Problem 1: (14 pts)

a) State whether the following statements are TRUE or FALSE. **(5pts)**
(0.5 pts each; if all correct, 5 more points as bonus)

- [...TRUE.....] KVL is based on conservation of energy.
- [...TRUE.....] KCL is based on conservation of charge.
- [...TRUE.....] Summation of voltage drops across *any* closed loop is zero.
- [...TRUE.....] Subtraction of voltage drops across *any* closed loop is zero.

- [...TRUE.....] Non-linear resistors are memoryless components.
- [...FALSE...] If a circuit can be algebraically analyzed with node analysis, but not with mesh analysis; the circuit must contain at least one component that is not current controlled. (one component that is not voltage controlled)
- [..FALSE....] Superposition principle is valid for all types circuits including memoryless, dynamic, linear, non-linear circuits. (not valid for non-linear circuits)

- [...FALSE...] Tellegen's Theorem is only valid for resistive circuits. (valid for all circuits)
- [...TRUE.....] Finding (i,v) characteristic of a linear memoryless circuit is another method of finding its Thevenin equivalent.
- [...TRUE.....] The branch current and branch voltage of a short circuited branch can be both zero. (Think of the (i,v) char. of $v_s = 0V$ source)

b) Answer the following short questions: (9 pts)

i) Determine the range of k for the component given in Figure i to be active. (3pts)

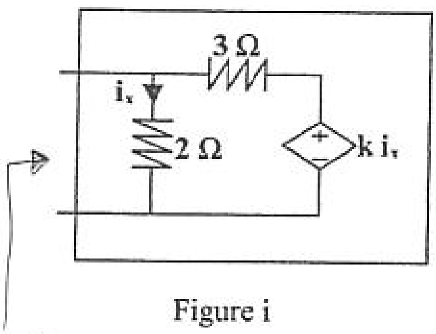


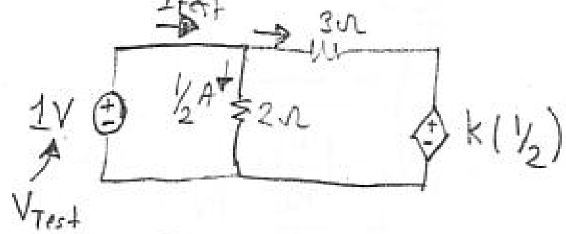
Figure i

Rin

Rin < 0 → Active

$\boxed{k > 5}$ → Active

Answer:



$$I_{test} = \frac{1}{2} + \frac{1 - k/2}{3}$$

$$I_{test} = \frac{5-k}{6}$$

$$R_{in} = \frac{V_{Test}}{I_{test}} = \frac{1}{\frac{5-k}{6}}$$

ii) The component shown in Figure ii-a is a current controlled circuit element that can be non-bilateral. Show that the component in Figure ii-b is always bilateral. (3 pts)

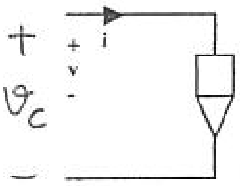


Figure ii-a

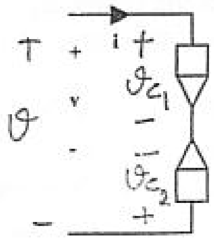


Figure ii-b

Answer :

$v_c = f(i)$: current controlled component.

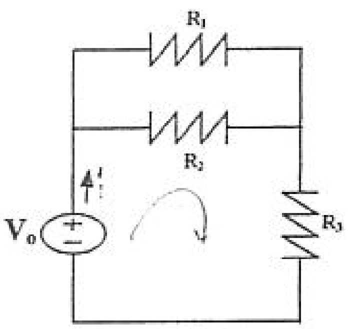
$v = v_{c_1} - v_{c_2} = f(i) - f(-i) \rightarrow \boxed{v = f(i) - f(-i)}$ (1)

If v_* and i_* satisfy (1) $\rightarrow (-v_*, -i_*)$ satisfy (1)

$(v_*, i_*) \in (i, v)$ chart $\rightarrow (-v_*, -i_*) \in (i, v)$ chart $\rightarrow$ bilateral.

iii) Find the dual of the following circuit. (No need to draw circuit graph or indicate current/voltage orientations) (3 pts)

