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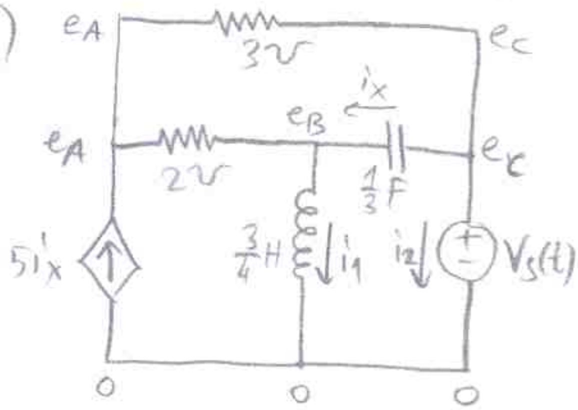
EE202 – CIRCUIT THEORY
HW #1

1. (1c in ZPS 1)

(i)


e_A, e_B, e_C, i_1, i_2 are unknowns.

KCL at e_A:

$3(e_A-e_C)+2(e_A-e_B)-5i_x=0$

$5e_A-2e_B-3e_C=5i_x$

KCL at e_B: $2(e_B-e_A)-i_x+i_1=0$

$-2e_A+2e_B+i_1=i_x$

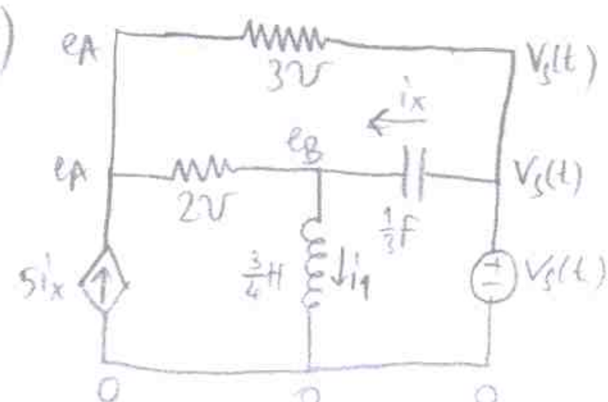
$i_x=\frac{1}{3}(\dot{e}_C-\dot{e}_B)$, KCL at e_C: $i_x+3(e_C-e_A)+i_2=0$

$\Longrightarrow$ $5e_A-2e_B-3e_C=5i_x=\frac{5}{3}(\dot{e}_C-\dot{e}_B)$, We also know that

$-2e_A+2e_B+i_1=i_x=\frac{1}{3}(\dot{e}_C-\dot{e}_B)$, $e_C=V_s(t)$

$-3e_A+3e_C+i_2=-i_x=\frac{1}{3}(\dot{e}_B-\dot{e}_C)$, $e_B=\frac{3}{4}\dot{i}_1$

$$\begin{bmatrix} 5 & -2+\frac{5}{3}D & -3-\frac{5}{3}D & 0 & 0 \\ -2 & 2+\frac{1}{3}D & -\frac{1}{3}D & 1 & 0 \\ -3 & -\frac{1}{3}D & 3+\frac{1}{3}D & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{4}D & 0 \end{bmatrix}\begin{bmatrix} e_A \\ e_B \\ e_C \\ i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ V_s(t) \\ 0 \end{bmatrix}$$

(ii)


e_A, e_B are unknowns

KCL at e_A:

$3(e_A-V_s(t))+2(e_A-e_B)-5i_x=0$

$5e_A-2e_B=5i_x+3V_s(t)$

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KCL at e_B: $2(e_B - e_A) + i_1 - i_x = 0$

$\Rightarrow -2e_A + 2e_B + i_1 = i_x$, from terminal equations:

$\frac{3}{4}\dot{i}_1 = e_B$, $\frac{1}{3}(\dot{V}_s(t) - \dot{e}_B) = i_x$

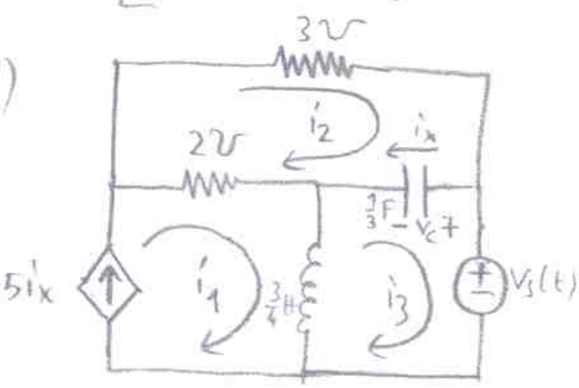
$\Rightarrow 5e_A - 2e_B = \frac{5}{3}\dot{V}_s(t) - \frac{5}{3}\dot{e}_B + 3V_s(t)$

$-2\dot{e}_A + 2\dot{e}_B = \frac{1}{3}\ddot{V}_s(t) - \frac{1}{3}\ddot{e}_B - \frac{4}{3}e_B$

$-2\dot{e}_A + \frac{1}{3}\ddot{e}_B + 2\dot{e}_B + \frac{4}{3}e_B = \frac{1}{3}\ddot{V}_s(t)$, $-6\dot{e}_A + \ddot{e}_B + 6\dot{e}_B + 4e_B = \ddot{V}_s(t)$

$$\Rightarrow \begin{bmatrix} 5 & -2+\frac{5}{3}D \\ -6D & D^2+6D+4 \end{bmatrix} \begin{bmatrix} e_A \\ e_B \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ddot{V}_s(t) + \begin{bmatrix} \frac{5}{3} \\ 0 \end{bmatrix} \dot{V}_s(t) + \begin{bmatrix} 3 \\ 0 \end{bmatrix} V_s(t)$$

(iii)



i_1, i_2, i_3 are unknowns.

$5i_x = i_1$; $i_2 - i_3 = i_x$

KVL at mesh 2:

$\frac{i_2}{3} + V_c + \frac{i_2 - i_1}{2} = 0$

KVL at mesh 3: $V_s(t) + \frac{3}{4}(\dot{i}_3 - \dot{i}_1) - V_c = 0$

$\Rightarrow V_s(t) + \frac{3}{4}\dot{i}_3 - \frac{3}{4}\dot{i}_1 = V_c = \frac{i_1}{2} - \frac{5i_2}{6} \Rightarrow \frac{3}{4}\dot{i}_1 + \frac{i_1}{2} - \frac{5i_2}{6} - \frac{3}{4}\dot{i}_3 = V_s(t)$

$\frac{i_1}{5} = i_x = i_2 - i_3 \Rightarrow \frac{i_1}{5} - i_2 + i_3 = 0$, $i_x = \frac{1}{3}\dot{V}_c = \frac{1}{6}\dot{i}_1 - \frac{5}{18}\dot{i}_2$

$\Rightarrow \frac{i_1}{5} - \frac{1}{6}\dot{i}_1 + \frac{5}{18}\dot{i}_2 = 0$

$$\begin{bmatrix} \frac{1}{2}+\frac{3}{4}D & -\frac{5}{6} & -\frac{3}{4}D \\ \frac{1}{5} & -1 & 1 \\ \frac{1}{5}-\frac{1}{6}D & \frac{5}{18}D & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_s(t) \\ 0 \\ 0 \end{bmatrix}$$

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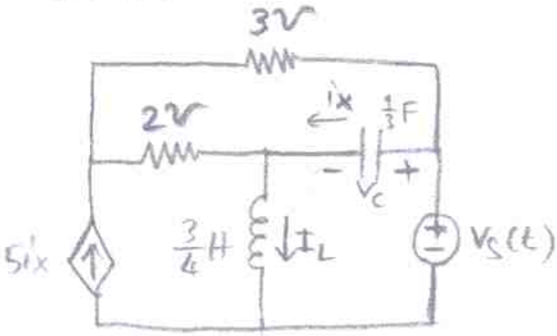

