— EE 202 —
Exam 2

Summer
2006/2007

① Find inverse Laplace transform of:

$$F(s) = \frac{10\,(s^2+119)}{(s+5)(s^2+10s+169)}$$

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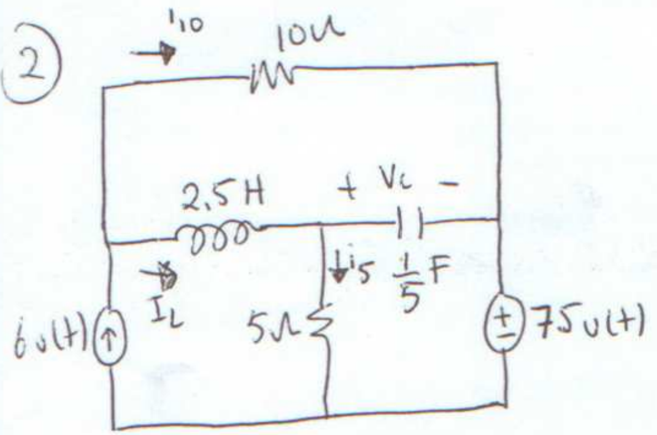
② 

$I_L(0^-) = I_0$
$V_C(0^-) = V_0$

a) Assume there is no initial energy in the circuit. Find value of $I_L(0^+)$ and $V_C(0^+)$, $i_{10}(0^+)$, $i_5(0^+)$

b) Using s-domain analysis find $I_L(t)$. (Assume zero initial conditions.)

c) What should be an initial condition set for $\{V_C(0^-), I_L(0^-)\}$ so that only the mode with the pole at −2 is excited, in the zero-input solution?

Exam 2
Solutions

① $\mathcal{L}^{-1}\{F(s)\} = ?$ $\quad F(s) = \dfrac{10(s^2+119)}{(s+5)(s^2+10s+169)} = \dfrac{A}{s+5} + \dfrac{Bs+C}{(s+5)^2+12^2}$

$$\boxed{A = \frac{10\cdot(119+25)}{144} = 10}$$

Now, let's find B comparing coef's of s^2 after equalization of denom.

$$\frac{10(s^2+119)}{(s+5)\left((s+5)^2+12^2\right)} = \frac{10}{(s+5)} + \frac{Bs+C}{(s+5)^2+12^2}$$

$$(s+5)^2+12^2 \qquad (s+5)$$

then $10s^2 = 10s^2 + Bs^2 \longrightarrow \boxed{B=0}$

then compare coef of "1".

$$1190 = 169\cdot 10 + 5\cdot C \longrightarrow \boxed{C = -2.50 = -100}$$

then $F(s) = \dfrac{10}{s+5} - \dfrac{100}{(s+5)^2+(12)^2} = \dfrac{10}{s+5} - \dfrac{100}{12}\dfrac{12}{(s+5)^2+(12)^2}$

$$f(t) = \left(10e^{-5t} - \frac{100}{12}\cdot e^{-5t}\sin 12\right)u(t)$$

Method 2:

$$\frac{10(s^2+119)}{(s+5)\left((s+5)^2+12^2\right)} = \frac{10}{s+5} + \frac{K}{s+5+j12} + \frac{K^*}{s+5-j12}$$

$$K = \frac{10\left[(-5-j12)^2+119\right]}{(-j12)\ (-j24)} = \frac{10\left[\cancel{25}-\cancel{144}+j120+\cancel{119}\right]}{-144.2}$$

$$= \frac{-j\,\cancel{120}^{5}.\cancel{10}^{5}}{\cancel{144}_{\cancel{12}\,6}.\cancel{2}} = -j\frac{25}{6}$$

$$= \frac{25}{6}e^{-j\frac{\pi}{2}}$$

$$F(s) = \frac{10}{s+5} + \frac{\frac{25}{6}e^{-j\frac{\pi}{2}}}{s+5+j12} + \frac{\frac{25}{6}e^{+j\pi/2}}{s+5-j12}.$$

$$f(t) = \left[10e^{-5t} + \frac{25}{6}e^{-j\frac{\pi}{2}}e^{-(5+j12)t} + \frac{25}{6}e^{+j\frac{\pi}{2}}e^{-(5-j12)t}\right]u(t)$$

$$f(t) = \left(10e^{-5t} + 2.\ \mathrm{Re}\left\{\frac{25}{6}e^{-j\frac{\pi}{2}}e^{-(5+j12)t}\right\}\right)u(t)$$

$$= \left[10e^{-5t} + \frac{50}{6}e^{-5t}\cos\left(12t+\frac{\pi}{2}\right)\right]u(t)$$

$$= \left(10e^{-5t} - \frac{50}{6}e^{-5t}\sin(12t)\right)u(t)$$

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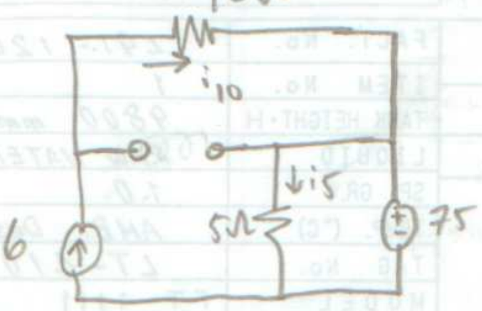
2) a)



