ector into

- and estsystems. In and throughthe find the sessions in mons. For a
- arty theory,"
- Acoust.,
- dge, 1985.
- ood Cliffs,
- HE NJ, 1977. Hill Englewood
- Chffs, NJ,
- Birs. 1969.
- Processing,
- as follows
- manues and X is
- he solution is the

- (a) Find the matrix W.
- (b) What properties does the matrix W have?
- (c) What is the inverse of W?
- 2.2. Prove or disprove each of the following statements:
- (a) The product of two upper triangular matrices is upper triangular.
- (b) The product of two Toeplitz matrices is Toeplitz.
- (c) The product of two centrosymmetric matrices is centrosymmetric.
- 2.3. Find the minimum norm solution to the following set of underdetermined linear equations,

$$\begin{bmatrix} 1 & 0 & 2 & -1 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2.4. Consider the set of inconsistent linear equations Ax = b given by

$$\left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \end{array}\right] = \left[\begin{array}{c} 1 \\ 1 \\ 0 \end{array}\right]$$

- (a) Find the least squares solution to these equations.
- (b) Find the projection matrix PA.
- (c) Find the best approximation $\hat{\mathbf{b}} = \mathbf{P}_A \mathbf{b}$ to \mathbf{b} .
- (d) Consider the matrix

Find the vector $\mathbf{h}^2 = \mathbf{P}_A^{\perp} \mathbf{h}$ and show that it is orthogonal to $\hat{\mathbf{b}}$. What does the matrix \mathbf{P}_A^{\perp} represent?

2.5. Consider the problem of trying to model a sequence x(n) as the sum of a constant plus a complex exponential of frequency ω_0 [5]

$$\hat{x}(n) = c + ae^{jn\omega_0}$$
; $n = 0, 1, ..., N-1$

where c and a are unknown. We may express the problem of finding the values for c and a as one of solving a set of overdetermined linear equations

$$\begin{bmatrix} 1 & 1 \\ 1 & e^{j\phi_0} \\ \vdots & \vdots \\ 1 & e^{j(N-1)\omega_0} \end{bmatrix} \begin{bmatrix} c \\ a \end{bmatrix} = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

- (a) Find the least squares solution for c and a.
- (b) If N is even and a_N = 2πk/N for some integer k, find the least squares solution for c and a.

sed form

of the setting pare your

fellowing

mmetric or

matrix?

z matrix

nive definite

(a) Find the eigenvalues and eigenvectors of A.

(b) Are the eigenvectors unique? Are they linearly independent? Are they orthogonal?

(c) Diagonalize A, i.e., find V and D such that

$$V^H \Lambda V = D$$

where **D** is a diagonal matrix.

2.14. Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \left[\begin{array}{cc} 1 & -1 \\ 2 & 4 \end{array} \right]$$

2.15. Consider the following 3 × 3 symmetric matrix

$$\mathbf{A} = \left[\begin{array}{rrr} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{array} \right]$$

(a) Find the eigenvalues and eigenvectors of A.

(b) Find the determinant of A.

(c) Find the spectral decomposition of A.

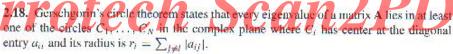
(d) What are the eigenvalues of A + I and how are the eigenvectors related to those of A?

2.16. Suppose that an $n \times n$ matrix A has eigenvalues $\lambda_1, \ldots, \lambda_n$ and eigenvectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$.

(a) What are the eigenvalues and eigenvectors of A²?

(b) What are the eigenvalues and eigenvectors of A⁻¹?

2.17. Find a matrix whose eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 4$ with eigenvectors $\mathbf{v}_1 = 1$ $[3, 1]^T$ and $\mathbf{v}_2 = [2, 1]^T$.



1. Prove this theorem by using the eigenvalue equation $Ax = \lambda x$ to write

$$(\lambda - a_{ii})x_i = \sum_{j \neq i} a_{ij}x_j$$

and then use the triangular inequality,

$$\begin{aligned} |\lambda_{i}| > |X_{J}| & |A_{J+i}| \\ |\lambda_{i}| > |A_{J+i}| & |A_{J+i}| & |A_{J+i}| \\ |\lambda_{i}| > |A_{J+i}| & |A_{J+i}| & |A_{J+i}| & |A_{J+i}| \\ |\lambda_{i}| > |A_{J+i}| & |A_{$$

Use this theorem to establish the bound on λ_{max} given in Property 7.

3. The matrix

$$\begin{bmatrix}
 4 & 1 & 2 \\
 2 & 3 & 0 \\
 3 & 2 & 6
 \end{bmatrix}$$

 $\left| \sum_{j \neq i} a_{ij} x_j \right| \le \sum_{j \neq i} \left| a_{ij} x_j \right|$