Answer :

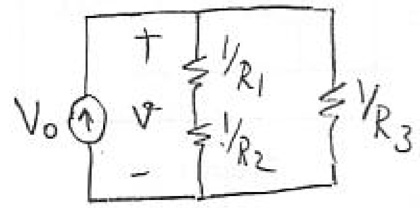


Dual: Series $\rightarrow$ Parallel

Voltage Source $\rightarrow$ Current Source

Mesh $\rightarrow$ Node

KVL: $V_o = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} i + R_3 i$



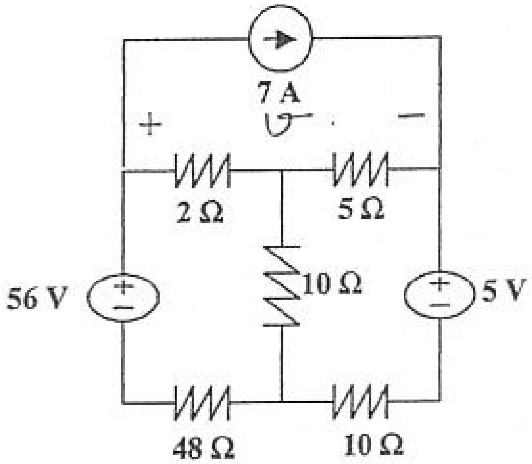
KCL: $V_o = \frac{v}{(1/R_1 + 1/R_2)} + \frac{v}{1/R_3}$

$V_o = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} v + R_3 v$

Same equation if $i \leftrightarrow v$

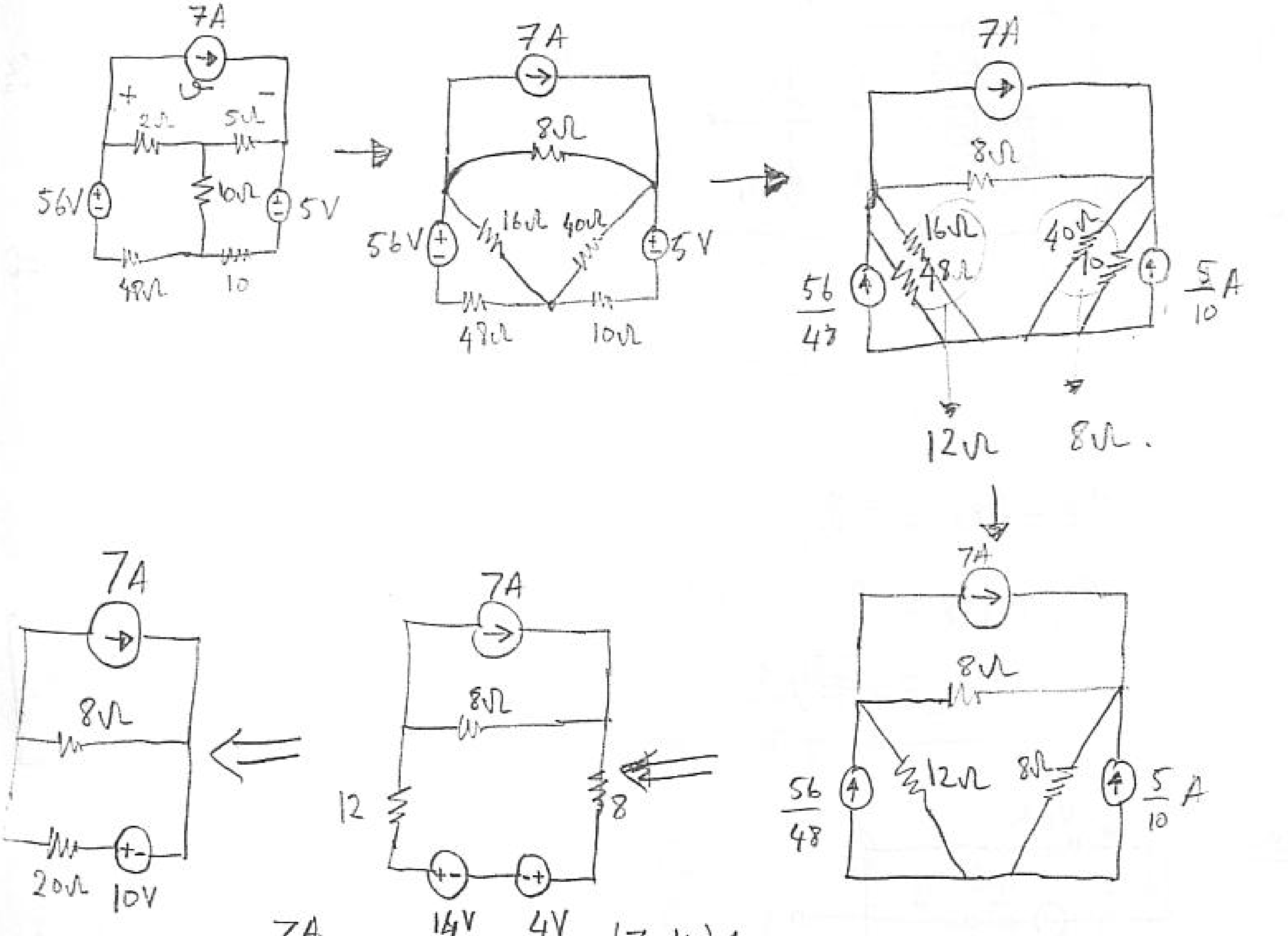
Problem 2: (14 pts)

