underline{s}_{\theta_k})}$$

GLRT results in

$$K' + \sum_{k=1}^{2} \left\| \underbrace{\left(\underline{\underline{I}} - \frac{\underline{\underline{M}}_k^{-1/2} \underline{s}_\theta \underline{s}_\theta^H \underline{\underline{M}}_k^{-1/2}}{\underline{s}_{\theta_k}^H \underline{\underline{M}}_k^{-1} \underline{s}_{\theta_k}} \right)}_{\underline{\underline{P}}^{\perp}_{s_{\theta_k}}} \underline{\underline{M}}_k^{-1/2} \underline{r}_k \right\|^2 \underset{H_1}{\overset{H_0}{\gtrless}} \text{Threshold.}$$

$\Downarrow$

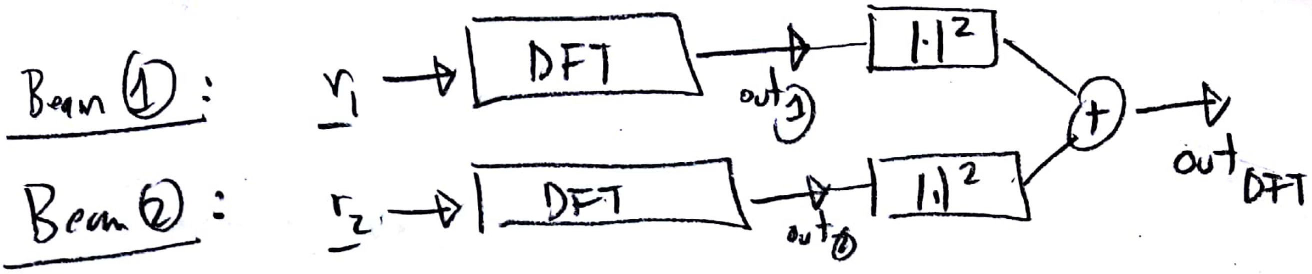
$$\sum_{k=1}^{2} \frac{\underline{r}_k^H \underline{\underline{M}}_k^{-1/2} \underline{\underline{M}}_k^{-1/2} \underline{s}_\theta \underline{s}_\theta^H \underline{\underline{M}}_k^{-1/2} \underline{\underline{M}}_k^{-1/2} \underline{r}_k}{\underline{s}_{\theta_k}^H \underline{\underline{M}}_k^{-1} \underline{s}_{\theta_k}} \underset{H_0}{\overset{H_1}{\gtrless}} \text{Threshold}$$

$\uparrow \underline{\underline{P}}_{s_{\theta_k}}$

$\Downarrow$

$$\sum_{k=1}^{2} \left| \underline{r}_k^H \underline{\underline{M}}_k^{-1} \underline{s}_\theta \right|^2 \Big/ \underline{s}_{\theta_k}^H \underline{\underline{M}}_k^{-1} \underline{s}_{\theta_k} \underset{H_0}{\overset{H_1}{\gtrless}} \text{Threshold.}$$

2 DFT Detector:



$$\sum_{k=1}^{2} \left|\underline{r}_k^H \underline{s}_{DFT_k}\right|^2 \Big/ \underbrace{\left(\underline{s}_{DFT_k}^H \underline{s}_{DFT_k}\right)}_{\substack{\text{constant} \\ \text{(unless windowed)}}} \underset{H_0}{\overset{H_1}{\gtrless}} \text{Threshold.}$$

$$\underline{s}_{DFT_k} = \underbrace{(\text{window}) \otimes (\underline{s}_{\theta_{test_k}})}_{\substack{\text{windowed} \\ \text{DFT} \\ \text{vector}}}$$

θ_{Test_k} ≜ Tested DFT bin.

3 Kelly's Detector:

Ricci eq. (2.21)

$$t_{Kelly} = \frac{\left|\underline{r}^H \underline{\underline{S}}^{-1} \underline{s}_\theta\right|^2}{\left(\underline{s}_\theta^H \underline{\underline{S}}^{-1} \underline{s}_\theta\right)\left(1 + \underline{r}^H \underline{\underline{S}}^{-1} \underline{r}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \text{Threshold.}$$

$$\boxed{\underline{\underline{S}} = \sum_{k=1}^{K} \underline{r}_k \underline{r}_k^H}$$

GLRT ⇒ $\frac{1}{1-t_{Kelly}} = \cancel{t}\ \hat{t}$ (eq. 2.20)

↳ GLRT stats: $(K+1)\ln\left(\frac{1}{1-t_{Kelly}}\right) \underset{H_0}{\overset{H_1}{\gtrless}}$ Threshold.

Assume K_1 and K_2 are the number auxillary/secondary (3)
data for Kelly's test with 2 beams, Test becomes:

$$(K_1+1)\ln\left(\frac{1}{1-t^{(1)}_{Kelly}}\right)+(K_2+1)\ln\left(\frac{1}{1-t^{(2)}_{Kelly}}\right)\gtrless\gamma$$

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(4) Multi-ind beam ACE, AMF:

It seems that we can apply the same reasoning to convert single beam ACE, AMF detectors to 2 beams:

that is

$$(K_1+1)\ln\left(\frac{1}{1-t_{ACE/AMF}}\right)+(K_2+1)\ln\left(\frac{1}{1-t_{ACE/AMF}}\right)\gtrless\gamma$$