EE 503
HW #5
Due: Jan. 7th, 2011

We examine the noise cancellation application of Wiener filtering in this homework. Please read *Section 7.2.3 Noise Cancellation* of Hayes before starting the homework.

In the following figure, *w[n]* is white noise with zero mean and unit variance. The sequences $w_1[n]$ and $w_2[n]$ are correlated random processes with zero mean and with variances of $\sigma_{w_1}^2$ and 1 respectively. In the set-up shown in the figure, the desired sequence *d[n]* is corrupted by the noise $w_1[n]$. Our goal is to estimate $w_1[n]$ from the auxiliary information $w_2[n]$ and cancel it as shown in the figure.

<u>Part A:</u>

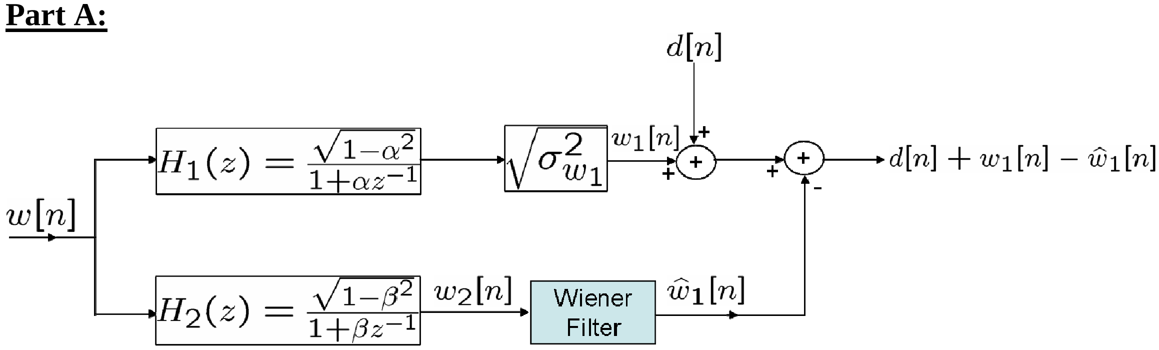

Figure 1: Noise Cancellation Configuration

A1. Execute the following in Matlab command line

>> load handel

In the work place, you should see "y" and "Fs" as Matlab variables. After adjusting the volume of your speakers to a comfortable level, execute the following:

>> sound (y,Fs);

You should hear the famous movement of Handel. In this homework, we use Handel sequence for noise cancellation experiment.

Now, execute the following

>> yzm = y – mean(y);

and plot yzm sequence and visually observe that it is a zero mean sequence. Now listen yzm using the sound command, there should not any noticeable difference. The desired sequence shown as *d[n]* in Figure 1 is the "yzm" sequence.

Calculate the variance of yzm sequence, call this variance as σ_d^2. The following is the definition of sample SNR for noise corrupted signal, *d[n]* + *w₁[n]*:

$$SNR = \frac{\sigma_d^2}{\sigma_{w_1}^2}$$

A2. Analytically calculate the auto-correlation of $w_2[n]$ and the cross-correlation of $w_1[n]$ and $w_2[n]$ in terms of α and β. (Report your calculations.)

A3. Set $\sigma_{w_1}^2$ such that sample SNR is 5 dB. Set $\alpha = -0.8$ and $\beta = -0.5$ and let the Wiener filter have P=5 taps (4^{th} order filter).

a. Listen $d[n] + w_1[n]$ at this SNR level.
b. Estimate the auto-correlation of $w_2[n]$ and the cross-correlation of $w_1[n]$ and $w_2[n]$ using xcorr command of Matlab. Compare your estimates with the ones you have found in step A2.
c. Implement the P tap Wiener filter using estimated coefficients. Examine the variance of the noise term effecting d[n] before and after noise cancellation, that is compare the variance of $w_1[n]$ and $w_1[n] - \hat{w}_1[n]$.
d. Listen the noise cancelled signal.

A4. Repeat part A3 for $\alpha = -0.8$, $\beta = \{0.9, 0.85, 0.5, 0.25, -0.25, -0.5, -0.85, -0.9\}$. (Present the results of part A3d in your report). How can you explain the change in error with β ?

A5. Repeat part A3 for $\alpha = -0.8$, $\beta = 0.25$, for P={2, 4, 8, 16, 32}. (Present the results of part A3d in your report).

A6. Repeat part A3 for $\alpha = -0.8$, $\beta = 0.25$, P=8 and SNR = {5, 0 , -5 -10} dB. (Present the results of part A3d in your report).

Everything should work fine up to this point. But, in practice, we face a more difficult problem.

Part B:

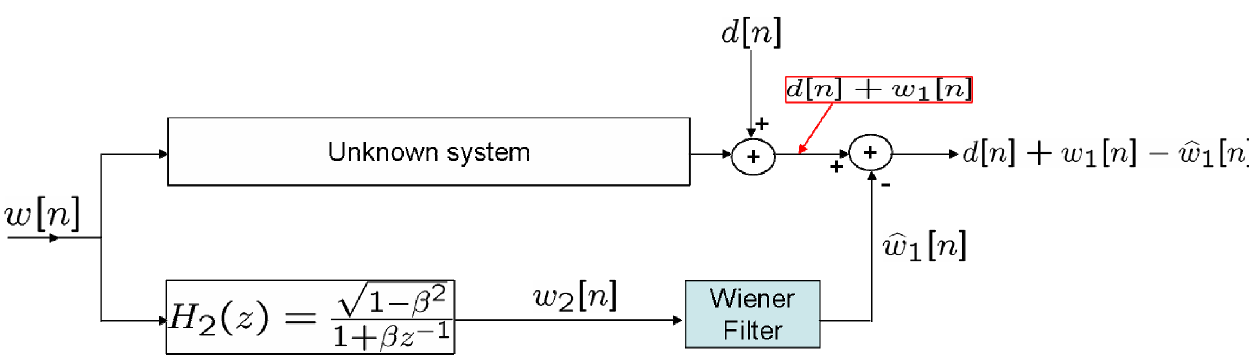

Figure 2: Practical Scheme

In practice, it is not possible to observe $w_1[n]$ directly, but we can only observe $d[n] + w_1[n]$. If we could have observed $w_1[n]$ and $d[n] + w_1[n]$ individually, then we can subtract $w_1[n]$ from the observation, so there is no need to estimate $\hat{w}_1[n]$!!!

Figure 2 shows the more practical scheme for noise cancellation. In this scheme different from the earlier one, we can only observe $d[n]+w_1[n]$, not $w_1[n]$.

B1. Let the unknown system be the $H_1[z]$ given part A. Set $\alpha=-0.8$ and $\sigma_{w_1}^2$ such that sample SNR is 5 dB. Set $\beta=-0.5$. Let the Wiener filter have P=5 taps (4th order filter).

a. Estimate the auto-correlation of $w_2[n]$ and the cross-correlation of $(d[n]+w_1[n])$ and $w_2[n]$. Compare the cross-correlation estimate with the one found in part A. Do you expect a similar result ? Why ?
b. Implement P tap Wiener filter and examine the variance of the noise term effecting d[n] before and after noise cancellation, that is compare the variance of $w_1[n]$ and $w_1[n]-\hat{w}_1[n]$.
c. Compare the result of part b with the corresponding result when $w_1[n]$ is available instead of $d[n]+w_1[n]$.

B2. Repeat part A6 and report your results.

Yet there is another difficulty that we face in practice.

Part C:

In practice, it is not possible to observe $w_2[n]$ alone that is without the presence $d[n]$. Even though by a sufficient isolation of the desired source from the second receiver (auxiliary channel), the leakage of $d[n]$ into the auxiliary signal (signal at the input of the Wiener filter) is in many scenarios unavoidable.

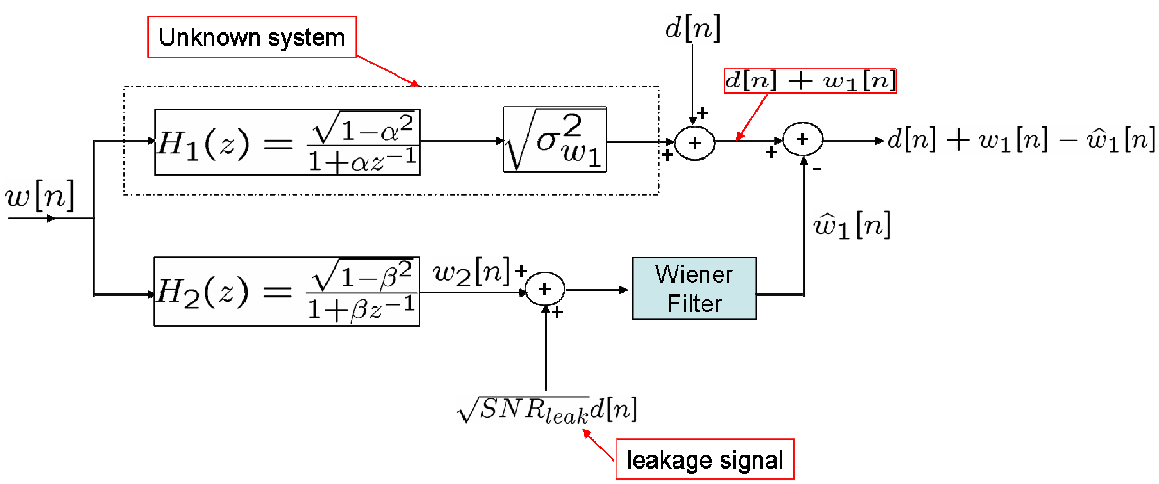

Figure 3: Practical scheme with leakage term

Note that when SNR_{leak} is 0 (that is, $SNR_{leak}=-\infty$ dB), Figure 3 is the same as Figure 2 given before.

C1. Let the unknown system be $H_1[z]$ given part A. Set $\alpha = -0.8$ and $\sigma^2_{w_1}$ be such that the sample SNR is 5 dB. Let SNR_{leak} = *-10* dB and the Wiener filter have P=5 taps (4th order filter).

a. Estimate the auto-correlation of $(w_2[n] + \sqrt{SNR_{leak}}\,d[n])$ and the cross-correlation of $(d[n]+w_1[n])$ and $(w_2[n] + \sqrt{SNR_{leak}}\,d[n])$. Compare the cross-correlation estimate with the one found in part A. Do you expect a similar result?
b. Implement P tap Wiener filter and examine the variance of the noise term effecting d[n] before and after noise cancellation, that is compare the variance of $w_1[n]$ and $w_1[n] - \hat{w}_1[n]$.
c. Compare the result of part b with the corresponding result given in Part A.

C2. Repeat part C1 for SNR_{leak} = *{-5,0,5}* dB. Compare your results with the corresponding results given in Part A. Report your comparisons.