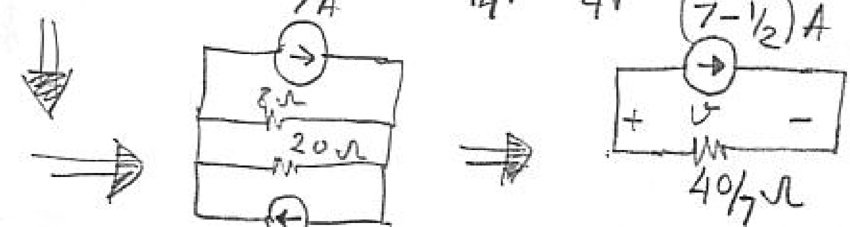
Find the power supplied by 7 A current source using *source transformation* method.



$P = v \cdot 7$ Watts

$v = ?$



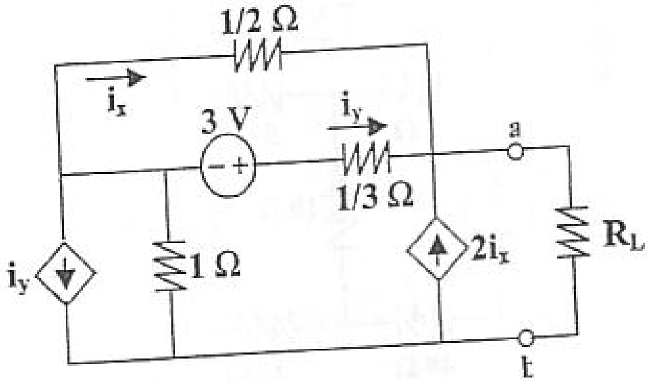


$v = \frac{-13}{2} \cdot \frac{40}{7} = \frac{-260}{7}$ Volt.

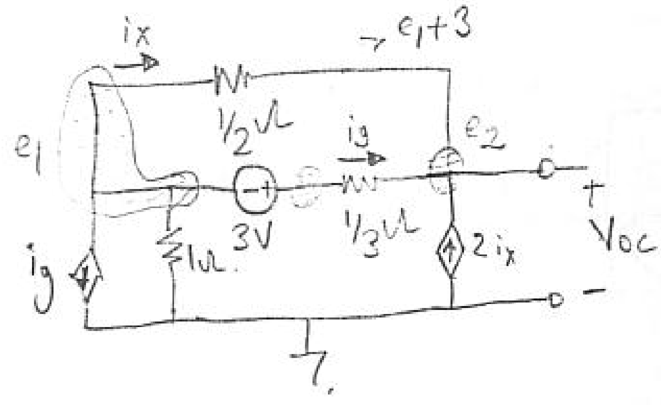
$\boxed{P = v\,7 = -260 \text{ Watts}}$

Problem 3: (16 pts)

Determine the value of R_L for maximum power transfer to a-b terminals.