$\{4,5,6\} \rightarrow$ tree branches

$\{1,2,3\} \rightarrow$ co-tree branches

$\{I_L, V_c\}$ are state variables

$FC_2: \; I_L = i_x + i_2 = i_x + 2V_2 = \frac{1}{3}\dot{V}_c + 2V_2$

$FL_1: \; V_2 = V_4 + V_C = \frac{1}{3} i_4 + V_c \implies i_4 = 3V_2 - 3V_c$

$FC_1: \; 5i_x = i_2 + i_4 = 2V_2 + i_4 = 5V_2 - 3V_c$

$\implies \frac{5}{3}\dot{V}_c = 5V_2 - 3V_c \implies I_L = \frac{1}{3}\dot{V}_c + 2V_2 = \frac{1}{3}\dot{V}_c + \frac{2}{5}\left(\frac{5}{3}\dot{V}_c + 3V_c\right)$

$= \dot{V}_c + \frac{6}{5}V_c$

$FL_2: \; V_s(t) = V_c + \frac{3}{4}\dot{I}_L$

$$\begin{bmatrix} \dot{V}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -\frac{6}{5} & 1 \\ -\frac{4}{3} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{4}{3} \end{bmatrix} V_s(t)$$

Let $\underline{x} = \begin{bmatrix} V_c \\ I_L \end{bmatrix}$ Then $\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}$ where $\underline{A} = \begin{bmatrix} -\frac{6}{5} & 1 \\ -\frac{4}{3} & 0 \end{bmatrix}$, $\underline{B} = \begin{bmatrix} 0 \\ \frac{4}{3}V_s(t) \end{bmatrix}$

$$|\lambda I - A| = \begin{vmatrix} \lambda + \frac{6}{5} & -1 \\ \frac{4}{3} & \lambda \end{vmatrix} = \lambda^2 + \frac{6}{5}\lambda + \frac{4}{3} = \left(\lambda + \frac{9 - J\sqrt{219}}{15}\right)\left(\lambda + \frac{9 + J\sqrt{219}}{15}\right)$$

Natural frequencies are $\frac{-9 + J\sqrt{219}}{15}$ and $\frac{-9 - J\sqrt{219}}{15}$ s^{-1}.

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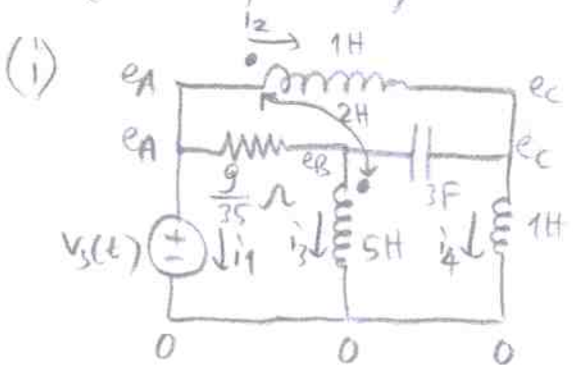
2. (1d in ZPS-1)

(i)



$\{e_A, e_B, e_C, i_1, i_2, i_3, i_4\}$ are unknowns

KCL at e_A:

$$\frac{35}{9}(e_A - e_B) + i_2 + i_1 = 0$$

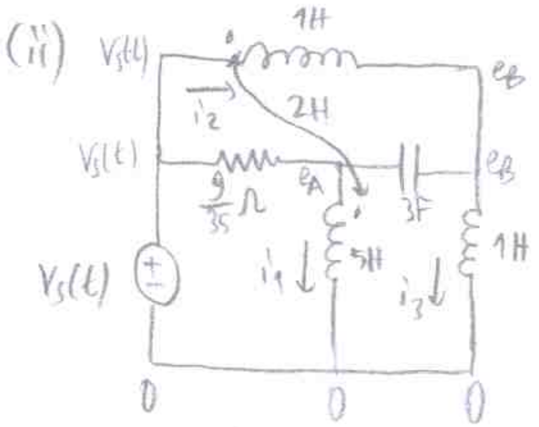
KCL at e_B: $\frac{35}{9}(e_B - e_A) + i_3 + 3(\dot{e}_B - \dot{e}_C) = 0$

