# — Quiz #2 —

March 06, 200

## Complex Numbers

① $z_1 = 12 + i4$ ;
$z_2 = 4 + i7$ ; $\Big\}$ Find a) $z_1^2$ c) $z_1 z_2^* + z_1^* z_2$ e) $z_1^{10}$

6pts b) $\dfrac{z_1}{z_2^2}$ d) $e^{z_1}$ f) $z_2^{z_1}$

② Show that if $f(t) = e^{-t}\cos t$ then $f(t) = \mathrm{Re}\{e^{-t+jt}\}$.

Using the result shown; evaluate the indefinite

4pts integral:

$$I(t) = \int e^{-z}\cos z \, dz.$$

(i.e find $I(t)$ such that $\dfrac{\partial}{\partial t} I(t) = e^{-t}\cos t$ )

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① $z_1 = \sqrt{160}\,\angle \tan^{-1} 1/3$ ; $z_2 = \sqrt{65}\,\angle \tan^{-1} 7/4$

a) $z_1^2 = 160\,\angle 2\tan^{-1} 1/3$ [illegible] or $(12+i4)^2 = 128 + i96$.

b) $z_1/z_2^2 = \dfrac{\sqrt{160}\,\angle \tan^{-1} 1/3}{65\,\angle 2\tan^{-1} 7/4} = \dfrac{\sqrt{160}}{65}\,\angle \tan^{-1} 1/3 - 2\tan^{-1} 7/4$ or

$z_1 = \dfrac{4(3+i)}{-33+i56} = \dfrac{-4(3+i)(+33+i56)}{56^2+33^2}$

c) $z_1 z_2^* + z_1^* z_2 = 2\mathrm{Re}\{z_1 z_2^*\} = 2\sqrt{160}\sqrt{65}\cos(\tan^{-1} 1/3 - \tan^{-1} 7/4)$.

d) $e^{z_1} = e^{12+4i} = e^{12}(\cos 4 + i\sin 4)$

e) $z^{10} = (160)^5\,\angle 10\tan^{-1} 1/3$

f) $z_2^{z_1} = (4+i7)^{(12+i4)} = (\sqrt{160})^{12+i4}\, e^{(i\tan^{-1} 1/3)(12+i4)} = (\sqrt{160})^{12}\, e^{-4\tan^{-1} 1/3}\, (160)^{2i}\, e^{i12\tan^{-1} 1/3}$

$\sqrt{160}\,\angle \tan^{-1} 1/3$

$(e^{\ln 160})^{2i} = \cos(\ln 160^2) + i\sin(\ln 160^2)$

② $I(t) = \int e^{-\tau}\cos\tau \, d\tau.$

$$= \int \mathrm{Re}\left\{e^{-\tau + j\tau}\right\} d\tau.$$

$$= \mathrm{Re}\left\{\int e^{-\tau + j\tau} \, d\tau.\right\}$$

$$= \mathrm{Re}\left\{\frac{1}{-1+j} e^{(-1+j)t}\right\}$$

$$= \mathrm{Re}\left\{\frac{-1-j}{2} e^{jt}\right\} e^{-t}$$

$$= \frac{e^{-t}\,\mathrm{Re}\left\{-\cos t + \sin t + j(-\cos t - \sin t)\right\}}{2}$$

$$= \frac{e^{-t}(-\cos t + \sin t)}{2}$$

Check:

$$\frac{d\,I(t)}{dt} = -e^{-t}\left(\frac{-\cos t + \sin t}{2}\right) + \frac{e^{-t}(\sin t + \cos t)}{2}$$

$$= e^{-t}\cos t \quad ✓$$