V_{oc}:



KCL at Supernode: $i_y + e_1 + \dfrac{e_1-e_2}{1/2} + \dfrac{e_1+3-e_2}{1/3} = 0$

KCL at e_2: $\dfrac{e_2-e_1}{1/2} + \dfrac{e_2-(e_1+3)}{1/3} - 2i_x = 0$

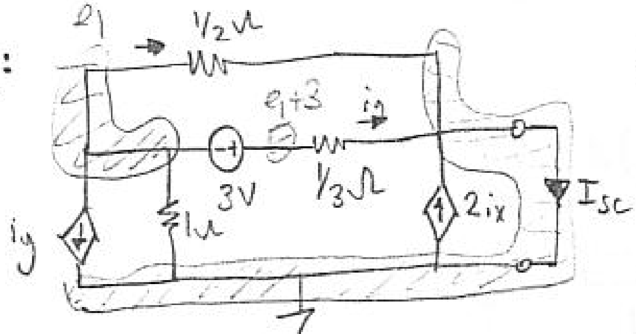
$i_y = \dfrac{e_1+3-e_2}{1/3}$ $\qquad i_x = \dfrac{e_1-e_2}{1/2}.$

$$9e_1 - 8e_2 = -18$$
$$-9e_1 + 9e_2 = 9$$
$$e_2 = -9\text{ V}$$

$V_{oc} = e_2 = -9V$

$$\boxed{R_{Th} = \frac{V_{oc}}{I_{sc}} = 1\Omega}$$

I_{sc}:



$\dfrac{e_1}{1/2} + i_y + e_1 + \dfrac{e_1+3}{1/3} = 0 \rightarrow 9e_1 = -18$

$i_y = \dfrac{e_1+3}{1/3}$ $\qquad e_1 = -2V$

$i_x = e_1/\frac{1}{2}$

$I_{sc} = \dfrac{e_1}{1/2} + \dfrac{e_1+3}{1/3} + 2i_x = 9e_1 + 9 = -9\text{ A}.$

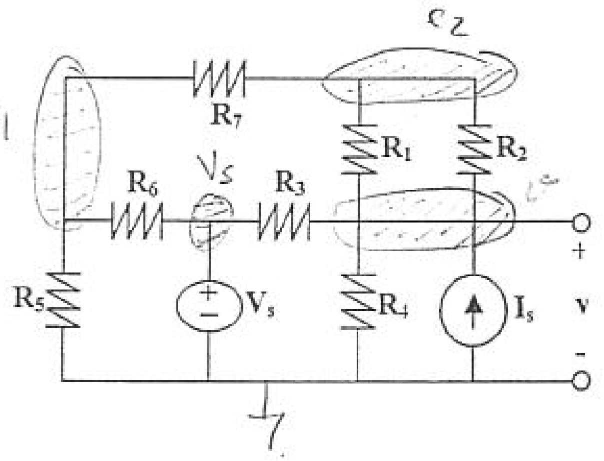
Problem 4: (21 pts)

- Is the following statement TRUE or FALSE ? (1 pt)

[..TRUE..] A circuit with "n" nodes and "k" voltage sources can be completely analyzed by (n-k) node equations with (n-k) node voltages as unknowns.

- Write the node equations for the solution of the circuits given below. Do not introduce more variables than asked.

i) Write 3 equations with 3 unknowns to find v. Do not solve or simplify the equations. (5pts)



Unknowns $\{e_1, e_2, v\}$.