KCL at e_C: $3(\dot{e}_C - \dot{e}_B) + i_4 - i_2 = 0$

$$\begin{bmatrix} e_A - e_C \\ e_B \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} \dot{i}_2 \\ \dot{i}_3 \end{bmatrix}, \quad e_C = \dot{i}_4, \quad e_A = V_s(t)$$

$$\begin{bmatrix} \frac{35}{9} & -\frac{35}{9} & 0 & 1 & 1 & 0 & 0 \\ -\frac{35}{9} & \frac{35}{9}+3D & -3D & 0 & 0 & 1 & 0 \\ 0 & -3D & 3D & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & -D & -2D & 0 \\ 0 & 1 & 0 & 0 & -2D & -5D & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -D \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_A \\ e_B \\ e_C \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_s(t) \end{bmatrix}$$

(ii)



$\{e_A, e_B\}$ are unknowns

KCL at e_A: $\frac{35}{9}(e_A - V_s(t)) + i_1 + 3(\dot{e}_A - \dot{e}_B) = 0$

KCL at e_B: $3(\dot{e}_B - \dot{e}_A) + i_3 - i_2 = 0$

$$\begin{bmatrix} e_A \\ V_s(t) - e_B \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix}, \quad e_B = \dot{i}_3$$

$e_A = 5\dot{i}_1 + 2\dot{i}_2$, $V_s(t) - e_B = 2\dot{i}_1 + \dot{i}_2$, $\frac{35}{9}e_A - \frac{35}{9}V_s(t) + i_1 = 3(\dot{e}_B - \dot{e}_A) = i_2 - i_3$

$$\Rightarrow \frac{35}{9}\dot{e}_A - \frac{35}{9}\dot{V}_s(t) = -\dot{i}_1 + \dot{i}_2 - \dot{i}_3$$

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$$\dot{i}_1 = e_A + 2e_B - 2V_s(t) \quad , \quad \dot{i}_2 = 5V_s(t) - 2e_A - 5e_B$$

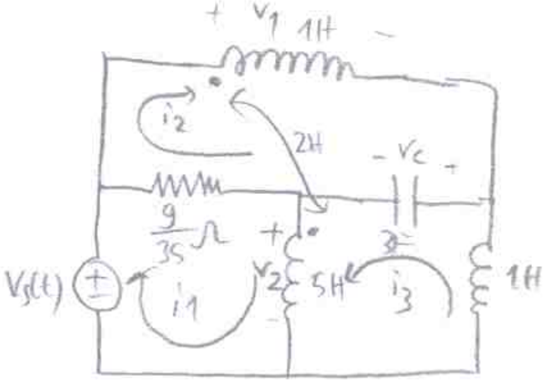
$$\Longrightarrow \frac{35}{9}\dot{e}_A - \frac{35}{9}\dot{V}_s(t) = -\dot{i}_1 + \dot{i}_2 - \dot{i}_3 = -3e_A - 8e_B + 7V_s(t)$$

$$\frac{35}{9}\dot{e}_A + 3e_A + 8e_B = \frac{35}{9}\dot{V}_s(t) + 7V_s(t)$$

$$3(\ddot{e}_B - \ddot{e}_A) = \dot{i}_2 - \dot{i}_3 = 5V_s(t) - 2e_A - 6e_B$$

$$\begin{bmatrix} -3D^2+2 & 3D^2+6 \\ \frac{35}{9}D+3 & 8 \end{bmatrix}\begin{bmatrix} e_A \\ e_B \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{35}{9} \end{bmatrix}\dot{V}_s(t) + \begin{bmatrix} 5 \\ 7 \end{bmatrix}V_s(t)$$

(iii)