at $t=0^+$

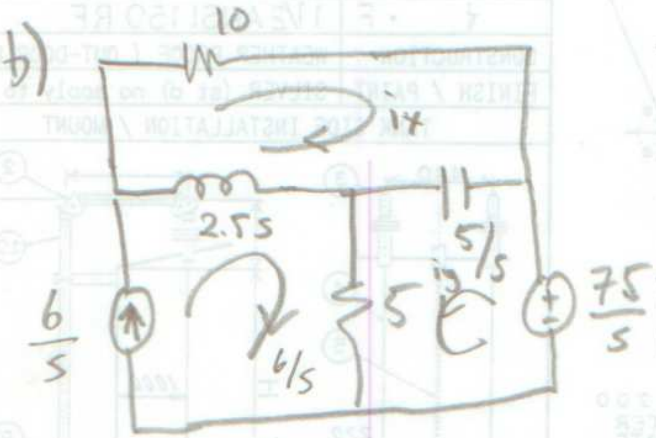
$i_{10}(0^+) = 6\ A$

$i_5(0^+) = 15\ A.$

$v_c(0^+) = v_c(0^-) = 0\ V$

$I_L(0^+) = I_L(0^-) = 0\ A$

b)



$\boxed{\text{KVL}: i_x}$

$$0 = 10 i_x + \frac{5}{s}(i_x + i_y) + 2.5s\left(i_x - \frac{6}{s}\right)$$

$\boxed{\text{KVL}: i_y}$

$$\frac{75}{s} = \frac{5}{s}(i_y + i_x) + 5\left(i_y + \frac{6}{s}\right).$$

$$\begin{bmatrix} 10 + \frac{5}{s} + \frac{5}{2}s & 5/s \\ \frac{5}{s} & 5 + \frac{5}{s} \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} = \begin{bmatrix} 15. \\ \frac{45}{s} \end{bmatrix}$$

$$\Delta(s) = 50 + \frac{25}{s} + \frac{25}{2}s + \frac{50}{s} + \cancel{\frac{25}{s^2}} + \frac{25}{2} - \cancel{\frac{25}{s^2}}$$

$$= 62.5 + \frac{75}{s} + 12.5 s.$$

$$= 12.5\left(5 + \frac{6}{s} + s\right) = \frac{12.5}{s}\left(s^2 + 5s + 6\right)$$

$$\left| \begin{bmatrix} 15 & 5/s \\ 45/s & s+\frac{5}{s} \end{bmatrix} \right| = 15\left(s+\frac{5}{s}\right) - \frac{5}{s}\cdot\frac{45}{s} \qquad 4$$

$$= 75\left[1+\frac{1}{s}-\frac{3}{s^2}\right]$$

$$\Delta_1(s) = \frac{75}{s^2}\left[s^2+s-3\right]$$

$$\boxed{i_x(s) = \frac{\Delta_1(s)}{\Delta(s)} = \frac{75/s^2\left[s^2+s-3\right]}{\frac{12.5}{s}\cdot\left(s^2+5s+6\right)}}$$

$$i_x(s) = \frac{6}{s}\cdot\frac{(s^2+s-3)}{(s^2+5s+6)}$$

$$= \frac{6}{s}\cdot\frac{s^2+s-3}{(s+2)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = -3 \ ; \quad B = 3 \ ; \quad C = +6.$$

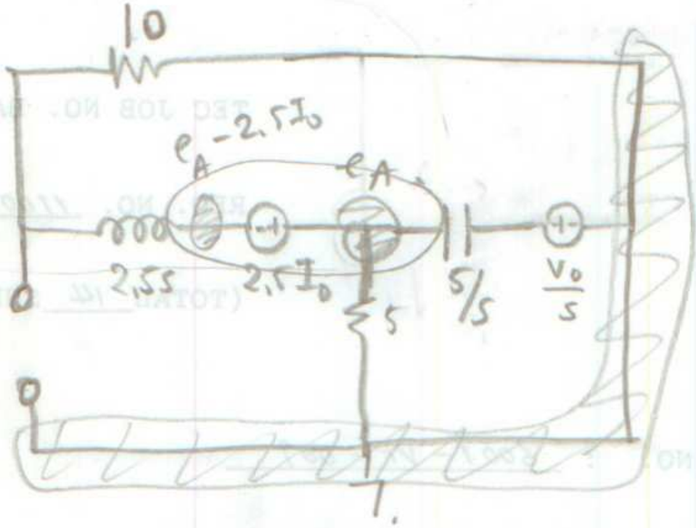
$$i_x(t) = \left(-3 + 3e^{-2t} + 6e^{-3t}\right)u(t)$$

$$i_L(t) = 6u(t) - i_x(t)$$
$$= \left[9 - 3e^{-2t} - 6e^{-3t}\right]u(t)$$

Note
$i_x(0^+) = 6$ A
$i_L(0^+) = 0$ A
as found in part a

c) Zero input.

(5)



$$\frac{e_A - V_0/s}{5/s} + \frac{e_A}{5} + \frac{e_A - 2.5I_0}{2.5s+10} = 0$$

$$e_A\left(\frac{s}{5} + \frac{1}{5} + \frac{2}{5(s+4)}\right) = \frac{V_0}{5} + \frac{I_0}{s+4}.$$

$$e_A\frac{(s^2+5s+6)}{5(s+4)} = \frac{V_0 s + 4V_0 + 5I_0}{5(s+4)}.$$

$$e_A = \frac{V_0 s + 4V_0 + 5I_0}{s^2+5s+6} = \frac{-(V_0+5I_0)}{s+3} + \frac{2V_0+5I_0}{s+2}.$$

When $V_0 = -5I_0 \longrightarrow e_A = \frac{V_0}{s+2}$

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \end{bmatrix} k.$$