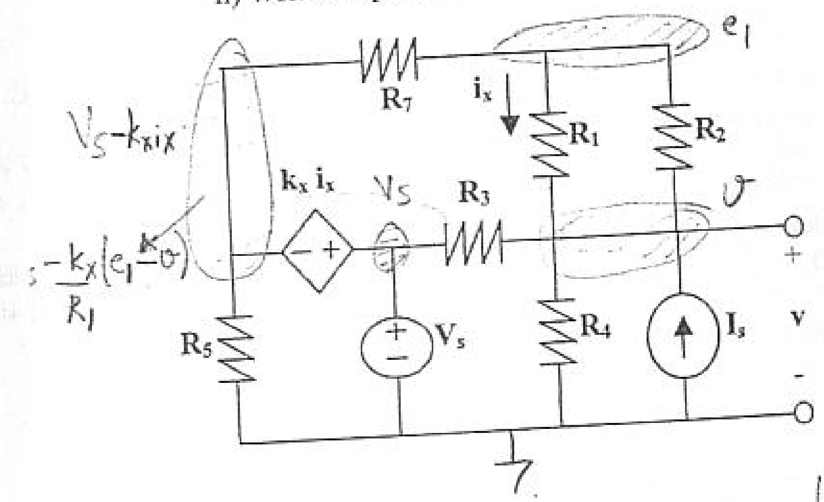
KCL at e_1: $$\frac{e_1}{R_5} + \frac{e_1 - V_s}{R_6} + \frac{e_1 - e_2}{R_7} = 0$$

KCL at e_2: $$\frac{e_2 - v}{R_1} + \frac{e_2 - v}{R_2} + \frac{e_2 - e_1}{R_7} = 0$$

KCL at v: $$\frac{v - e_2}{R_1} + \frac{v - e_2}{R_2} + \frac{v}{R_4} + \frac{v - v_s}{R_3} - I_s = 0$$

ii) Write 2 equations with 2 unknowns to find v. Do not solve for v. (7 pts)

Unknowns: $\{e_1, \vartheta\}$



$$i_x = \frac{e_1 - \vartheta}{R_1}$$

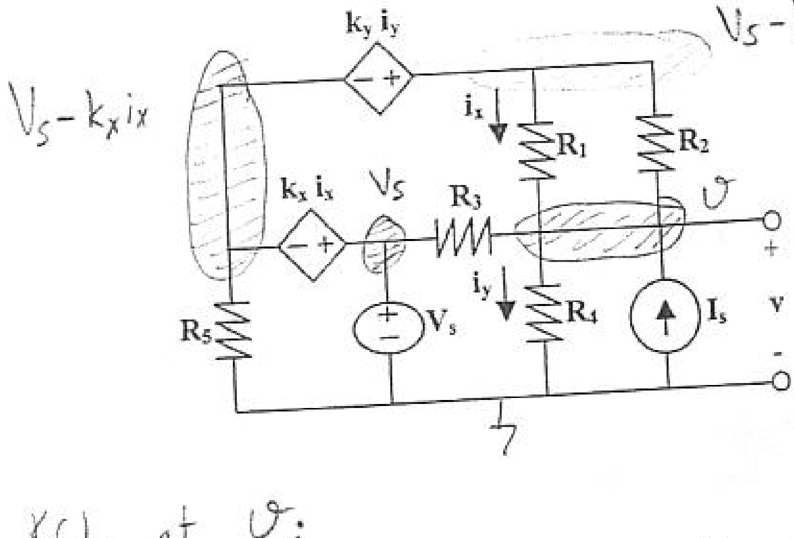
KCL at e_1: $$\frac{e_1-\vartheta}{R_1} + \frac{e_1-\vartheta}{R_2} + \frac{e_1 - \left(V_s - \frac{k_x}{R_1}(e_1-\vartheta)\right)}{R_7} = 0$$

KCL at ϑ: $$\frac{\vartheta - e_1}{R_1} + \frac{\vartheta - e_1}{R_2} + \frac{\vartheta - \vartheta_s}{R_3} + \frac{\vartheta}{R_4} - I_s = 0$$

iii) Write only one node equation to find v. Solve for v. (8 pts)

Unknown: $\{\vartheta\}$

$V_s - k_x i_x + k_y i_y$



$$\boxed{i_y = \vartheta / R_4}$$

$$i_x = \frac{\vartheta_s - k_x i_x + k_y i_y - \vartheta}{R_1}$$

$$(R_1 + k_x) i_x = \vartheta_s - \vartheta + \frac{k_y}{R_4}\vartheta.$$

KCL at ϑ:

$$\frac{\vartheta - (\vartheta_s - k_x i_x + k_y i_y)}{R_1} + \frac{\vartheta - (\vartheta_s - k_x i_x + k_y i_y)}{R_2} + \frac{\vartheta - \vartheta_s}{R_3} + \frac{\vartheta}{R_4} - I_s = 0$$

