Feb 27, 200

EE 202

Quiz #1

1) A 4^{th} order circuit has the following response to two different sets of initial conditions.

$$\underline{x}(0) = \begin{bmatrix} 1 \\ ? \\ ? \\ ? \end{bmatrix} \longrightarrow \underline{x}(t) = \begin{bmatrix} e^{-3t} + te^{-3t} \\ ? \\ ? \\ ? \end{bmatrix} \Bigg\} \text{ Set \#1.}$$

$$\underline{x}(0) = \begin{bmatrix} ? \\ 1 \\ ? \\ ? \end{bmatrix} \longrightarrow \underline{x}(t) = \begin{bmatrix} ? \\ \cos t + 4\sin t \\ ? \\ ? \end{bmatrix} \Bigg\} \text{ Set \#2.}$$

a) What are the natural frequencies of this circuit?

b) Give the type or form of response to the external input

$\longrightarrow$ i) $v_{input}(t) = \cos 3t$

$\hookrightarrow$ ii) $v_{input}(t) = \cos t$

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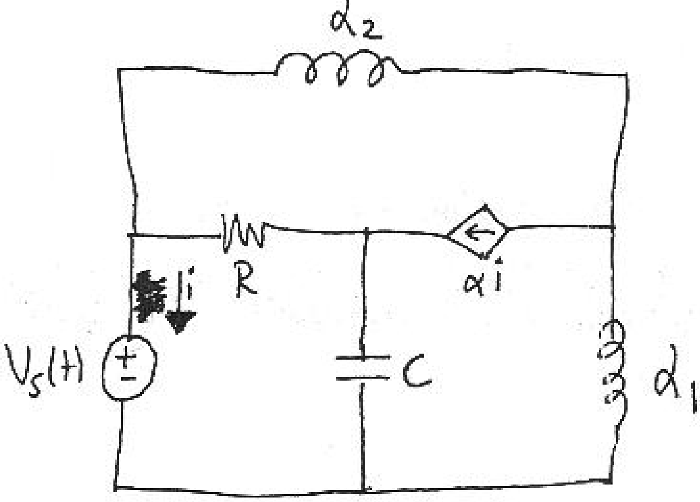
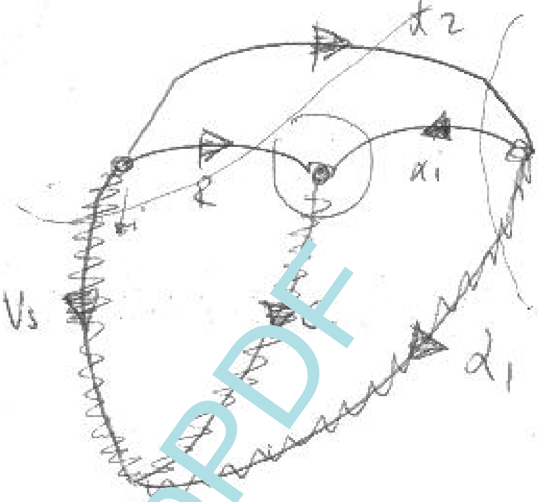
a) 4 natural frequencies are $= \{-3, -3, j, -j\}$

b) i) Response $\Rightarrow c_1 e^{-3t} + c_2 t e^{-3t} + c_3 \cos t + c_4 \sin t + A\cos 3t + B\sin 3t$

ii) Response $\Rightarrow c_1 e^{-3t} + c_2 t e^{-3t} + c_3 \cos t + c_4 \sin t + At\cos t + Bt\sin t$

② Write State Equations for the following circuit:



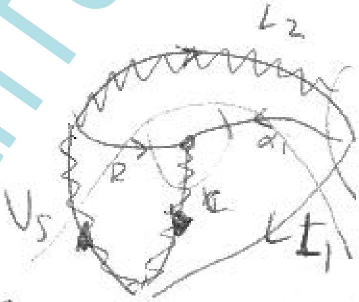


(Choice 1)

State Var's

For (Choice 1): $\{V_c, I_{L_2}\}$

For (Choice 2): $\{V_c, I_{L_1}\}$



(Choice 2)

Choice 1:

$$C\dot{V}_c = i_R + \alpha i$$

$$= \frac{V_s - V_c}{R} + \alpha[-i_R - i_{L_2}]$$

$$\boxed{C\dot{V}_c = \frac{(1-\alpha)[V_s - V_c]}{R} - \alpha i_{L_2}}$$

$$L\dot{I}_{L_2} = V_s - V_{L_1}$$

$$= V_s - L_1 \dot{I}_{L_1}$$

$$= V_s - L_1\frac{d(I_{L_2} - \alpha i)}{dt}$$

$$= V_s - L_1 \dot{I}_{L_2} + \alpha L_1[-\dot{i}_R - \dot{i}_{L_2}]$$

$$= V_s - L_1(1+\alpha)\dot{I}_{L_2} - \frac{\alpha L_1}{R}(\dot{V}_s - \dot{V}_c)$$

$$\boxed{L_1(2+\alpha)\dot{I}_{L_2} = V_s - \frac{\alpha L_1}{R_1}\dot{V}_s + \frac{\alpha L_1}{R}\left[\frac{1-\alpha}{RC}(V_s - V_c) - \frac{\alpha}{C} i_{L_2}\right]}$$

Choice 2:

$$C\dot{V}_c = i_R + \alpha i = \frac{V_s - V_c}{R} + \alpha(-i_R - \alpha i - i_{L_1})$$

$$i = -i_R - \alpha i - i_{L_1} \rightarrow i = \frac{-i_R - i_{L_1}}{(1+\alpha)}$$

$$\boxed{C\dot{V}_c = \frac{1}{1+\alpha}\frac{V_s - V_c}{R} - \frac{\alpha}{1+\alpha} i_{L_1}}$$

$$L\dot{I}_{L_1} = -L\dot{I}_{L_2} + V_s$$

$$= -L\left(\frac{d}{dt}(\alpha i + I_{L_1})\right) + V_s$$

$$= -L\dot{I}_{L_1} + \frac{L\alpha}{1+\alpha}[\dot{i}_R + \dot{I}_{L_1}] + V_s$$

$$\frac{L(2+\alpha)}{1+\alpha}\dot{I}_{L_1} = \frac{L\alpha}{1+\alpha}\,\frac{\dot{V}_s - \dot{V}_c}{R} + V_s$$

found above.