$\{i_1, i_2, i_3\}$ are unknowns

KVL at mesh 1: $-V_s(t) + \frac{9}{35}(i_1 - i_2) + V_2 = 0$

KVL at mesh 2: $\frac{9}{35}(i_2 - i_1) + V_1 + V_c = 0$

KVL at mesh 3: $V_c + V_2 + \dot{i}_3 = 0$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} \dot{i}_2 \\ \dot{i}_1 + \dot{i}_3 \end{bmatrix} , \quad i_2 + i_3 = 3\dot{V}_C \quad , \quad \begin{aligned} V_1 &= 2\dot{i}_1 + \dot{i}_2 + 2\dot{i}_3 \\ V_2 &= 5\dot{i}_1 + 2\dot{i}_2 + 5\dot{i}_3 \end{aligned}$$

$$\Longrightarrow 5\dot{i}_1 + \frac{9}{35}i_1 + 2\dot{i}_2 - \frac{9}{35}i_2 + 5\dot{i}_3 = V_s(t) \quad , \quad \frac{9}{35}(i_2 - i_1) + V_1 = -V_c = V_2 + \dot{i}_3$$

$$\frac{9}{35}(i_2 - i_1) - \dot{i}_3 = 3\dot{i}_1 + \dot{i}_2 + 3\dot{i}_3 \Longrightarrow 3\dot{i}_1 + \frac{9}{35}i_1 + \dot{i}_2 - \frac{9}{35}i_2 + 4\dot{i}_3 = 0$$

$$i_2 + i_3 = 3\dot{V}_c = -3(\dot{V}_2 + \ddot{i}_3) = -15\ddot{i}_1 - 6\ddot{i}_2 - 18\ddot{i}_3 \Longrightarrow 15\ddot{i}_1 + 6\ddot{i}_2 + i_2 + 18\ddot{i}_3 + i_3 = 0$$

$$\begin{bmatrix} 5D + \frac{9}{35} & 2D - \frac{9}{35} & 5D \\ 3D + \frac{9}{35} & D - \frac{9}{35} & 4D \\ 15D^2 & 6D^2+1 & 18D^2+1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_s(t) \\ 0 \\ 0 \end{bmatrix}$$

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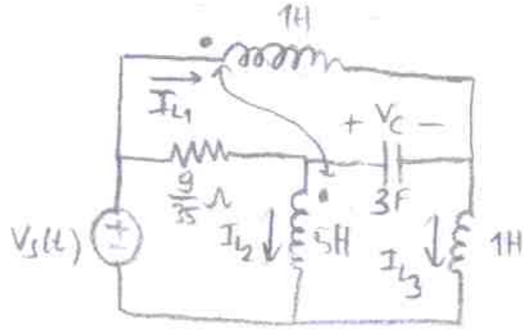

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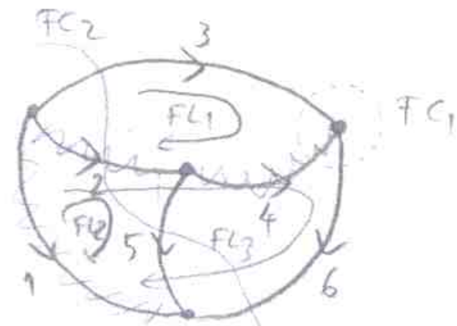

$\{1,2,4\} \rightarrow$ tree branches

$\{3,5,6\} \rightarrow$ co-tree branches

$\{V_C, I_{L_1}, I_{L_2}, I_{L_3}\} \rightarrow$ state variables

$FC_1: \quad 3\dot{V}_C + I_{L_1} = I_{L_3}$

$FL_1: \quad V_3 = V_2 + V_C = \frac{9}{35} i_2 + V_C$

$FC_2: \quad I_{L_1} + i_2 = I_{L_2} + I_{L_3} \implies V_3 = \frac{9}{35}(I_{L_2} + I_{L_3} - I_{L_1}) + V_C$