$$\boxed{i_x = \frac{R_4 \vartheta_s + \vartheta(k_y - R_4)}{R_4 (R_1 + k_x)}}$$

$$\vartheta = \frac{\vartheta_s / R_1 - \frac{k_x \vartheta_s}{R_1 R_4 (R_1 + k_x)} + \frac{\vartheta_s}{R_2} - \frac{k_y \vartheta_s}{R_2 R_4 (R_1 + k_x)} + \vartheta_s / R_3 + I_s}{\left(1/R_1 + \frac{k_x (k_y - R_4)}{\ldots} - \frac{k_y}{\ldots}\right) + \left(\frac{1}{\ldots} + \frac{k_x (k_y - R_4)}{\ldots} - \frac{k_y}{\ldots}\right) + \frac{1}{R_3} + 1/R_4}$$

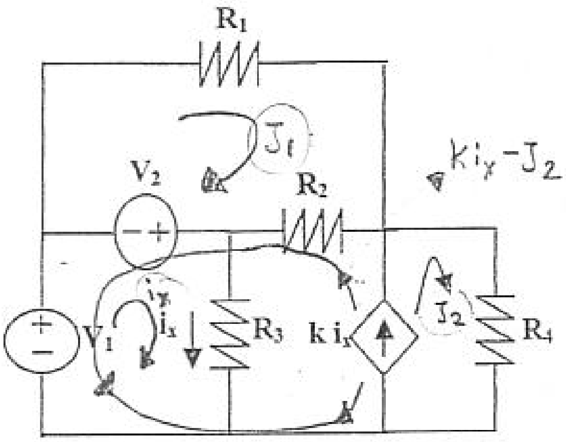
Problem 5: (21 pts)

- Is the following statement TRUE or FALSE ?

[..TRUE..] A circuit with "m" meshes and "k" current sources can be completely analyzed by (m-k) mesh equations with (m-k) unknowns.

- Write down the mesh equations for the circuits given below. One of the mesh currents in each circuit is given. Determine the other current variables and express the solution of circuit in terms of mesh equations.

i) Write 3 mesh equations to find current i_x. Do not solve or simplify the equations. (5 pts)



Note: Selection of current variables significantly simplifies the problem. Make sure that you have fully understood the method.

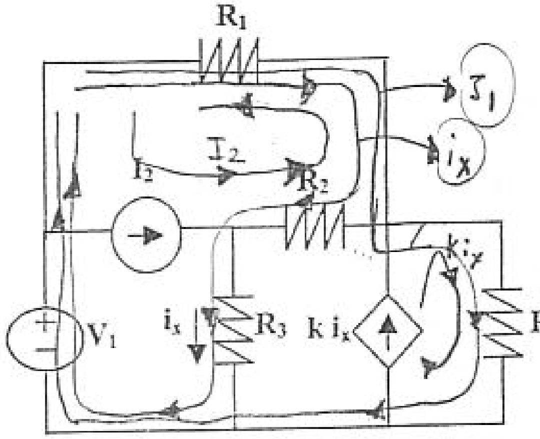
Unknowns: $\{J_1, J_2, i_x\}$

Mesh J_1: $\quad v_2 + R_1 J_1 + R_2 (J_1 + ki_x - J_2) = 0$

Mesh i_x: $\quad -v_1 - v_2 + R_3 i_x = 0$

Mesh $(ki_x - J_2)$ union J_2: $\quad R_2 (ki_x - J_2 + J_1) + v_2 + v_1 - R_4 J_2 = 0$

ii) Write 2 mesh equations with 2 unknowns to find i_x. (7 pts)

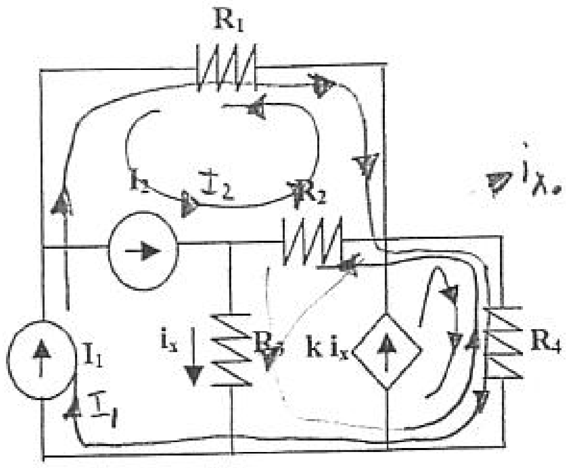


Unknowns: $\{J_1, i_x\}$

Mesh J_1: $R_1(J_1+i_x-I_2)+R_4(ki_x+J_1)-V_1=0$

Mesh i_x: $R_1(J_1+i_x-I_2)+R_2(i_x-I_2)+R_3 i_x - V_1=0$

iii) Write only one equation to find i_x. Solve for i_x. (8 pts)



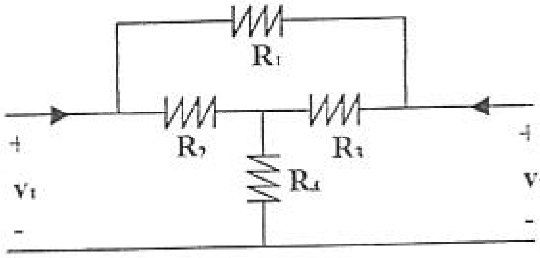
Unknown : i_x

Mesh i_x: $R_3 i_x + R_4(i_x - k i_x - I_1) + R_2(i_x - I_2) = 0$

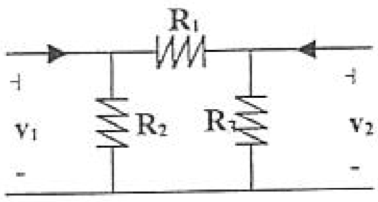
$$i_x = \frac{R_4 I_1 + R_2 I_2}{R_2 + R_3 + R_4(1-k)}$$

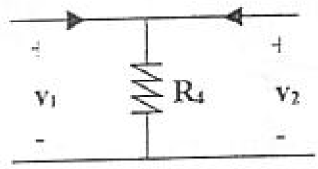
Problem 6: (14 pts)

i) Find the resistance parameters of the following two port: (6 pts)



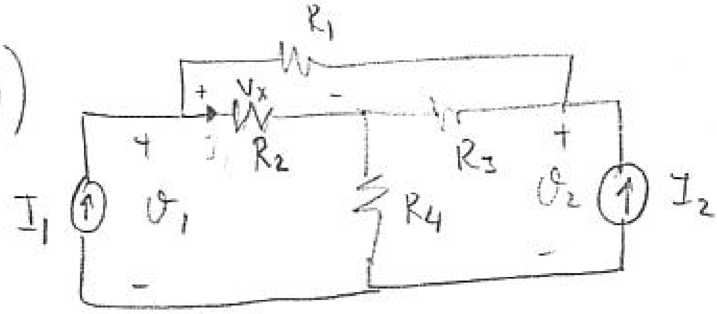
ii) Show that the two port in part i) is the series combination of the following two ports: (4 pts)





iii) Find the resistance parameters of the two ports in part ii). Verify that resistance parameters found in part i) is the addition of parameters found in part ii). (4 pts)

i)



$I_2 = V_x$.

By superposition

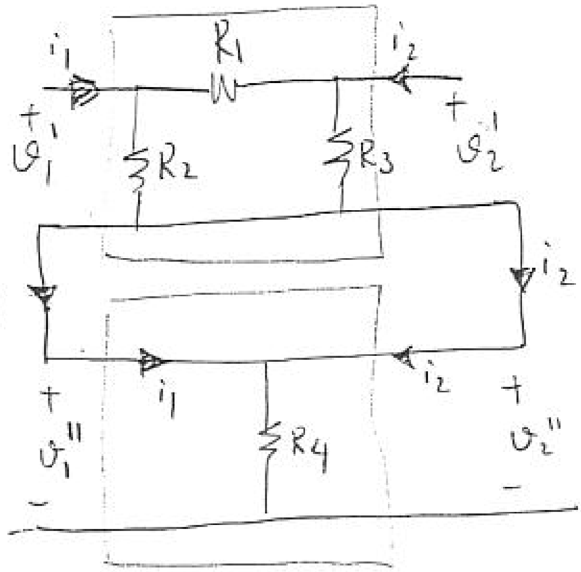
$$\begin{cases} v_1 = I_1\left[(R_1+R_3)//R_2 + R_4\right] + I_2\left[\dfrac{R_2R_3}{R_1+R_2+R_3} + R_4\right] \\ v_2 = I_1\left[\dfrac{R_2R_3}{R_1+R_2+R_3} + R_4\right] + I_2\left[(R_1+R_2)//R_3 + R_4\right] \end{cases}$$