$FL_2: \quad V_2 + V_5 = V_s(t) = \frac{9}{35}(I_{L_2} + I_{L_3} - I_{L_1}) + V_5$

$$\begin{bmatrix} V_3 \\ V_5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} \dot{I}_{L_1} \\ \dot{I}_{L_2} \end{bmatrix} \implies \begin{bmatrix} \dot{I}_{L_1} \\ \dot{I}_{L_2} \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} V_3 \\ V_5 \end{bmatrix}$$

$$\dot{I}_{L_1} = 5V_3 - 2V_5 = \frac{9}{7}(I_{L_2} + I_{L_3} - I_{L_1}) + 5V_C - 2V_s(t) + \frac{18}{35}(I_{L_2} + I_{L_3} - I_{L_1})$$

$$= \frac{9}{5}(-I_{L_1} + I_{L_2} + I_{L_3}) + 5V_C - 2V_s(t)$$

$$\dot{I}_{L_2} = -2V_3 + V_5 = \frac{-18}{35}(I_{L_2} + I_{L_3} - I_{L_1}) - 2V_C + V_s(t) - \frac{9}{35}(I_{L_2} + I_{L_3} - I_{L_1})$$

$$= \frac{27}{35}(I_{L_1} - I_{L_2} - I_{L_3}) - 2V_C + V_s(t)$$

$FL_3: \quad V_2 + V_C + \dot{I}_{L_3} = V_s(t) \implies \dot{I}_{L_3} = V_s(t) - V_C - \frac{9}{35}(I_{L_2} + I_{L_3} - I_{L_1})$

$$\begin{bmatrix} \dot{V}_C \\ \dot{I}_{L_1} \\ \dot{I}_{L_2} \\ \dot{I}_{L_3} \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 & 1/3 \\ 5 & -9/5 & 9/5 & 9/5 \\ -2 & 27/35 & -27/35 & -27/35 \\ -1 & 9/35 & -9/35 & -9/35 \end{bmatrix} \begin{bmatrix} V_C \\ I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix} V_s(t)$$

Let $\underline{X} = \begin{bmatrix} V_C \\ I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix}$ and $\underline{\underline{A}} = \begin{bmatrix} 0 & -1/3 & 0 & 1/3 \\ 5 & -9/5 & 9/5 & 9/5 \\ -2 & 27/35 & -27/35 & -27/35 \\ -1 & 9/35 & -9/35 & -9/35 \end{bmatrix}$

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$$\begin{vmatrix} \lambda & \frac{1}{3} & 0 & -\frac{1}{3} \\ -5 & \lambda+\frac{9}{5} & \frac{-9}{5} & \frac{-9}{5} \\ 2 & \frac{-27}{35} & \lambda+\frac{27}{35} & \frac{27}{35} \\ 1 & \frac{-9}{35} & \frac{9}{35} & \lambda+\frac{9}{35} \end{vmatrix} = 0 \quad , \quad \begin{vmatrix} 3\lambda & 1 & 0 & -1 \\ -25 & 5\lambda+9 & -9 & -9 \\ 70 & -27 & 35\lambda+27 & 27 \\ 35 & -9 & 9 & 35\lambda+9 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3\lambda & 1 & 0 & -1 \\ 10 & 5\lambda & 0 & 35\lambda \\ -35 & 0 & 35\lambda & -105\lambda \\ 35 & -9 & 9 & 35\lambda+9 \end{vmatrix} = 0 \quad , \quad \begin{vmatrix} 3\lambda & 1 & 0 & -1 \\ 2 & \lambda & 0 & 7\lambda \\ 1 & 0 & -\lambda & 3\lambda \\ 35 & -9 & 9 & 35\lambda+9 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3\lambda & 0 & 0 & -1 \\ 2 & 8\lambda & 0 & 7\lambda \\ 1 & 3\lambda & -\lambda & 3\lambda \\ 35 & 35\lambda & 9 & 35\lambda+9 \end{vmatrix} = 0 \quad , \quad \lambda \begin{vmatrix} 3\lambda & 0 & 0 & -1 \\ 2 & 8 & 0 & 7\lambda \\ 1 & 3 & -\lambda & 3\lambda \\ 35 & 35 & 9 & 35\lambda+9 \end{vmatrix} = 0$$