$$r_{11} = (R_1+R_3)//R_2 + R_4$$

$$r_{22} = (R_1+R_2)//R_3 + R_4$$

$$r_{12} = r_{21} = \frac{R_2R_3}{R_1+R_2+R_3} + R_4$$

b)



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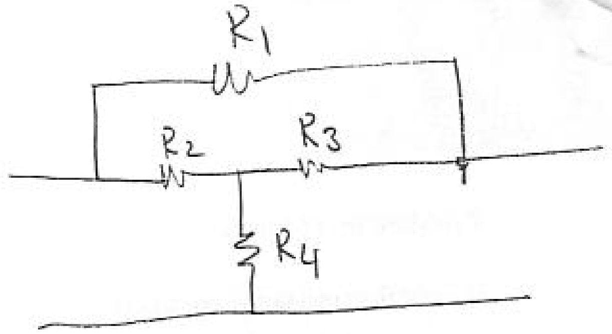
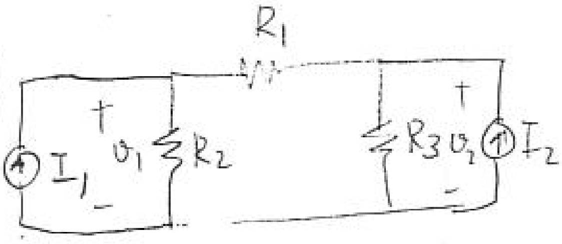
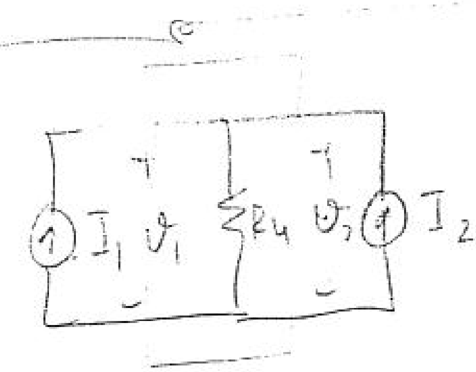


c)



$$v_1 = I_1 \underbrace{\left[R_2 // (R_1+R_3)\right]}_{r_{11}} + I_2 \underbrace{\left[\frac{R_3 R_2}{R_1+R_2+R_3}\right]}_{r_{12}}$$

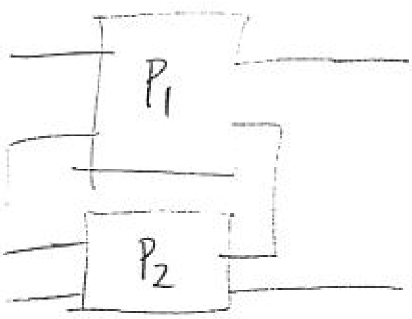
$$v_2 = I_1 \underbrace{\left[\frac{R_2 R_3}{R_1+R_2+R_3}\right]}_{r_{21}} + I_2 \underbrace{\left[(R_1+R_2) // R_3\right]}_{r_{22}}$$



$$v_1 = R_4 I_1 + R_4 I_2$$

$$v_2 = R_4 I_1 + R_4 I_2$$

$$r_{11} = r_{12} = r_{21} = r_{22} = R_4 .$$



$$R_{P_1} + R_{P_2} = \begin{bmatrix} R_2 // (R_1+R_3) + R_4 & \frac{R_3 R_2}{R_1+R_2+R_3} + R_4 \\ \frac{R_2 R_3}{R_1+R_2+R_3} + R_4 & (R_1+R_2)//R_3 + R_4 \end{bmatrix}$$

$=$ $=$

$R_{P_1} + R_{P_2}$ matches the parameters in part i.