$$\lambda \begin{vmatrix} 3\lambda & 0 & 0 & -1 \\ 2 & 8 & 0 & -\lambda \\ 1 & 3 & -\lambda & 0 \\ 35 & 35 & 9 & 9 \end{vmatrix} = 0 \quad , \quad 3\lambda^2 \begin{vmatrix} 8 & 0 & -\lambda \\ 3 & -\lambda & 0 \\ 35 & 9 & 9 \end{vmatrix} + \lambda \begin{vmatrix} 2 & 8 & 0 \\ 1 & 3 & -\lambda \\ 35 & 35 & 9 \end{vmatrix} = 0$$

$$\Longrightarrow \lambda(\lambda+1)(35\lambda^2+64\lambda+6) = 0 \quad , \quad \lambda = 0, -1, \frac{-32+\sqrt{814}}{35}, \frac{-32-\sqrt{814}}{35}$$

Natural frequencies are $0, -1, \frac{-32+\sqrt{814}}{35} \approx -0.1$, $\frac{-32-\sqrt{814}}{35} \approx -1.73\ s^{-1}$.

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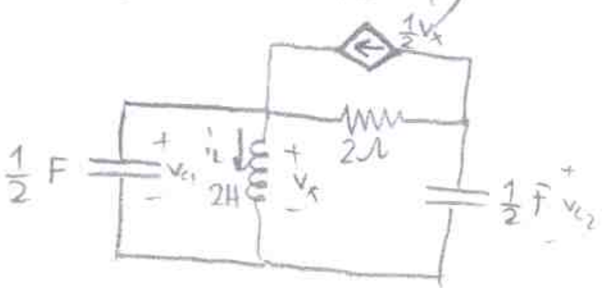
EE202 - CIRCUIT THEORY
HW #1

3. (2a in ZPS-1)



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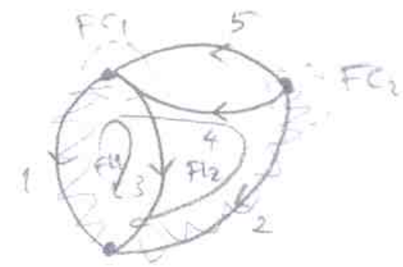

1,2 $\rightarrow$ tree branches ; 3,4,5 $\rightarrow$ co-tree branches
v_{c_1}, v_{c_2}, i_L are unknowns.

$$FC_1: \ \frac{1}{2}\dot{v}_{c_1} + i_L = i_4 + \frac{1}{2}v_x = \frac{v_4}{2} + \frac{v_x}{2}$$

$$FL_1: \ v_{c_1} = v_x \ , \ FC_2: \ \frac{1}{2}\dot{v}_{c_2} + i_4 + \frac{1}{2}v_x = 0 \ , \ \dot{v}_{c_2} = -v_x - v_4$$

$$FL_2: \ v_4 + v_{c_1} = v_{c_2} \ , \ v_x = 2\dot{i}_L$$

$$\Longrightarrow \dot{v}_{c_1} = 2\dot{i}_L + v_4 - 2i_L \ , \ v_{c_1} = 2\dot{i}_L \ , \ \dot{v}_{c_2} = -2\dot{i}_L - v_4 = -v_{c_1} + (v_{c_1} - v_{c_2}) = -v_{c_2}$$

$$\dot{v}_{c_1} = v_{c_1} + (v_{c_2} - v_{c_1}) - 2i_L = v_{c_2} - 2i_L$$

$$\begin{bmatrix} \dot{v}_{c_1} \\ \dot{v}_{c_2} \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{c_1} \\ v_{c_2} \\ i_L \end{bmatrix} \ , \quad \begin{vmatrix} \lambda & -1 & 2 \\ 0 & \lambda+1 & 0 \\ -\frac{1}{2} & 0 & \lambda \end{vmatrix} = \lambda^2(\lambda+1) - \frac{1}{2}(-2)(\lambda+1) = (\lambda+1)(\lambda^2+1)$$

Natural frequencies are $-1, j, -j$ where $j = \sqrt{-1}$

$$-\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \Longrightarrow \left. \begin{matrix} -\gamma_1 = \gamma_2 - 2\gamma_3 \\ -\gamma_3 = \frac{1}{2}\gamma_1 \end{matrix} \right) \Longrightarrow \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = k \begin{bmatrix} 2 \\ -4 \\ -1 \end{bmatrix}$$

Real initial condition exciting the modes is :

$$\begin{bmatrix} v_{c_1}(0^-) \\ v_{c_2}(0^-) \\ i_L(0^-) \end{bmatrix} = k \begin{bmatrix} 2 \\ -4 \\ -1 \end{bmatrix}, \ (k \in \mathbb{R})$$

Note that other eigenvectors corresponding to j and $-j$ are not real vectors

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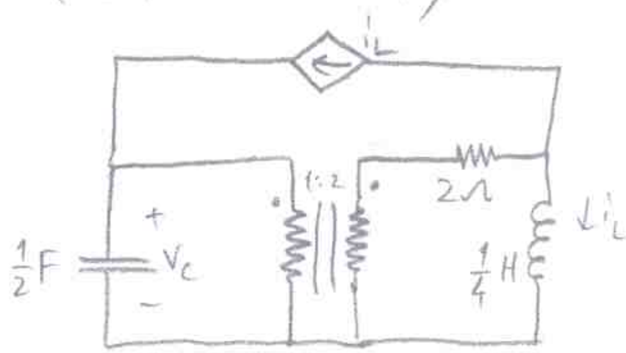
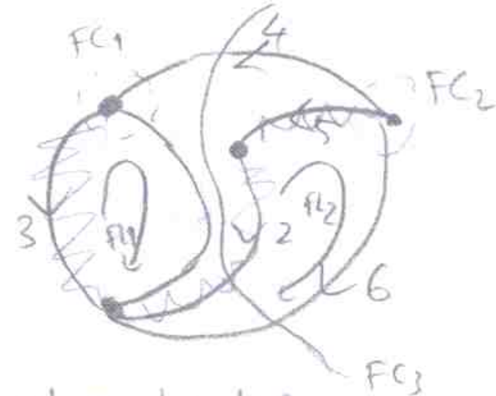
HW # 1

4. (2d in ZPS-1)



$\rightarrow$



$\{2,3,5\} \rightarrow$ tree branches
$\{1,4,6\} \rightarrow$ co-tree branches
$\{v_c, i_L\}$ are unknowns

$FC_1: \frac{1}{2}\dot{v}_c + i_1 = i_L$, $FL_1: v_1 = v_c$

$FC_2: i_5 = -2i_L$, $\frac{v_5}{2} = -2i_L$, $v_5 = -4i_L$

$FL_2: v_5 + v_2 = \frac{1}{4}\dot{i}_L \Rightarrow -4i_L = v_5 = \frac{1}{4}\dot{i}_L - v_2$

$FC_3: i_2 = -2i_L$, $\frac{v_1}{v_2} = \frac{1}{2}$, $\frac{i_1}{i_2} = -2 \Rightarrow i_1 = -2i_2 = 4i_L$

$v_2 = 2v_1 = 2v_c$, $\dot{v}_c = 2(i_L - i_1) = -6i_L$, $\dot{i}_L = 4(v_2 + v_5) = 8v_c - 16i_L$

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ 8 & -16 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} \Rightarrow \begin{vmatrix} \lambda & 6 \\ -8 & \lambda+16 \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + 16\lambda + 48 = 0$$

$\Leftrightarrow (\lambda+4)(\lambda+12) = 0$, $\lambda = -4, \lambda = -12$; $\boxed{-4, -12}$ are natural frequecies.

$$\lambda = -4 \Rightarrow -4\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ 8 & -16 \end{bmatrix}\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \Rightarrow 2\gamma_1 = 3\gamma_2$$

$$\lambda = -12 \Rightarrow -12\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ 8 & -16 \end{bmatrix}\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \Rightarrow 2\gamma_1 = \gamma_2$$

Initial conditions exciting the modes are : $\begin{bmatrix} V_c(0^-) \\ I_L(0^-) \end{bmatrix} = k \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $(k \in \mathbb{R})$

and $\begin{bmatrix} V_c(0^-) \\ I_L(0^-) \end{bmatrix} = k \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $(k \in \mathbb{R})$